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Introducing the

Two Sigma Factor Lens

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Abstract

Asset allocators have taken an increasing interest in risk factors for the analysis of everything from their overall portfolios to their individual managers. Most of the literature on the topic discusses *why* allocators should apply a risk factor approach. This paper instead focuses on *how* to construct a functional lens suiting institutional investors' analytical needs. Specifically, we present a framework for constructing a parsimonious set of actionable risk factors that individually describe independent risks common across many asset class returns yet collectively explain much of the cross-sectional and time-series risk in typical institutional investor portfolios.

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I. Introduction

For many years, investors relied on the assumption that combining different asset classes within a portfolio was an effective way to maximize risk-adjusted returns. A key issue with that assumption, however, is that different asset classes may be exposed to the same systematic sources of risk, or risk factors, which may lead an investor to believe they are more diversified than is actually the case. In contrast, examining a portfolio through a risk factor lens may allow investors to better understand overlapping sources of risk across multiple asset classes and more efficiently manage their portfolios' overall risk exposures and expected return.

In this paper, we provide an overview of the Two Sigma Factor Lens, designed for analyzing multi-asset portfolios and derived from returns of broad, liquid asset class proxy indexes. This lens is intended to be:

- **Holistic**, by capturing the large majority of cross-sectional and time-series risk for typical institutional portfolios;
- **Parsimonious**, by using as few factors as possible;
- **Orthogonal**, with each risk factor capturing a statistically uncorrelated risk across assets;
- **Actionable**, such that desired changes to factor exposure can be readily translated into asset allocation changes.

Finally, we discuss methods for constructing and assessing the Two Sigma Factor Lens that can be extended to produce additional risk factors for new sub-asset-classes or cross-sectional risks that may not currently be captured by the lens.[1] This factor lens, and our ongoing work to expand it, form the foundations of the Venn™ platform.[2]

Academic theories of common risk factors that drive returns across investible assets date back to the Capital Asset Pricing Model,[3] which posited a single-factor view of global asset returns driven by the average risk-aversion of investors who collectively hold the market portfolio. Stephen Ross's 1976 paper on Arbitrage Pricing Theory expanded the idea of asset-pricing models to encompass multiple (unspecified) risk factors that a heterogeneous mix of investors would require compensation to hold. In recent years, there has been a flurry of work tying academic theories of multi-factor asset pricing to practical factor lenses that can allow investors to measure more accurately the risks and returns in typical multi-asset institutional portfolios.[4]

Factor lenses can provide several key advantages in simplifying the investment process for institutional investors seeking to build more efficient portfolios. A

1 Methods outlined in this paper for building a functional risk factor lens for major asset classes can be extended to produce risk factors for investment styles and key sub-asset classes. This is an area of continuing research and development as Two Sigma builds out this lens to accommodate a broader array of institutional investments.
2 Venn is an analytics platform provided by Two Sigma Investor Solutions, LP, to support portfolio management and manager evaluation needs of allocators.
3 Treynor (1961); Sharpe (1964); Lintner (1965); Mossin (1966).
4 Ilmanen (2011); Shepard (2011); Asl and Etula (2012); Podkaminer (2013); Bass, Gladstone, and Ang (2017).

Exhibit 1 | Risk Factor Descriptions

Category	Risk Factor	Description
Core Macro	Interest Rates	Exposure to the time value of money (inflation risk and future interest rate changes)
	Equity	Exposure to the long-term economic growth and profitability of companies
	Credit	Exposure to corporate default and relative asset illiquidity risks
	Commodities	Exposure to changes in prices for hard assets, which can be driven by economic shifts
Secondary Macro	Foreign Currency	Exposure to moves in developed-market currency values versus the portfolio's local currency
	Emerging Markets	Exposure to the sovereign and economic risks of emerging markets
	Equity Short Volatility	Negative exposure to the changes in equity market volatility
	Local Inflation	Exposure to inflation-linked rates relative to fixed nominal rates within the currency area

parsimonious lens can help consolidate and quantify exposure to common risk and return drivers across thousands of individual global assets to only a handful of key factors, greatly reducing the number of forward-looking risk and return estimates needed as inputs to asset allocation decisions. The unified view of a factor lens can also help ensure that risk and return estimates across multiple assets, based on their individual factor exposures, are well calibrated, ensuring similar risk exposures correspond to similar expected returns. This use of a factor model can mitigate the tendency of standard mean-variance optimization techniques to suggest extreme allocations to any asset with inadvertently higher or lower risk-adjusted return expectations.[5] Finally, by further dividing risk factors into those that appear to carry long-term return premia and those that do not, a factor lens can help investors control for or minimize uncompensated risks in their portfolio while improving their allocation across the risk factors that have been shown to drive long-term returns.

