Stars and Planets I
ASTR 220A Winter 2025
Homework 3
Due January 28

1 (35 points). Show that in a classical thermalized ionized ideal gas that the energy transport by conduction is done primarily by electrons and not ions.

(a) First, assume that the gas has an electron number density, n_e, and a temperature T. Write down equations for the kinetic energy, E_K, energy density, u_e, heat capacity per unit volume, C_e, and thermal velocity, v_e, for the electrons.

(b) Electron-electron collisions are not efficient at transferring energy (they essentially swap energy like billiard balls so the gas is essentially the same before and after the collision). Therefore, we will focus on electron-ion collisions. For this interaction, the mean-free path is

$$l = \frac{1}{n_i \sigma}, \tag{1}$$

where n_i is the number density of ions and σ is the cross section of the electron-ion collision. Use the this equation with the equations from part (a) to write down an equation for the flux density of heat across a surface, $j(x)$.

(c) Assuming that the cross section is defined (roughly) by the radius where the Coulomb potential is equal to the thermal energy for a particle,

$$\frac{Ze^2}{4\pi\epsilon_0 r} = kT, \tag{2}$$

where Z is the charge of the ion, determine the equation for the thermal conductivity of electrons, K_e, as a function of n_e, n_i, Z, T, and constants. Here K_e is defined by

$$j(x) = -K_e \frac{dT}{dx}. \tag{3}$$

(d) There is a similar equation for the thermal conductivity of ions where the particle masses and densities are swapped. Determine the ratio of the two thermal conductivities, K_i/K_e to show under reasonable assumptions that $K_i \ll K_e$.

2 (65 points). We will be building on your work from the first Homework where you made a color-magnitude diagram (CMD) for a cluster. You will look at a different cluster, but you can re-use some of your previous code to make things go faster. Functions are particularly useful!

The goal is to determine the fraction of binary stars in the Praesepe (Bee Hive) cluster. This cluster takes up a large area on the sky, so you will need to set a large radius in your query (perhaps as large as 10 degrees).

(a) You want to determine what stars are in the Praesepe cluster. Like in Homework 1, go through the location, parallax, and proper motion functions several times, iterating (i.e., using the output of one round as the input for the next) a few times. Be conservative for each step, only excluding stars that are clearly *not* part of the cluster. Continue until you converge on your final sample. Show all plots related to this.

(b) Make a CMD for the cluster using the **apparent** magnitude in the G band vs. the $BP-RP$ color. Plot the y axis inverted so smaller number (corresponding to more luminous stars) are at the top. You should see a really, really nice main sequence of stars! You will also see a few other stars (white dwarfs, subgiants, etc) that are not on the main sequence, but in the next part we will focus on the main sequence.

(c) If you look carefully, you should see several stars "above" the main sequence (slightly more luminous), across the entire main sequence. The stars *directly to the red* of the main sequence at its most luminous end are subgiants that have moved off the main sequence and are in the process of becoming red supergiants — these are not the stars we care about. If you selected your cluster stars carefully, they should all be cluster members and not contaminants. Instead, the stars just above the main sequence (at all colors) are binary stars, where the combined light boosts the measured brightness. On the CMD, show regions selecting the main sequence stars and the binary main sequence stars. This does not need to be fancy; a hand-drawn region is fine.

(d) Count the number outlier stars, $N_{\rm outlier}$ and the number of main sequence stars (only main sequence), $N_{\rm MS}$. These numbers don't have to be especially precise, and you don't need to be perfect in your selection. We are aiming for a rough estimate. The binary fraction is

$$f_{\rm binary} = \frac{N_{\rm outlier}}{N_{\rm MS}}. \tag{4}$$

Report $N_{\rm outlier}$, $N_{\rm MS}$, and $f_{\rm binary}$.

(e) For each "outlier" star, determine the difference between its magnitude and the "non-outlier" main sequence magnitude at the same color. You will need to create a function that approximates the non-outlier main sequence. Plot this function on your CMD to ensure it is a good fit. Plot a histogram of the magnitude differences. Make sure there are enough bins to see the peak and width of the distribution. Plot a histogram of the multiplicative factor between the outlier luminosities and the non-outlier main sequence luminosity (in G). Calculate that value directly from the magnitude residuals. What can you say about the luminosity of the outlier stars? What does this mean for your binary fraction calculated above?