# Stars and Planets I
## AY 220A Winter 2025
## Homework 5
## Due February 18

1 (50 points). Using MESA, examine how the structure of a star depends on the details of its energy generation. Evolve two models for each of 0.8 $M_\odot$ and 6 $M_\odot$ stars, where one model for each star has only *p–p* fusion and the other only fuses through the CNO cycle. Start with pre-main sequence models and evolve them to the end of the main sequence (no more H in the core). Compare the models in the ways explained below. In each case use physical arguments to explain the differences between the two models. Don't worry about getting all the small differences correct (e.g., wiggles in the HR diagram); focus on the large-scale differences between the models.

(a) Plot the evolutionary tracks on an HR diagram. Mark the zero-age main sequence for all models. Mark the end of the main sequence for all models.

(b) At the zero-age main sequence (or perhaps slightly after "zero-age main sequence"), plot the interior structure of each star. Specifically, plot $\rho$, $T$ and $\epsilon_{\rm nuc}$ as a function of $M_r$ (mass coordinates) where simulations of the same mass are on the same plots. Mark any convective regions. Do the same at the end of the main sequence. Each plot should directly compare the two models for each mass by having them on the same plot.

(c) Plot the H, He, C, N, and O abundance as a function of $M_r$ at the end of the main sequence. Again, directly compare the two models. Be careful to plot things in a clear way.

(d) Plot the evolution of $\rho_c$ and $T_c$ from the zero-age main sequence to hydrogen exhaustion for all models. Do this by making a "$\rho$-$T$" diagram where lines show the stars evolution. Indicate the ZAMS and TAMS for each model.

(e) Now having all plots, explain in words the major differences in evolution. This should include differences in central temperature, how energy is transported, luminosity, radius, and core radius.

2 (50 points). So far, we have used *Gaia* to primarily look at samples of stars. We will now use it to first find a very select sample of stars and then investigate one particular star, a black hole.

First, retrieve the sample of all *Gaia* astrometric binaries. You may already have this from the previous homework. Select only systems where the binary goodness-of-fit parameter is <5.

(a) From the Thiele-Innes parameters, we can calculate the angular orbital separation for a binary system, $a_0$, with the relations,

$$u = (A^2 + B^2 + F^2 + G^2)/2 \tag{1}$$

$$v = AG - BF \tag{2}$$

$$a_0 = \left(u + \sqrt{u^2 - v^2}\right)^{1/2}. \tag{3}$$

All Thiele-Innes parameters have units of mas from *Gaia*, which means $a_0$ also has units of mas.

We can then use $a_0$, the parallax ($\mu$), and the orbital period ($P$) along with Kepler's third law to determine the astrometric mass function,

$$m_f = 1 \left(\frac{a_0}{1\ \text{mas}}\right)^3 \left(\frac{\mu}{1\ \text{mas}}\right)^{-3} \left(\frac{P}{1\ \text{year}}\right)^{-2} \text{M}_\odot. \tag{4}$$

Explain in words why the prefactor for this equation is 1.

Make a log-log scatter plot of $a_0 \times d$ (in units of AU), where $d$ is the distance, as a function of $P$ (in units of days). Make stars far above the the cluster of points distinct with large points. These stars tend to have dark companions.

Plot a histogram of the mass functions for the binary sample.

(b) If one star in the binary emits no light, the astrometric mass function (which deviates by the spectroscopic mass function by a factor of $\sin^3 i$) is

$$m_f = \frac{M_2^3}{(M_1 + M_2)^2}, \tag{5}$$

where $M_1$ is the observed star and $M_2$ is the dark companion.

Assuming $M_1 = 1$ M$_\odot$, calculate $M_2$. Make a histogram of $M_2$, with a second histogram zooming in for $M_2 > 1.3$ M$_\odot$.

(c) Objects that emit little or no light will be either faint normal stars (like M dwarfs), white dwarfs, neutron stars, or black holes. White dwarfs cannot have a mass above the Chandrasekhar mass limit, and thus restricting the sample to those with $M_2 > 1.4$ M$_\odot$ should exclude all single white dwarfs. This restriction should also remove most faint normal stars with the possible exception of having an extremely massive luminous star.

Verify that your list contains Gaia DR3 4373465352415301632. Print the separation, period, and $M_2$ for this star.

This star is known as Gaia-BH1. You have discovered a black hole.