# -SOTA-
# Document-level Neural Machine Translation

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April 2020

# Plan

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# Overview

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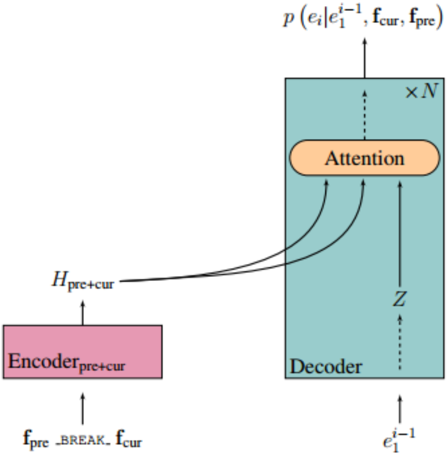
# Concatenation Approaches

Concatenation approaches to DLNMT consist in feeding a standard encoder-decoder architecture with a concatenation of sentences.



# Concatenation Approaches

For instance:

- [Tiedemann and Scherrer, 2017] firstly introduced this approach proposing an RNN-based model that incorporate the preceding sentence by prepending it to the current one, separated by a <CONCAT> token. They propose two methods:
  - **2-TO-2**: the previous and the current sentences are translated together. The translation of the current sentence is then obtained by only retaining the tokens following the concatenation token.
  - **2-TO-1**: only the current sentence is translated.
- [Agrawal et al., 2018, Scherrer et al., 2019] investigated the concatenation approach with the Transformer as base model, extending the number of context sentences both on the source (s:-3,+1) and the target (t:-2) side.

# Separate Encoding Approaches

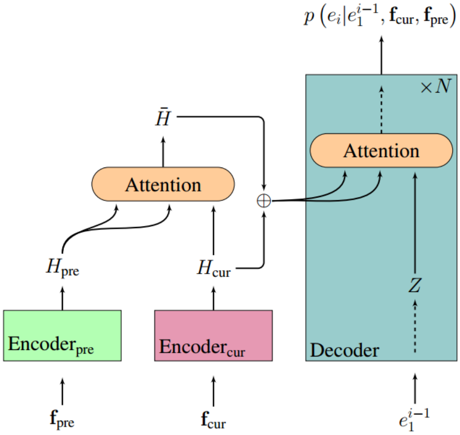
Separate encoding approaches to DLNMT consist in encoder-decoder models that encode the current and context sentences separately. This can be undertaken by:

- **Multiple encoders** working in parallel for the current and previous sentence. E.g. [Wang et al., 2017].
- **Multiple encoders with shared weights**. In this case, the parallel-working encoders not only have the same architecture, but also the same weights. E.g. [Voita et al., 2018].
- **Two-pass approaches**, in which the encoder makes a first sentence-level encoding pass of the source, and a second in which it encodes contextual information too. See Slide 13.
  - Remark: a powerful feature of two-pass approaches is their ability to exploit **future** target-side context.

# Separate Encoding Approaches

Once the encoding of the current and the context sentences has been carried out, they can be integrated in different ways:

- **Outside** the decoder.
  - $(+)$ symbol represents a gate, a sum or a concatenation.



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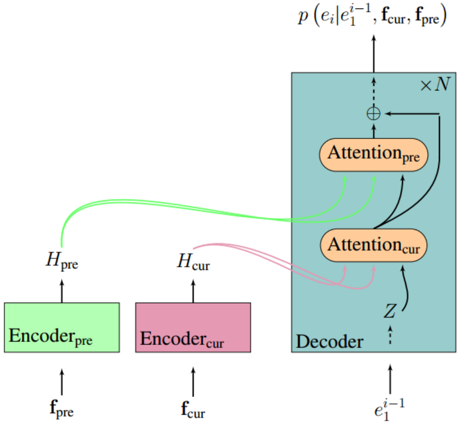
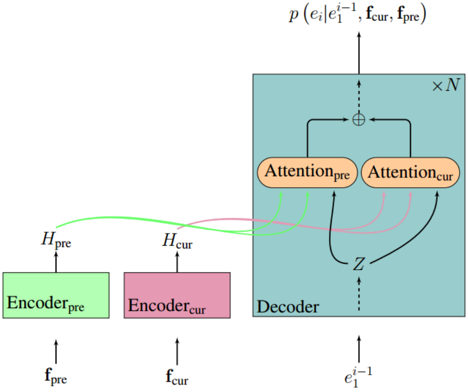
- **Inside** the decoder, **sequentially**.



