Stellar Astrophysics
AY 220A Winter 2025
Homework 4
Due February 11

1 (30 points). The Hayashi track is for pre-main sequence stars that are fully convective. There are two ends to a star on the Hayashi track. First, the star can travel down the Hayashi track until it starts fusion. Alternatively, the star can travel down the Hayashi track until it stops being fully convective. In the latter case, we look for when the star becomes dominated by radiative diffusion. For stars that stop being fully convective, they move along the **Henyey Track**, collapsing until they start hydrogen fusion, hitting the ZAMS. We want to find out which stars go on the Henyey Track.

(a) First, we want to determine an equation for the radius of a collapsing pre-main sequence star as a function of time and mass. Assume that the proto-star has a mass, M, starting radius, R_0, and starting luminosity, L_0. Using the Virial theorem, we know

$$L = -\frac{1}{2}\frac{dE_g}{dt}. \tag{1}$$

Separately, since

$$L = 4\pi R^2 \sigma T^4, \tag{2}$$

we see that for stars on the Hayashi track (where $T \approx$ const)

$$\frac{dL}{L} = 2\frac{dR}{R}. \tag{3}$$

Starting with Equation 1, find an equation for dL/dt that depends on M, R, and derivatives of R. Then use Equation 3 to derive an equation for d^2R/dt^2. Combining both of those results, derive an equation for the radius that is a function of the form

$$R = R_0 \left(1 + f(M, L_0, R_0)\right)^{-\alpha}, \tag{4}$$

where f is a function of the initial luminosity, initial radius, and mass, and α is a number.

(b) The convective luminosity for these stars can be approximated as

$$L_{\rm conv} = 0.034 L_\odot \left(\frac{M}{M_\odot}\right)^{4/7} \left(\frac{R}{R_\odot}\right)^2. \tag{5}$$

There is a useful approximation for Equation 4 so that

$$R \approx \left(\frac{1.3 \times 10^8 \text{ years}}{t}\right)^{\alpha} \left(\frac{M}{M_\odot}\right)^{10/21} R_\odot. \tag{6}$$

Note that α needs to be input from your answer to (a). Combining Equations 5 and 6, you can determine a convective luminosity as a function of time and mass. Compare the convective luminosity to the radiative luminosity for massive stars,

$$L_{\rm rad} = L_\odot \left(\frac{M}{M_\odot}\right)^3, \tag{7}$$

to determine the convective timescale (i.e., when the convective luminosity is less than the radiative luminosity) as a function of mass.

(c) Repeat part (b), but now use the (more accurate) equation for radiative luminosity for low-mass stars,

$$L_{\rm rad} = L_\odot \left(\frac{M}{M_\odot}\right)^{5.5} \left(\frac{R}{R_\odot}\right)^{-1/2} . \tag{8}$$

(d) Next, we need to think about the timescale for collapse. This should be the Kelvin-Helmholtz time. Note that

$$t_{\rm KH} = E/L \tag{9}$$

$$\propto \frac{M^2}{R} L^{-1} \tag{10}$$

$$t_{\rm KH} = 10^7 \text{ years} \left(\frac{M}{M_\odot}\right)^{2} \left(\frac{R}{R_\odot}\right)^{-1} \left(\frac{L}{L_\odot}\right)^{-1} . \tag{11}$$

For stars with $M > M_\odot$, we know $L \propto M^3$. For stars with $M < M_\odot$, $L \propto M^{11/2} R^{-1/2}$. For each mass regime, derive an equation for $t_{\rm KH}$ as a function of M and R.

(e) Using the timescales in (b), (c), and (d), determine the mass above which stars go on the Henyey track. That is, determine which masses have $t_{\rm conv} > t_{\rm KH}$. Be sure to check both mass regimes.

3 (40 points). A way to separate stars, brown dwarfs, and planets is based on their fusion. If there is sufficient hydrogen burning to halt collapse (i.e., a main sequence), we can consider that object a star. If an object doesn't have enough hydrogen fusion to halt collapse, but does halt collapse earlier from deuterium burning, we can consider it a brown dwarf. And then planets would be objects that cannot halt collapse because of fusion.

