Universidad Pontificia Bolivariana

Fundada en 1936

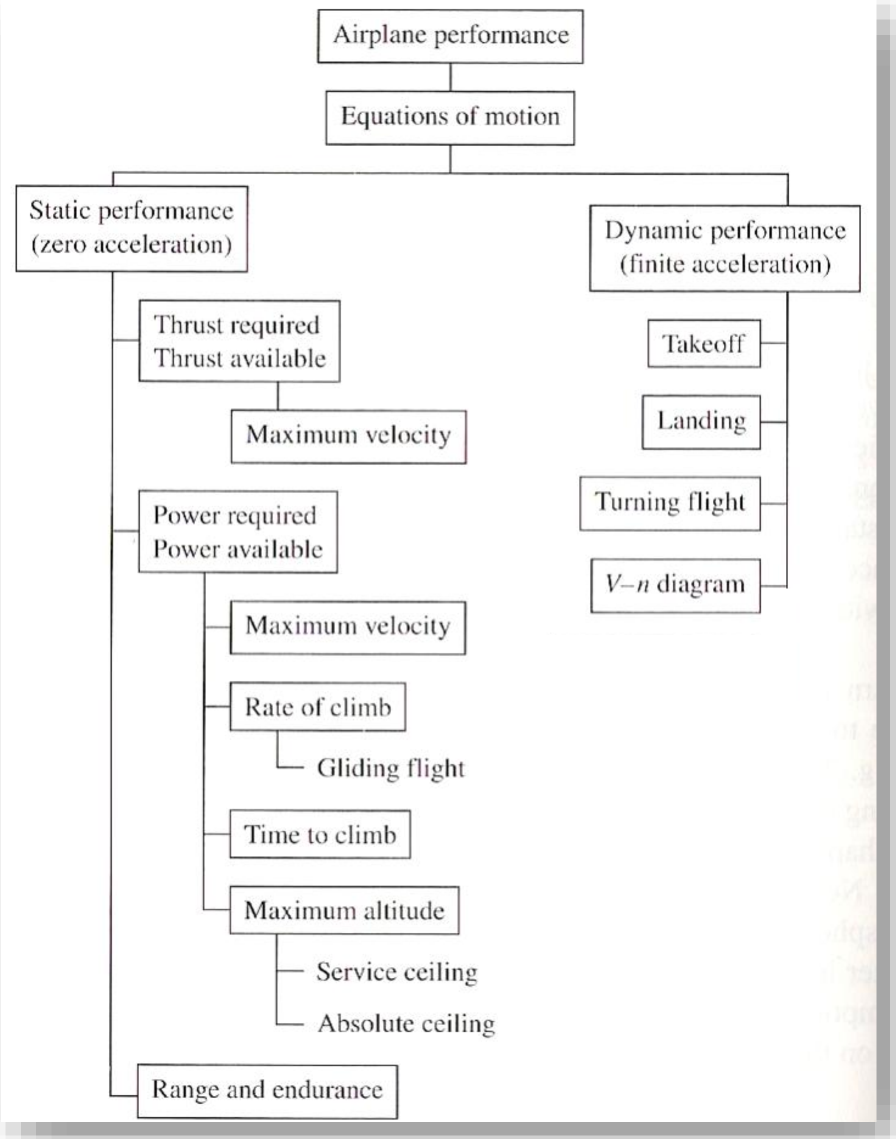
AIRCRAFT DESIGN I

Aeronautical Engineering

School of Engineering

UPB

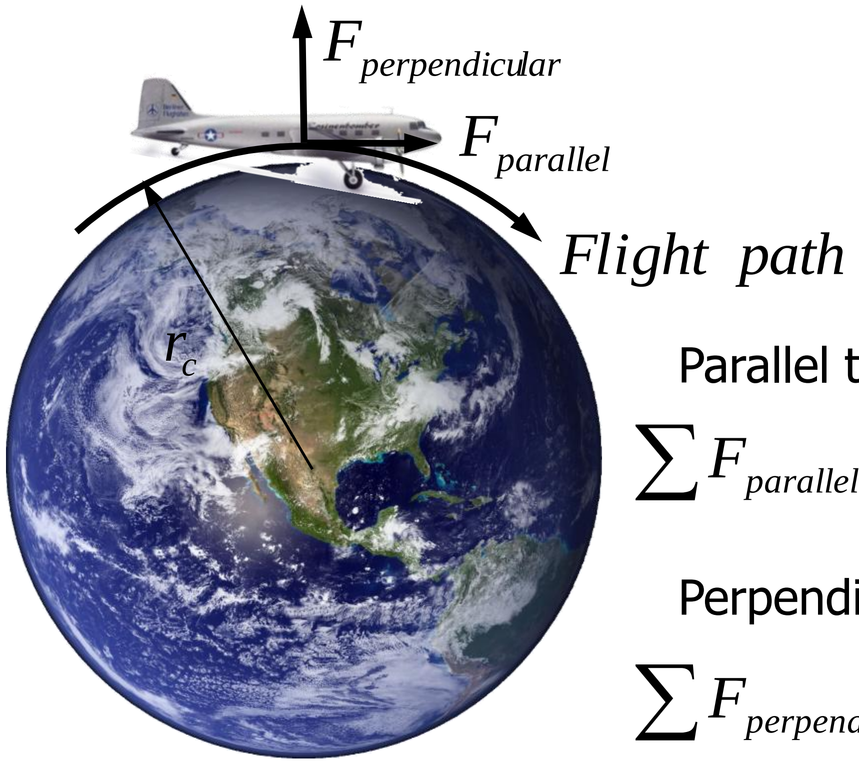
AIRCRAFT PERFORMANCE



EQUATIONS OF MOTION

When an object (such as an aircraft) is moving along a curved path, the motion is called curvilinear as opposed to motion along a straight line, which is rectilinear

Applying *Newton's* second law for either case ($F = m \times a$)



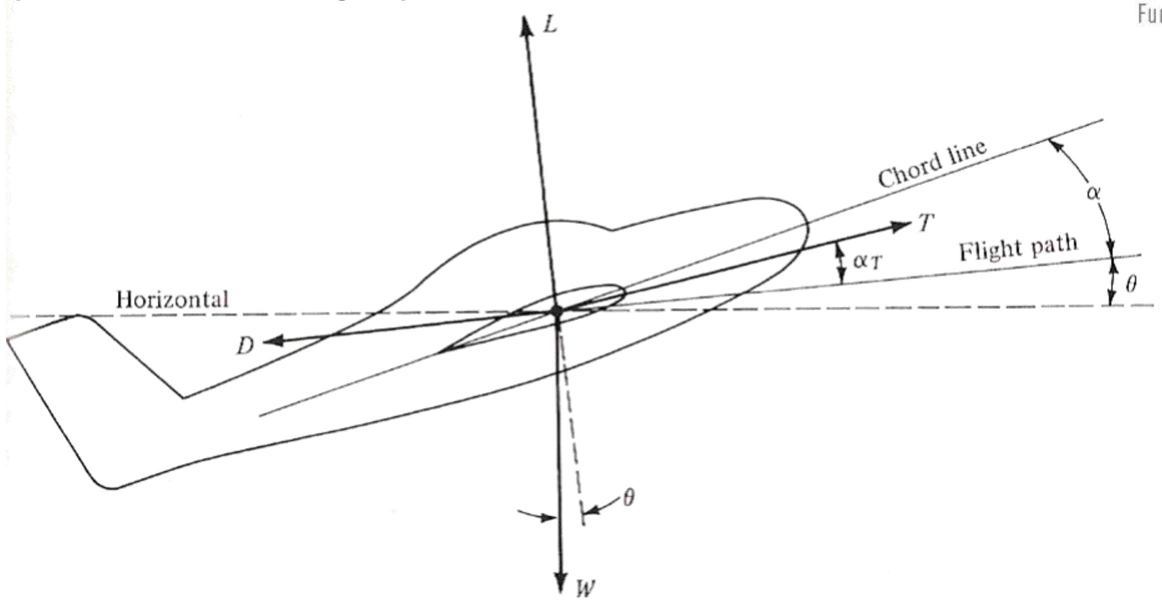
Parallel to flight path:

$$\sum F_{parallel} = m \cdot a = m \frac{dV}{dt}$$

Perpendicular to flight path:

$$\sum F_{perpendicular} = m \cdot a_N = m \frac{V^2}{r_c}$$

In accordance to the figure the forces parallel and perpendicular to the flight path can be seen to be:



Parallel forces:

$$\sum F_{parallel} = T \cos \alpha_T - D - W \sin \theta$$

Perpendicular forces:

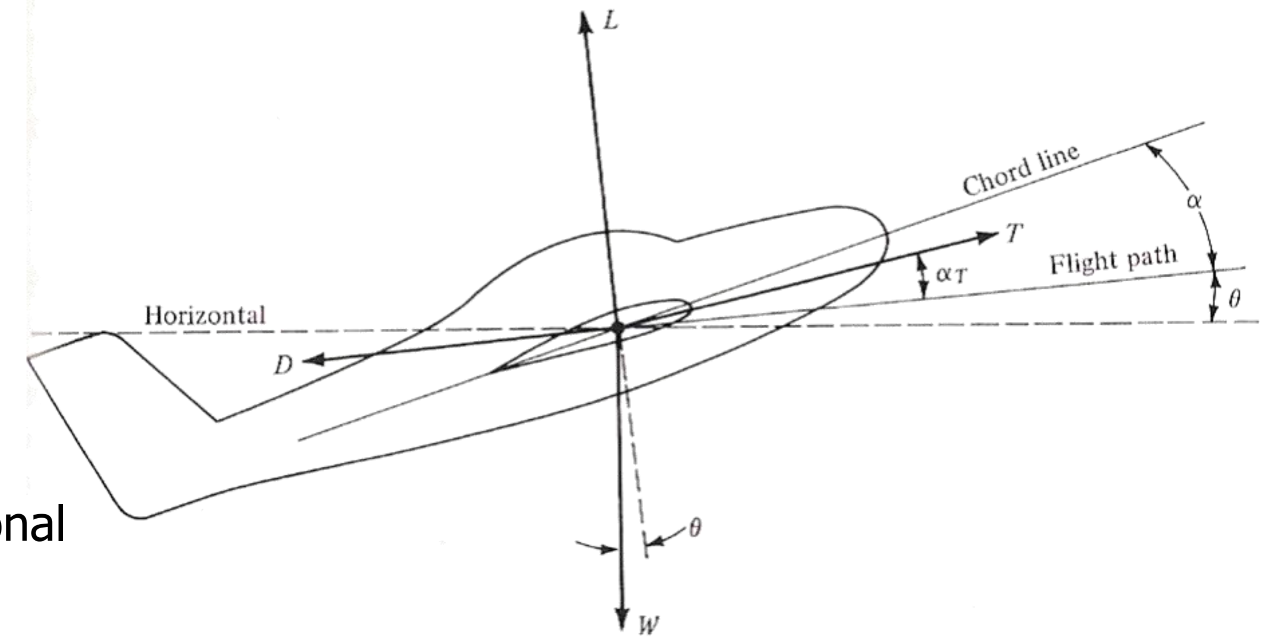
$$\sum F_{perpendicular} = L + T \sin \alpha_T - W \cos \theta$$

EQUATIONS OF MOTION

The ***equations of motion*** for an aircraft in translational flight can be obtained with the combination of the last four equations:

Parallel to flight path: $\boxed{T\cos\alpha_T - D - W\sin\theta = m\dfrac{dV}{dt}}$

Perpendicular to flight path: $\boxed{L + T\sin\alpha_T - W\cos\theta = m\dfrac{V^2}{r_c}}$

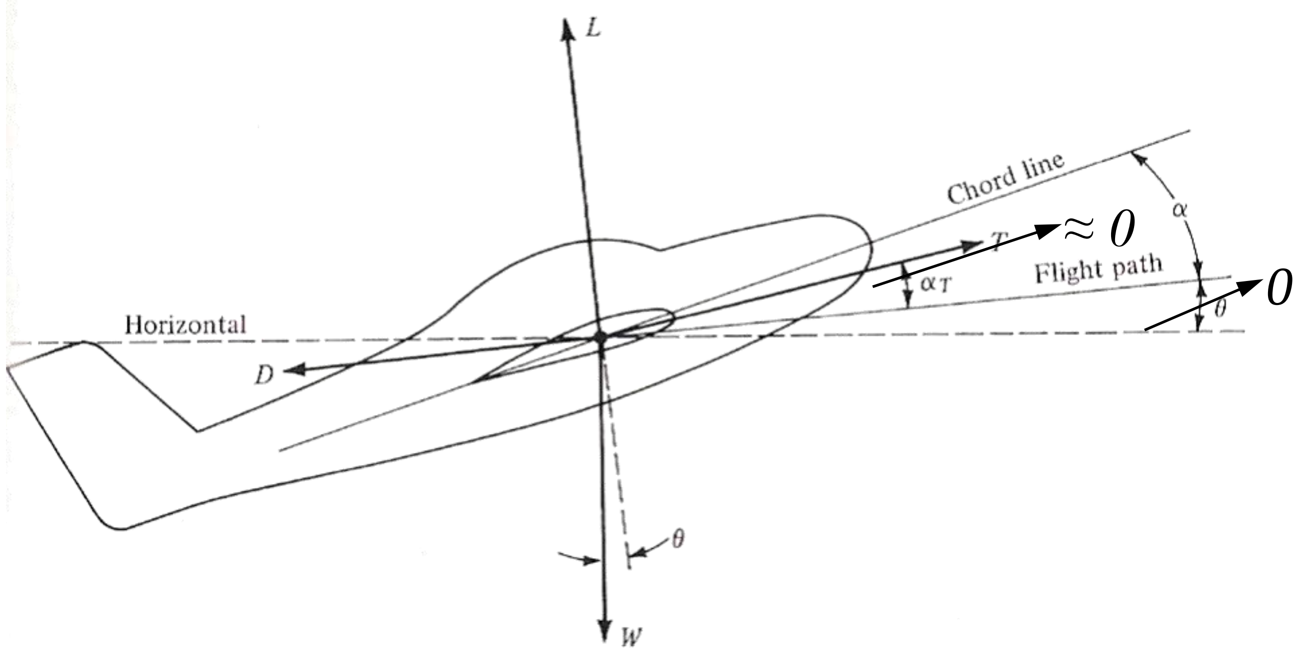


These two equations describe the general two-dimensional translational motion of an aircraft in accelerated flight.

For static performance it is necessary to have an unaccelerated flight condition ($a = 0$)

EQUATIONS OF MOTION

For most conventional aircrafts, α_T is small enough that $\cos\alpha_T \approx 1$ and $\sin\alpha_T \approx 0$, thus the equations can be written as:



Parallel forces: $$T\cos\alpha_T = D$$
$$T = D$$

Perpendicular forces: $$L + T\sin\alpha_T = W$$
$$L = W$$

This two last equations are the ***equations of motion for level, un-accelerated flight***

In level, un-accelerated flight, the aerodynamic **drag** is balanced by the **thrust** of the engine, and the aerodynamic **lift** is balanced by the **weight** of the aircraft

THRUST REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

Considering an aircraft in a steady, level flight at a given velocity. For this condition, the aircraft's power plant must produce a net thrust that is equal to the drag.

The thrust required to obtain a certain velocity can be calculated to be as:

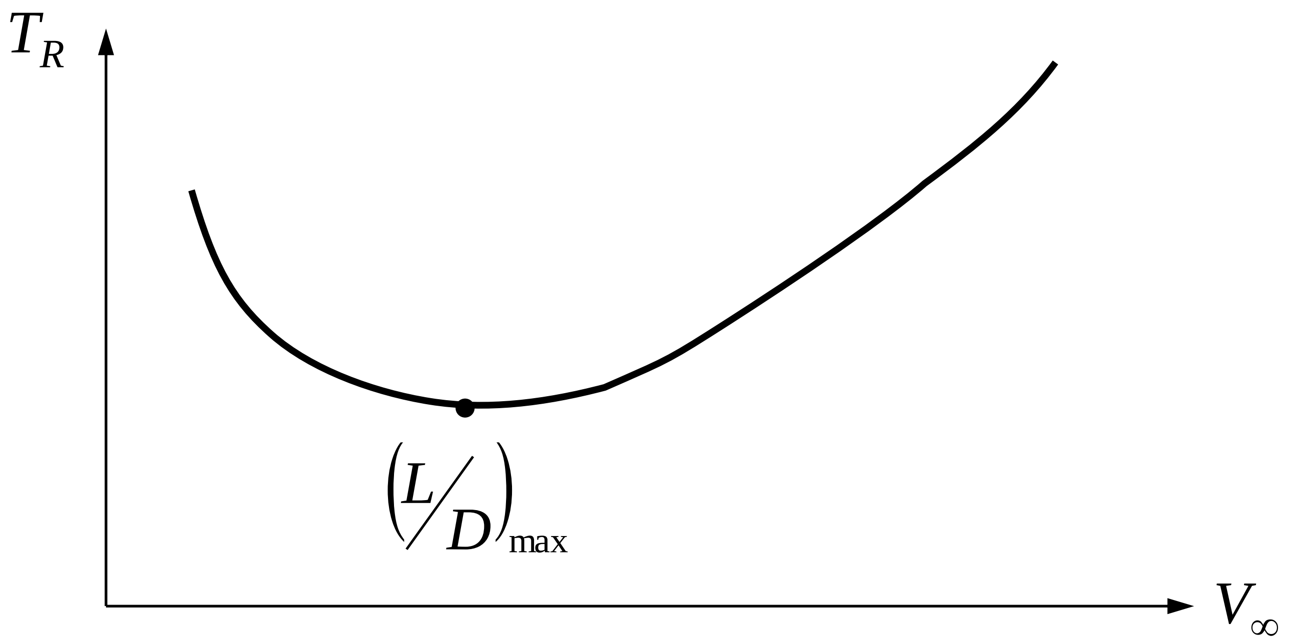
$$T = D = q_\infty S\, C_D$$

and $L = W = q_\infty S\, C_L$

Dividing these two equations yields $\dfrac{T}{W} = \dfrac{C_D}{C_L}$

Then, the required thrust for an aircraft, to fly at a given velocity in level, the unaccelerated flight is:

$$\boxed{T_R = \frac{W}{C_L / C_D} = \frac{W}{L/D}}$$



THRUST REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

To construct the thrust required curve, it is proceeding as follows:

1. Chose values for V_∞

2. For this values of V_∞, calculate the lift coefficient from:

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$

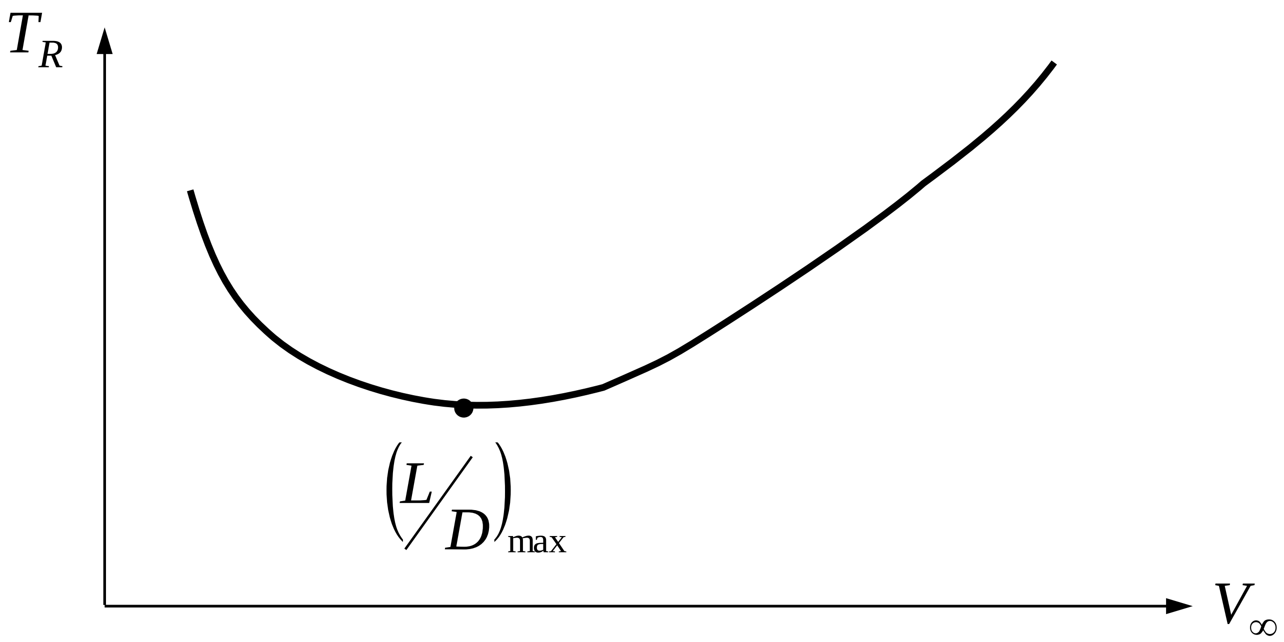
ρ_∞ – is given by the altitude
S – is the wing area given for the design aircraft
C_L – is that value needed for the lift to balance the weight (W) of the aircraft

