L03LogicalEquivalences

January 26, 2017

1 Lecture 3

1.1 Recap of Lecture 2

- introduction to propositional logic
- propositions: can take T or F value
- logical operators (aka logical connectives): used to build propositions from other propositions
- truth tables: express truth values of compound propositions

- common logical operators

  - NEGATION: $\neg P$,

  - DISJUNCTION: $P \vee Q$,

  - CONJUNCTION: $P \wedge Q$.

- operators for conditional statements

  - IMPLICATION: $P \implies Q$
  - EQUIVALENCE: $P \iff Q$

1.2 Recognizing conditional statements

Many different ways of expressing the implication $P \implies Q$ in English. Familiarize yourself.

List in the textbook: - if P, then Q - P implies Q - if P, Q - P only if Q
- Q unless $\neg P$ - Q when P - Q if P
- Q whenever P - P is sufficient for Q - Q follows from P - Q is necessary for P - a necessary condition for P is Q - a sufficient condition for Q is P

Clicker Question 1:

Match each English sentence to its logical meaning:

English sentences:

1. Q only if P
2. P is a necessary and sufficient condition for Q
3. Q holds whenever $\neg P$

4. unless P, $\neg Q$ holds

Possible logical meanings (same can repeat!):

- a) $Q \implies P$
- b) $P \implies Q$
- c) $P \iff Q$
- d) $\neg P \implies Q$

Answer:
1 –> a
2 –> c
3 –> d
4 –> a

1.3 Operators and Truth Tables

** Clicker Question 2**: Suppose we are writing down, the truth table for a compound proposition of the propositional variables P and Q. How many rows does it have? * a) 1 rows * b) 2 rows * c) 4 rows * d) 8 rows * e) it depends on the number of operators in the compound proposition

** Answer **: 4 rows, as there are two truth assignments for P and true for Q, yielding 4 possible cases.

Clicker Question 3):

How many distinct binary logical operators are there? *Hint*: each operator corresponds to a different truth table. * a) 4 operators * b) 8 operators * c) 16 operators * d) 64 operators * e) infinitely many

Answer: 16 operators.

Each operator corresponds to a truth table with 4 rows and is completely specified by the truth assignment to these rows. For each row, there are 2 choices. Hence, there are $2^4 = 16$ possible operators.

How many binary operators have we seen so far?: List compiled by students - $\vee$ - $\wedge$ - $\implies$ - exclusive or - $\iff$ - false -the negations of the above operators Total: 12

The remaining ones are: - the operator that just returns the first term. - the operator that just returns the second term. - their negations

Check out the whole list here: https://en.wikipedia.org/wiki/Truth_table#Binary_operations

1.4 A Little Formal Detail: Precedence of Operators

Last lecture I have tried to be careful and avoid writing things like

$$\neg P \vee Q$$

because they are ambiguous.

Does it mean

$$(\neg P) \vee Q$$

or

$$\neg(P \vee Q)?$$

Precedence of Operators: negation beats dis/conjunction beats conditionals

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$ $\vee$	2 3
$\rightarrow$ $\leftrightarrow$	4 5

2 Logical Equivalences

Last time: Two compound propositions are logically equivalent if they take the same truth value under all truth settings of the propositional variables, i.e., they have the same truth table.

Example: $P \implies Q$ is logically equivalent to $\neg P \vee Q$.

We established this by computing the truth tables of both compound propositions.

3 Logical Equivalences

Last time: Two compound propositions are logically equivalent if they take the same truth value under all truth settings of the propositional variables, i.e., they have the same truth table.

Different definition: Two propositions R and S are logically equivalent if

$$R \iff S$$

is a tautology (always true).

Example:

$$[(\neg P) \vee Q] \iff [P \implies Q]$$

is a tautology

3.1 L02's ending question

Clicker Question: How many T's are there in the truth table for the compound proposition:

$$[(\neg P) \vee Q] \iff [P \implies Q]$$

a. 0

b. 1

c. 2

d. 3

e. 4

Answer: Statement is a tautology, so all 4 rows will be true.

3.2 Why Logical Equivalences?

1. simplify expressions
2. develop boolean algebra and boolean calculus
3. reason about logic formally

3.3 Important Logical Equivalences

1. Basic equivalences
   - Identity laws
   - domination laws
   - idempotent laws
   - negation laws
   - double negation laws
2. Properties of conjunctions and disjunctions

- Commutative Laws
- Associative Laws
- Distributive Laws

3.4 Next Lecture: More Logical Equivalences

- De Morgan's Laws

- Equivalences Involving Conditional Statemetns