Using MESA, you will determine the mass limits for these different regimes. You will need to run multiple models with different masses to examine if the object undergoes a deuterium and/or hydrogen main sequence. For these models, you will want to make sure to track the D and ^{7}Li abundance.

To determine if an object has a main sequence, we can look at its luminosity as a function of time. If there is a main sequence, that object will have relatively constant luminosity over some period (a plateau).

(a) Run solar-metallicity models with masses of 0.01 and 0.2 $M_\odot$ from collapse for roughly 10^{10} years. For the former model, the `make_brown_dwarf` test run is helpful. Plot the log luminosity as a function of log age (covering the range 10^6–$10^{9.5}$ years), marking the mass for each model. Remark on the general behavior of each model and how it matches our predictions above.

(b) Now examine the abundance of D and ^{7}Li. Make a copy of the plot from (a), marking the point where the D and ^{7}Li are depleted by a factor of 100 for each model. Make it clear which element is which.

(c) Make a copy of the plot from (b), but now add enough models where you can determine the star/brown dwarf divide to at least 0.02 $M_\odot$ and the brown dwarf/planet divide to at least 0.005 $M_\odot$. Note that very low-mass MESA models can take a long time to run. Plot all planet, brown dwarf, and planet tracks with different line styles (e.g., solid, dashed). Again, plot the points of D and ^{7}Li depletion. Write down the divisions in mass you found in both solar masses and Jupiter masses. Does our initial definition make sense?

These models can take a very long time to complete. I *highly* suggest you start early. You also do not need to run each model to completion. If you set up your plotting and make plots as you run models, you should be able to determine when the models have run long enough and can end the simulation there.

3 (30 points). We want to examine binary stars from *Gaia* in a different way from the previous homework. *Gaia* produced a catalog of stars where a two-body orbital solution is preferred to a single-star solution. This catalog is `gaiadr3.nss_two_body_orbit`. Download the entire catalog (only 443,205 stars), including `nss_solution_type`, eccentricity, period, the Thiele-Innes parameters (and errors), and inclination (and error).

(a) Make a scatter diagram of the eccentricity as a function of period. Ignore all stars without a measured eccentricity or where it is identically zero. Explain the features in this plot. Why does it cover the periods it does? Why are there regions that do not have (m)any stars?

(b) *Gaia* provides several different kinds of orbital fits. The catalog only includes inclination for the "EclipsingBinary" solutions. However, you can determine the inclination from the Thiele-Innes parameters, which are directly constrained by the astrometric data. They are:

$$A = a_0(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i) \tag{12}$$

$$B = a_0(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i) \tag{13}$$

$$F = -a_0(\sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i) \tag{14}$$

$$G = -a_0(\sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i) \tag{15}$$

where A, B, F, and G are the Thiele-Innes parameters, a_0 is the semi-major axis of the photocenter, ω is periastron longitude measured from the ascending node, Ω is the position angle of the ascending node, and i is the inclination angle.

For each star that has the A, B, F, and G parameters (and errors), fit for the orbital parameters, especially i. There are four equations with four unknowns, so this is perfectly constrained up to the uncertainties. Plot a histogram of $\cos i$, and explain why its shape is what it is.

(c) Plot a color-magnitude diagram showing a random set of 200,000 *Gaia* stars from the main catalog. You can make S/N cuts, but do not cut on things like parallax that could significantly bias the sample. Overplot the binaries that are within 5 degrees of being face on and within 5 degrees of being edge on, each with a different color/symbol. Discuss any differences between the three groups.

(d) EXTRA CREDIT! Extra 10 points.

TESS stares at a particular part of the sky for about a month at a time. That means that an eclipsing system with a period $<$60 days likely had either a primary or secondary eclipse measured within that time. Select the set of stars that are edge on within 5 degrees that have periods $<$60 days. From this sample, select a single bright star. From MAST (or another database), download the corresponding *TESS* light curve.

Plot the *TESS* light curve. Mark any eclipses you see by eye. If you see more than one, note the time between them (which is likely either the period or half the period).