3. Calculate C_D from the drag polar of the aircraft:

$$C_D = C_{D,0} + \frac{C_L^2}{\pi \cdot e \cdot AR}$$

C_L – is the lift coefficient

4. Form the ratio C_L/C_D
5. Calculate thrust required from: $T_R = \frac{W}{C_L/C_D} = \frac{W}{L/D}$



THRUST REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

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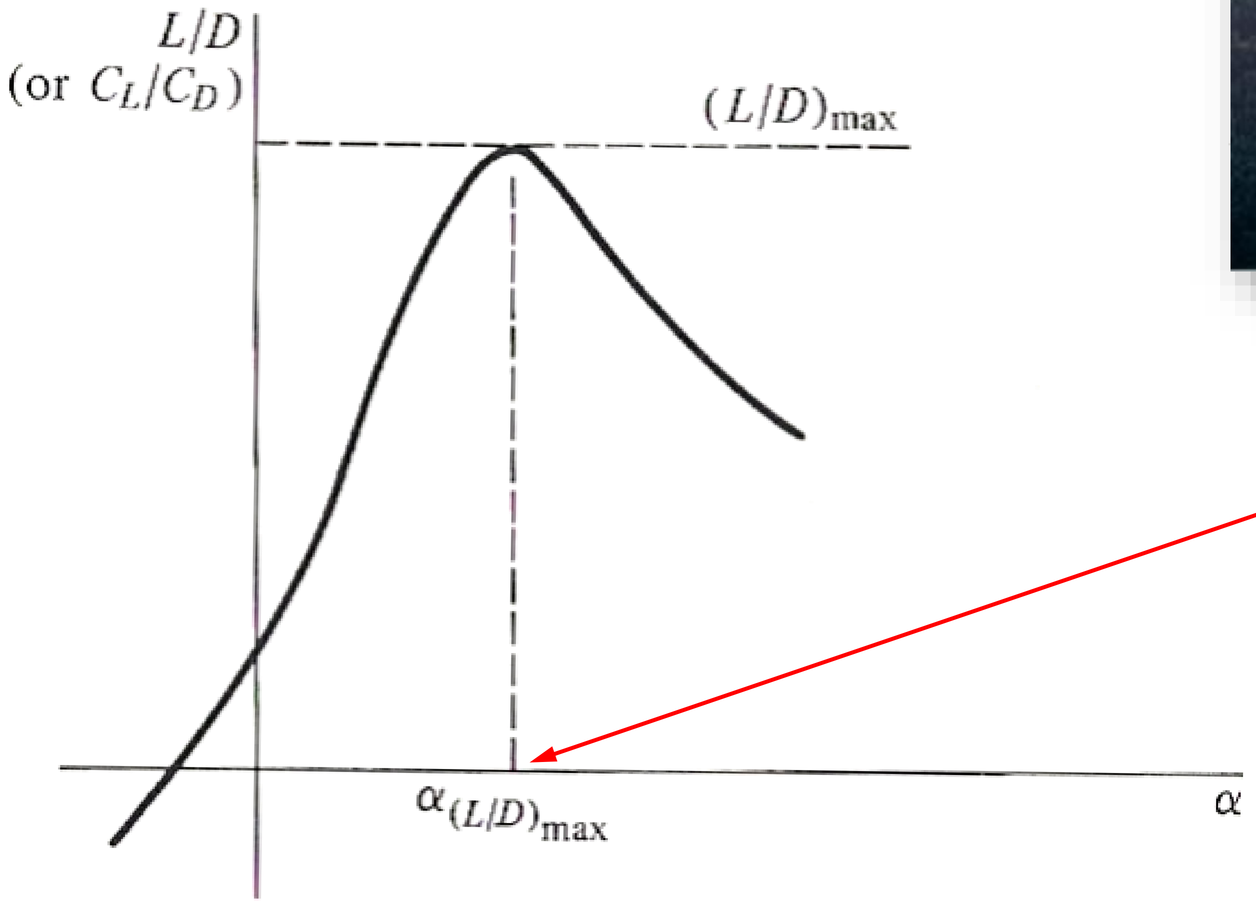
L/D is a measure of the aerodynamic efficiency and a function of the angle of attack as shown in the following figure:



For most conventional subsonic aircraft, L/D reaches a maximum at some specific value of α, usually on the order of 2 to 5°

When an aircraft is flying at a velocity for minimum T_R, it is simultaneously flying at the angle of attack for maximum L/D

THRUST REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

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The total drag of an aircraft is the sum of the zero-lift drag and the drag due to lift. The corresponding drag coefficients are $C_{D,0,}$ and $C_{D,i}$, respectively.

At a given condition T_R will then be:

$$T_R = D = q_\infty S\, C_D = q_\infty S\left(C_{D,0} + C_{D,i}\right) = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi \cdot e \cdot AR}\right)$$

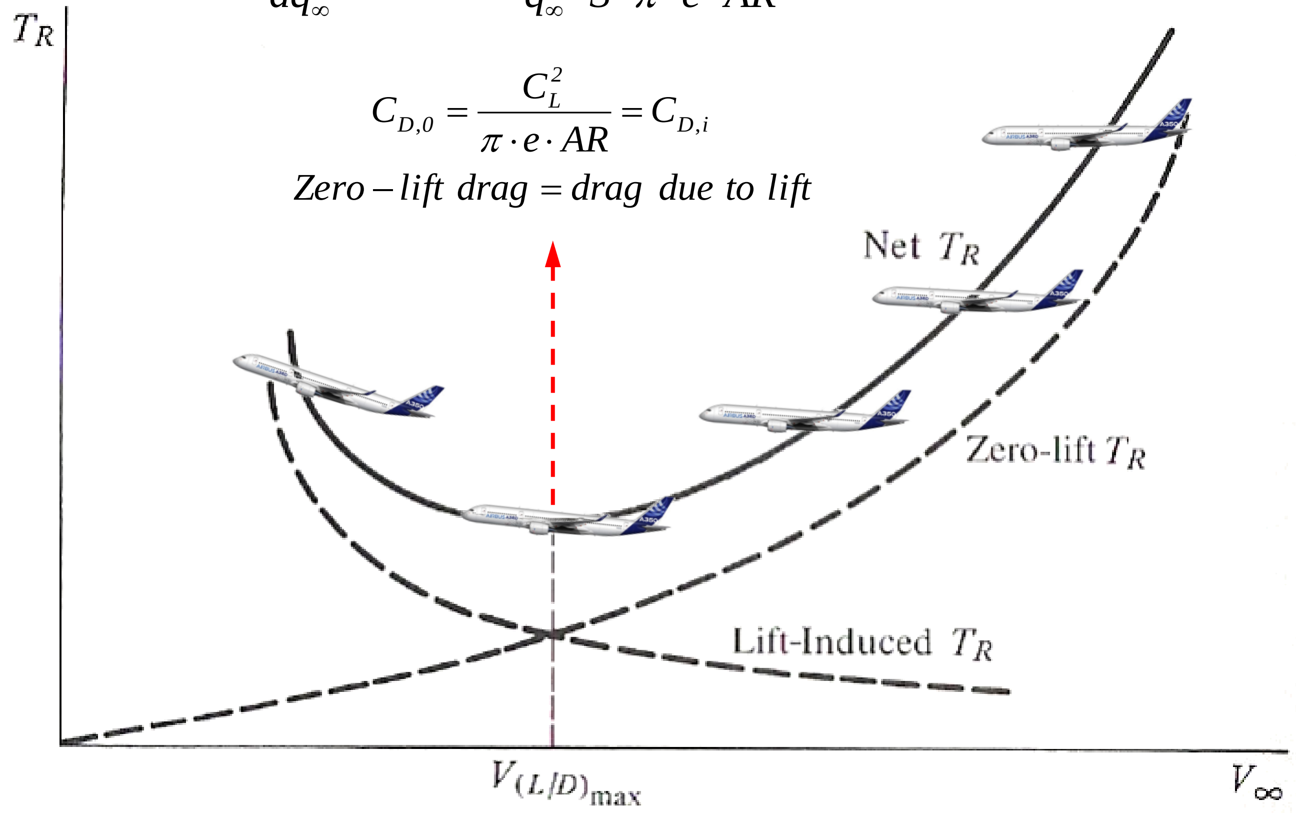
$$T_R = \underbrace{q_\infty S\, C_{D,0}}_{Zero-lift\ T_R} + \underbrace{q_\infty S\frac{C_L^2}{\pi \cdot e \cdot AR}}_{Lift-induced\ T_R}$$

It is noted that the trust required can be considered as the sum of the ***zero-lift thrust required*** (thrust required to balance zero-lift drag) and ***lift-induced thrust required*** (thrust required to balance drag due to lift)

$$\frac{dT_R}{dq_\infty} = S\, C_{D,0} - \frac{W^2}{q_\infty^2 \cdot S \cdot \pi \cdot e \cdot AR} = 0$$

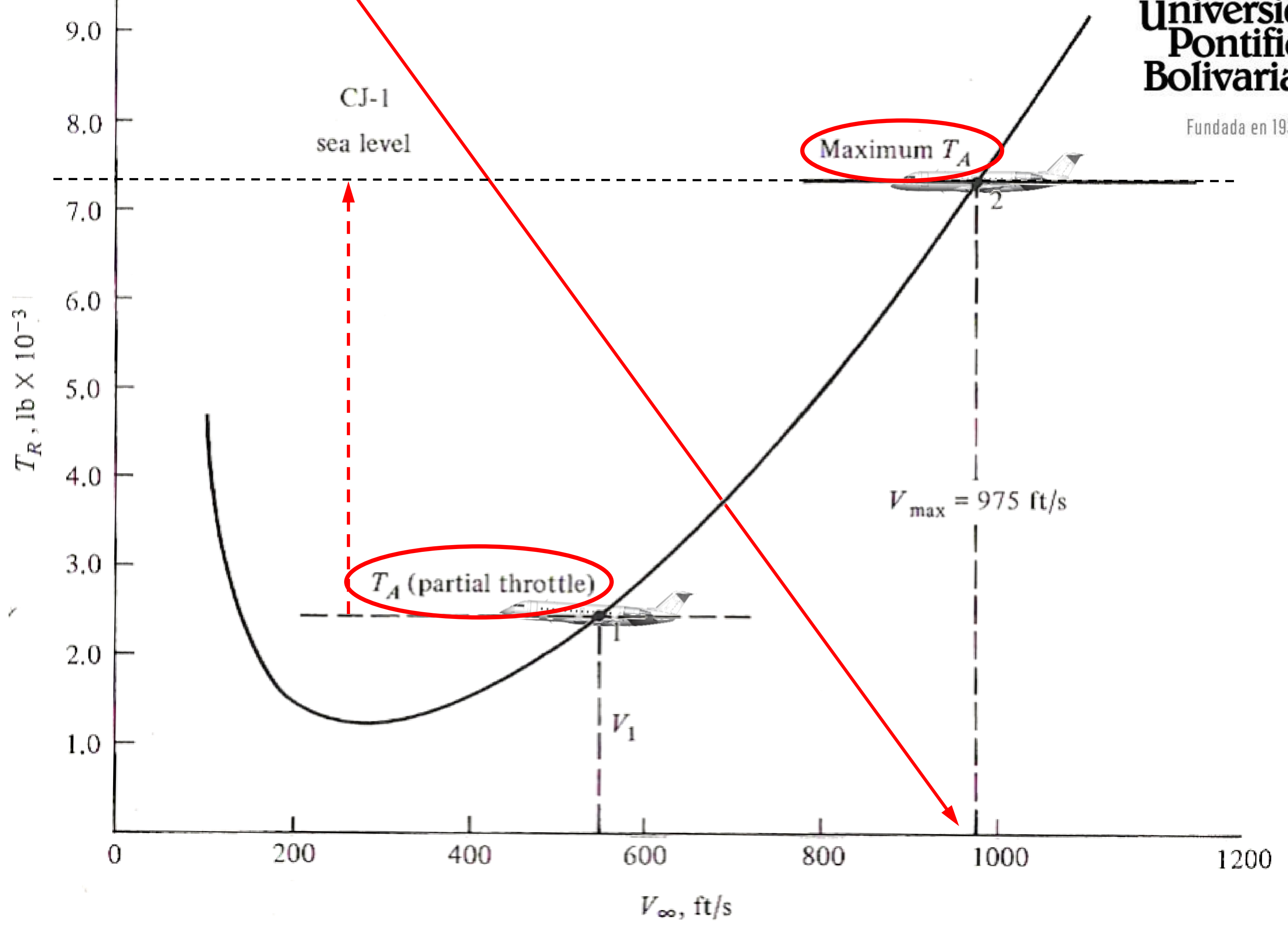
$$C_{D,0} = \frac{C_L^2}{\pi \cdot e \cdot AR} = C_{D,i}$$

$$Zero-lift\ drag = drag\ due\ to\ lift$$



THRUST AVAILABLE AND MAXIMUM VELOCITY

$$V_{max} = \sqrt{\frac{\left(\frac{T_A}{W}\right)_{max}\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T_A}{W}\right)_{max}^{2} - \frac{4C_{D,0}}{\pi e AR}}}{\rho_\infty C_{D,0}}}$$



POWER REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

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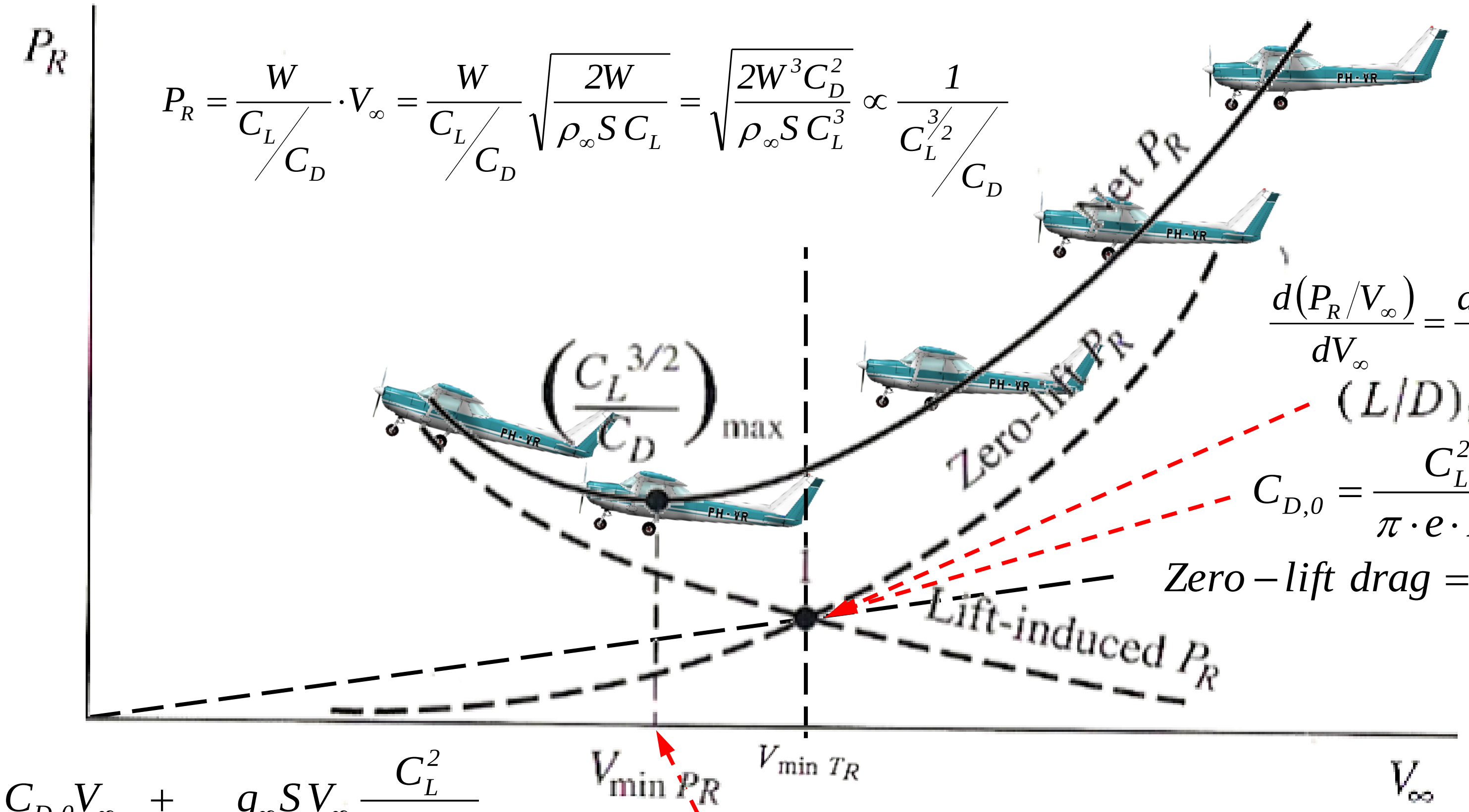
$$P_R = T_R \cdot V_\infty = \frac{W}{C_L/C_D} \cdot V_\infty$$

The power required can be split into the respective contributions needed to overcome zero-lift drag and drag due to lift

This could be expressed in an equation form as follows:

$$P_R = \frac{W}{C_L/C_D} \cdot V_\infty = \frac{W}{C_L/C_D} \sqrt{\frac{2W}{\rho_\infty S\, C_L}} = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S\, C_L^3}} \propto \frac{1}{C_L^{3/2}/C_D}$$



$$\frac{d(P_R/V_\infty)}{dV_\infty} = \frac{d(T_R V_\infty/V_\infty)}{dV_\infty} = \frac{dT_R}{dV_\infty} = 0$$

$(L/D)_{\max}$

$$C_{D,0} = \frac{C_L^2}{\pi \cdot e \cdot AR} = C_{D,i}$$

$$Zero-lift\ drag = drag\ due\ to\ lift$$

$$P_R = T_R \cdot V_\infty = D \cdot V_\infty = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi\, e\, AR}\right)V_\infty = \underbrace{q_\infty S\, C_{D,0} V_\infty}_{Zero-lift\ power\ required} + \underbrace{q_\infty S\, V_\infty \frac{C_L^2}{\pi\, e\, AR}}_{Lift-induced\ power\ required}$$

$$\frac{dP_R}{dV_\infty} = \frac{3}{2}\rho_\infty V_\infty^2 S\, C_{D,0} - \frac{W^2/(0{,}5\rho_\infty V_\infty^2 S)}{\pi\, e\, AR} = \frac{3}{2}\rho_\infty V_\infty^2 S\left[C_{D,0} - \frac{W^2/(0{,}75\rho_\infty^2 V_\infty^4 S^2)}{\pi\, e\, AR}\right]$$

$$C_{D,0} = \frac{1}{3} C_{D,i}$$

RELATIONS BETWEEN $C_{D,0}$ AND $C_{D,i}$

The relations between $C_{D,0}$ and $C_{D,i}$ depend purely on the conditions for maximum.

