

How to Turn your Knowledge Graph Embeddings into Generative Models

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Paper



Code



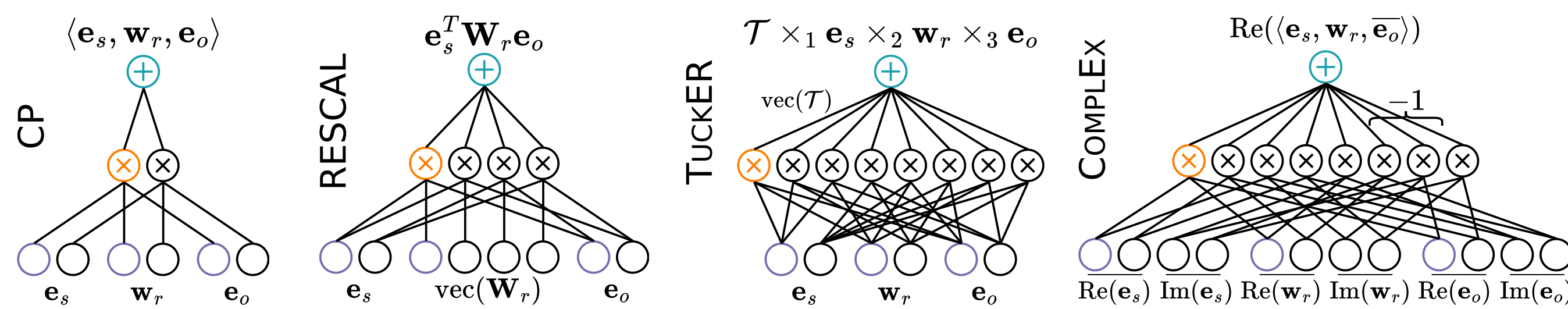
TL;DR

“We reinterpret KGE models into generative models of triples, making them to scale to large knowledge graphs, reliable with logical constraints and supporting sampling.”

Knowledge graph embedding (KGE) models

Issues?

- 1 Scores are difficult to interpret, combine, compare [1].
How to measure the confidence of predictions?
- 2 Link predictions violate logical constraints.
How to guarantee the satisfaction of constraints?
- 3 KGE models are expensive to learn.
How to scale to KGs with millions of entities?



Interpreting the score functions of KGE models as constrained computational graphs: circuits [2]

1 From KGE models to probabilistic circuits (PCs)

We convert score functions (i.e., circuits) into **probabilistic circuits** (PCs) [2, 3] without additional memory requirements.

$$p(s, r, o) = \frac{1}{Z} \phi_{pc}(s, r, o) \quad s.t. \quad \phi_{pc}(s, r, o) \geq 0$$



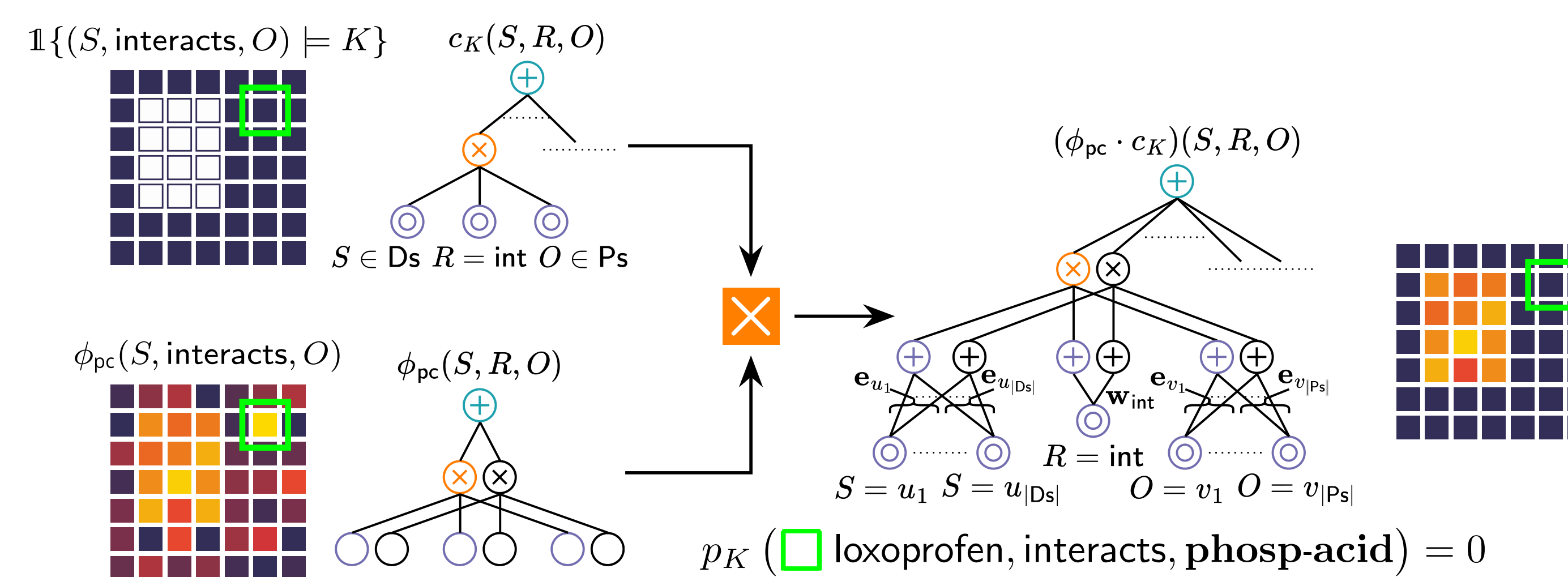
Generative KGE circuits (GeKCs) obtained via:

Non-negative restriction make the embeddings and computational unit activations non-negative;

or **Squaring the score function** square the circuit [4] without restricting the parameters domain, e.g.,

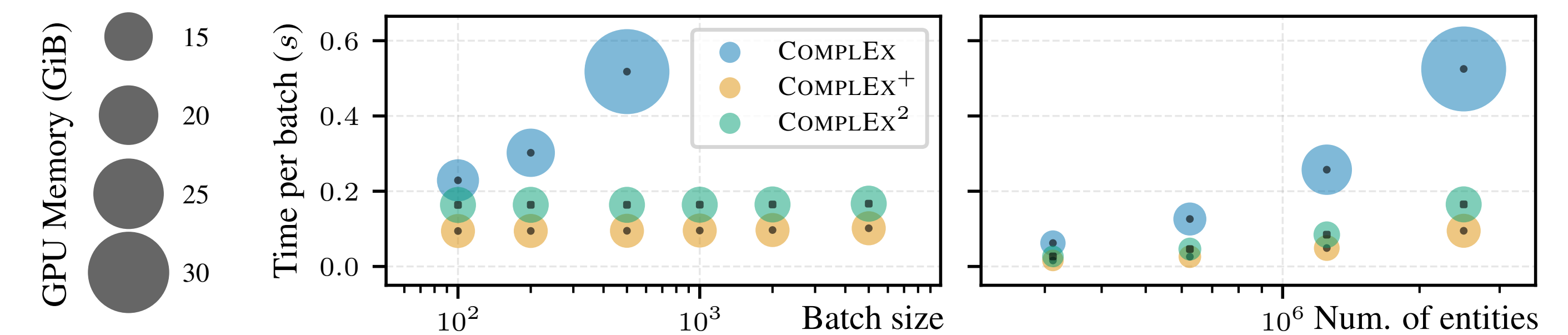
$$\phi_{CP^2}(s, r, o) = \left(\sum_{i=1}^d \mathbf{e}_{si} \mathbf{w}_{ri} \mathbf{e}_{oi} \right)^2 = \sum_{i=1}^d \sum_{j=1}^d \mathbf{e}_{si} \mathbf{e}_{sj} \mathbf{w}_{ri} \mathbf{w}_{rj} \mathbf{e}_{oi} \mathbf{e}_{oj}$$

2 Integration of constraints with guarantees



[5]

3 Scaling to KGs with millions of entities



Link prediction benchmarks

Model	FB15k-237		WN18RR		ogbl-biogk	
	PLL	MLE	PLL	MLE	PLL	MLE
CP	0.310	—	0.105	—	0.831	—
CP ⁺	0.237	0.230	0.027	0.026	0.496	0.501
CP ²	0.315	0.282	0.104	0.091	0.848	0.829
ComplEx	0.342	—	0.471	—	0.829	—
ComplEx ⁺	0.214	0.205	0.030	0.029	0.503	0.516
ComplEx ²	0.334	0.300	0.420	0.391	0.858	0.840

Bonus Sampling triples

$$\text{KTD}(\mathbb{P}, \mathbb{Q}) = \left\| \mathbb{E}_{\mathbf{x} \sim \mathbb{P}} [\varphi(\psi(\mathbf{x}))] - \mathbb{E}_{\mathbf{z} \sim \mathbb{Q}} [\varphi(\psi(\mathbf{z}))] \right\|^2$$

References

- [1] Erik Arakelyan, Pasquale Minervini, and Isabelle Augenstein. *Adapting Neural Link Predictors for Complex Query Answering*. 2023. arXiv: 2301.12313 [cs.LG].
- [2] Antonio Vergari, Nicola Di Mauro, and Guy Van den Broeck. “Tractable probabilistic models: Representations, algorithms, learning, and applications”. In: *Tutorial at the 35th Conference on Uncertainty in Artificial Intelligence (UAI)* (2019).
- [3] Yoojung Choi, Antonio Vergari, and Guy Van den Broeck. “Probabilistic Circuits: A Unifying Framework for Tractable Probabilistic Modeling”. In: (2020).
- [4] Antonio Vergari et al. “A Compositional Atlas of Tractable Circuit Operations for Probabilistic Inference”. In: *Advances in Neural Information Processing Systems 34 (NeurIPS)*. Curran Associates, Inc., 2021, pp. 13189–13201.
- [5] Kareem Ahmed et al. “Semantic probabilistic layers for neuro-symbolic learning”. In: *Advances in Neural Information Processing Systems 35 (NeurIPS)*. Vol. 35. Curran Associates, Inc., 2022, pp. 29944–29959.