We believe that the Two Sigma Factor Lens offers advantages relative to other empirically-based factor lenses in the current literature. First, the lens was designed to exploit the more responsive pricing and lower expected trading costs of highly liquid capital asset markets such as global equities and high-quality sovereign bonds,[6] which represent key return drivers of diversified institutional portfolios. Second, the lens allows marginal analysis of returns for less liquid assets and sub-asset classes, isolating and quantifying their potential value as portfolio additions. This marginal analysis can help identify assets that might not carry any additional return premium for a diversified portfolio.[7] Finally, the lens was designed to provide an intuitive and stable mapping of its independent risk factors to the traditional asset classes from which they are derived. This can allow investors to more easily translate between a factor-based portfolio analysis and

5 Jobson and Korkie (1980); Michaud (1989).
6 PricewaterhouseCoopers (2015); Two Sigma internal transaction cost estimates.
7 Marginal analysis of the net-of-transaction-cost returns to less liquid assets is outside the scope of this discussion, but will be the subject of future research.

asset allocations.

The rest of this paper is structured as follows: Section II describes statistical and economic evidence for common drivers of systematic risk across market assets; Section III outlines key aspects of the construction of the Two Sigma Factor Lens; Section IV provides illustrative quantitative and qualitative tests of the Two Sigma Factor Lens that we believe are broadly applicable to the use of any factor lens for particular analytical tasks; and Section V concludes.

II. Understanding Sources of Systematic Risk

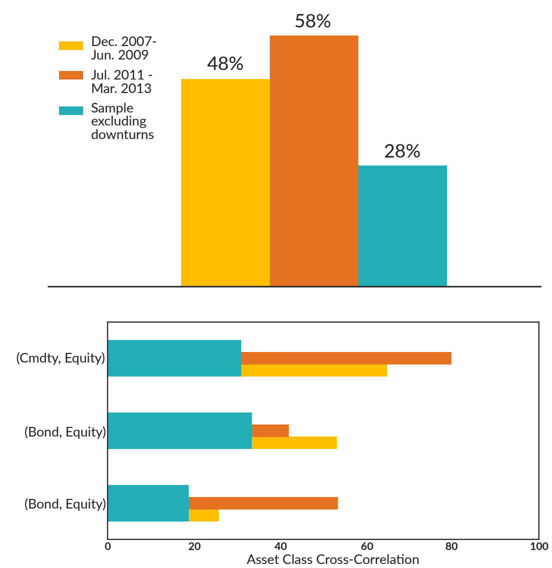
Historically, asset classes have played an important role in the investment process. However, the typical increase in correlations during adverse markets and the mixture of underlying systematic risks make asset classes imperfect candidates for risk analysis. Indeed, the correlations across a simple portfolio comprised of three asset classes, global equity (MSCI All Country World Index), commodities (S&P GSCI), and global investment grade bonds (Barclays Global Aggregate Bond Index) increased significantly during economic downturns as shown in **Exhibit 2**.[8] During recessionary periods, represented here by the U.S. financial crisis (Dec. 2007 - Jun. 2009) and the European sovereign debt crisis (Jul. 2011 - Mar. 2013), the average correlation across the three asset classes was 0.48 and 0.58, respectively, nearly twice as large as the average correlations when such downturn periods are excluded (0.28).

The higher correlations across asset classes in "bad times" suggest the existence of underlying, overlapping forces driving their risk and return dynamics. For example, overlapping exposure of equities, commodities, and credit spreads to the changes in global macroeconomic growth and investor risk aversion can drive positive correlations across these asset classes. Decomposing asset returns into fundamental risk factors that proxy for these common effects could help provide greater insight on how assets will perform under different market environments.

There are many examples in academic finance literature of fundamental risk factors that drive asset class returns. Nominal interest rates, as embodied in high quality sovereign bonds, have been shown to have a strong fundamental connection to the level and uncertainty of inflation, as high and volatile inflation can corrode the future value of fixed coupon payments.[9] Equities' value derives from future real cash flows to global businesses, which can be fundamentally linked to rates of economic growth.[10] Aggregate investor risk aversion also provides a common risk factor across equities and other assets exposed to economic growth, such as corporate bonds[11] and commodities. Risk aversion may even be a stronger fundamental explanatory factor for those risky assets, accounting for their high degree of correlation in market downturns and their higher common variance than economic volatility alone would suggest.[12]

While nominal rates and equities can be more simply mapped to underlying

Exhibit 2 | Asset Class Correlations



Correlations across equities, bonds, and commodities in both downturn months (orange and yellow bars) and months excluding the downturn periods (blue bars) from January 2003 to April 2018. Please see data appendix for details on downturn period selection and index data used.

