Lab 6a
Pole Placement
for the Inverted Pendulum
ELENG128

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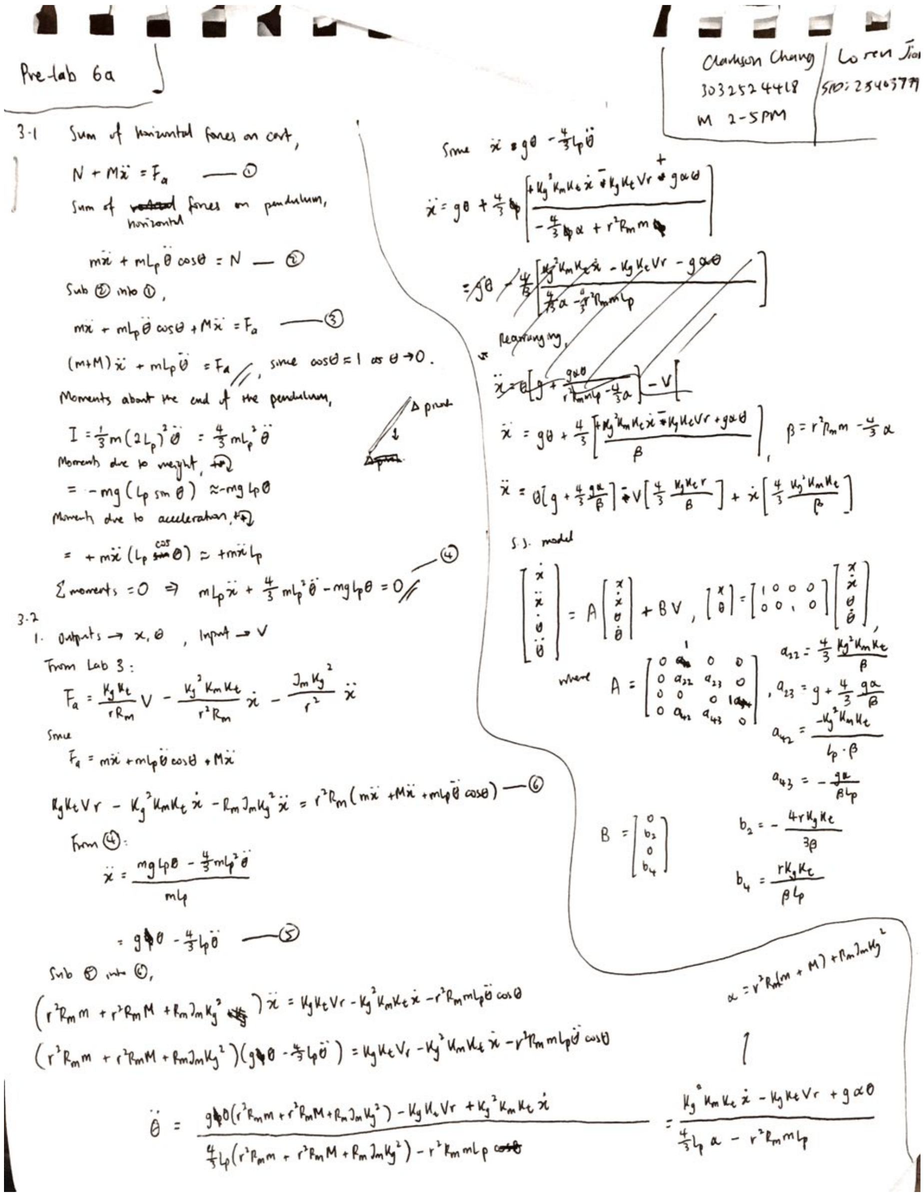
1 Objective

The objective of this lab is to achieve simultaneous control of both the angular position of the pendulum and horizontal position of the cart on the track using full-state feedback. We will be considering small angle perturbations and sine wave reference tracking of the cart position. Note that the system is a SIMO a Single Input Multiple Output system, since we are trying to control both the position of the cart and the angle of the pendulum by using only the motor voltage.

2 Pre-lab assignments



Pre-lab 6a

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M 2-5PM

3.1 Sum of horizontal forces on cart,

$$N + M\ddot{x} = F_a \quad \text{—①}$$

Sum of ~~vertical~~ horizontal forces on pendulum,

$$m\ddot{x} + mL_p\ddot{\theta}\cos\theta = N \quad \text{—②}$$

Sub ② into ①,

$$m\ddot{x} + mL_p\ddot{\theta}\cos\theta + M\ddot{x} = F_a \quad \text{—③}$$

$$(m+M)\ddot{x} + mL_p\ddot{\theta} = F_a$$, since $\cos\theta = 1$ as $\theta \to 0$.