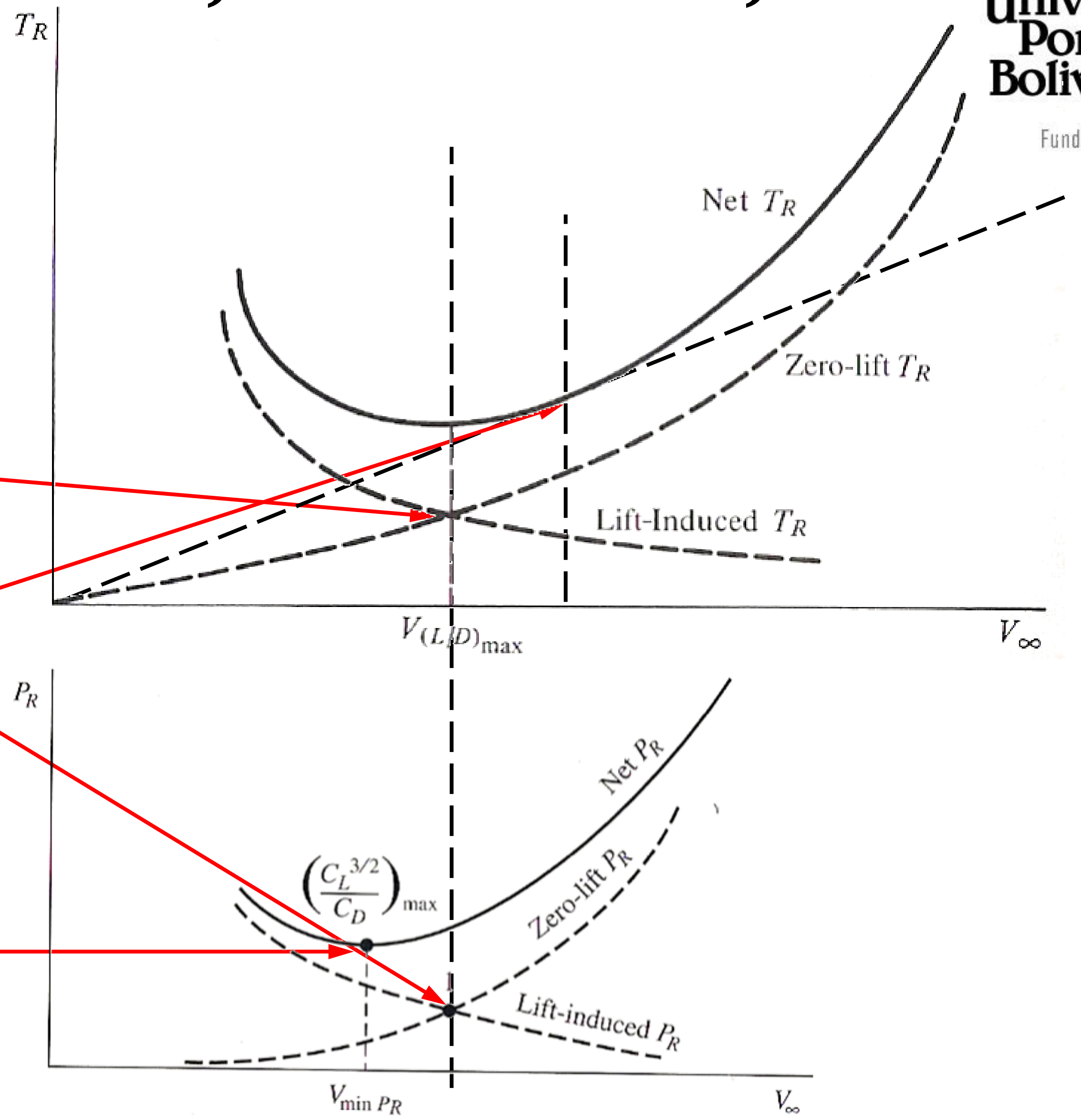
Hence, the value of the maximum C_L/C_D, is obtained from the last equation as:

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{\sqrt{C_{D,0}\pi e AR}}{2C_{D,0}}$$

In the same vein, it can be shown that,

$$\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{\left(\frac{1}{3}C_{D,0}\pi e AR\right)^{1/4}}{\frac{4}{3}C_{D,0}}$$

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{\left(3C_{D,0}\pi e AR\right)^{3/4}}{4C_{D,0}}$$



ALTITUDE EFFECTS ON POWER REQUIRED

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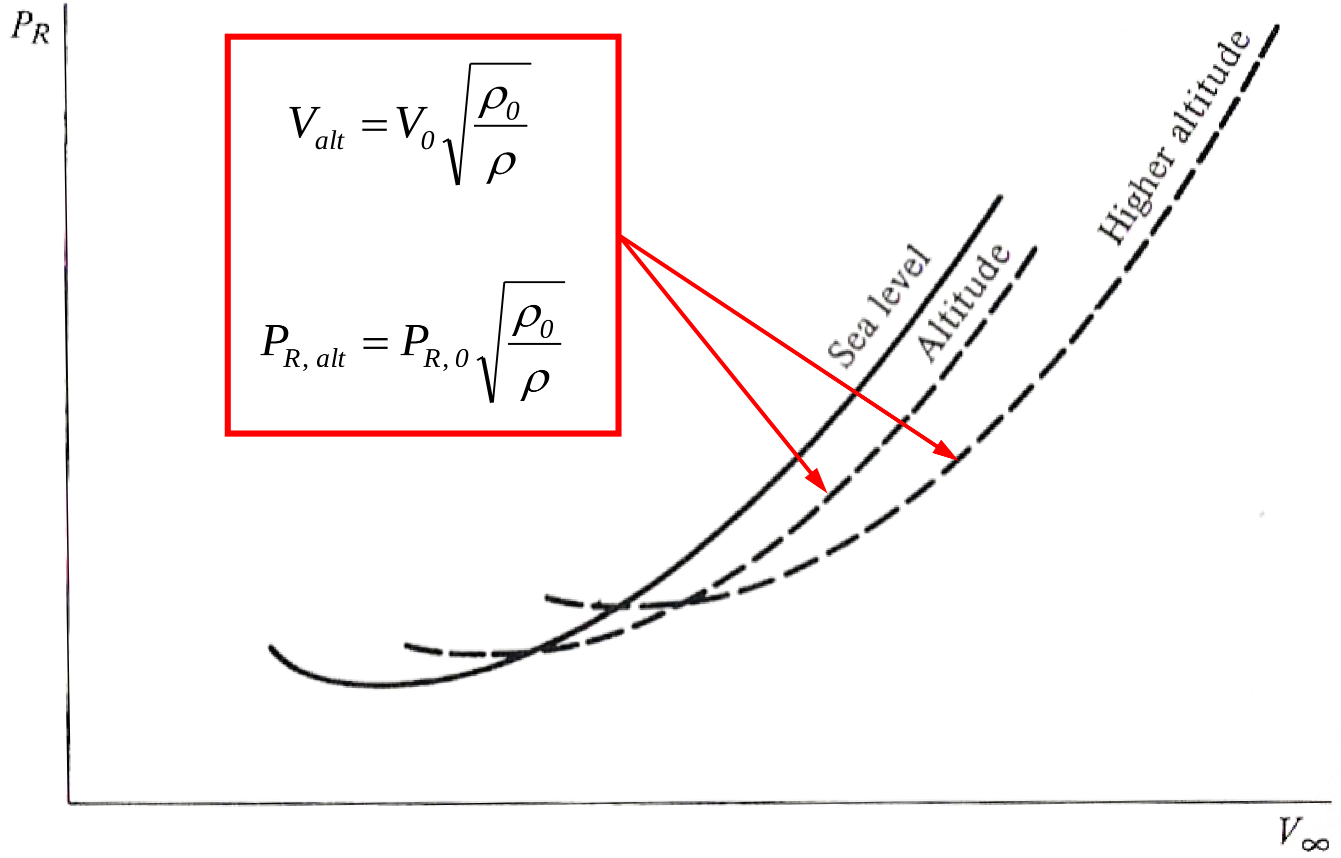
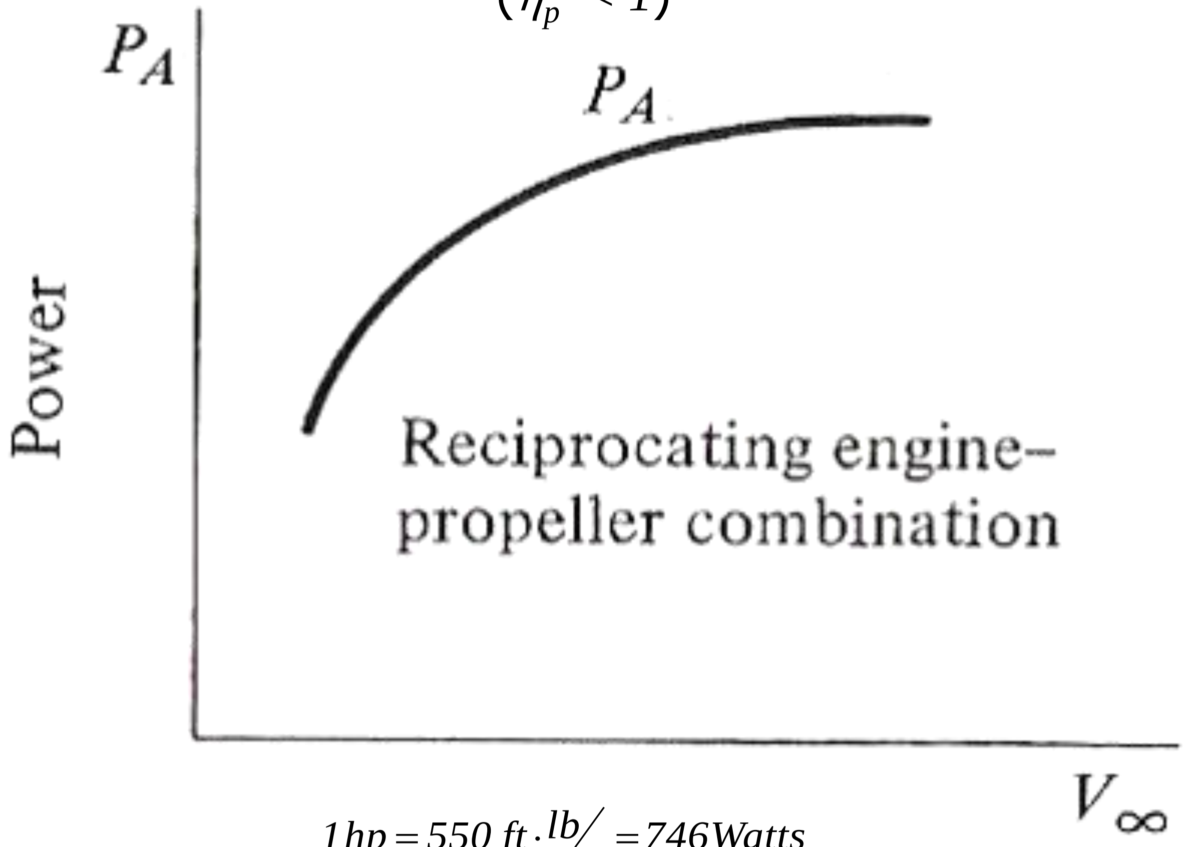
The complete P_R curve for any given altitude (different from the $S.L.$ condition) tends to experience an upward and right translation and a slight clockwise rotation.



POWER AVAILABLE

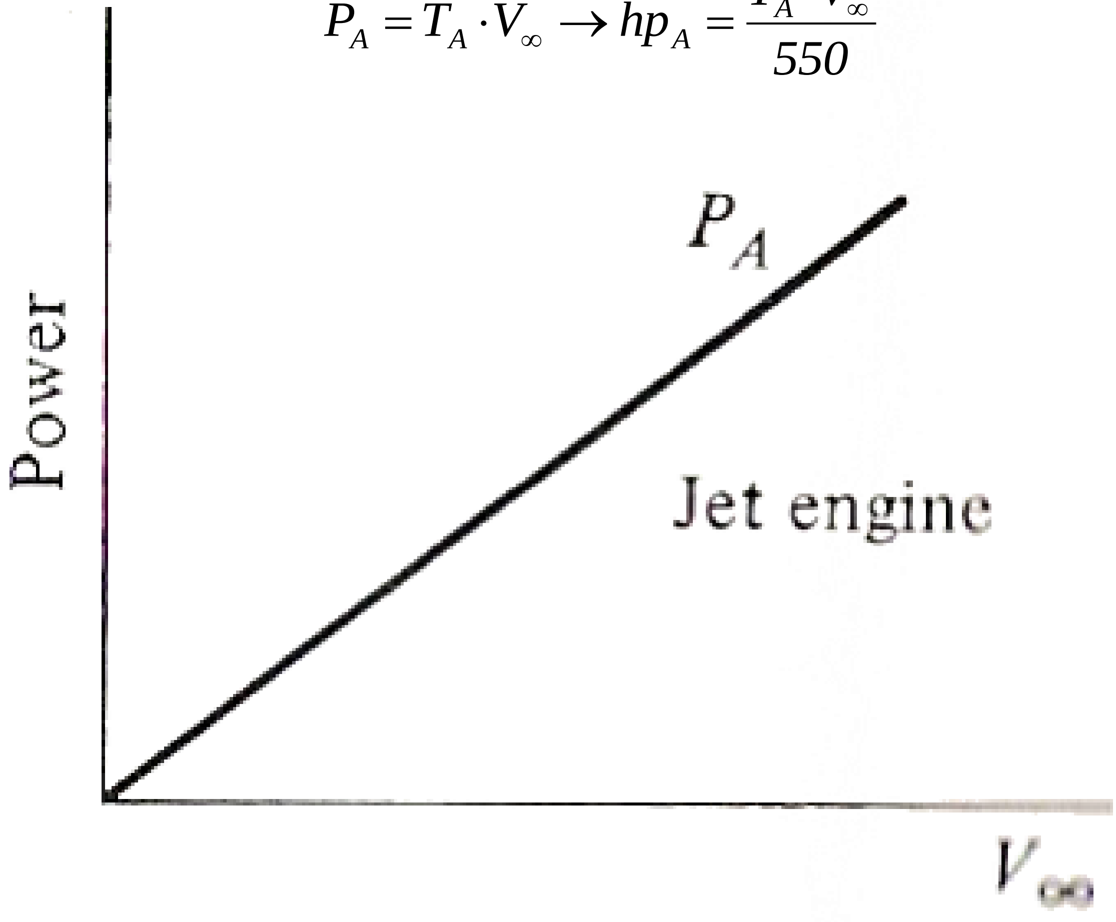
$$P_A = \eta_p \cdot P$$
$$(\eta_p < 1)$$



$$1\,hp = 550\; ft \cdot {}^{lb}\!/_{s} = 746 Watts$$

$$hp_A = \eta_p \cdot bhp$$

$$P_A = T_A \cdot V_\infty \rightarrow hp_A = \frac{T_A \cdot V_\infty}{550}$$



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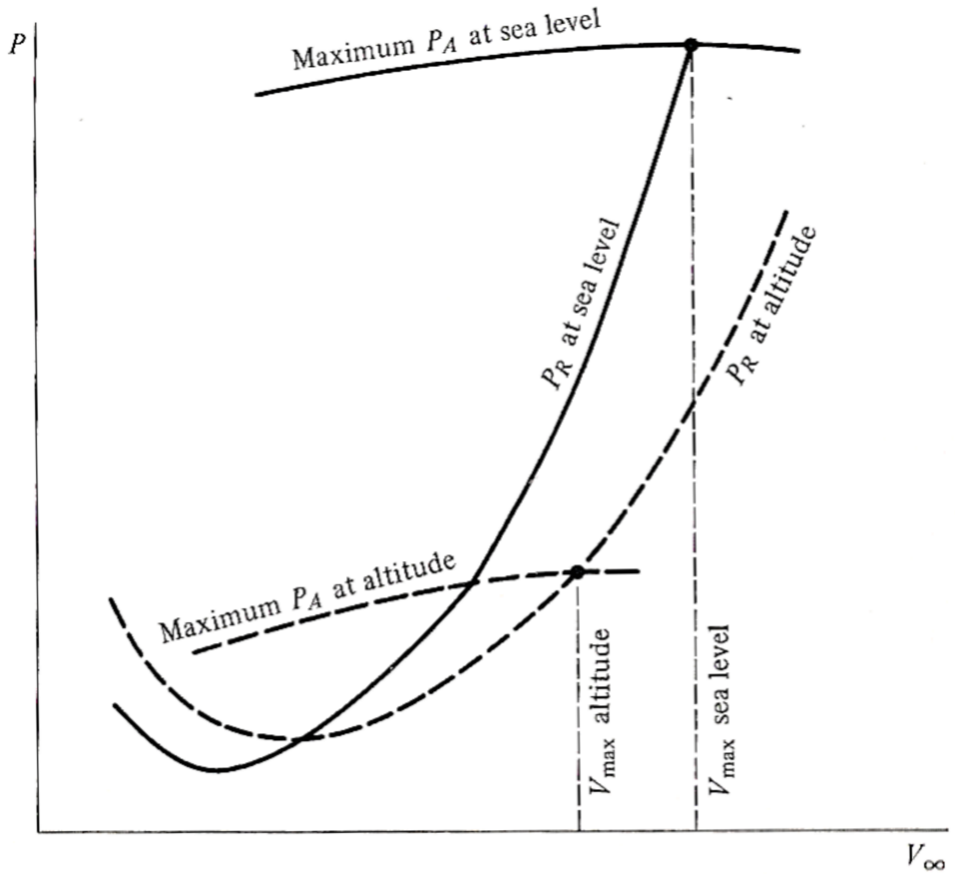
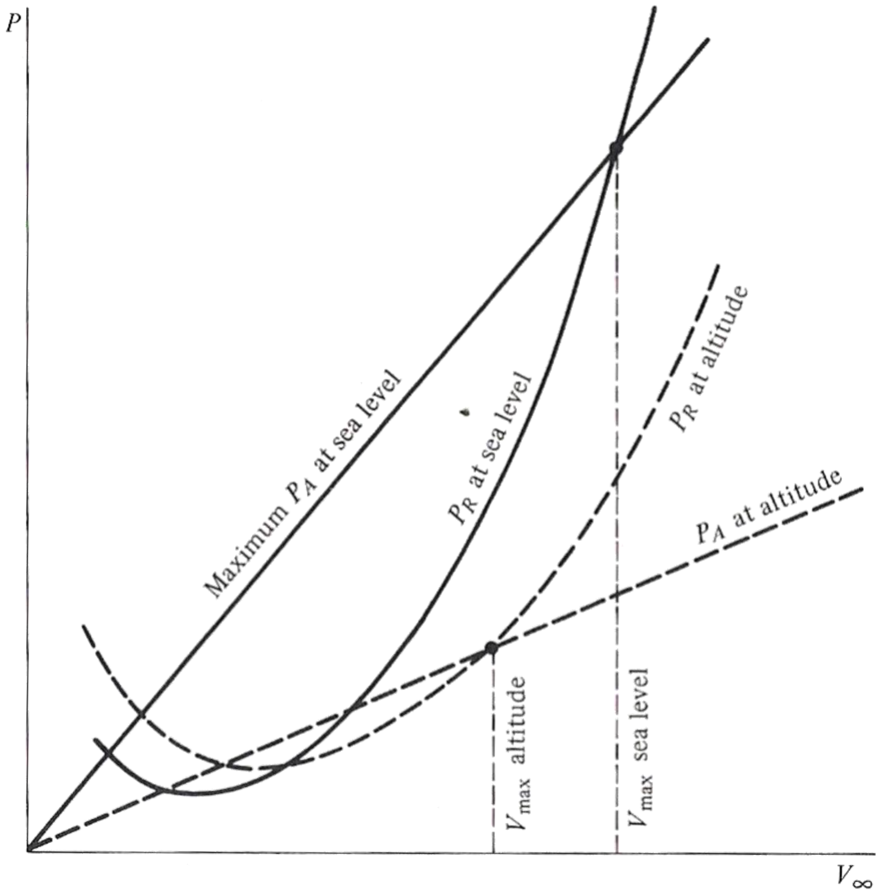
ALTITUDE EFFECTS ON POWER REQUIRED AND AVAILABLE

The impact on aircraft performance due to altitude effects are:



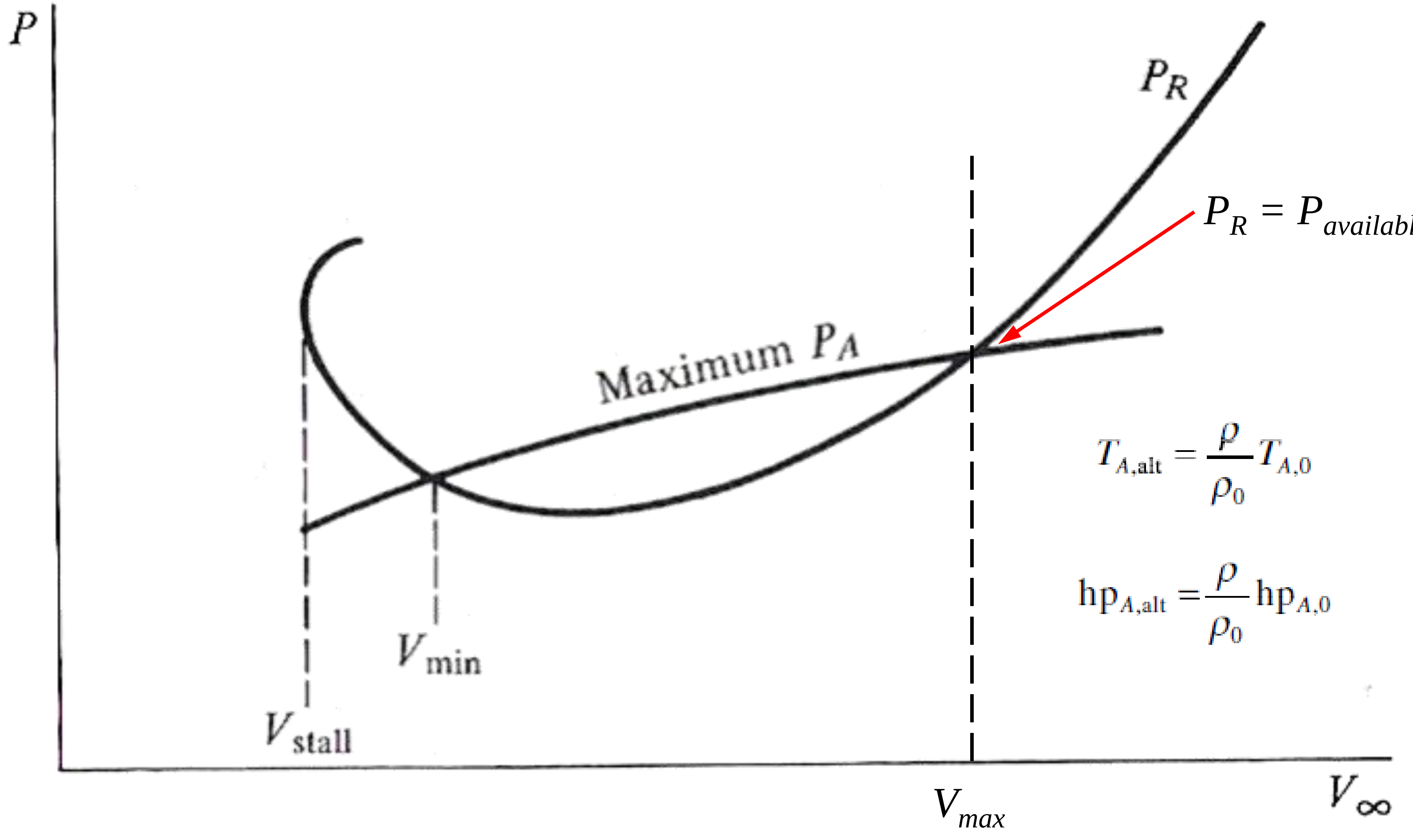
Propeller-driven aircraft



Jet-propelled aircraft

ALTITUDE EFFECTS ON POWER REQUIRED AND AVAILABLE

It is to note that $V_{\max}$ and also $V_{\min}$ limits vary

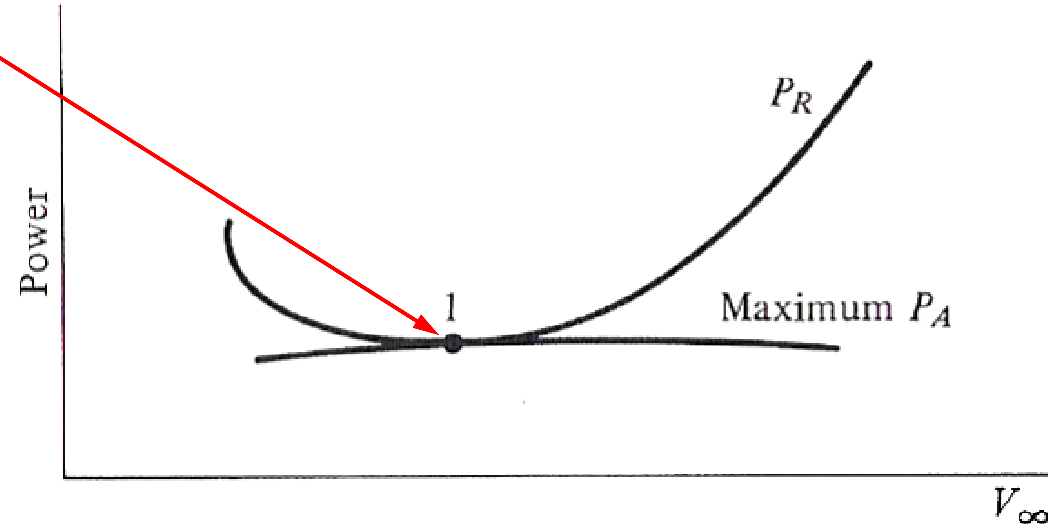


$P_R = P_{available}$

$$T_{A,\text{alt}} = \frac{\rho}{\rho_0} T_{A,0}$$

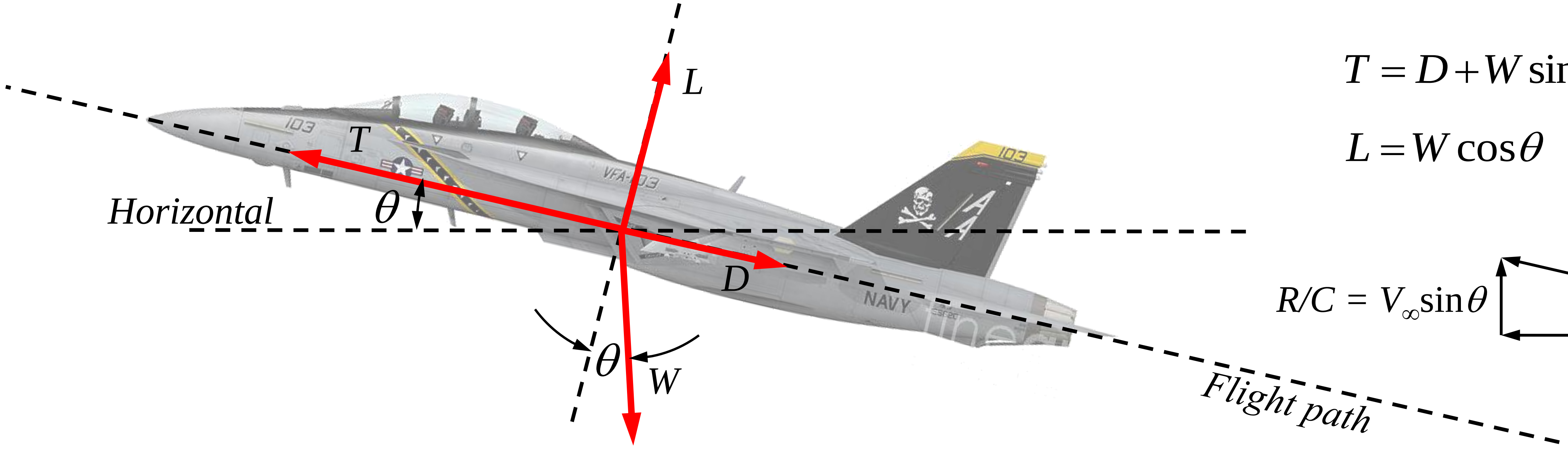
$$\text{hp}_{A,\text{alt}} = \frac{\rho}{\rho_0} \text{hp}_{A,0}$$

Absolute (maximum) ceiling



RATE OF CLIMB

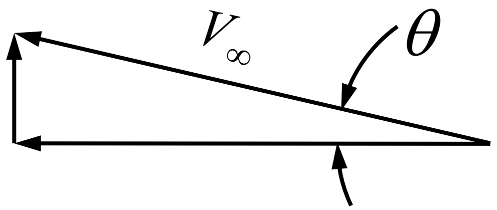
Considering an aircraft in steady, unaccelerated, climbing flight



$$T = D + W \sin\theta$$

$$L = W \cos\theta$$

$$R/C = V_\infty \sin\theta$$



$$T \cdot V_\infty = D \cdot V_\infty + W \cdot V_\infty \sin\theta$$

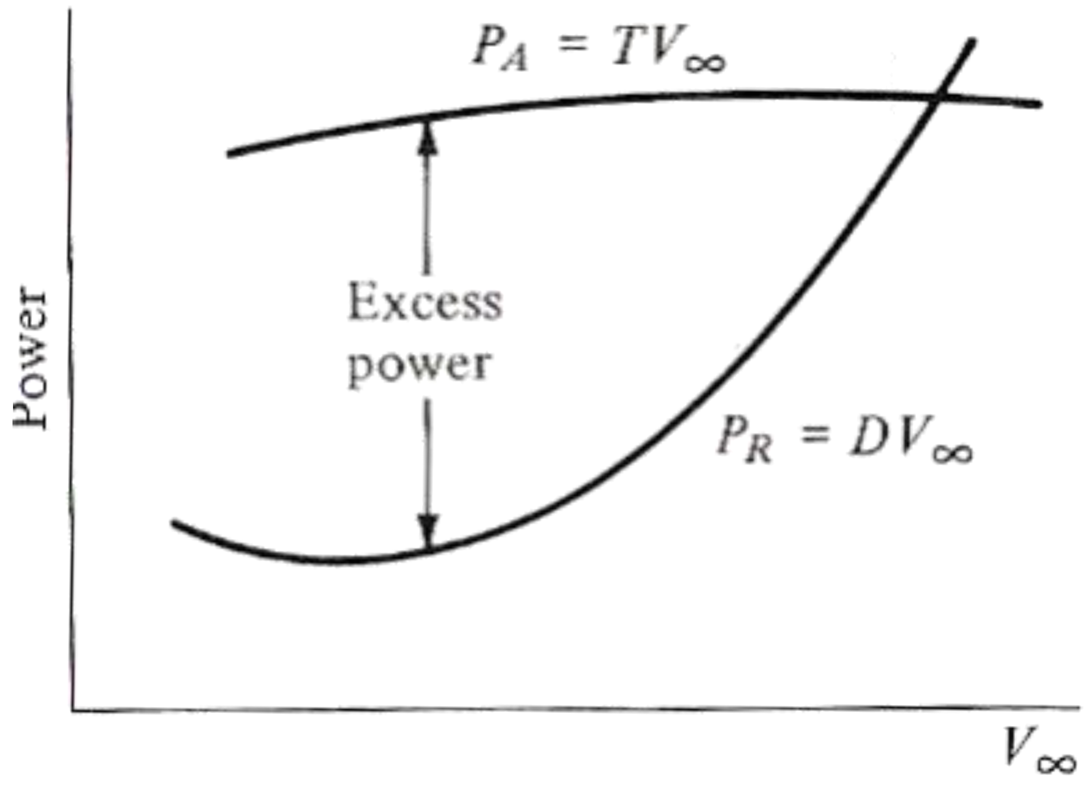
$$T \cdot V_\infty - D \cdot V_\infty = excess\ \ power$$

$$\frac{T \cdot V_\infty - D \cdot V_\infty}{W} = V_\infty \sin\theta$$

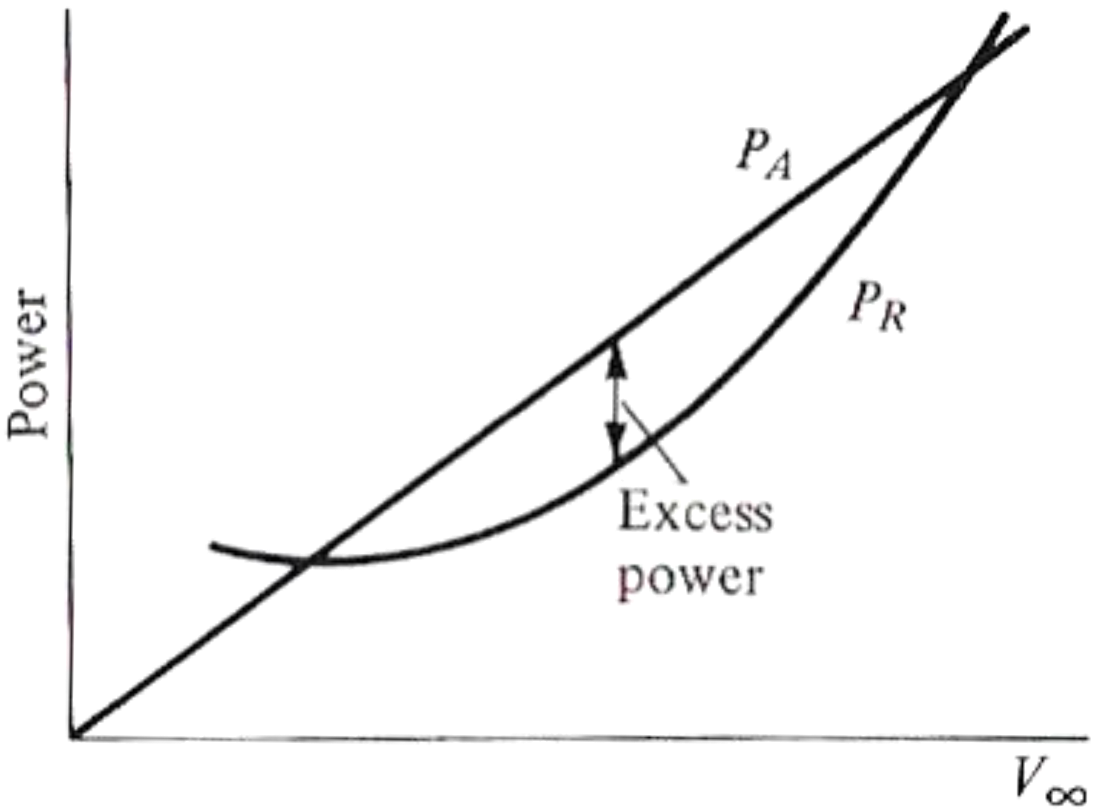
$$R/C = \frac{T \cdot V_\infty - D \cdot V_\infty}{W} = \frac{P_A - P_R}{W} = \frac{excess\ \ power}{W}$$

RATE OF CLIMB

Propeller-driven aircraft



Jet-propelled aircraft



$$R/C = \frac{T \cdot V_\infty - D \cdot V_\infty}{W} = \frac{P_A - P_R}{W} = \frac{excess\ \ power}{W}$$

RATE OF CLIMB

$$(R/C)_{\max} = \frac{\max.\ excess\ power}{W}$$

The $(R/C)_{\max}$ can be obtained by differentiation

From Anderson's *Aircraft Performance and Design*, it can be seen that:

For propeller-driven aircraft:

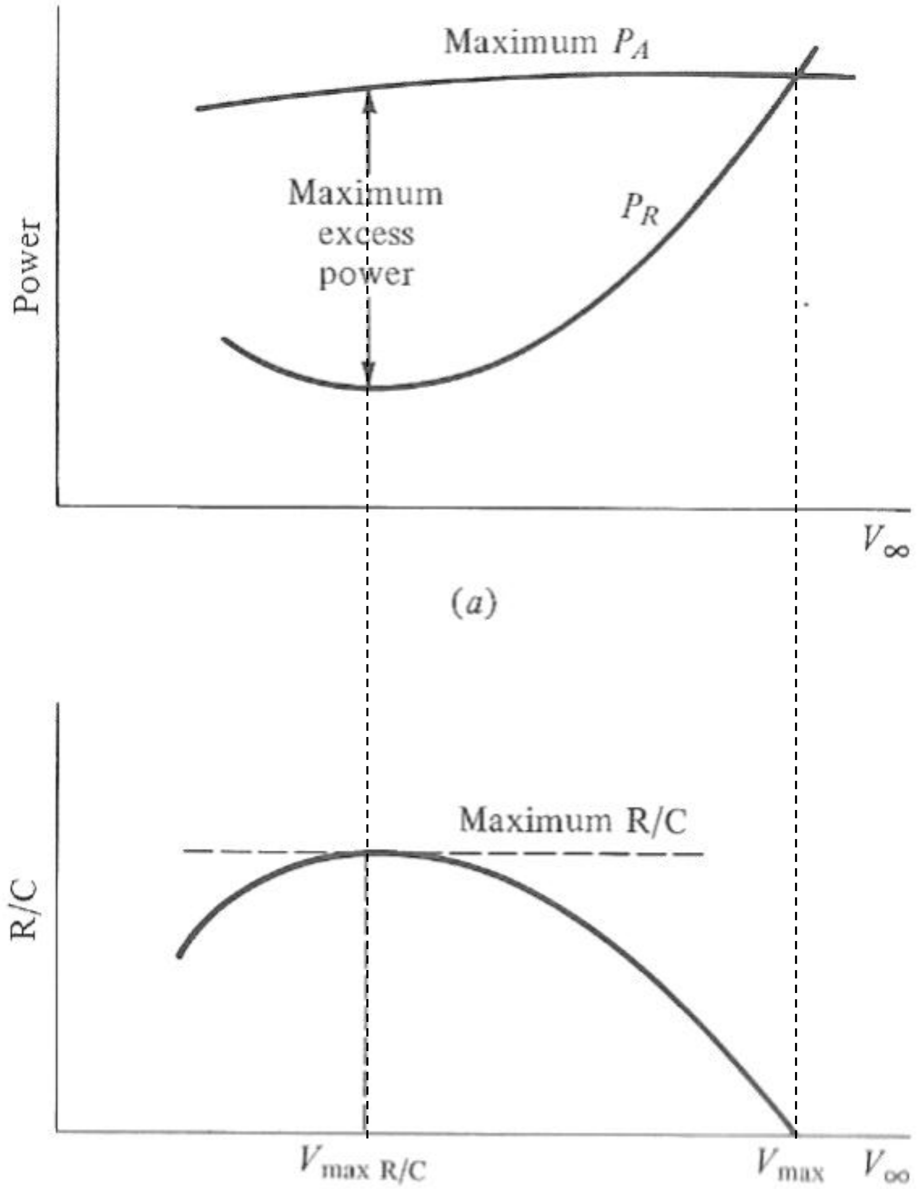
$$(R/C)_{\max} = \left(\frac{\eta_p P}{W}\right)_{\max} - 0{,}8776\sqrt{\frac{W/S}{\rho_\infty C_{D,0}}}\frac{1}{\left(L/D\right)_{\max}^{3/2}}$$

For jet-propelled aircraft:

$$(R/C)_{\max} = \left[\frac{(W/S)Z}{3\rho_\infty C_{D,0}}\right]^{1/2}\left(\frac{T}{W}\right)_{\max}^{3/2}\left[1 - \frac{Z}{6} - \frac{3}{2\left(\frac{T}{W}\right)_{\max}^2\left(\frac{L}{D}\right)_{\max}^2 Z}\right]$$

where:

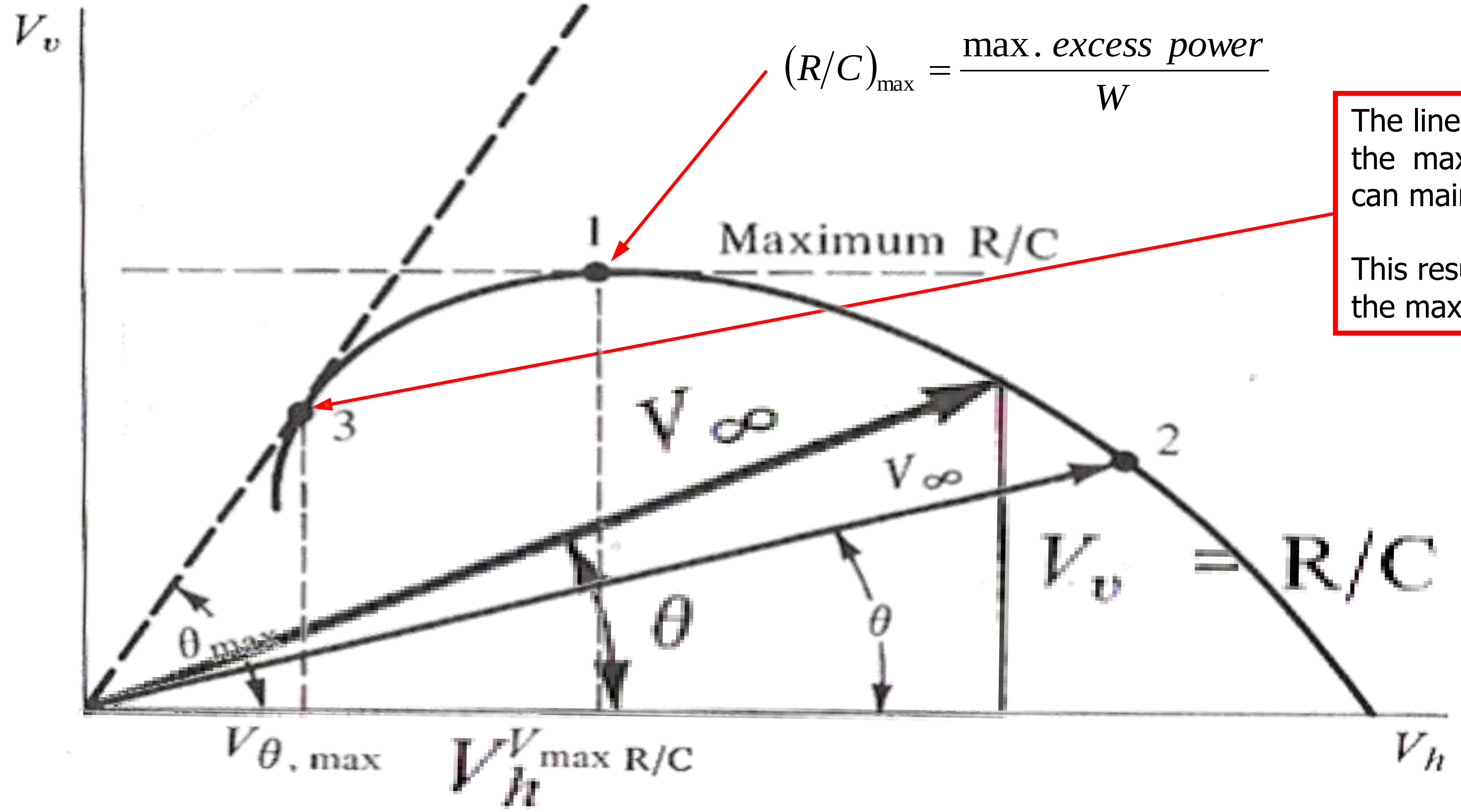
$$Z = 1 + \sqrt{1 + \frac{3}{\left(\frac{L}{D}\right)_{\max}^2\left(\frac{T}{W}\right)_{\max}^2}}$$



RATE OF CLIMB

Another useful method to find the $(R/C)_{max}$ is by a *Hodograph* diagram, which is a graph of the aircraft's vertical velocity V_v ($R/C = V_v$) versus its horizontal velocity.