8 "Downturns" refer to the business cycle peak-to-trough months defined by the NBER and the Euro Area Business Cycle Dating Committee and cover the periods December 2007 to June 2009 (NBER) and July 2011 to March 2013 (Euro Area), respectively. "Sample Excluding Downturns" cover the period from January 2003 to November 2017, excluding the previously mentioned "Downturns."
9 Fisher (1930); Yohe and Karnosky (1969); Fama (1990).
10 Gordon (1962); Sharpe (1964); Chen, Roll, and Ross (1986).
11 Merton (1974).
12 Mehra and Prescott (1985); Campbell (1991).

economic risks, many other asset classes and sub-asset classes have relatively more complex fundamental drivers. For example, corporate credit bonds are sensitive not only to changes in nominal interest rates (similar to high quality sovereign bonds), but also in economic growth and investor risk aversion (similar to equities). Corporate credits are sensitive, as well, to the unique negatively skewed return profile of default risk and even, potentially, to aggregate investors aversion to the relative illiquidity of the asset class.[13]

It might seem then that the ideal risk factor lens should look through to the fundamental drivers such as economic growth and inflation, asset liquidity, and aggregate investor risk aversion. However, these latent factors are not directly observable or investible. Instead, what we can directly observe are the asset prices driven by these systematic risks. Some factor research surmounts this problem by building compound risk factors that approximate the fundamental drivers through statistical approaches such as Principal Component Analysis or observed proxies for macroeconomic series (e.g., Chen, Roll, and Ross (1986)). However, these approaches can run into implementation issues as they may generate factor-mimicking portfolios with high turnover and many assets, or rely heavily on smoothed macroeconomic data with long reporting lags.[14]

← unobserved

To generate a factor lens that should retain intuitive and stable relationships to both asset classes and the underlying drivers of market risk, we have taken the approach of exploiting a natural hierarchy across global assets. The most liquid asset classes, like global equities and high quality sovereign bonds, not only have deep markets and near-instantaneous price updating, but also represent relatively clean exposure to fundamental drivers such as economic growth uncertainty and inflation uncertainty, respectively. Returns from asset classes with more complex underlying drivers, such as corporate bonds, may be decomposed into the components that appear statistically driven by exposure to the more liquid asset class risk factors and an orthogonal component representing the unique, marginal risk factor related to that asset class. Highlights of the Two Sigma Factor Lens construction are provided in the next section.

III. Construction of the Two Sigma Factor Lens

The Two Sigma Factor Lens seeks to capture the common risk factors driving returns across the majority of global assets, with a hierarchy that starts from the more liquid and high capacity factors that form the foundation of most institutional investors' portfolios. We denote the most prevalent risk factors, which stem from holding globally diversified long positions in equities, sovereign bonds, corporate credit, and commodities, as the "Core Macro" factors in our lens. To these, we add four "Secondary Macro" factors that can cut across multiple asset classes to explain additional concentrated risks that frequently arise in diversified portfolios: Foreign Currency, Emerging Markets, Equity Short Volatility, and Local Inflation.

13 Longstaff, Mithal, and Neis (2005); Bao, Pan, and Wang (2011).
14 Kroencke (2017).

The identification and construction process — described in the next sub-section — helps make this final set of risk factors *holistic*, *parsimonious*, *orthogonal*, and *actionable*. We believe these four attributes, as discussed in Section IV, characterize any factor lens that is ideally suited for portfolio and manager analysis.

III.i Identifying Risk Factors

Our lens starts with a global Interest Rates factor, as proxied by global sovereign bonds of seven to ten years' maturity, and a global Equity factor, as proxied by a global equity market index. We chose these base risk factors for their deep market liquidity and fundamental connection to global asset risk drivers: nominal rates are a key input in estimating discounted cash flow valuations for assets, while equities represent the most liquid diversified asset class sensitive to macroeconomic growth and aggregate investor risk aversion.

We treat other major asset classes, such as corporate bonds and commodities, as compound assets with contributions from the Interest Rates and Equity factors plus their own unique, marginal risk factors. In an attempt to isolate the risk unique to the Credit factor, we apply a statistical procedure outlined in the next section to extract the rolling risk contributions of Interest Rates and Equity factors to multiple indexes of corporate bond returns, then treat the orthogonal, or residual, returns as the Credit risk factor in our lens. We use the same statistical procedure (extracting higher-order Interest Rates and Equity risks) in developing an orthogonal Commodities risk factor. This completes the Core Macro factors that we believe can explain most of the risk in a typical institutional portfolio.[15]

risk factor = intercept + errors
= Y - BX

Y: returns of index
X: includes returns of interests and equity factors

Once a lens incorporates core risks to holding long exposure to globally diversified asset classes, several identifiable risk factors may still arise due to positions held in foreign or inflation-linked securities and negatively skewed investment strategies. The lens's Secondary Macro factors attempt to identify and quantify the most prominent of such risk exposures.