# Separate Encoding Approaches

Once the encoding of the current and the context sentences has been carried out, they can be integrated in different ways:

- **Outside** the decoder.
  - $(+)$ symbol represents a gate, a sum or a concatenation.
- **Inside** the decoder, **sequentially**.
- **Inside** the decoder, **in parallel**.



# Separate Encoding Approaches

**Architecture**

The encoder-decoder architectures depicted above can be both RNN-based (until 2017) or Transfomer-based (after 2017), as for any approach to DLNMT. However, often some modifications are applied. For example:

- In the case of RNN-based architectures, integration inside the decoder can be undertaken without attention by simply concatenating context representations to the cell state of the deocdrr's RNN [Wang et al., 2017].
- beside contextual representation of words, the context encoder can also higher level representations such as sentence or document representations. This representations can also be attended by the decoder [Miculicich et al., 2018, Maruf et al., 2019] or added to the word-representations [Tan et al., 2019].
- Parallel integration inside the decoder can also happen within a single multi-head attention that takes as values and queries the concatenations of the current and context sentence representations [Voita et al., 2019b]

# Separate Encoding Approaches

**Including target-side context**

Despite some have considered including past target-side context harmful because of the *error propagation* problem [Zhang et al., 2018], most recent works have showed it to be of utmost importance for making the most out of context. Past works have successfully included target-side context information in different ways:

- Translating past sentences (usually 1) along with the current one, and then discarding them, as in concatenation approaches [Bawden et al., 2018].
- By making the decoder attend the target-side hidden representations or embeddings of previously decoded sentences [Miculicich et al., 2018, Voita et al., 2019b, Maruf et al., 2019, Zheng et al., 2020].

# Separate Encoding Approaches

| Reference | Context | Two-Pass Approach | Outside Integr. | Inside Integr. | Lang. Pair |
|---|---|---|---|---|---|
| [Wang et al., 2017] | s:-3 | | aut... | ...aut | Zh→En |
| [Voita et al., 2018] | s:-1 | | yes | | En→Ru |
| [Zhang et al., 2018] | s:-2 | | yes | sequential | Zh→En |
| [Miculicich et al., 2018] | s:-3; t:-3 | | yes | | Zh/Es→En |
| [Maruf et al., 2019] | s:all; t:all | optional | yes | | En→De |
| [Zheng et al., 2020] | s:all; t:all | yes | yes | | Zh/En→En/De |
| [Jean et al., 2017] | s:-1 | | | parallel | En→De/Fr |
| [Bawden et al., 2018] | s:-1; t:-1 | | | parallel | En→Fr |
| [Fu et al., 2019] | s:all | yes | | parallel | En/Zh→De/En |
| [Tan et al., 2019] | s:all | yes | | parallel | Zh/De→En |
| [Voita et al., 2019b] | s:-3; t:-3 | yes | | parallel* | En→Ru |

# Positional Embedding Schema

For separate encoding approaches as well as concatenation approaches, the standard positional encoding proposed by [Vaswani et al., 2017] is insufficient because the DNMT system needs to tell context sentences from the current one. For this reason, many strategies have been proposed in the literature, such as:

1. Adding a **sentence distance embedding** to context sentences, that tell the model how far away they are from the current sentence [Voita et al., 2019b].
2. Assign **positional embeddings progressively** to the current sentence, then to the previous one, and so on, so that far away sentences have high values of positional embedding [**?**].
3. Adding a **segment embedding**, similar to classical positional encoding but for the position of the sentence/segment within the document [Zheng et al., 2020].

Cache approaches to DLNMT consist in encoder-decoder models that are equipped with one or more caches that store context information. The information stored can belong to both **source side or target side, past and future**.