$$(R/C)_{max} = \frac{\max.\ excess\ power}{W}$$

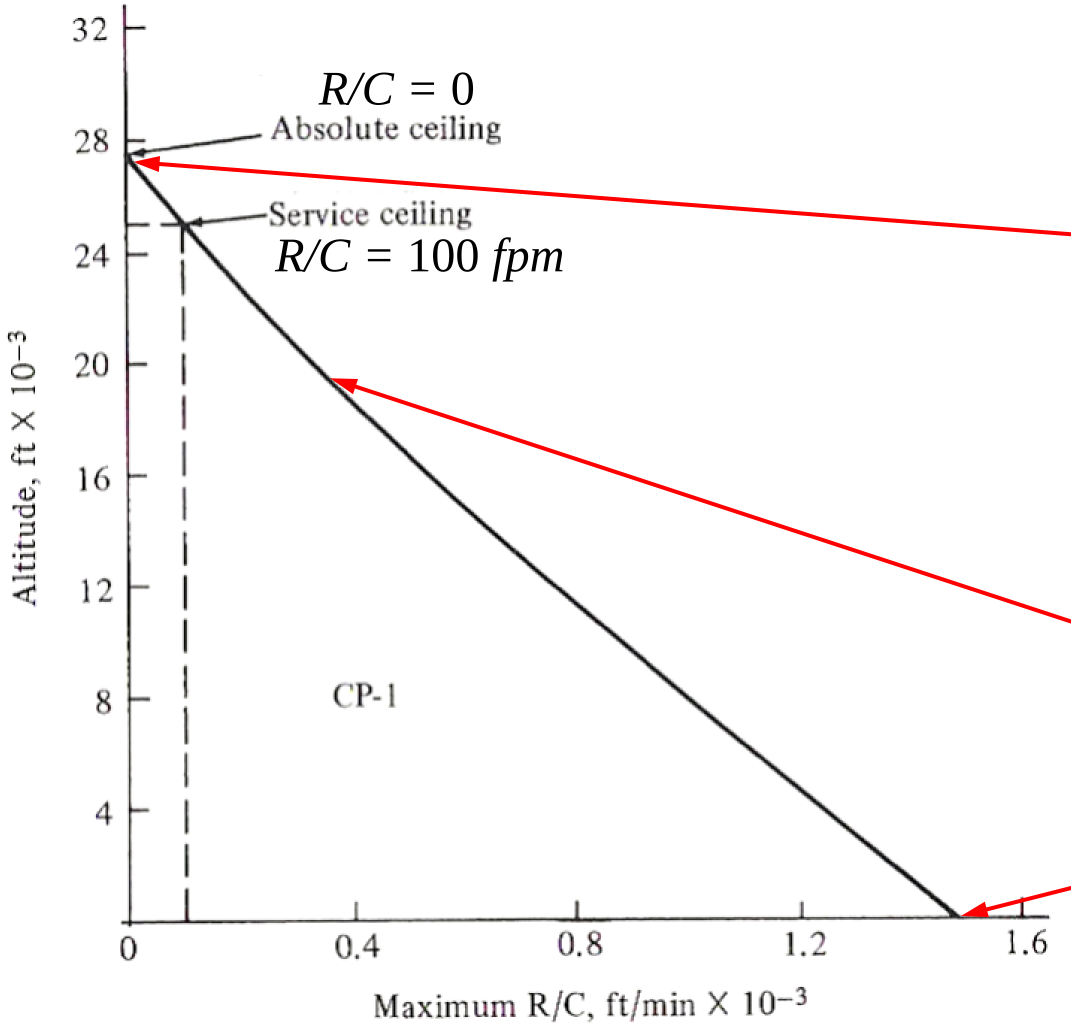


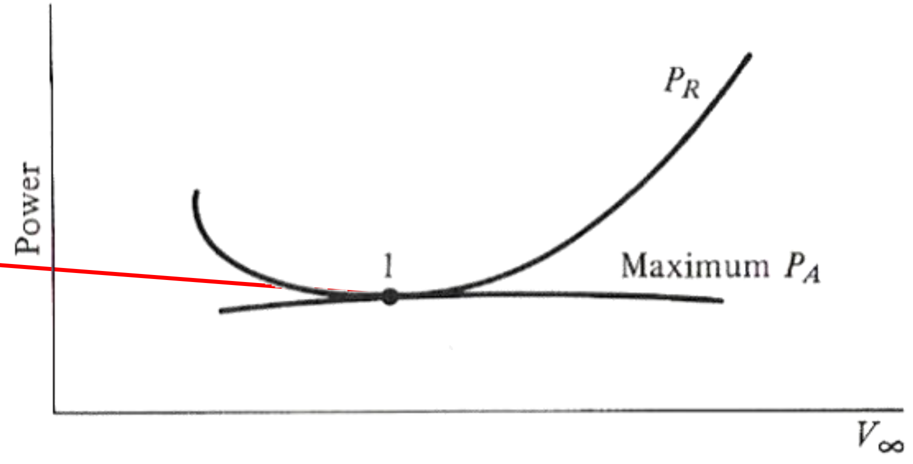
The line (at point 3) tangent to the *hodograph* gives the maximum climb angle for which the aircraft can maintain steady flight (θ_{max})

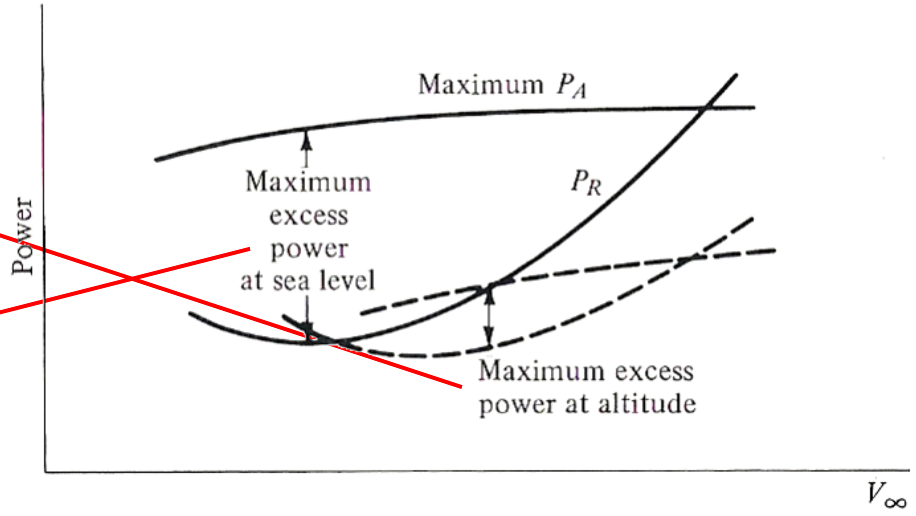
This results in that maximum R/C does not occur at the maximum climb angle

ABSOLUTE AND SERVICE CEILINGS

Considering a propeller-driven aircraft, as altitude increases, the maximum excess power decreases (which results in a decrease of R/C), as shown in the figures:







ABSOLUTE AND SERVICE CEILINGS

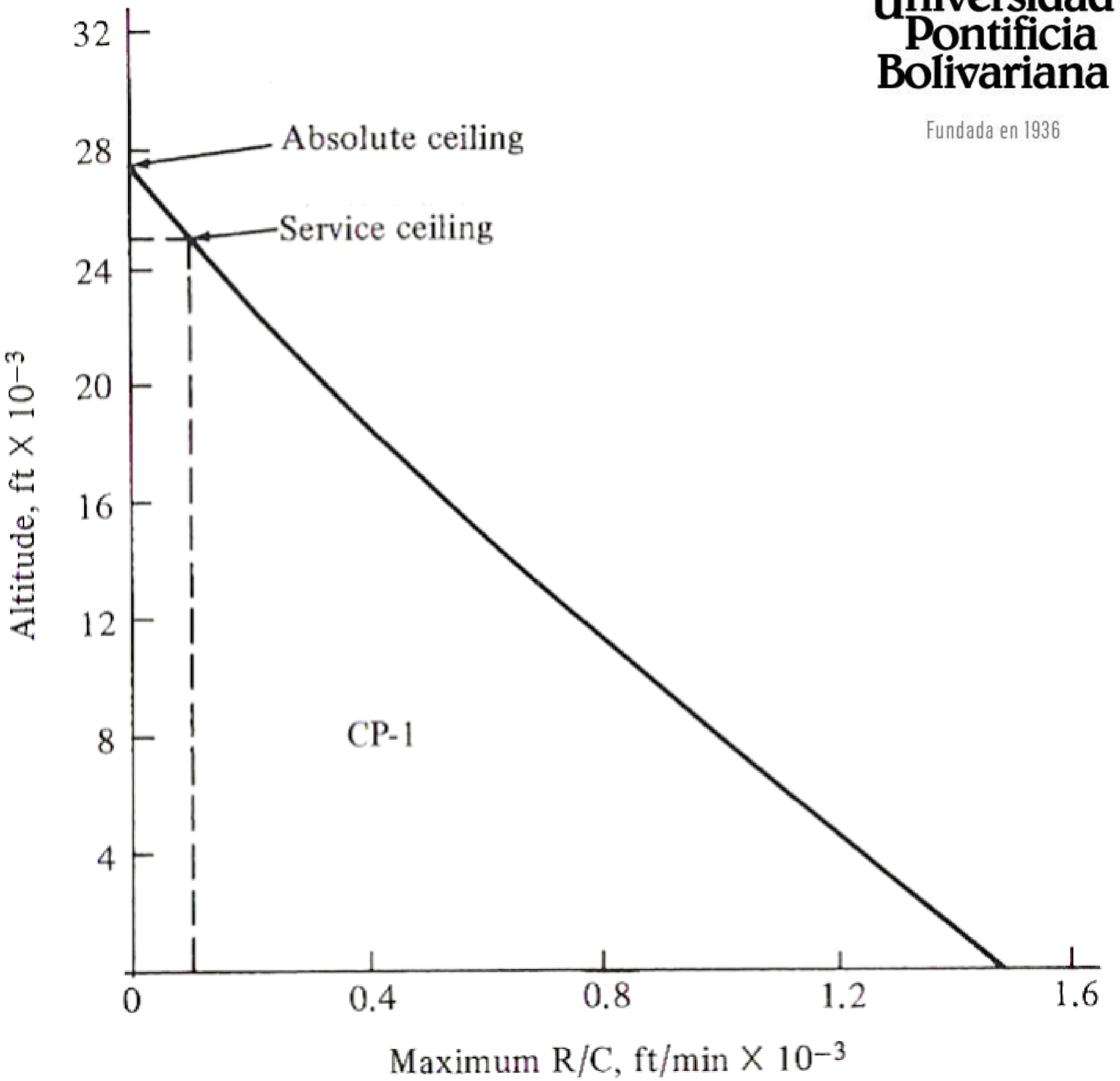
The absolute and service ceiling can be determined as follows:

1. Calculate the values of maximum R/C for several different altitudes

2. Plot maximum rate of climb versus altitude

3. Extrapolate the curve to 100 fpm and 0 to find the service and absolute ceilings, respectively

Absolute ceiling	0 fpm
Service ceiling	
Commercial/Piston-propeller	100 fpm
Commercial/jet	500 fpm
Military at maximum power	100 fpm
Combat ceiling	
Military/Subsonic/maximum power	500 fpm at M<1
Military/Supersonic/maximum power	1,000 fpm at M>1
Cruise ceiling	
Military/Subsonic/max.cont. power	300 fpm at M<1
Military/Supersonic/max.cont. power	1,000 fpm at M>1



TIME TO CLIMB

The rate of climb is defined as the vertical velocity of the aircraft. Velocity is simply the time rate of change of distance, the distance here being the altitude h

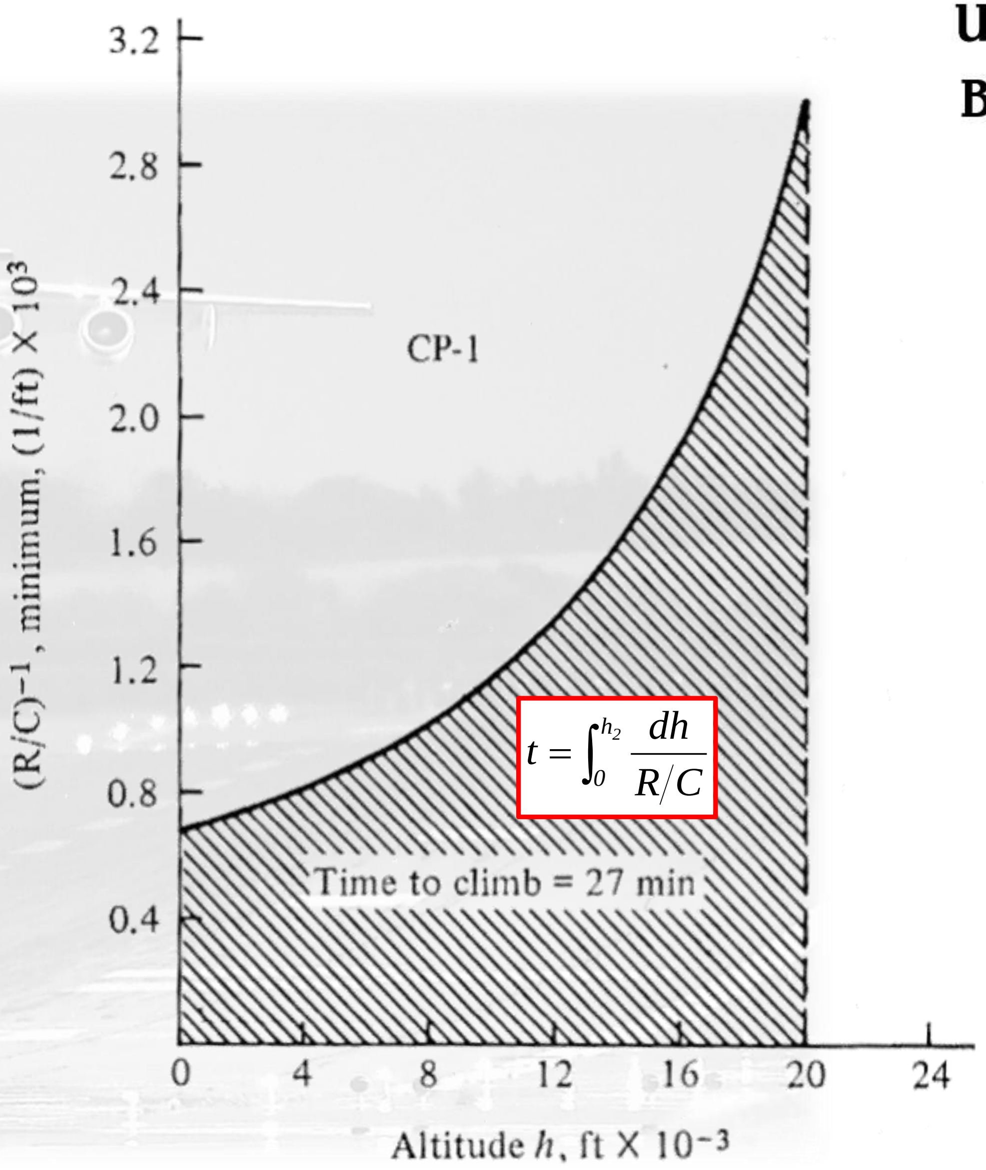
Hence, $R/C = dh/dt$, therefore: $dt = \dfrac{dh}{R/C}$

The time to climb from one altitude h_1 to another h_2 is obtained by integrating:

$$t = \int_{h_1}^{h_2} \frac{dh}{R/C}$$

To calculate t graphically, first plot $(R/C)^{-1}$ versus h

The area under the curve from $h = 0$ to $h = h_2$ is the time to climb to altitude h_2



GLIDING (DESCENT) FLIGHT

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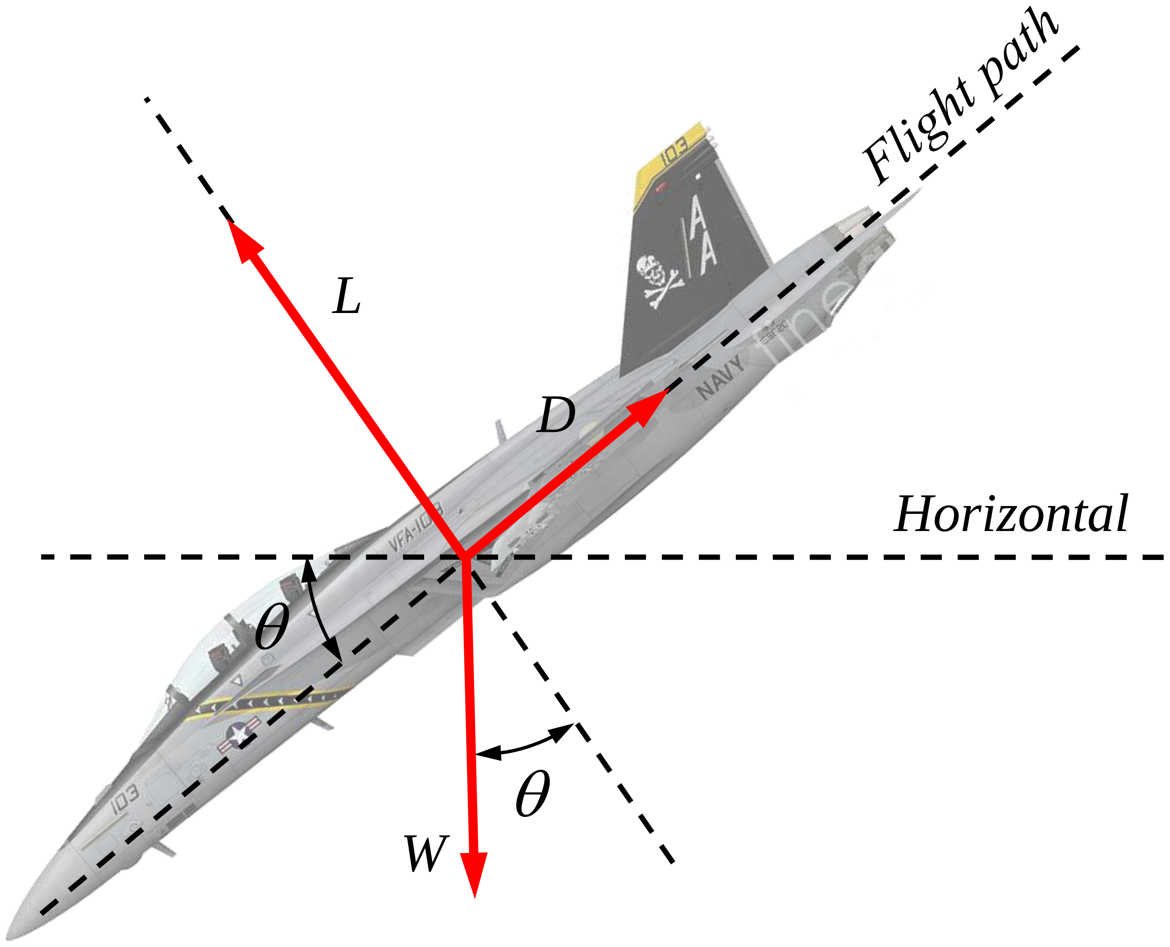
Considering an aircraft in a **power-off** glide ($T = 0$)