III.ii Constructing Risk Factors

While we treat equities and nominal interest rates as risk factors in their own right, most other risk factors are not directly observable. Therefore, we propose a construction process using rolling weighted linear regressions to attempt to decompose existing market proxies into additional, marginal risk factors. In the nomenclature explained in Section IV, this process promotes the orthogonality of the Two Sigma Factor Lens.

To present a simple example, an investor can identify unique asset class risk from an index of commodities by extracting the level of Interest Rates and Equity risk embedded in the asset. This is intended to hedge out sensitivities of commodities

15 See Exhibit 4 later in the paper for an application of the Core Macro factors to explain the majority of time-series risk in representative institutional investor portfolios.

to fundamental drivers such as changes in risk-free discount rates and aggregate investor risk aversion, resulting in a purified Commodities risk factor reflecting exposure to the particular supply and demand characteristics of this asset class.[16] Since exposures of less liquid assets to our primary risk factors can vary over time, this residualization process is performed on a rolling basis using three years of daily returns data with an exponentially-weighted regression that emphasizes recent performance. To account for daily short-term lead-lag effects across the asset classes, the exposures to more liquid risk factors are calculated using five-day rolling returns, with Newey-West (1987) adjustments to the estimated beta loadings.

Our Credit factor is constructed similarly; however, we perform separate residualization procedures in our effort to isolate a unique credit risk factor for each of investment grade and high yield credits in both the U.S. dollar and European corporate markets, due to the widely varying relative levels of Interest Rates and Equity risk embedded in higher quality investment grade and riskier high yield bonds. We found, however, that these four residualized credit factors showed a high level of covariance, suggesting a common risk process driving the sub-asset classes that is potentially attributable to their shared default risk and relatively lower liquidity. Thus we feel these can be reasonably combined into a single Credit risk factor at equal risk weights.

For the Secondary Macro factors, we seek to identify key risks that can cut across multiple asset classes held in institutional investor portfolios, many of which may not carry a long-term return premium. Foreign Currency risk, for example, is embedded in any foreign securities held without hedging their base currency risk.[17] At a high level, our Foreign Currency risk factor construction involves calculating returns to holding a diversified basket of G10 currencies relative to the applicable base currency, with the currencies weighted by the relative GDP of their respective economies. Our Emerging Markets risk factor also cuts across multiple asset classes, and is constructed from an equal-risk-weighted combination of the relative returns to hard currency emerging market credit bonds versus developed market credit bonds and the relative returns of currency-hedged emerging market equities to developed market equities. Equity Short Volatility risk, which can appear in numerous alternative strategies,[18] is approximated using the returns to a rolling strategy of selling monthly put options on the S&P 500 index. To promote the orthogonality of our factor set, these Secondary Macro risk factors are residualized against all four of the Core Macro factors.

The approach to factor construction outlined above involves many layers of complex decisions. For example, one must decide which return series to use as proxies for each asset class or risk factor. We have generally leaned toward using indexes that include as diverse an array of securities as possible while still possessing similar characteristics and strong covariance over time, i.e., looking

16 Technically speaking, in a linear regression setting, the resulting risk factor is the estimated regression constant plus the residual ($\hat{a}_i + \varepsilon_{it}$), which is equivalent to the index total return net of the estimated factor impacts ($Y_{it} - \Sigma_{k=1}^{n} \hat{B}_k X_{kt}$, where Y_{it} is the return to proxy index i at time t and X_{kt} is the return to risk factor k at time t).
17 For more detailed analysis on foreign currency risk and the potential lack of a return premium, see Boudoukh et al. (2015).
18 Fung and Hsieh (2001); Mitchell and Pulvino (2001).

for both fundamental and statistical support for a common risk factor across the underlying securities. In addition, the regression model implicitly assumes that any proxy index is a linear combination of risk factors, which may miss potential non linear relationships that can arise in downturns. We have tried to mitigate this risk in the factor lens by explicitly including risk factors with more skewed returns that capture non linear return dynamics, such as Credit and Equity Short Volatility. This approach also assumes that the relationships between factors and assets changes at a steady pace that can be captured with rolling regressions. However, the use of rolling historical periods for the residualization process adds another layer of complexity, i.e., how to define the look-back window and potential losses in the estimates' stability.[19] Our approach with the Two Sigma Factor Lens represents just one path through this wilderness of decisions, though one we aim to support with the empirical test results outlined in the next section.

IV. Tests of the Two Sigma Factor Lens

As mentioned previously, we believe a functional risk lens for typical institutional investor portfolios should satisfy four criteria: *holistic*, *parsimonious*, *orthogonal*, and *actionable*. This section describes these characteristics and provides practical examples to help clarify the concepts. As we attempt to show the relevance of the Two Sigma Factor lens to typical institutional portfolios, the analyses below are performed using representative proxy-based portfolios for three types of institutional investors: endowments and foundations, insurance company general accounts, and pension funds.[20]

Holistic

By holistic, we mean that the lens should explain a large majority of the variation in portfolio returns. Assume that a typical institutional portfolio were analyzed using an asset class lens, rather than a risk factor lens, and that a risk decomposition is performed by running regressions on returns from a broad set of asset classes in the portfolio. The holistic nature of the lens can be measured using the percentage of returns variation explained, or r-squared, of these regressions. **Exhibit 3** shows an example of this exercise, with the t-statistics for each asset class loading (OLS betas) and r-squared of the regressions. The large r-squared values suggest that applying an asset class lens would be holistic, as they explain the vast majority of monthly risk.

