Homework 3

ASTR 220A Winter 2025

Due January 28

Question 1

Show that in a classical thermalized ionized ideal gas that the energy transport by conduction is done primarily by electrons and not ions.

(a)

Assume that the gas has an electron number density, n_e, and a temperature T. Write down equations for the kinetic energy, E_K, energy density, u_e, heat capacity per unit volume, C_e, and thermal velocity, v_e, for the electrons.

Solution:

The kinetic energy of an electron in an ideal gas is given by:

$$E_K = \frac{3}{2}kT$$

The energy density is:

$$u_e = \frac{3}{2}n_e kT$$

The heat capacity per unit volume is:

$$C_e = \frac{du_e}{dT} = \frac{3}{2}n_e k$$

Assuming an average speed v_e, we use:

$$\frac{1}{2}m_e v_e^2 = \frac{3}{2}kT$$

Solving for v_e:

$$v_e = \sqrt{\frac{3kT}{m_e}}$$

(b)

Electron-electron collisions are not efficient at transferring energy, so we focus on electron-ion collisions. The mean-free path is given by:

$$l = \frac{1}{n_i \sigma}$$

where n_i is the number density of ions. Using this with the previous equations, derive an equation for the flux density of heat, $j(x)$.

Solution:

Using the energy flux equation:

$$j(x) = -\frac{1}{2} v_e l \frac{du}{dx}$$

Applying the chain rule:

$$\frac{du}{dx} = \frac{dT}{dx} \cdot \frac{du}{dT} = C_e \frac{dT}{dx} = \frac{3}{2} n_e k \frac{dT}{dx}$$

Substituting v_e and l:

$$j(x) = -\frac{1}{2} \left(\frac{3kT}{m_e} \right)^{1/2} \cdot \frac{1}{n_i \sigma} \cdot \frac{3}{2} n_e k \frac{dT}{dx}$$

Simplifying:

$$j(x) = -\frac{1}{2} \frac{n_e}{n_i} \left(\frac{3kT}{m_e} \right)^{1/2} k \frac{dT}{dx}$$

(c)

Assuming that the cross-section is determined by the Coulomb potential,

$$\frac{Ze^2}{4\pi\varepsilon_0 r} = kT$$

solve for the thermal conductivity of electrons, K_e, where:

$$j(x) = -K_e \frac{dT}{dx}$$

Solution:

Solving for the cross-section:

$$r = \frac{Ze^2}{4\pi\varepsilon_0 kT}$$

The collision cross-section:

$$\sigma = \pi r^2 = \frac{\pi Z^2 e^4}{16\pi^2 \varepsilon_0^2 k^2 T^2}$$

Thus, the thermal conductivity is:

$$K_e = \frac{8\pi\varepsilon_0^2}{Ze^4} k^{7/2} T^{5/2} \frac{n_e}{n_i} \sqrt{\frac{3}{m_e}}$$

(d)

Derive the ratio of the thermal conductivities of ions and electrons, K_i/K_e.

Solution:

$$\frac{K_i}{K_e} = \sqrt{\frac{m_e}{m_i}} \frac{n_i^2}{n_e^2}$$

Assuming $n_i \approx n_e$:

$$\frac{K_i}{K_e} \approx \sqrt{\frac{m_e}{m_i}}$$

Since $m_e \ll m_i$, it follows that:

$$K_i \ll K_e$$

Question 2

We will be building on your work from the first Homework where you made a color-magnitude diagram (CMD) for a cluster. You will look at a different cluster, but you can re-use some of your previous code to make things go faster. Functions are particularly useful!