Along the flight path: $D = W \sin\theta$

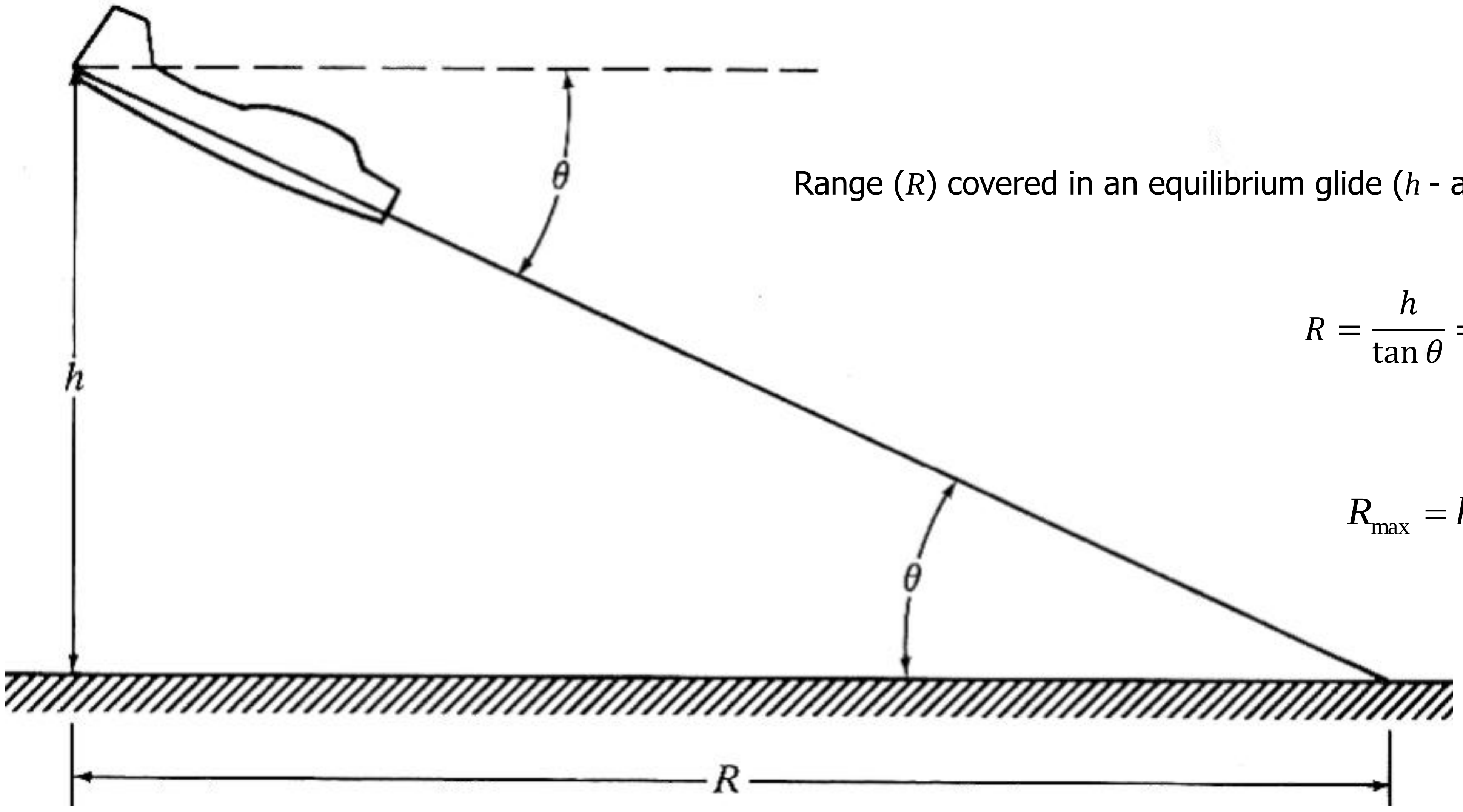
Perpendicular to the flight path: $L = W \cos\theta$

Dividing the last equations: $\dfrac{\sin\theta}{\cos\theta} = \dfrac{D}{L}$

$$\tan\theta = \frac{1}{\left(L/D\right)}$$



GLIDING FLIGHT



Range (R) covered in an equilibrium glide (h - altitude):

$$R = \frac{h}{\tan\theta} = h\left(\frac{L}{D}\right)$$

$$R_{\max} = h\left(\frac{L}{D}\right)_{\max}$$

RANGE AND ENDURANCE – PROPELLER-DRIVEN AIRCRAFT

One of the critical factors influencing range and endurance is the specific fuel consumption (engine characteristic)

SFC – the weight of fuel consumed per unit of power per unit of time

$$SFC = \frac{lb of fuel}{(bhp)(h)}$$

$cPdt$ represents the differential change in weight of the fuel due to consumption over the short time period dt

The weight of the fuel:

$$W_1 = W_0 - W_f$$

$$dW_f = dW = -cPdt$$

$$dt = -\frac{dW}{cP}$$

RANGE AND ENDURANCE – PROPELLER-DRIVEN AIRCRAFT

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Endurance – to stay in the air for the longest time, common sense says that it must use the minimum number of pounds of fuel per hour ($t = E\ [sec]$)

$$\int_0^E dt = -\int_{W_0}^{W_1} \frac{dW}{cP} \dashrightarrow \qquad E = -\int_{W_0}^{W_1} \frac{dW}{cP} = \int_{W_1}^{W_0} \frac{dW}{cP} \longrightarrow \qquad E = \frac{\eta_p}{c} \frac{C_L^{3/2}}{C_D} \sqrt{2\rho_\infty S} \left(W_1^{-1/2} - W_0^{-1/2} \right)$$

Range – to cover the longest distance (i.e., in nautical miles), common sense says that it must use the minimum number of pounds of fuel per mile

$$V_\infty dt = -\frac{V_\infty dW}{cP} \dashrightarrow \qquad ds = -\frac{V_\infty dW}{cP} \dashrightarrow \qquad \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty dW}{cP}$$

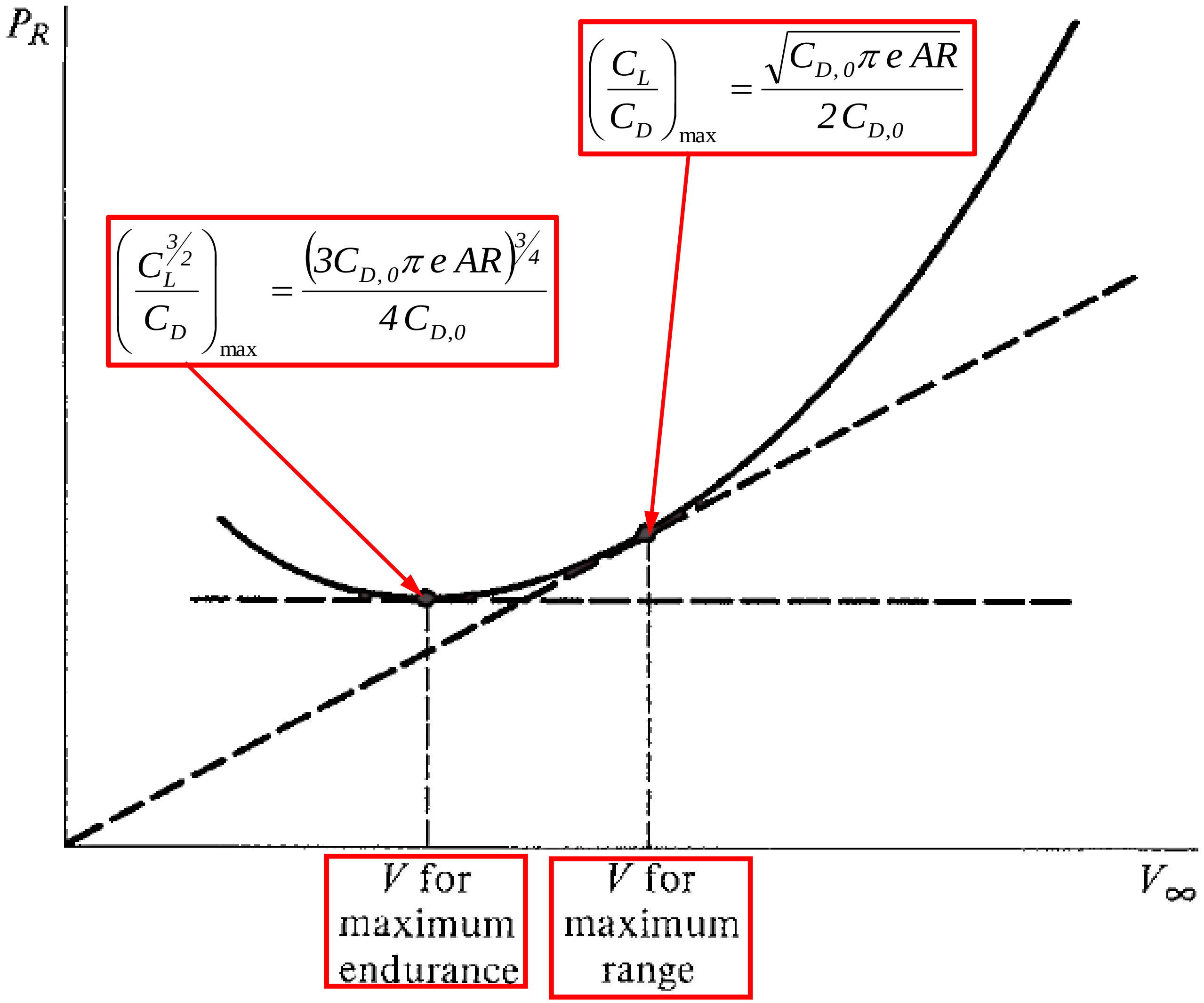
One of the critical factors influencing range and endurance is the specific fuel consumption (engine characteristic)

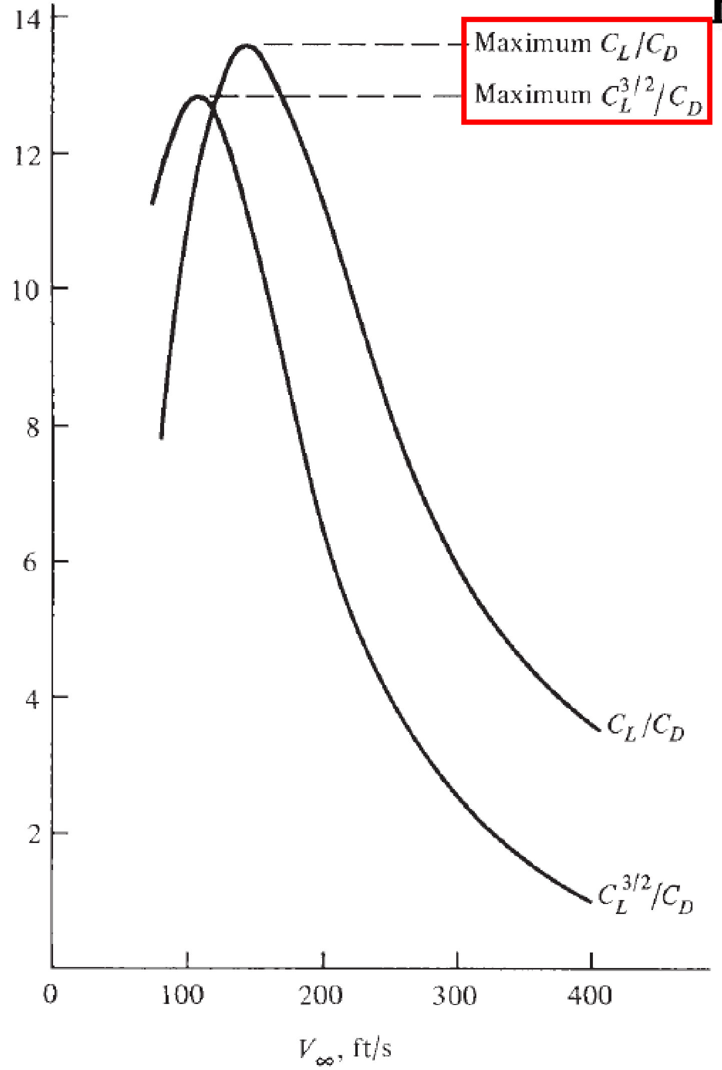
SFC – the weight of fuel consumed per unit of power per unit of time $SFC = \dfrac{lb\ of\ fuel}{(bhp)(h)}$

$cPdt$ represents the differential change in weight of the fuel due to consumption over the short time period dt

$$R = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP} \longrightarrow \qquad R = \frac{\eta_p}{c} \frac{C_L}{C_D} \ln \frac{W_0}{W_1}$$

RANGE AND ENDURANCE – PROPELLER-DRIVEN AIRCRAFT





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RANGE AND ENDURANCE – JET AIRCRAFT

One of the critical factors influencing range and endurance is the thrust-specific fuel consumption (engine characteristic)

$TSFC$ – the weight of fuel consumed per unit of power per unit of time

$$SFC = \frac{lb\ of\ fuel}{(lb\ of\ thrust)(h)}$$

c_t is the thrust-specific fuel consumption in consistent units

If dW is the elemental change in weight of the aircraft due to fuel consumption over a time increment dt, then:

$$dW = -c_t T_A dt$$

$$dt = -\frac{dW}{c_t T_A}$$

RANGE AND ENDURANCE – JET AIRCRAFT

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Endurance

Integrating between $t = 0$, where $W = W_0$, and $t = E$, where $W = W_1$, and also that $T_A = T_R = D$ and $W = L$, yields:

$$E = \int_{W_1}^{W_0} \frac{dW}{c_t T_A} = \int_{W_1}^{W_0} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W} \longrightarrow E = \frac{1}{c_t} \frac{C_L}{C_D} \ln \frac{W_0}{W_1}$$

Range

Noting that for steady, level flight, the engine throttle has been adjusted such that $TA = T_R$ and recalling that

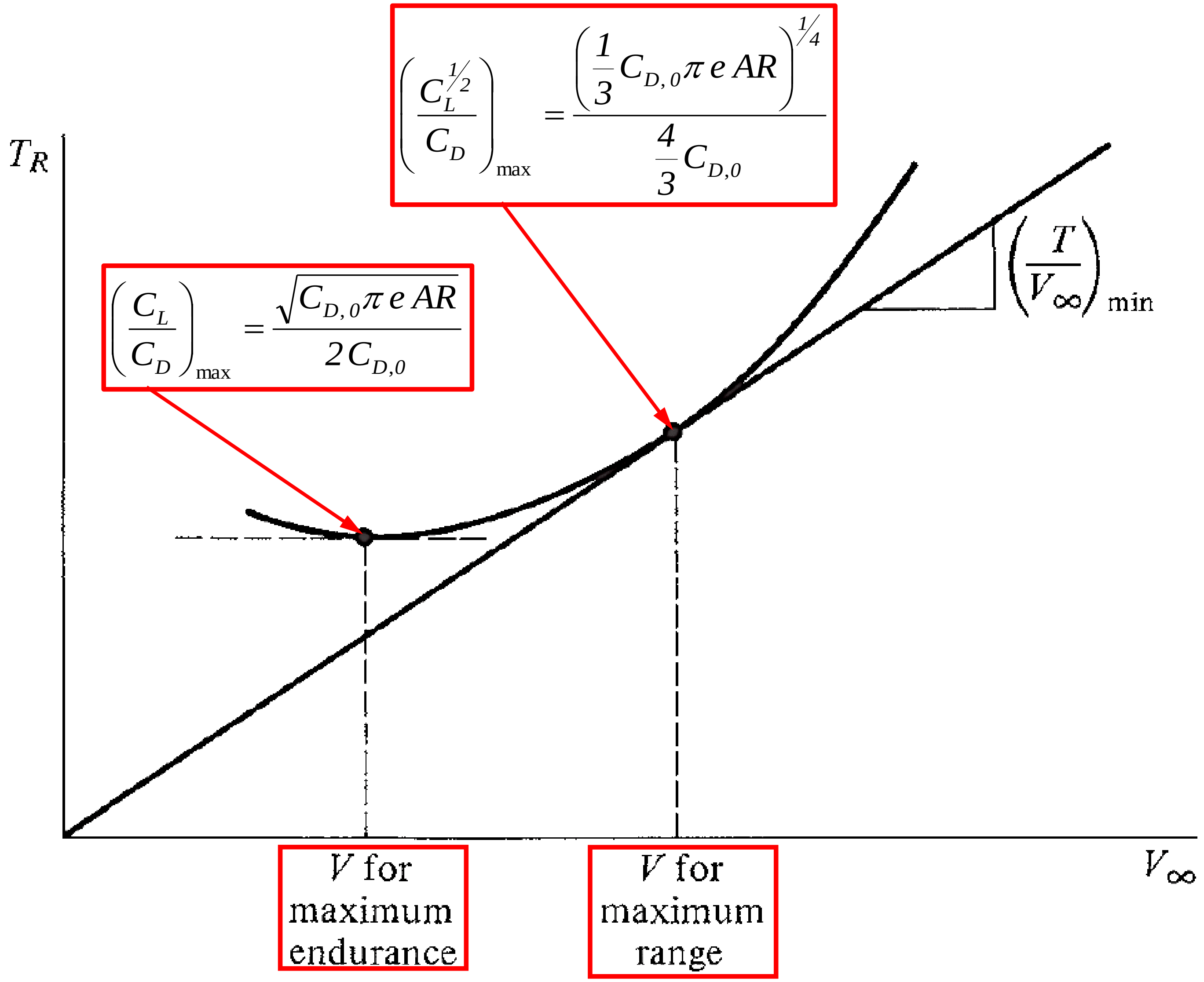
$$R = -\int_{W_0}^{W_1} \frac{V_\infty}{c_t} \frac{C_L}{C_D} \frac{dW}{W} \longrightarrow R = 2 \cdot \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}}{C_D} \frac{1}{c_t} \left(W_0^{1/2} - W_1^{1/2} \right)$$

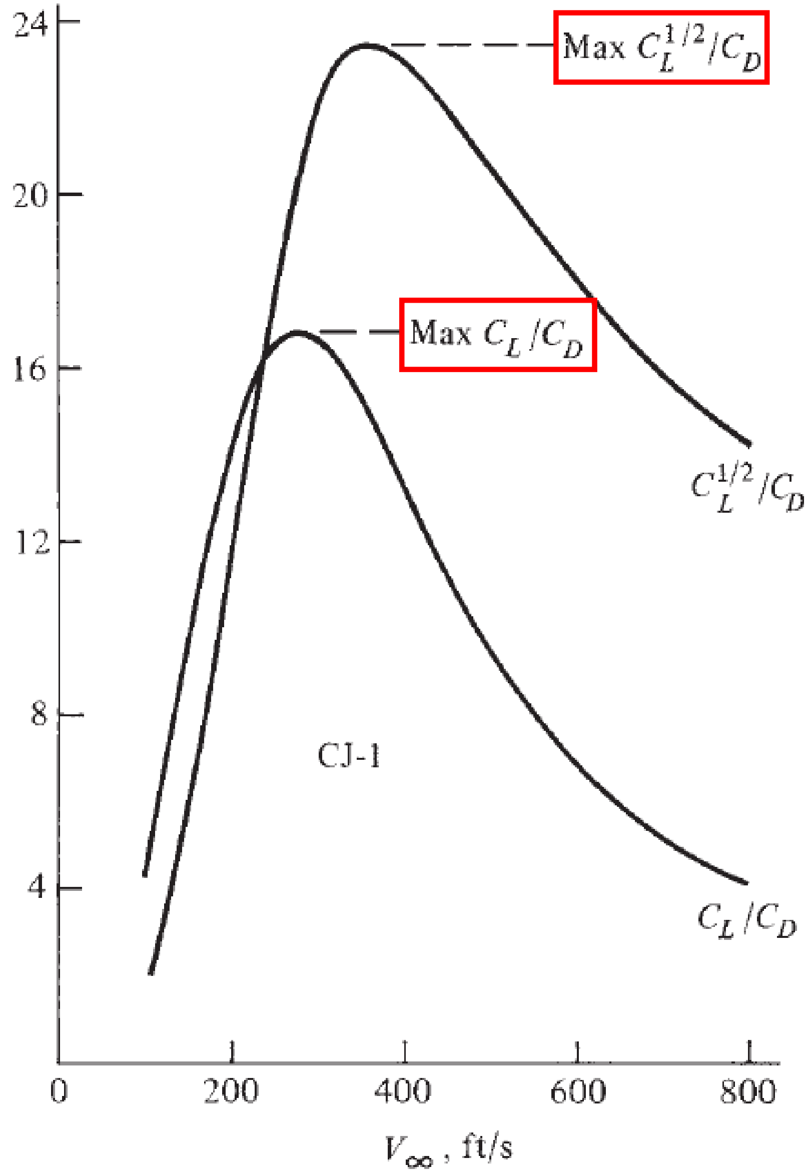
One of the critical factors influencing range and endurance is the thrust-specific fuel consumption (engine characteristic)

$TSFC$ – weight of fuel consumed per unit power per unit time $\quad SFC = \dfrac{lb\ of\ fuel}{(lb\ of\ thrust)(h)}$

c_t is the thrust-specific fuel consumption in consistent units

RANGE AND ENDURANCE – JET AIRCRAFT



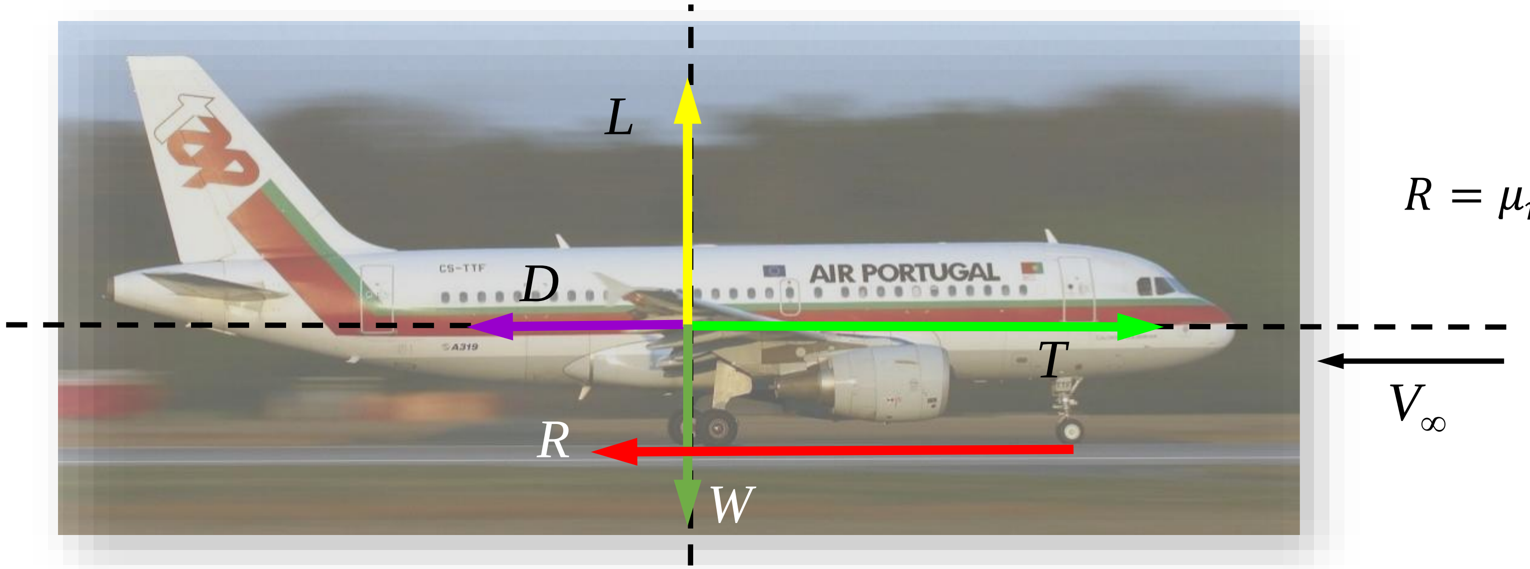


TAKE-OFF PERFORMANCE

Summing forces parallel to the ground and employing *Newton's* second law

$$F = T - D - R = T - D - \mu_r(W - L) = m\frac{dV}{dt}$$

This equation gives the local instantaneous acceleration of the aircraft dV/dt as a function of T, D, W, and L



$$R = \mu_r(W - L)$$

TAKE-OFF PERFORMANCE

T and W are reasonably constant for the Take-Off case (ground roll).