However, the Two Sigma Factor Lens in **Exhibit 4** shows that similar explanatory power is possible with only four core macro factors. The lower r-squared for the representative insurance company general account portfolio suggests that the factor lens may be improved further to capture unique risk factors in fixed income instruments, such as asset-backed securities with significant convexity and the behavior of the very long end of the yield curve. Yet overall, the high r-squared values show that the risk factor lens also appears holistic, but with the additional

Exhibit 3 | Explaining Returns Using an Asset Class Lens

Asset Class	Proxy	E & F	Insurance	Pension
Interest Rates	Barcl Glb Agg	-1.8	-2.3	-2.4
	Barcl Glb Sov	0.0	5.5	1.2
Equity	EM Equity	-0.7	-2.5	-3.4
	MSCI ACWI	22.3	7.8	31.8
Credit	EM Credit	-1.3	2.7	0.7
	EU Corp HY	-0.2	0.9	-1.6
	EU Corp IG	0.5	-3.0	0.7
	US Corp HY	1.0	1.8	3.6
	US Corp IG	-0.1	4.4	0.3
Commodities	S&P GSCI	3.7	0.3	4.7
Inflation	EU Inflation	-1.1	-1.0	-1.6
	US Inflation	2.0	2.4	3.2
	R^2 (%)	**95.5**	**94.1**	**98.1**
	# obs	180	180	180

The table reports the r-squareds and independent variable t-statistics from regressions of returns to representative portfolios for endowments and foundations (E&F), insurance company general accounts, and pension funds on a set of asset class indexes. Please see the data appendix for representative portfolio details and underlying data sources.

Exhibit 4 | Explaining Returns Using the Two Sigma Factor Lens

Risk Factor	E & F	Insurance	Pension
Interest Rates	-0.6	15.3	2.7
Equity	28.2	7.5	29.0
Credit	2.9	5.6	3.5
Commodities	8.0	2.0	8.7
R^2 (%)	**92.6**	**80.1**	**94.6**
# obs	180	180	180

The table reports the r-squareds and independent variable t-statistics from regressions of returns to representative portfolios for endowments and foundations (E&F), insurance company general accounts, and pension funds on the core macro factors from the Two Sigma Factor Lens. Please see the data appendix for representative portfolio details and underlying data sources.

19 Rolling betas require the empirical estimation of a rolling correlation matrix that could be unstable (Engle (2002)).
20 Asset class weights for representative portfolios are based upon average and median reported portfolio allocations in Pensions & Investments Research Center through fiscal year 2016. Please see the data appendix for more details on representative portfolio construction.

benefit of *parsimony* (as discussed in the next sub-section). This regression analysis provides a simpler view than the asset class lens by reducing the confusion of overlapping risk from highly correlated asset classes, allowing a deeper understanding of the risk exposures for each representative portfolio.[21]

Parsimonious

A functional lens should also be parsimonious, in that it should focus on those select risk factors that drive the majority of risk and return in institutional investor portfolios. Focusing on the small set of key risks shown to materially affect performance can limit the distracting noise that a more granular risk lens might offer. Given real-world constraints on data availability and managerial bandwidth, bringing too many factors to an analysis with relatively few return data points can increase the risk of spurious results.

Narrowing down the number of relevant factors is challenging. However, simple statistical tools can help the investor select the number of factors that explain, with confidence, a large percentage of the variation in returns.[22] One of these tools is Principal Component Analysis (PCA), which extracts uncorrelated principal components (PCs, or statistical risk factors), each explaining a percentage of the variation of large panels of returns.[23]

A PCA applied to the asset classes in Exhibit 3 suggests that five out of thirteen principal components explain about 90 percent of the variation in asset classes.[24] Even though PCs narrow down the number of relevant factors, they say little about their economic intuition, thus making the underlying risk less clearly identifiable. In comparison with the explanatory power of the four core macro factors from the Two Sigma Factor Lens shown in Exhibit 4, we believe there is insufficient advantage in parsimony from a PCA approach if the cost is sacrificing much of the intuition.