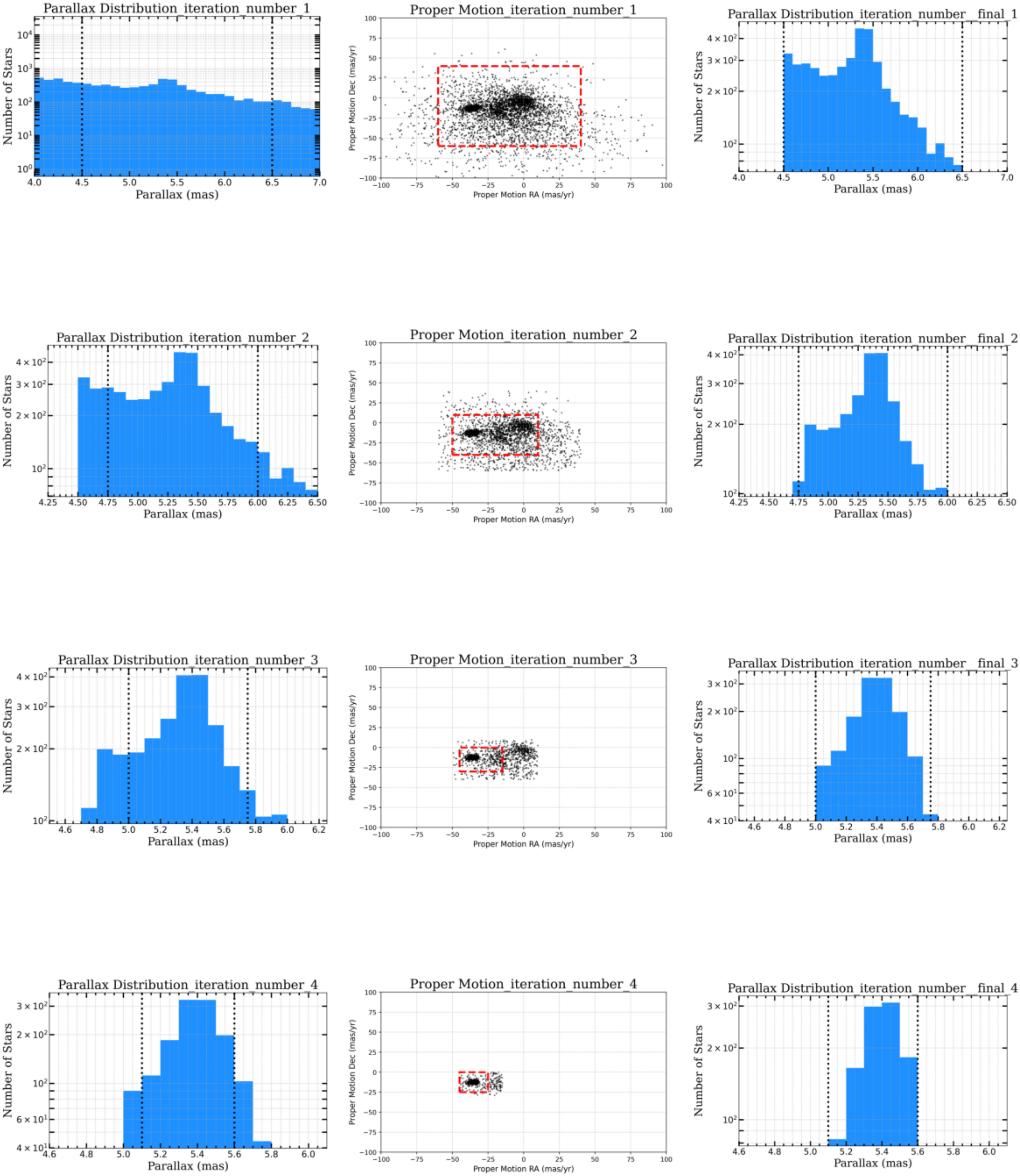
The goal is to determine the fraction of binary stars in the Praesepe (Bee Hive) cluster. This cluster takes up a large area on the sky, so you will need to set a large radius in your query (perhaps as large as 10 degrees).

(a)

Determine what stars are in the Praesepe cluster. Like in Homework 1, go through the location, parallax, and proper motion functions several times, iterating a few times. Be conservative for each step, only excluding stars that are clearly not part of the cluster. Continue until you converge on your final sample. Show all plots related to this.

Solution:

Below are the plots for the cluster stars selection iteration. see *run_cluster_iterations.py* in hw3.zip for the code.

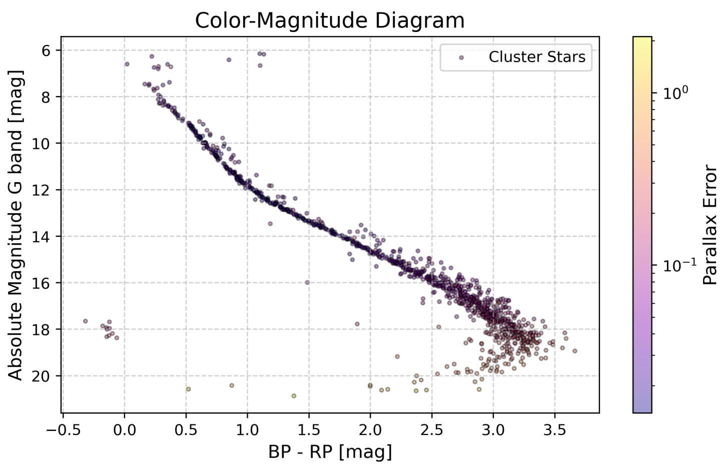


Parallax Distribution_iteration_number_1
Proper Motion_iteration_number_1
Parallax Distribution_iteration_number_final_1
Parallax Distribution_iteration_number_2
Proper Motion_iteration_number_2
Parallax Distribution_iteration_number_final_2
Parallax Distribution_iteration_number_3
Proper Motion_iteration_number_3
Parallax Distribution_iteration_number_final_3
Parallax Distribution_iteration_number_4
Proper Motion_iteration_number_4
Parallax Distribution_iteration_number_final_4
Number of Stars
Parallax (mas)
Proper Motion Dec (mas/yr)
Proper Motion RA (mas/yr)

(b)

Make a CMD for the cluster using the apparent magnitude in the G band vs. the $BP - RP$ color. Plot the y axis inverted so smaller numbers (more luminous stars) are at the top.

