L15MoreStrongInduction

April 5, 2017

1 Lecture 15: More Strong Induction

1.1 Recap of Previous Lectures

- Principle of Mathematical Induction:

1.2 Strong Induction

Question: When to choose weak vs strong induction?
Answer: depends on the problem, i.e., depends on what inductive step we can prove.

1.3 Examples

1.4 Let's try one

Exercise 9 from Rosen 5.2: Show that $\sqrt{2}$ is irrational by strong induction, using the hypothesis:

$$P(n): \ \forall b \in \mathbb{Z}_{\geq \nvdash}, \ \sqrt{2} \neq \frac{n}{b}$$

1.5 Question

Suppose than $P(n)$ is a predicate. Determine for which non-negative integers n the statement $P(n)$ must be true if

a) $P(0)$ is true; for all $n \geq 0$, $P(n) \implies P(n+2)$;

The Principle of Induction.

Let $P(n)$ be a predicate. If

- $P(0)$ is true, and
- $P(n)$ IMPLIES $P(n+1)$ for all nonnegative integers, n,

then

- $P(m)$ is true for all nonnegative integers, m.

PMI

Principle of Strong Induction. Let $P(n)$ be a predicate. If

- $P(0)$ is true, and
- for all $n \in \mathbb{N}$, $P(0), P(1), \ldots, P(n)$ *together* imply $P(n+1)$,

then $P(n)$ is true for all $n \in \mathbb{N}$.

Strong PMI

A *recurrence relation* for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots, a_{n-1}$, for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer. A sequence is called a *solution* of a recurrence relation if its terms satisfy the recurrence relation. (A recurrence relation is said to *recursively define* a sequence. We will explain this alternative terminology in Chapter 5.)

recurrence

b) $P(1)$ is true; for all $n \geq 1$, $P(n) \implies P(2n)$;

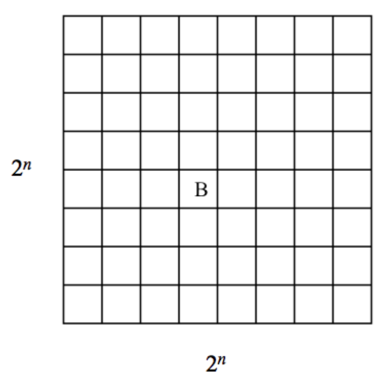
c) $P(0) \wedge P(1)$ is true; for all $n \geq 0$, $(P(n) \wedge P(n+1) \implies P(n+2)$;

d) $P(0)$ is true; for all $n \geq 0$, $P(n) \implies P(n+2) \wedge P(n+3))$.

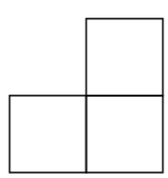
1.6 Recurrence Relations

1.7 Back to Tromino Tiling

A tiling board:



A tromino:



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In [ ]:
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Question: can you remove a central square from the board and still tile the board with trominoes?