Orthogonal

One key advantage of both PCA and the Two Sigma Factor Lens is the relatively lower correlations across constituent factors. When an allocator examines the risk factor breakdown of a fund or portfolio, high correlations across any pair of factors will tend to obscure their joint contribution to overall volatility. For example, if credit spreads were used as a risk factor alongside equity returns, a portfolio might appear diversified when the risk allocation across those two factors appeared relatively equal. However, both factors have historically shown a high correlation driven by common exposure to investor risk aversion. Each risk factor in an ideal lens should capture a unique source of systematic risk, that is, they should be orthogonal. The construction process for the Two Sigma Factor Lens outlined in previous sections was designed to generate relatively independent risk factors (by construction) that isolate distinct risks in the markets. Thus we believe that an

21 The pairwise correlations across the 13 asset class proxies range between -0.22 and +0.85.
22 The set of risk factors that explain at least 80 percent of the variation in portfolio returns could be an effective risk lens. However, it is the investor's choice to decide this level of confidence.
23 Machine learning offers more sophisticated tools, such as LASSO or ridge regressions. See Friedman et al. (2001).
24 In this example we implement the PCA on standardized raw data, but it can also be applied on either the covariance or the correlation matrix. The latter approach is in theory equivalent to using standardized raw data, but in practice it will depend on missing data.

allocator seeing relatively dispersed exposure to multiple factors in the Two Sigma Factor Lens can be more assured that their portfolio is diversified among many independent sources of risk and return.

Exhibit 5 shows a comparison of average cross-correlations of monthly returns for asset class proxy indexes and the Two Sigma Factor Lens, both in "normal" markets and in cyclical downturns. Given the wide variation in correlations across assets over time, as seen in the exhibit, it is difficult to completely minimize positive cross-correlations of risk factors in both normal and down markets without sacrificing the stability of factor-mimicking asset portfolios. Thus even the relatively orthogonal factor lens on the right shows rising correlations in down markets, though not to the extreme values seen in asset class proxies.

The potential value of a factor lens with lower average cross-correlations lies not only in the greater independence of risk estimates across the individual factors, but also in the greater degree of statistical power when performing returns-based regression analysis.[25] Going back to our example in Exhibit 4, an allocator would have explained most of the variation in portfolio returns with a holistic, parsimonious, and orthogonal set of risk factors. Given the greater independence of the individual factors relative to the asset classes in Exhibit 3, the typical t-statistics for explanatory factors are also higher than for the individual asset classes, showing what we believe is the greater statistical power of returns-based analysis with the Two Sigma Factor Lens.

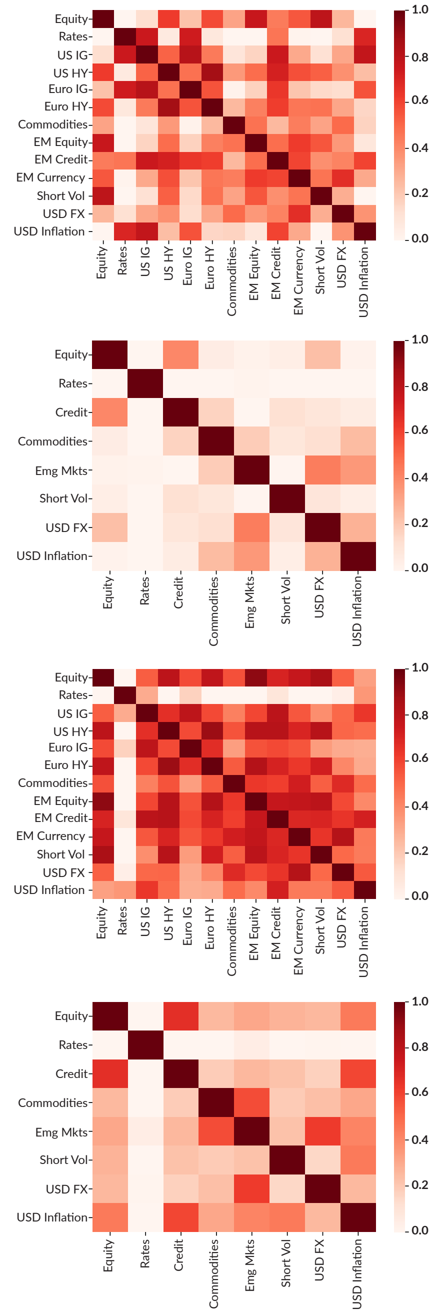
Actionable

Finally, we believe the lens should allow investors to translate outputs from factor analysis into asset allocation insights; this requires the individual factors in the lens to be "investible." In other words, we believe a factor lens is actionable only if there exists a relatively stable relationship between the factors and a readily investible set of liquid assets.

For those factors that are not captured by a unique market asset, risk factors can be approximated using factor-mimicking portfolios. For example, contaminating risks of rates and equity found in credit indices require the construction of a factor-mimicking portfolio for unique credit risk that is long a basket of credit indices and short both equity and nominal rates indices. This approach holds true for other factors. If a factor has shown a stable loading or statistical relationship to a set of tradable instruments, an allocator may invest in the factor for the long-term by holding a relatively static portfolio of assets.