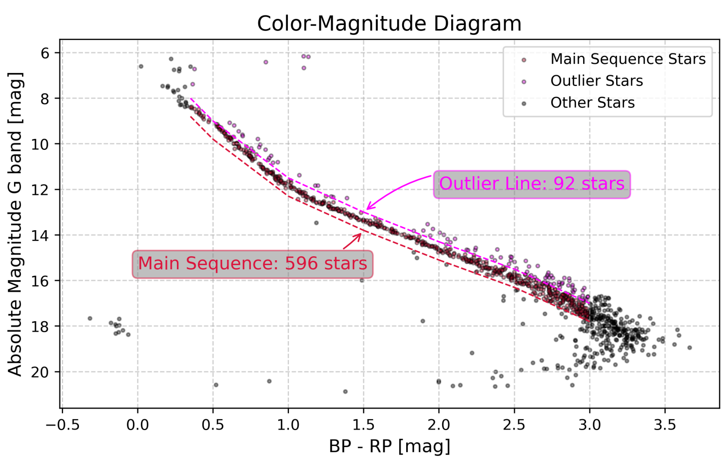
Solution:



(c)

Identify binary stars in the CMD.

Solution:



(d)

Count the number of outlier stars, $N_{outlier}$, and the number of main sequence stars, N_{MS}. Compute the binary fraction:

$$f_{binary} = \frac{N_{outlier}}{N_{MS}}$$

$$f_{binary} = \frac{86}{583}$$

$$f_{binary} = \frac{N_{outlier}}{N_{MS}}$$

Solution:

$$f_{binary} = \frac{N_{outlier}}{N_{MS}}$$

$$f_{binary} = \frac{92}{596}$$

$$f_{binary} = \frac{N_{outlier}}{N_{MS}}$$

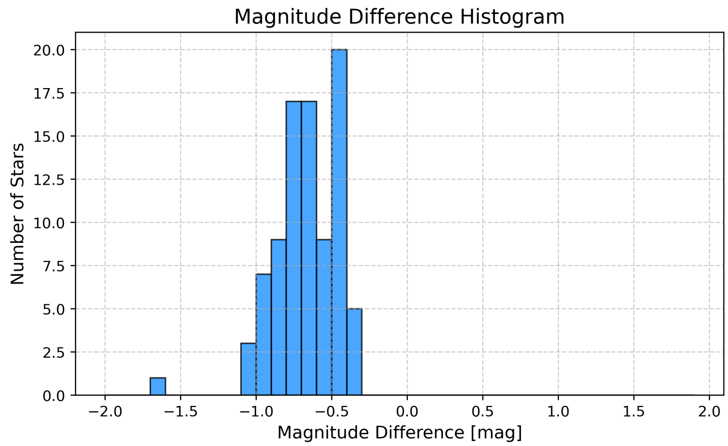
$N_{outlier}$	N_{MS}	f_{binary}
92	**596**	**0.16**

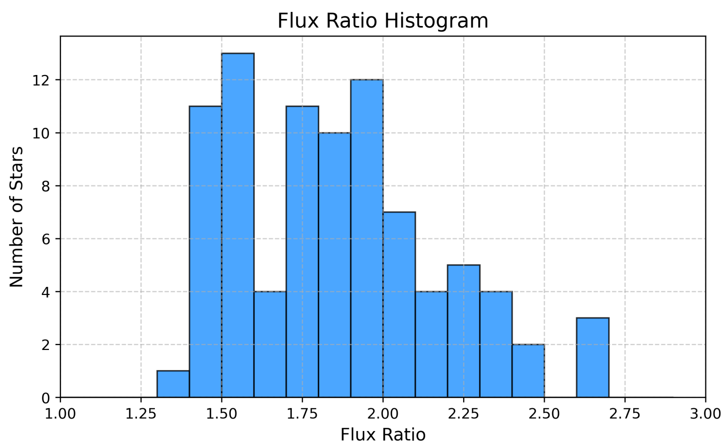
Table 1: Binary fraction Results

(e)

Analyze the luminosity of outlier stars and compute their impact on the binary fraction.

Solution:





The flux Ratio shows that a large of outlier stars are about twice as luminous as their main sequence counterpart of the same color, indicating that they are binary star systems. Out of 92 outliers 63 have flux ratios > 1.7 and are likely to be binary stars. The mean of the distribution of Flux ratios is ≈ 2 since it would be difficult to have a stable system of more than two stars.