However, both L and D vary with velocity, since

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad D = \frac{1}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} + \phi\frac{C_L^2}{\pi e AR}\right)$$

The term ϕ sustains for a phenomenon called the ***ground effect***, which is the cause of the tendency for an aircraft to flare, or "float", above the ground near the instant of landing.

The reduced drag in the presence of ground effect is accounted for ϕ ($\phi \le 1$). An approximation for ϕ, based on aerodynamic theory ($McCormick$), is given as:

$$\phi = \frac{\left(16h/_b\right)^2}{1 + \left(16h/_b\right)^2}$$

where:
h – is the height of the wing above the ground
b – is the wingspan

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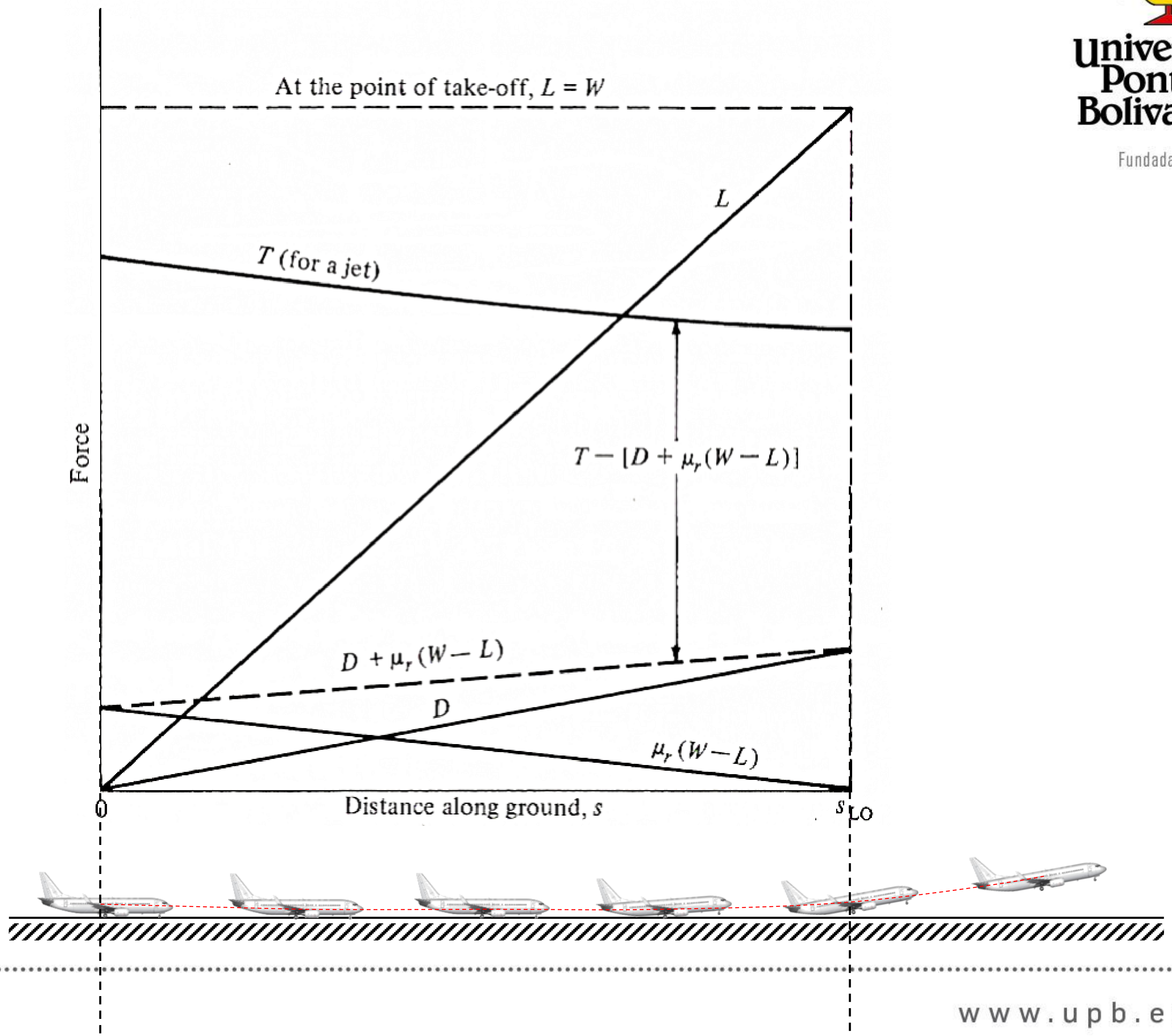
TAKE-OFF PERFORMANCE

Typical variation of the forces with distance along the ground during Take-Off with respect to V^2



TAKE-OFF PERFORMANCE

A simple but approximate expression for the lift-off distance (s_{LO}) can be obtained as follows:

Assuming that:

- T is constant
- An average value for the sum of drag and resistance forces, such that produces the proper lift-off distance s_{LO}

With these assumptions, the effective constant force acting on the aircraft during its Take-Off ground roll could be considered as $F_{eff} = T - [D + \mu_r(W - L)]_{av} = const.$

Knowing that: $s = \dfrac{V^2 m}{2F}$

and considering F_{eff} , $V = V_{\mathrm{LO}}$ and $m = W/g$, then:

$$s_{LO} = \frac{V^2 m}{2F} = \frac{V_{LO}^2 \left(W/g\right)}{2\{T - [D + \mu_r(W - L)]_{av}\}}$$

TAKE-OFF PERFORMANCE

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To ensure a margin of safety during Take-Off, the lift-off velocity is typically 20% higher than the stalling velocity, hence:

$$V_{LO} = 1.2V_{stall} = 1.2\sqrt{\frac{2W}{\rho_\infty S C_{L,\max}}}$$

Which for s_{LO} gives:

$$s_{LO} = \frac{1.44W^2}{g\rho_\infty S C_{L,max}\{T - [D + \mu_r(W - L)]_{av}\}}$$

There is a suggestion that the average force stated in the last equation can be set to its instantaneous value at a velocity equal to $0.7\ V_{\mathrm{LO}}$:

$$[D + \mu_r(W - L)]_{av} = [D + \mu_r(W - L)]_{0.7V_{LO}}$$

Experience also has shown that:

Type of airfield	Asphalt	Hard grass	Cut grass	Long grass	Unprepared
μ_r	0.02	0.04	0.05	0.10	0.10 - 0.30

TAKE-OFF PERFORMANCE

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If it is assumed that the thrust is much larger than either D or R during Take-Off (in accordance with the last figure), then:

$$s_{LO} = \frac{1.44W^2}{Tg\rho_\infty SC_{L,\max}}$$

This equation illustrates some important physical trends:

1. The s_{LO} is very sensitive to W^2
2. s_{LO} is dependent of ρ_∞, that is, $$s_{LO} \propto \frac{1}{\rho_\infty}$$

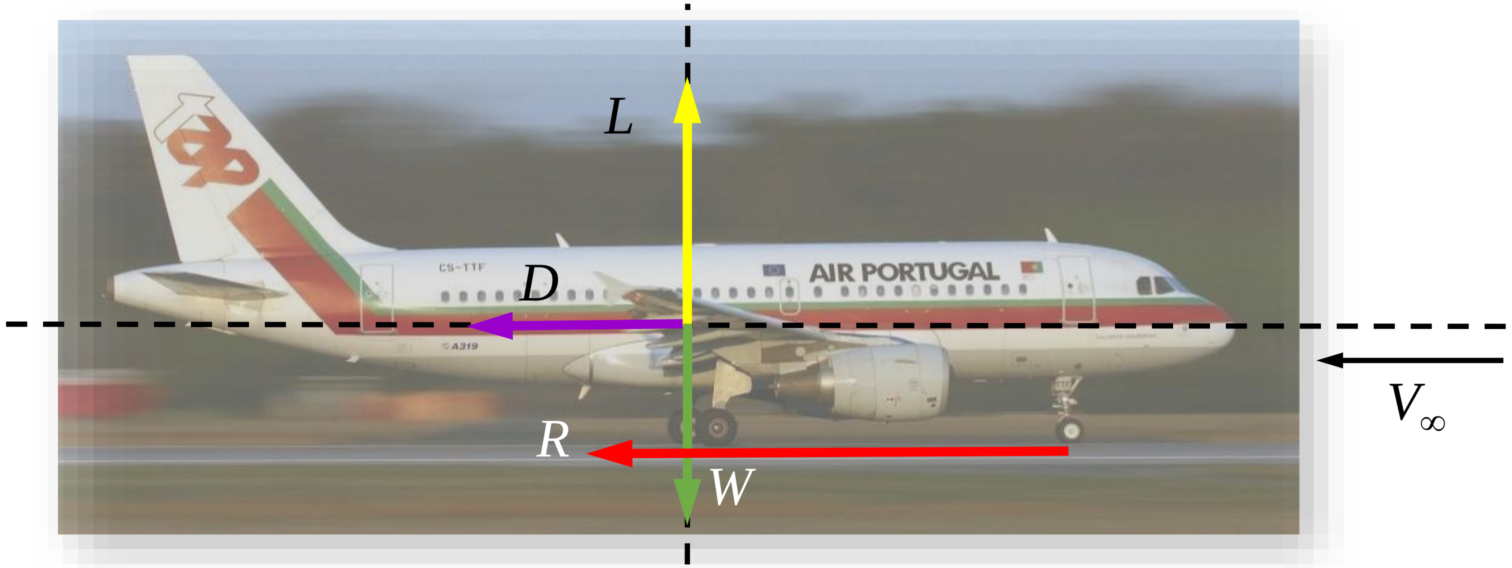
   This is why on hot days, when the air density is less than on cooler days, a given aircraft requires a longer ground roll to get off the ground. Also, longer lift-off distances are required at airports which are located at higher altitudes

3. s_{LO} can be decreased by increasing the S, increasing $C_{L,\ max}$ and increasing the thrust, all of which makes common sense

FAR defines the total Take-Off distance as the sum of the ground roll distance (s_{LO}) and the distance to clear a 35 *ft* height (jet-powered civilian aircraft) or a 50 *ft* height (all other aircrafts)

LANDING PERFORMANCE

Considering the force diagram during the landing:



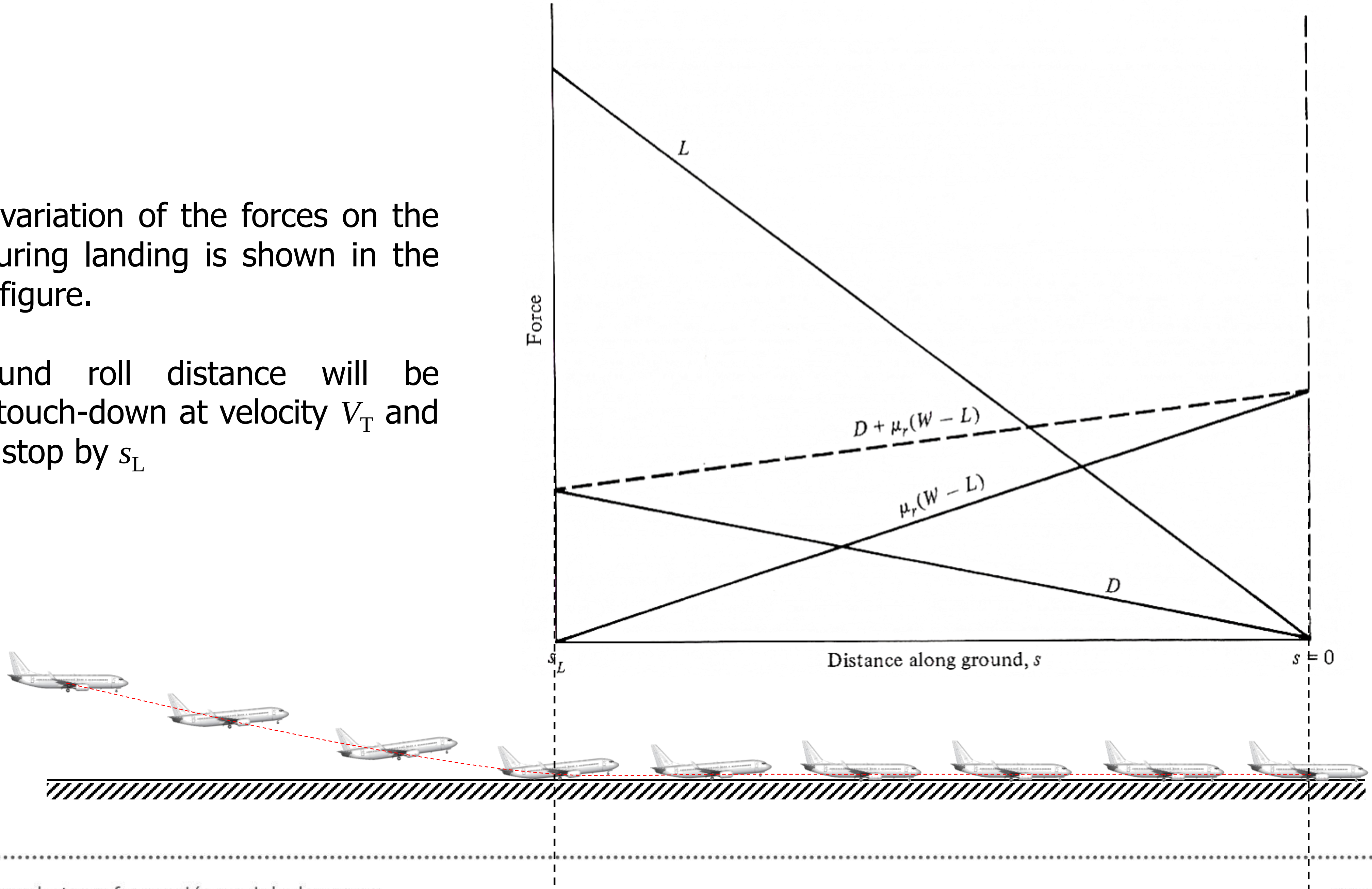
To minimize the distance required to come to a complete stop, it is assumed that the pilot has to decrease the thrust to zero at touchdown, and therefore the equation of motion for the landing ground roll is:

$$-D-\mu_r(W-L)=m\frac{dV}{dt}$$

LANDING PERFORMANCE

A typical variation of the forces on the aircraft during landing is shown in the following figure.

The ground roll distance will be between touch-down at velocity V_T and complete stop by s_L



LANDING PERFORMANCE

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Assuming an average constant value for $D + \mu_r(W - L)$ which effectively yields the correct ground roll distance at landing s_L

It can be assumed that $[D + \mu_r(W - L)]_{av}$ is equal to its instantaneous value evaluated at $0.7V_T$

$$F = -[D + \mu_r(W - L)]_{av} = -[D + \mu_r(W - L)]_{0.7V_T}$$

Integrating the between the touchdown point, where $s = s_L$ and $t = 0$, and the point where the aircraft's motion stops, where $s = 0$, and time equals t, then

$$ds = \frac{F}{m}tdt \Rightarrow \int_{s_L}^{0} ds = \frac{F}{m}\int_{0}^{t} tdt \Rightarrow s_L = -\frac{F}{m}\frac{t^2}{2}$$

Knowing that,

$$t = \frac{Vm}{F} \Rightarrow s_L = -\frac{V^2}{2}\frac{m}{F}$$

This last equation gives the distance required to decelerate from an initial velocity V to zero velocity under the action of constant force F.

LANDING PERFORMANCE

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In the last equation of s_L, F is given by, $-[D + \mu_r(W - L)]_{0.7V_T}$, and V is V_T

Thus s_L will be:

$$s_L = \frac{V_T^2 \left({}^{W}\!/_{g} \right)}{2[D + \mu_r(W - L)]_{0.7V_T}}$$

To maintain a factor of safety:

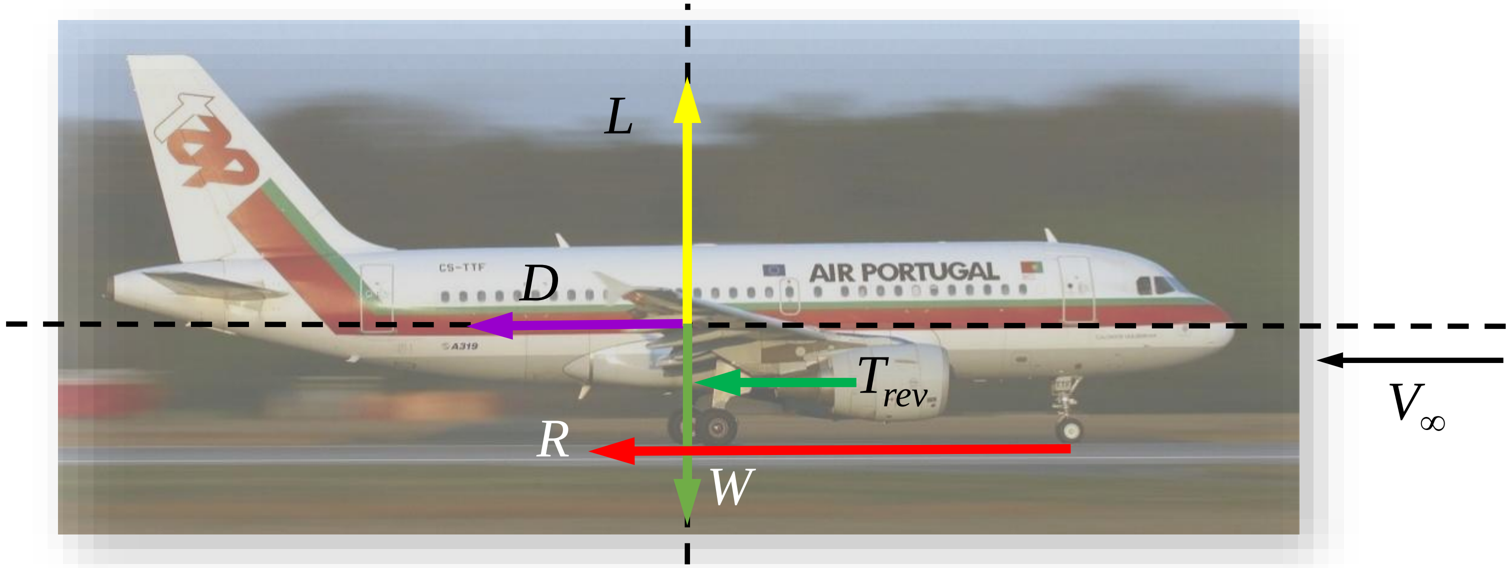
$$V_T = 1.3V_{stall} = 1.3\sqrt{\frac{2W}{\rho_\infty S C_{L,\max}}}$$

Substituting the value of V_T in the s_L equation yields:

$$s_L = \frac{1.69W^2}{g\rho_\infty S C_{L,max}[D + \mu_r(W - L)]_{0.7V_T}}$$

LANDING PERFORMANCE

Considering the force diagram during the landing:



To minimize the distance required to come to a complete stop, it is assumed that the pilot has to decrease the thrust to zero at touchdown, and therefore the equation of motion for the landing ground roll is:

$$-T_{rev} - D - \mu_r(W - L) = m\frac{dV}{dt}$$

LANDING PERFORMANCE

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Modern jet transports use thrust reversal during the landing ground roll.

With thrust reversals, the thrust vector is reversed and points in the drag direction, thus aiding the deceleration and shortening the ground roll.