A good measurement of the factor-mimicking portfolio's stability is the autocorrelation of the factor loadings on liquid underlying instruments. A stable loading is a persistent one, that is, one with a large autocorrelation coefficient. The monthly (21-day) trailing correlations of loadings for the Credit factor in the

Exhibit 5 | Correlations Across Asset Classes and the Two Sigma Factor Lens



A comparison of correlations of monthly excess returns for asset classes (first and third from the top) and the Two Sigma Factor Lens (second and fourth). The top two charts show correlations for January 2003 through April 2018 excluding downturn periods, while bottom two charts show correlations for solely the downturn periods as defined in Exhibit 2. Please see the data appendix for details on labels and index data used.

25 Farrar and Glauber (1967)

Two Sigma Factor Lens on its implied short positions in equities and nominal rates fall between 0.91 and 0.98, suggesting that the factor-mimicking portfolio is stable through time and changing market environments. This suggests that an allocator could hold a steady exposure to the risk factor with relatively modest trading. Similar tests may be applied to each factor in the lens, providing statistical evidence of whether the lens as a whole is actionable.

V. Conclusion

Recent years have seen conferences, papers, books, and even entire journals dedicate bandwidth to discussing what Stephen Ross, a forefather of the asset allocation field, describes as the "frenzy of factor-focused investing".[26] Such heightened attention appears to be a healthy development, in that the "frenzy"—more charitably seen as a growing enthusiasm for promising research—aims to apply more quantitative tools to the management of assets. The goal of this emerging approach is to help asset allocators quantify, analyze, and manage their portfolios in a more systematic, empirically rigorous manner.

The many dimensions of financial markets, asset classes, and individual portfolios make it impossible to define a unique risk lens that is applicable in every circumstance. Building a valid factor lens is, instead, a thoughtful exercise that involves the identification and construction of risk factors that possess specific characteristics suitable for one's unique purposes.

In that spirit, this paper advances the application of risk factor-based asset allocation by outlining a practical framework for systematically identifying and then constructing a workable set of risk factors—the Two Sigma Factor Lens. This framework emphasizes a discrete, but at times competing, set of criteria: that risk factors be holistic, parsimonious, orthogonal, and actionable. Collectively, these criteria can help guide the construction and expansion of a flexible risk factor lens to aid the wide variety of asset allocators' tasks from portfolio construction to risk management and manager evaluation. We intend to address many of these individual applications in future white papers and other short pieces.

This paper provides only an overview of certain aspects of the Two Sigma Factor Lens. It does not discuss many important assumptions, methodologies and other aspects of the lens. All information herein is subject to change without notice.

26 Ross (2017).

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Data Appendix

Exhibit 2

Exhibit 2 uses monthly returns for the full period from January 2003 through April 2018. All index returns data are from Bloomberg. "Downturns" refer to the business cycle peak-to-trough months defined by the National Bureau of Economic Research (NBER) and the CEPR Euro Area Business Cycle Dating Committee and cover the periods December 2007 to June 2009 (NBER) and July 2011 to March 2013 (CEPR), respectively.

The table below shows the proxy indices used for each asset class.

Asset Class	Index	Bloomberg Ticker
Equity	MSCI All-Country World Index	MXWD
Bonds	Bloomberg Barclays Global Aggregate Bond	LEGATRUU
Commodities	S&P GSCI Total Return	SPGSCITR

Exhibits 3 and 4

Representative portfolios are based on median and average asset allocations for institutional investor portfolios in each of the respective categories (endowments and foundations, insurance company general accounts, and pension funds). Asset allocation data is from Pensions & Investments Research Center, as of November 2017 using data from the 2016 fiscal year-end. For allocations among fixed income and natural resources sub-categories not broken out by Pensions & Investments, portfolio and index weights are our own estimates.

The table below provides representative portfolios asset allocations and proxy indices used. Returns data came from Bloomberg and has the Bloomberg ticker for the underlying index in the "Source" column unless otherwise noted.

			Weight in Representative Portfolios		
Asset Class	Index	Source	E&F	Insurance	Pension
US Equity	Russell 3000 Total Return	Bloomberg: RU30INTR	20%	10%	28%
Foreign Equity	MSCI World ex-US Net Return	Bloomberg: M1WOU	15%	5%	17%
Core Bonds	Bloomberg Barclays US Aggregate Bond	Bloomberg: LBUSTRUU	10%	20%	21%
Long-Term Bonds	Bloomberg Barclays US Long Government/Credit	Bloomberg: BFALTRUU	--	20%	--
High Yield Bonds	ICE B of AML US High Yield Master II	Bloomberg: H0A0	--	10%	5%
Emerging Market Bonds	JP Morgan EMBI Global	Bloomberg: JPEIGLBL	--	5%	3%
Asset-Backed Securities	Bloomberg Barclays US Aggregate ABS	Bloomberg: LUABTRUU	--	5%	--

