Chapter 1: Fundamentals[1]

Section 1.1: Introduction to Sets

In this section, we introduce *sets* as a collection of objects and go over the basic notation to describe them. We'll also learn how to represent standard sets (like the real numbers, counting numbers, empty set, etc.).

Learning Objectives

- State the definitions of set, cardinality, and finite and infinite sets.
- Use the roster or set-builder method to describe a set.
- Define the special sets $\mathbb{Z}, \mathbb{N}, \mathbb{R}, \mathbb{Q}$, and $\emptyset$.

Section 1.2: The Cartesian Product

Here we'll define the *Cartesian Product,* an operation that builds ordered pairs from elements of two given sets. This will form the groundwork for later topics: relations, functions, and coordinate systems.

Learning Objectives

- State the definitions of ordered pair, coordinates, and the Cartesian product of two sets.
- Visualize Cartesian products of sets of real numbers in the plane.

Section 1.3: Subsets

In this section we'll consider what it means for one set to be a *subset* of another. This discussion is necessary for proving key relationships between sets.

[1] Course objectives modified from Talbert, Robert. *Proof Techniques.* GitHub repository, 2023. Available at: https://github.com/RobertTalbert/proof. Accessed May 2025.

Learning Objectives

- State the definition of a subset.
- List all of the subsets of a given set.

Section 1.4: Power Sets

We'll define the *Power Set*, the set of all subsets of a given set. These sets are useful in counting arguments and combinatorics!

Learning Objectives

- State the definition of a power set.
- Determine the power set of a given set.

Section 1.5: Union, Intersection, Difference

We'll learn three simple ways to combine sets.

Learning Objectives

- State the definition of union, intersection, and difference.
- Determine union, intersection, and/or difference of given sets.

Section 1.6: Complement

Here we discuss an additional operation on a set, by fixing a universe and determining everything in that universe that's *not* in that set.

Learning Objectives

- Define universal set and complement.
- Determine the complement of a given set.

Section 1.7: Venn Diagrams

We'll explore an informal way to visualize relationships between sets.

Learning Objectives

- Sketch Venn diagrams for combinations of sets.

Chapter 2: Logic

Section 2.1: Statements

We define *statements*, mathematical expressions that are always true or always false. These form the foundation for the types of expressions we'll prove later in the semester.

Learning Objectives

- Define statement.
- Differentiate statements from open sentences or other expressions.
- Rephrase vague claims into precise statements.

Section 2.2: And, Or, Not

We examine *compound* statements, which are combined with logical operators, and explore the conditions under which they are true or false.

Learning Objectives

- Identify the conjunction, disjunction, and negation operators and use their notation correctly.
- Phrase the negation of a statement in a way that does not use the word "not".
- Construct truth tables for conjunctions, disjunctions, negations, and implications.

Section 2.3: Conditional Statements

We'll focus on a cornerstone of mathematical statements, the conditional or "if-then" statement.

Learning Objectives

- Identify the implication operator and its their notation correctly.
- Construct truth table for implications.
- Explain why a false hypothesis makes the implication true.
- Phrase a conditional statement in several different English formats.

Section 2.4: Biconditional Statements

We'll explore biconditional statements or "if and only if" statements.

Learning Objectives

- Define converse.
- Construct truth table for biconditionals.

Section 2.5: Truth Tables for Statements

Here we build truth tables for more complex expressions.

Learning Objectives

- Build truth tables for compound statements, including those involving three or more variables.
- Test logical equivalence with tables.

Section 2.6: Logical Equivalencies

In this section we'll consider statements that appear different but mean the same thing logically, because their truth tables are identical (logical equivalence). We'll discuss an important result on negating and/or statements, called DeMorgan's Laws.

Learning Objectives

- State DeMorgan's Laws
- Prove equivalence by algebra of propositions or tables.

Section 2.7: Quantifiers

In this section, we discuss statements that are true for all objects of a certain type (a universally quantified statement) or just for one object (an existentially quantified statement) . We'll see how every conditional statement can be written as a universally quantified statement, and we'll see how in general a quantified statement can be negated. We'll also explore statements with two quantifiers in them.

Learning Objectives

- Define the terms "universal quantifier" and "existential quantifier"; identify these quantifiers in practice and use correct notation to denote them.
- Determine whether a quantified statement is true or false.
- Phrase both universally and existentially quantified statements in several different English formats.
- Find a counterexample for a (false) universally quantified statement.
- Form the negation of universally quantified and existentially quantified statements; write these negations in forms that do not use the negation symbol.
- Rewrite a conditional statement as a universally quantified statement.

- Construct examples and non-examples of definitions that involve quantified statements.
- Phrase doubly-quantified statements in English, and determine if such statements are true or false.
- Form the negation of a doubly-quantified statement.

Section 2.10: Negating Statements

In proof writing, we often need to form the negation of a given statement. Here we practice negating complex statements.

Learning Objectives

- Correctly negate conditionals and quantified statements.
- Verify negations by truth tables or examples.
- Explain why the negation of a conditional statement is not another conditional statement.

Chapter 4: Direct Proof

Section 4.1: Theorems

In this section we learn what a proof is and introduce terminology for the statements we'll explore in later sections.

Section 4.2: Definitions

Here we discuss some key definitions that play a rule in many propositions.

Section 4.3: Direct Proof

We finally start constructing solutions to mathematical problems using direct proof of conditional statements.

Section 4.4: Using Cases

Sometimes we proof a statement is true by examining different cases.

Section 4.5: Treating Similar Cases

Similar to the previous section, we discuss how to simplify proofs where several cases are similar or redundant.

Chapter 5: Contrapositive Proof

Section 5.1: Contrapositive Proof

Section 5.2: Congruence of Integers

Section 5.3: Mathematical Writing

Chapter 6: Proof by Contradiction

Section 6.1: Proving Statements with Contradiction

Section 6.2: Proving Conditional Statements by Contradiction

Chapter 7: Proving Non-Conditional Statements

Section 7.1: If-and-Only-If Proof

Section 7.2: Equivalent Statements

Section 7.3: Existence Proofs; Existence and Uniqueness Proofs

Section 7.4: Constructive Versus Non-Constructive Proofs

Chapter 8: Proofs Involving Sets

Section 8.1: How to Prove $a \in A$

Section 8.2: How to Prove $A \subseteq B$

Section 8.3: How to Prove $A = B$

Chapter 9: Disproof

Section 9.1: Counterexamples

Section 9.2: Disproving Existence Statements

Section 9.3: Disproof by Contradiction

Chapter 10: Mathematical Induction

Section 10.1: Proof by Induction

Section 10.2: Proof by Strong Induction

Section 10.3: Proof by Smallest Counterexample

Section 10.5: Fibonacci Numbers

Chapter 11: Relations

Section 11.1: Relations

Section 11.2: Properties of Relations

Section 11.3: Equivalence Relations

Section 11.4: Equivalence Classes and Partitions

Section 11.5: The Integers Modulo n

Section 11.6: Relations Between Sets

Chapter 12: Functions

Section 12.1: Functions

Section 12.2: Injective and Surjective Functions

Section 12.3: The Pigeonhole Principle Revisited

Section 12.4: Composition

Section 12.5: Inverse Functions

Section 12.6: Image and Preimage

Chapter 14: Cardinality of Sets

Section 14.1: Sets with Equal Cardinalities

Section 14.2: Countable and Uncountable Sets

Section 14.3: Comparing Cardinalities