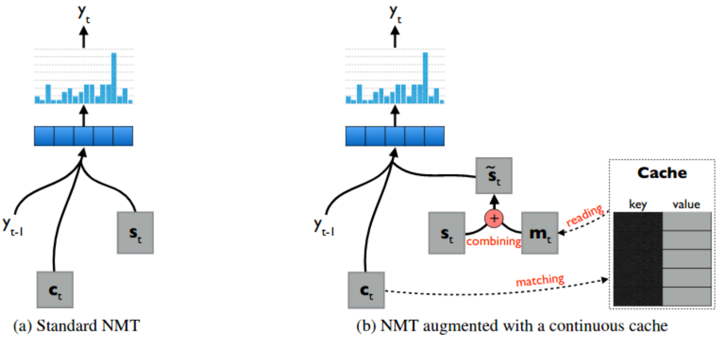
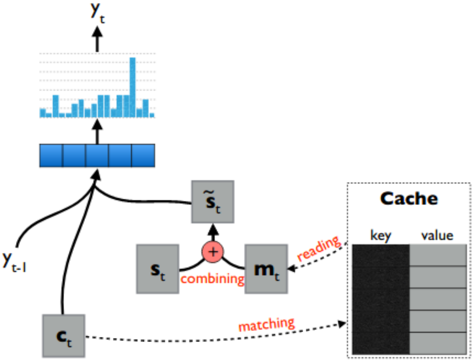



Figure: Continuous cache by [Tu et al., 2017]

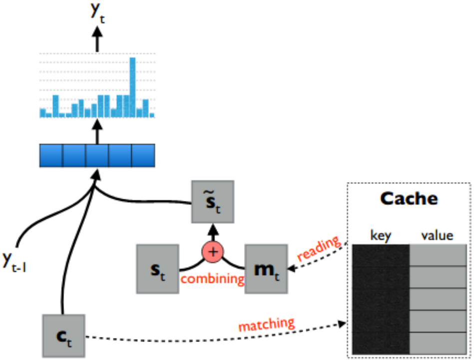
Every cache slot is a **key-value-indicator** triplet (the key and the indicator are often the same thing). With these variables, we can **read** or **write** caches.



# Cache Approaches

**Cache reading** involves:

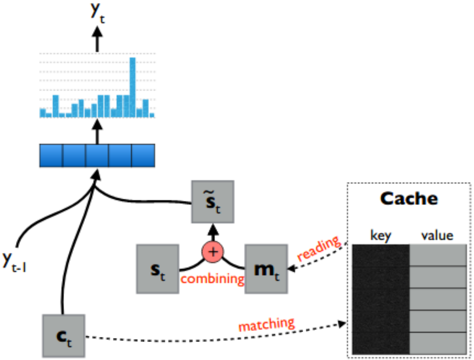
- Soft key matching
- Value reading
- Combining



# Cache Approaches

**Cache writing** can be undertaken after having translated one or more sentences. For every triplet:

- If the **indicator** already exists in the cache, we just update it's keys and values.
- Else, we write the triplet in an empty slot, after having emptied the oldest slot if the cache is full.



# Cache Approaches

| Reference | Caches | Size | Key (Indic.) | Value | Lang. Pair |
|---|---|---|---|---|---|
| [Tu et al., 2017] | single | $\leqslant 500$ | $c_t(\ y_{k<t})$ | $s_{k<t}$ | Zh→En |
| [Kuang et al., 2018] | dynamic<br>topic | 100<br>200 | $c_t$ | $y_{k<t}$<br>topic emb. | Zh→En |
| [Maruf and Haffari, 2018] | source<br>target | doc.size | $h_t$<br>$s_t$ | $sent.emb.$<br>$s_{k<t}$ | Fr/De/Et→En |

# On Parallel Corpora for Training

DLNMT systems require training on **document-level parallel corpora**. These corpora are usually released during workshops on machine translation like IWSLT and WMT, and hosted on open source web inventories. The most common ones, are extracted from:

- Movie subtitles (OpenSubtitles)
- TED talks (WIT3)
- News articles (LDC)
- Parliamentary interventions (Europarl)

# On Parallel Corpora for Training

Unfortunately, document-level parallel corpora are often insufficient to train DLNMT systems from scratch, although it is often possible to make them converge to a local optimum.

[Kim et al., 2019] pointed out that when constraining training on such small datasets, model comparison becomes misleading because gains in performance are mainly related to better regularization.

A popular solution to this problem is the **two-step training strategy**: [Tu et al., 2017, Voita et al., 2018, Miculicich et al., 2018]

1. Distinguish two integrated components in your model with params $\Theta = [\theta_S; \theta_D]$:
   - A self-standing sentence-level NMT system with parameters $\theta_S$.
   - Some context-handling modules with parameters $\theta_D$.
2. Train $\theta_S$ independently on a sentence-level parallel corpus $C_S$.
3. Train together $\theta_D$ and $\theta_S$ (finetune) on a document-level parallel corpus $C_D$.