Designating the reversed thrust by T_{rev}, then:

$$-T_{rev} - D - \mu_r(W - L) = m\frac{dV}{dt}$$

Assuming that the thrust reversal is constant, then:

$$s_L = \frac{1.69W^2}{g\rho_\infty SC_{L\_max}\{T_{rev} + [D + \mu_r(W - L)]_{0.7V_T}\}}$$

The lift on the aircraft can be destroyed by applying spoilers.

According to FAR, the total landing distance is the sum of the ground roll distance and the distance to achieve touchdown in a glide from $50.0\ ft$ height.

TURNING FLIGHT AND THE *V-n* DIAGRAM

A level turn is illustrated in the figure.

It is seen that the aircraft is banked at an angle ϕ to the vertical.

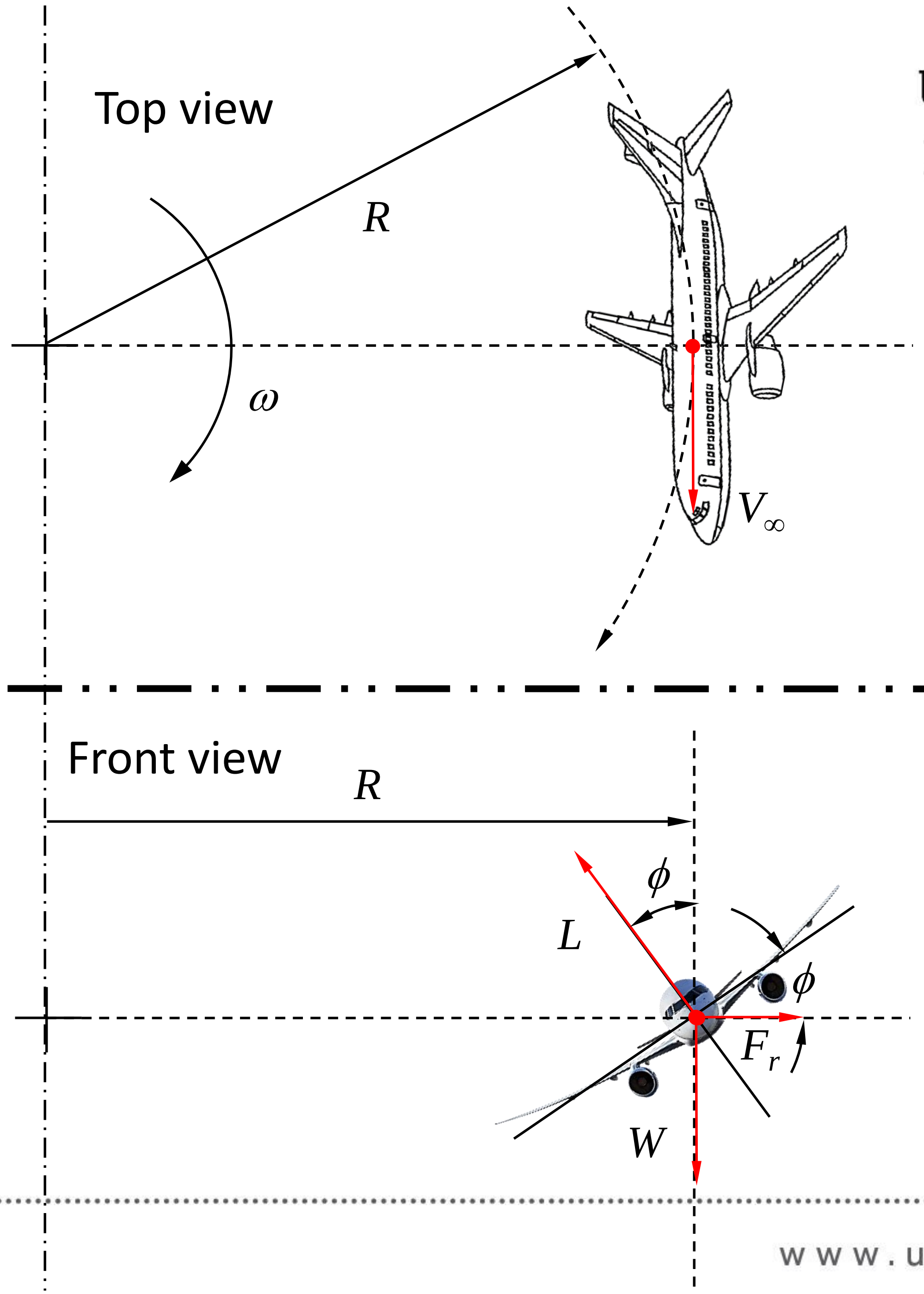
The **bank angle** ϕ, hence the lift L is such that the component of the lift in the vertical direction exactly equals the weight.

$$L\cos\phi = W$$

The points of interest here are the turn radius R and the turn rate $d\omega/dt$

$$F_r = \sqrt{L^2 - W^2}$$

$$F_r = m \cdot a_{rad} = m\frac{V_\infty^2}{R}$$



TURNING FLIGHT AND THE *V-n* DIAGRAM

A new term is introduced, **the load factor n, defined as** $n \equiv \dfrac{L}{W}$

The load factor is usually quoted in terms of g's

Having this into account, then the F_r equation can be re-written as:

$$F_r = W\sqrt{n^2 - 1}$$

The aircraft is moving in a circular path at velocity V_∞; therefore, the radial acceleration is given by

$$a_{rad} = \frac{V_\infty^2}{R}$$

From *Newton's* second law,

$$F_r = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

TURNING FLIGHT AND THE *V-n* DIAGRAM

Combining the two last equations of Fr, and solving for R

$$R = \frac{V_\infty^2}{g\sqrt{n^2 - 1}}$$

The angular velocity denoted by $\omega = d\theta/dt$, is called the **turn rate** and is given by V_∞/R, thus from the equation for R, it is obtained,

$$\omega = \frac{g\sqrt{n^2 - 1}}{V_\infty}$$

For the maneuvering performance of an aircraft, both military and civil, it is frequently advantageous to have the smallest possible R and the largest possible ω

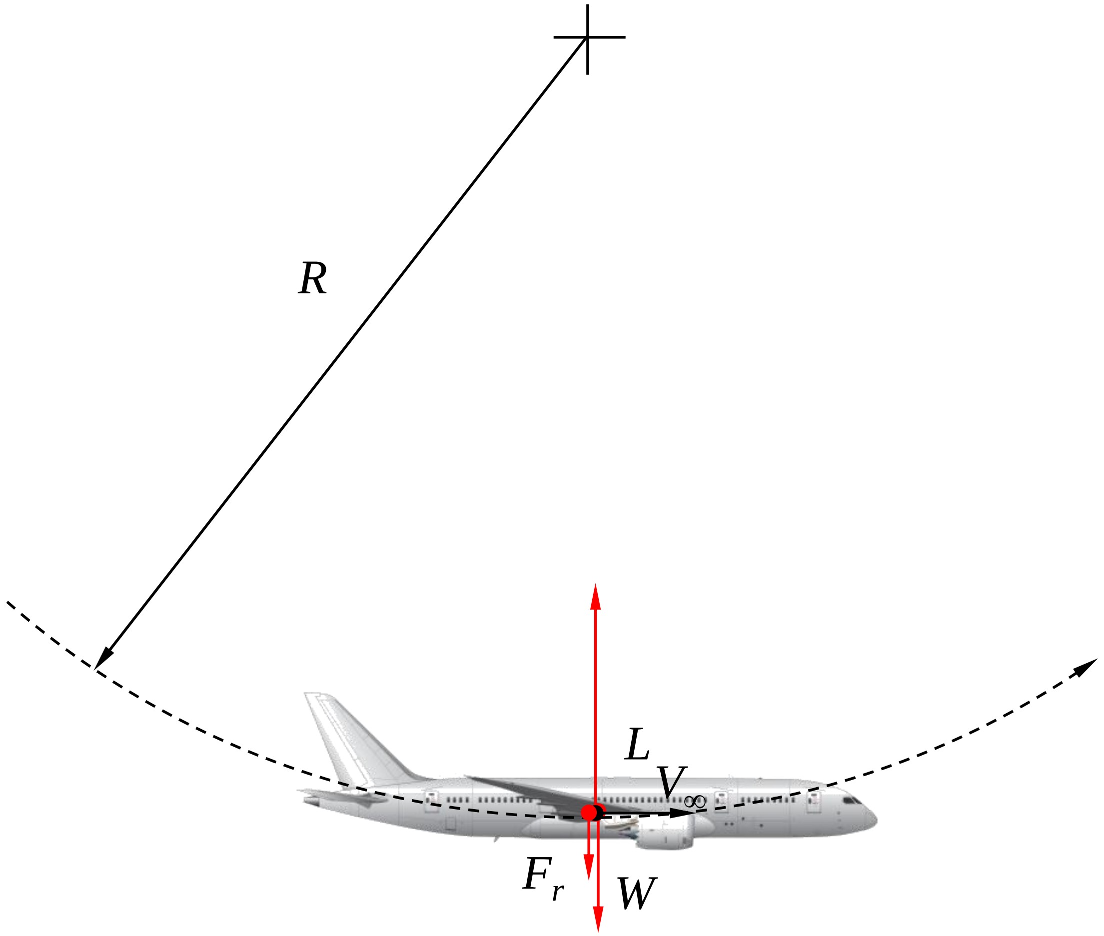
To maintain a small turn radius and a large turn rate, it is necessary to have

- The highest possible load factor
- The lowest possible velocity

TURNING FLIGHT AND THE *V-n* DIAGRAM

The pull-up maneuver is another case of turning flight, in which an aircraft initially in straight, level flight ($L = W$), suddenly experiences an increase in lift

Since $L > W$, the aircraft will begin to turn upward as sketched



$$\sum F_y = 0 \qquad L = F_r + W$$

$$F_r = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

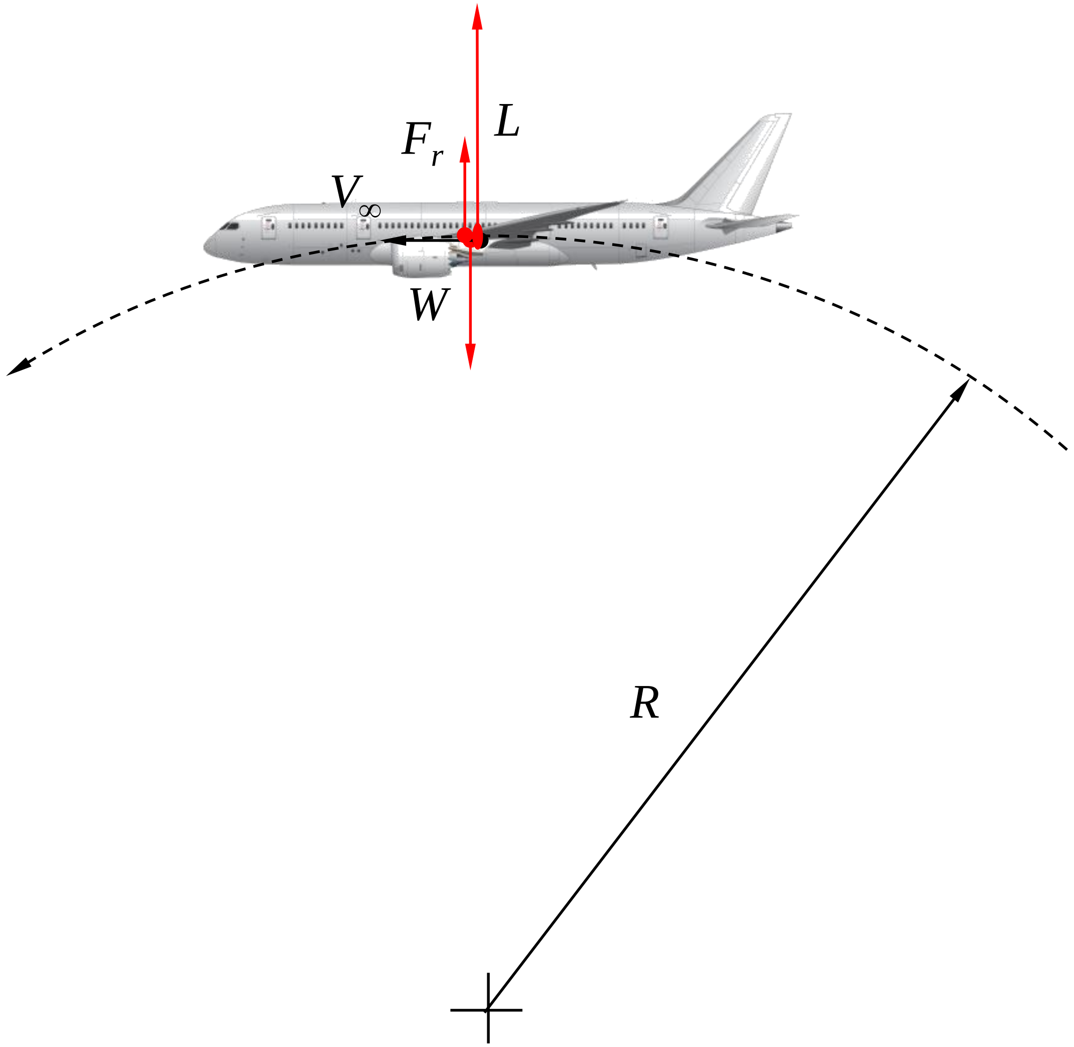
$$L = W + \frac{W}{g}\frac{V_\infty^2}{R} = W\left(1 + \frac{V_\infty^2}{gR}\right)$$

$$\frac{L}{W} = n \equiv 1 + \frac{V_\infty^2}{gR}$$

$$R = \frac{V_\infty^2}{g(n-1)} \qquad \omega = \frac{g(n-1)}{V_\infty}$$

TURNING FLIGHT AND THE ***V-n*** DIAGRAM

The pull-down maneuver is another case of turning flight, in which an aircraft initially in straight, level flight ($L = W$), suddenly rolls an inverted position, such that both L and W are pointing downwards.



$$\sum F_y = 0 \qquad W = L + F_r$$

$$F_r = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

$$L = W - \frac{W}{g}\frac{V_\infty^2}{R} = W\left(1 - \frac{V_\infty^2}{gR}\right)$$

$$\frac{L}{W} = n \equiv 1 - \frac{V_\infty^2}{gR}$$

$$R = \frac{V_\infty^2}{g(1-n)} \qquad \omega = \frac{g(1-n)}{V_\infty}$$

TURNING FLIGHT AND THE *V-n* DIAGRAM

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Considerations of turn radius and turn rate are critical to military fighter aircraft. Those aircraft with the smallest R and the largest ω will have definite advantages in air combat.

High-performance fighter aircraft are designed to operate at higher load factors (from -3.0 to 10)

When n is large, then $n + 1 \approx n$ and $n - 1 \approx n$, then the equations can be reduced to:

$$R = \frac{V_\infty^2}{gn} \qquad \omega = \frac{gn}{V_\infty}$$

Since, $L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$, then, $V_\infty^2 = \frac{2L}{\rho_\infty S C_L}$

Substituting these values into the equations for R and ω yields,

$$R = \frac{2L}{\rho_\infty S C_L g \left(L/W\right)} = \frac{2}{\rho_\infty C_L g}\frac{W}{S} \qquad \omega = \frac{gn}{\sqrt{\frac{2L}{\rho_\infty S C_L}}} = \frac{gn}{\sqrt{\frac{2n}{\rho_\infty C_L \left(W/S\right)}}} = g\sqrt{\frac{\rho_\infty C_L n}{2\left(W/S\right)}}$$

TURNING FLIGHT AND THE *V-n* DIAGRAM

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Here the important values are the knowledge of how low or minimum will be the turn radius and how high or maximum will be the turn rate.

$$R_{\min} = \frac{2}{\rho_\infty C_{L,\max}\, g} \frac{W}{S}$$

$$\omega_{\max} = g\sqrt{\frac{\rho_\infty C_{L,\max} n_{\max}}{2\left(W/S\right)}}$$

Another consideration is that at lower speeds, $n_{\max}$ is a function of $C_{\mathrm{L,\,max}}$ itself because,

$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 S\, C_L}{W} \quad \Rightarrow \quad n_{\max} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L,\max}}{\left(W/S\right)}$$

At higher speeds, $n_{\max}$ is limited by the structural design of the aircraft.

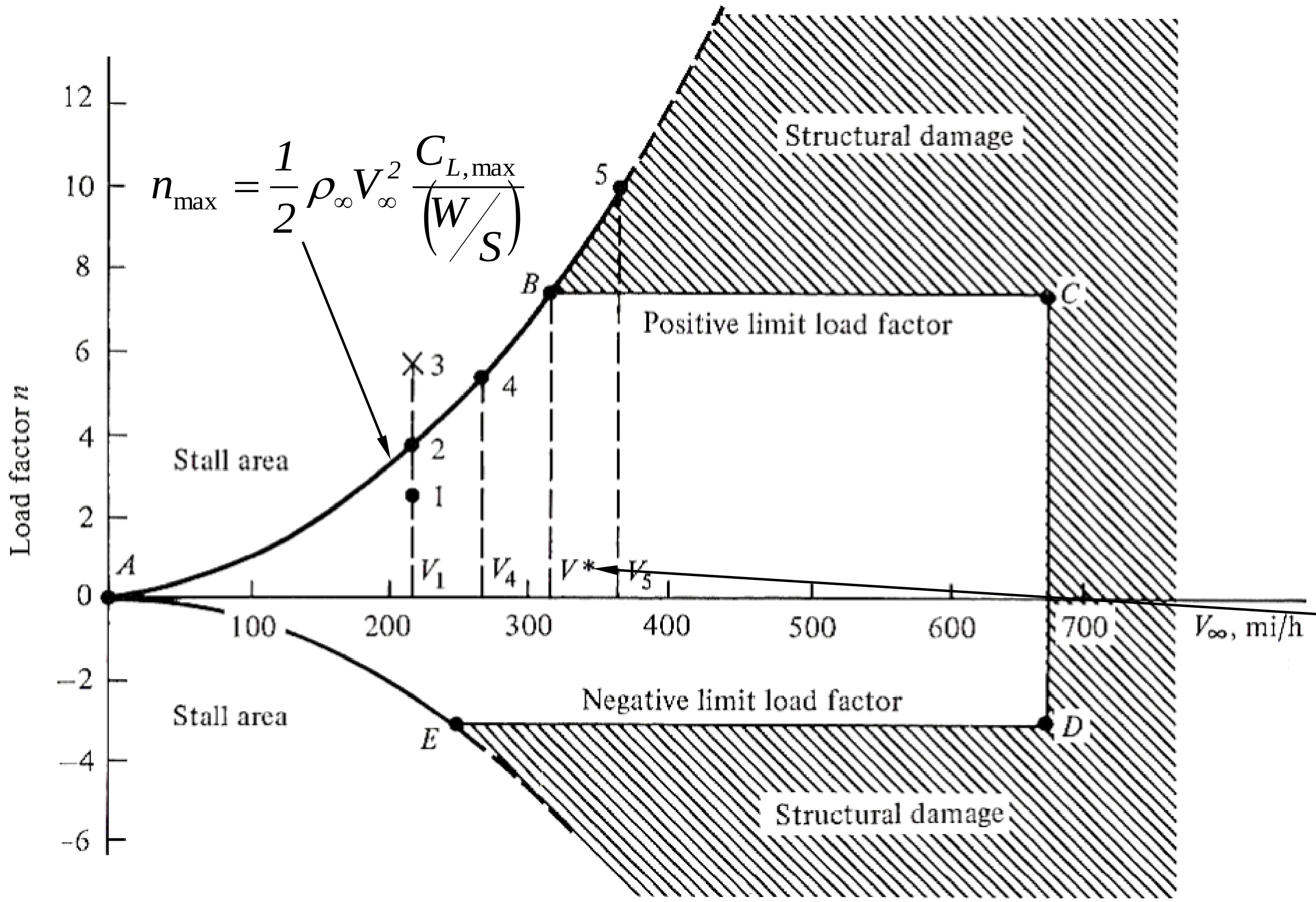
TURNING FLIGHT AND THE *V-n* DIAGRAM

Load factors values stablished by category:

Category		**Load factor - *n***	
		Negative	**Positive**
Airplanes	Transport	-1.0	+2.5 *(or up to* +3.8*)*
	Commuter	-1.52	+3.8
	Utility	-1.76	+4.4
	Acrobatic	-3.0 *(special cases* -10.0*)*	+6.0 *(special cases* +12.0*)*
Helicopters		-1.0	+3.5

TURNING FLIGHT AND THE *V-n* DIAGRAM

V-n diagram for a typical jet trainer aircraft



$$R_{\min} = \frac{2}{\rho_\infty C_{L,\max}\, g}\frac{W}{S}$$

$$\omega_{\max} = g\sqrt{\frac{\rho_\infty C_{L,\max} n_{\max}}{2\left(W/S\right)}}$$

$$V^* = \sqrt{\frac{2\,n_{\max}}{\rho_\infty C_{L,\max}}\frac{W}{S}}$$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