Municipal Bonds	Bloomberg Barclays Municipal Bond	Bloomberg: LMBITR	--	10%	--
Hedge Funds	HFRI Fund Weighted Composite	HFR website	20%	--	5%
Cash	ICE BofAML 3 Month Treasury Bills	Bloomberg: G0O1	2%	--	2%
Natural Resources	75% S&P Oil & Gas Exploration & Production Select Index Total Return 25% S&P Metals & Mining Select Index Total Return	Bloomberg; SPSIOPTR SPSIMMTR	5%	--	5%
Private Equity	Cambridge Associates US Private Equity	Cambridge Associates website	18%	--	7%
Private Real Estate	Cambridge Associates Real Estate	Cambridge Associates website	10%	15%	7%

Quarterly returns to the private equity and private real estate index proxies were interpolated linearly over all three months of each respective quarter to produce monthly returns for the representative portfolios.

The table below provides the index details used for the explanatory return series in the Exhibit 3 regressions on each of the representative institutional investor portfolios. All index returns data are from Bloomberg.

Label	Index Name	Bloomberg Ticker
Barcl Glb Agg	Bloomberg Barclays Global Aggregate	LEGATRUU
Barcl Glb Sov	Bloomberg Barclays Global Treasury USD Hedged	BTSYTRUH
EM Credit	Bloomberg Barclays Emerging Markets USD Aggregate	EMUSTRUU
EU Corp HY	Bloomberg Barclays Pan-European High Yield	LP01TREU
EU Corp IG	Bloomberg Barclays Euro Aggregate Corporate	LECPTREU
US Corp HY	Bloomberg Barclays US Corporate High Yield	LF98TRUU
US Corp IG	Bloomberg Barclays US Corporate	LUACTRUU
EM Equity	MSCI Emerging Markets Index	MXEF
MSCI ACWI	MSCI All-Country World Index	MXWD
S&P GSCI	S&P GSCI	SPGSCI
EU Inflation	Bloomberg Barclays Euro Government Inflation-Linked Bond All Maturities	BEIG1T
US Inflation	Bloomberg Barclays US Treasury Inflation-Linked Notes	LBUTTRUU

Exhibit 5

Exhibit 5 uses monthly returns for the full period from January 2003 through April 2018. All returns data are from Bloomberg.

"Downturns" refer to the business cycle peak-to-trough months defined by the National Bureau of Economic Research (NBER) and the CEPR Euro Area Business Cycle Dating Committee and cover the periods December 2007 to June 2009 (NBER) and July 2011 to March 2013 (CEPR), respectively.

The table below provides index details used for the asset class return series in the left-hand pair of correlation heatmaps.

Label	Index Name	Bloomberg Ticker
Equity	MSCI All-Country World Index 100% Hedged to USD	MXCXDMHR
Rates	Bloomberg Barclays Global Government 7 to 10 Years Hedged to USD	LGY7TRUH
US IG	Bloomberg Barclays US Corporate	LUACTRUU
US HY	Bloomberg Barclays US Corporate High Yield	LF98TRUU
Euro IG	Bloomberg Barclays Pan-European Aggregate Corporate Hedged to USD	LP05TRUH
Euro HY	Bloomberg Barclays Pan-European High Yield Hedged to USD	LP01TRUH
Commodities	Bloomberg Commodity Index	BCOMTR
EM Equity	MSCI Emerging Markets Net Return Index	M1EF
EM Credit	Bloomberg Barclays Emerging Markets USD Aggregate	EMUSTRUU
EM Currency	MSCI Emerging Markets Currency Index	MXEF0CX0
Short Vol	CBOE S&P 500 PutWrite Index	PUT
USD FX	Two Sigma proprietary series, see below for details	--
USD Inflation	Bloomberg Barclays US Government Inflation-Linked 7 to 10 Years	BCIT5T

The table below provides the labels used for individual factors from the Two Sigma Factor Lens in the right-hand pair of correlation heatmaps. These heatmaps are based upon the factor returns for a U.S. dollar investor; versions of the Two Sigma Factor Lens for investors with different base currencies may have different factors and/or realized correlations.

Label	Two Sigma Factor Name
Equity	Equity
Rates	Interest Rates
Credit	Credit
Commodities	Commodities
Emg Mkts	Emerging Markets
Short Vol	Equity Short Volatility
USD FX	Foreign Currency (vs USD)
USD Inflation	U.S. Inflation

Foreign Currency returns (such as USD FX in Exhibit 5) are proprietary Two Sigma calculations that replicate the returns to holding a GDP-weighted basket of G10 currencies relative to the base currency of the factor lens, including both spot price movements and cash rate differentials.

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