# Exploiting Monolingual Corpora

Another solution to the lack of vast document-level parallel corpora is leveraging on huge *monolingual* document-level corpora like BookCorpus [Zhu et al., 2015] and PG-19 [Rae et al., 2019]. In the literature, we can find various approaches to leverage monolingual corpora:

- **Back-translate** target-side corpus to augment dl corpus [Sugiyama and Yoshinaga, 2019].
- Train **context-aware language models** on target/source-side corpus, then:
  - Generate translations by fusioning the decoder and the LM's scores to candidate words [Martnez Garcia et al., 2019].
  - Initialize the econder (or decoder) of a DLNMT model [Li et al., 2019].
- Train **Automatic Post Editing** systems on target-side corpus (See next slide).

# Exploiting Monolingual Corpora

**Automatic Post Editing** (APE)
[Voita et al., 2019a] devised an APE system called DocRepair, that turns a sentence-level translation into a context-aware translation. DocRepair can work on top of whatever sentence-level MT system and is lightweight compared to NMT systems.

# Cache Approaches

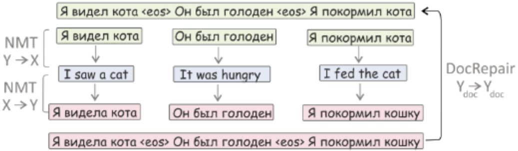



Figure 1: Training procedure of DocRepair. First, round-trip translations of individual sentences are produced to form an inconsistent text fragment (in the example, both genders of the speaker and the cat became inconsistent). Then, a repair model is trained to produce an original text from the inconsistent one.

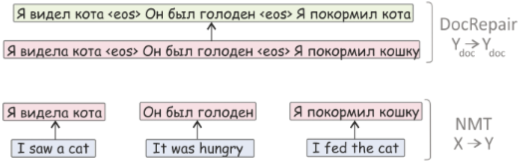



Figure 2: The process of producing document-level translations at test time is two-step: (1) sentences are translated independently using a sentence-level model, (2) DocRepair model corrects translation of the resulting text fragment.

# Others

**Approaches Including Additional Discourse Information as Input**

These approaches consist in concatenation approaches or separate encoding approaches that also integrate discourse-related information as additional input features. Examples of extra features are:

- Lexical chains of semantically similar words to promote word sense disambiguation [Rios Gonzales et al., 2017].
- Coreference chains to promote coreference resolution [Stojanovski and Fraser, 2018, Ohtani et al., 2019].

**Learning Approaches**

[Jean and Cho, 2019] looked at the problem from a learning perspective and designed a regularisation term to encourage a DLNMT model to exploit the additional context in a useful way . This regularisation term is applied at the token, sentence and corpus levels and is based on pair-wise ranking loss, that is, it helps to assign a higher log-probability to a translation paired with the correct context than to the translation without context.

**Possible Future Research Directions**

- build a large DL corpus for training systems, or find automatic approaches to generate synthetic data other than back-translation [Jean et al., 2019]
- design models exploiting full context;
- design models performing good for single-sentence translation [Zheng et al., 2020]
- design post-processing models that are lightweight and can be trained on little data [Kim et al., 2019]. Nonetheless, beware that While this kind of approach is easy to deploy, the two-stage generation process may result in error accumulation.
- pre-trained language model for DLNMT decoder.

Thank you for your attention!

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# Markov Decision Processes

## Reinforcement Learning

General class of algorithms that allow an agent to learn how to behave in a stochastic and possibly unknown environment by trial-and-error.

## Markov Decision Process (MDP)

stochastic dynamical system specified by $< \mathbb{S}, \mathbb{A}, \mathcal{P}, \mathcal{R}, \gamma >$

1. $(\mathbb{S}, \mathcal{S})$ is a measurable state space
2. $(\mathbb{A}, \mathcal{A})$ is a measurable action space
3. $\mathcal{P} : \mathbb{S} \times \mathbb{A} \times \mathcal{S} \to \mathbb{R}$ is a Markov transition kernel
4. $\mathcal{R} : \mathbb{S} \times \mathbb{A} \to \mathbb{R}$ is a reward function
5. $0 < \gamma < 1$ is the discount factor.

# Monte-Carlo Policy Gradient: Pseudocode

**Input:** Stochastic policy $\pi_\theta$, Initial parameters $\theta_0$, learning rate $\{\alpha_k\}$
**Output:** Approximation of the optimal policy $\pi_{\theta*} \approx \pi_*$

**repeat**
  Sample $M$ trajectories $h^{(m)} = \{(s_t^{(m)}, a_t^{(m)}, r_{t+1}^{(m)})\}_{t=0}^{T^{(m)}}$ under policy $\pi_{\theta_k}$
  Approximate policy gradient

$$\nabla_\theta J(\theta_k) \approx \frac{1}{M} \sum_{m=0}^{M} \sum_{u=0}^{T^{(m)}-1} \nabla_\theta \log \pi_{\theta_k} \left( s_u^{(m)}, a_u^{(m)} \right) \sum_{v \geqslant u}^{T^{(m)}-1} \gamma^{v-u} r_{v+1}^{(m)}$$

  Update parameters using gradient ascent $\theta_{k+1} = \theta_k + \alpha_k \nabla_\theta J(\theta_k)$
  $k \leftarrow k + 1$
**until** converged

# Episodic PGPE Algorithm: Pseudocode

**Input:** Controller $F_\theta$, hyper-distribution $p_\xi$, initial guess $\xi_0$, learning rate $\{\alpha_k\}$
**Output:** Approximation of the optimal policy $F_{\xi*} \approx \pi_*$

1: **repeat**
2: **for** $m = 1, \ldots, M$ **do**
3: Sample controller parameters $\theta^{(m)} \sim p_{\xi_k}$
4: Sample trajectory $h^{(m)} = \{(s_t^{(m)}, a_t^{(m)}, r_{t+1}^{(m)})\}_{t=0}^{T^{(m)}}$ under policy $F_{\theta^{(m)}}$
5: **end for**
6: Approximate policy gradient

$$\nabla_\xi J(\xi_k) \approx \frac{1}{M} \sum_{m=1}^{M} \nabla_\xi \log p_\xi \left(\theta^{(m)}\right) \left[G\left(h^{(m)}\right) - b\right]$$

7: Update hyperparameters using gradient ascent $\xi_{k+1} = \xi_k + \alpha_k \nabla_\xi J(\xi_k)$
8: $k \leftarrow k + 1$
9: **until** converged

# Truncated Multiple Importance Sampling Estimator

## Importance Sampling

Given a bounded function $f : \mathcal{Z} \to \mathbb{R}$, and a set of i.i.d. outcomes $z_1, \ldots, z_N$ sampled from $Q$, the importance sampling estimator of $\mu := \underset{z \sim P}{\mathbb{E}}[f(z)]$ is:

$$\widehat{\mu}_{\text{IS}} = \frac{1}{N} \sum_{i=1}^{N} f(z_i) w_{P/Q}(z_i), \tag{1}$$

which is an unbiased estimator, i.e., $\underset{z_i \overset{\text{iid}}{\sim} Q}{\mathbb{E}} [\widehat{\mu}_{IS}] = \mu$.

## Truncated Estimator With Balance Heuristic

$$\check{\mu}_{\text{BH}} = \frac{1}{N} \sum_{k=1}^{K} \sum_{i=1}^{N_k} \min \left\{ M, \frac{p(z_{ik})}{\sum_{j=1}^{K} \frac{N_j}{N} q_j(z_{ik})} \right\} f(z_{ik}). \tag{2}$$

# OPTIMIST2

## Theorem

*regretdiscretized Let $\mathcal{X}$ be a d-dimensional compact arm set with $\mathcal{X} \subseteq [-D, D]^d$. For any $\kappa \geqslant 2$, under Assumptions 1 and 2, OPTIMIST2 with confidence schedule $\delta_t = \dfrac{6\delta}{\pi^2 t^2 \left(1 + \lceil t^{1/\kappa} \rceil^d\right)}$ and discretization schedule $\tau_t = \lceil t^{\frac{1}{\kappa}} \rceil$ guarantees, with probability at least $1 - \delta$:*

$$Regret(T) \leqslant \Delta_0 + C_1 T^{\left(1 - \frac{1}{\kappa}\right)d} + C_2 T^{\frac{1}{1+\epsilon}} \cdot \left[ v_\epsilon \left( (2 + {}^{d}/_{\kappa}) \log T + d \log 2 + \log \frac{\pi^2}{3\delta} \right) \right]^{\frac{\epsilon}{1+\epsilon}},$$

*where $C_1 = \dfrac{\kappa}{\kappa - 1} LD$, $C_2 = (1 + \epsilon) \left( 2\sqrt{2} + \dfrac{5}{3} \right) \|f\|_\infty$, and $\Delta_0$ is the instantaneous regret of the initial arm $\boldsymbol{x}_0$.*