Moments about the end of the pendulum,

Δ pivot

$$I = \frac{1}{3}m(2L_p)^2\ddot{\theta} = \frac{4}{3}mL_p^2\ddot{\theta}$$

Moments due to weight,

$$= -mg(L_p\sin\theta) \approx -mgL_p\theta$$

Moments due to acceleration,

$$= +m\ddot{x}(L_p\cos\theta) \approx +m\ddot{x}L_p$$

$$\Sigma \text{ moments} = 0 \Rightarrow mL_p\ddot{x} + \frac{4}{3}mL_p^2\ddot{\theta} - mgL_p\theta = 0 \quad \text{—④}$$

3.2

1. Outputs $\to x, \theta$, Input $\to V$

From Lab 3:

$$F_a = \frac{K_gK_t}{rR_m}V - \frac{K_g^2K_mK_t}{r^2R_m}\dot{x} - \frac{J_mK_g^2}{r^2}\ddot{x}$$

Since

$$F_a = m\ddot{x} + mL_p\ddot{\theta}\cos\theta + M\ddot{x}$$

$$K_gK_tVr - K_g^2K_mK_t\dot{x} - R_mJ_mK_g^2\ddot{x} = r^2R_m(m\ddot{x} + M\ddot{x} + mL_p\ddot{\theta}\cos\theta) \quad \text{—⑥}$$

From ④:

$$\ddot{x} = \frac{mgL_p\theta - \frac{4}{3}mL_p^2\ddot{\theta}}{mL_p}$$

$$= g\theta - \frac{4}{3}L_p\ddot{\theta} \quad \text{—⑤}$$

Sub ⑤ into ⑥,

$$(r^2R_mm + r^2R_mM + R_mJ_mK_g^2)\ddot{x} = K_gK_tVr - K_g^2K_mK_t\dot{x} - r^2R_mmL_p\ddot{\theta}\cos\theta$$

$$(r^2R_mm + r^2R_mM + R_mJ_mK_g^2)(g\theta - \frac{4}{3}L_p\ddot{\theta}) = K_gK_tV_r - K_g^2K_mK_t\dot{x} - r^2R_mmL_p\ddot{\theta}\cos\theta$$

$$\ddot{\theta} = \frac{g\theta(r^2R_mm + r^2R_mM + R_mJ_mK_g^2) - K_gK_tVr + K_g^2K_mK_t\dot{x}}{\frac{4}{3}L_p(r^2R_mm + r^2R_mM + R_mJ_mK_g^2) - r^2R_mmL_p\cancel{\cos\theta}} = \frac{K_g^2K_mK_t\dot{x} - K_gK_tVr + g\alpha\theta}{\frac{4}{3}L_p\alpha - r^2R_mmL_p}$$

$$\alpha = r^2R_m(m+M) + R_mJ_mK_g^2$$

Since $\ddot{x} = g\theta - \frac{4}{3}L_p\ddot{\theta}$

$$\ddot{x} = g\theta + \frac{4}{3}L_p\left[\frac{-K_g^2K_mK_t\dot{x} + K_gK_tVr - g\alpha\theta}{-\frac{4}{3}L_p\alpha + r^2R_mmL_p}\right]$$

Rearranging,

$$\ddot{x} = g\theta + \frac{4}{3}\left[\frac{-K_g^2K_mK_t\dot{x} + K_gK_tVr + g\alpha\theta}{\beta}\right], \quad \beta = r^2R_mm - \frac{4}{3}\alpha$$

$$\ddot{x} = \theta\left[g + \frac{4}{3}\frac{g\alpha}{\beta}\right] + V\left[\frac{4}{3}\frac{K_gK_tr}{\beta}\right] + \dot{x}\left[\frac{4}{3}\frac{K_g^2K_mK_t}{\beta}\right]$$

S.S. model

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = A\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix} + BV, \quad \begin{bmatrix}x\\ \theta\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix},$$

where

$$A = \begin{bmatrix}0&1&0&0\\0&a_{22}&a_{23}&0\\0&0&0&1\\0&a_{42}&a_{43}&0\end{bmatrix}, \quad a_{22} = \frac{4}{3}\frac{K_g^2K_mK_t}{\beta}, \quad a_{23} = g + \frac{4}{3}\frac{g\alpha}{\beta}, \quad a_{42} = \frac{-K_g^2K_mK_t}{L_p\cdot\beta}, \quad a_{43} = -\frac{g\alpha}{\beta L_p}$$

$$B = \begin{bmatrix}0\\b_2\\0\\b_4\end{bmatrix}, \quad b_2 = -\frac{4rK_gK_t}{3\beta}, \quad b_4 = \frac{rK_gK_t}{\beta L_p}$$

Figure 1

Using MATLAB,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6.812 & 7.497 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 15.473 & 25.683 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1.523 \\ 0 \\ -3.4583 \end{bmatrix}$$

3.3.

1. Using MATLAB,

The eigenvalues $-7.56, -4.13, 0, 4.87$.

The system is unstable as there are RHP poles.

2. x increases without bounds, so the cart moves infinitely along the track. should be increasing in your step plot.

θ remains constant. The system will not behave this way in real life as the track is not of infinite length, and there are external forces such as friction and motor resistance

No output response?

3 a) $K = [k_1 \; k_2 \; k_3 \; k_4]$

$A_k = A - BK$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix} [k_1 \; k_2 \; k_3 \; k_4]$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_2k_1 & b_2k_2 & b_2k_3 & b_2k_4 \\ 0 & 0 & 0 & 0 \\ b_4k_1 & b_4k_2 & b_4k_3 & b_4k_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -b_2k_1 & a_{22}-b_2k_1 & a_{23}-b_2k_3 & -b_2b_4 \\ 0 & 0 & 0 & 1 \\ -b_4k_1 & a_{42}-b_4k_2 & a_{43}-b_4k_3 & -b_4k_4 \end{bmatrix} //$$

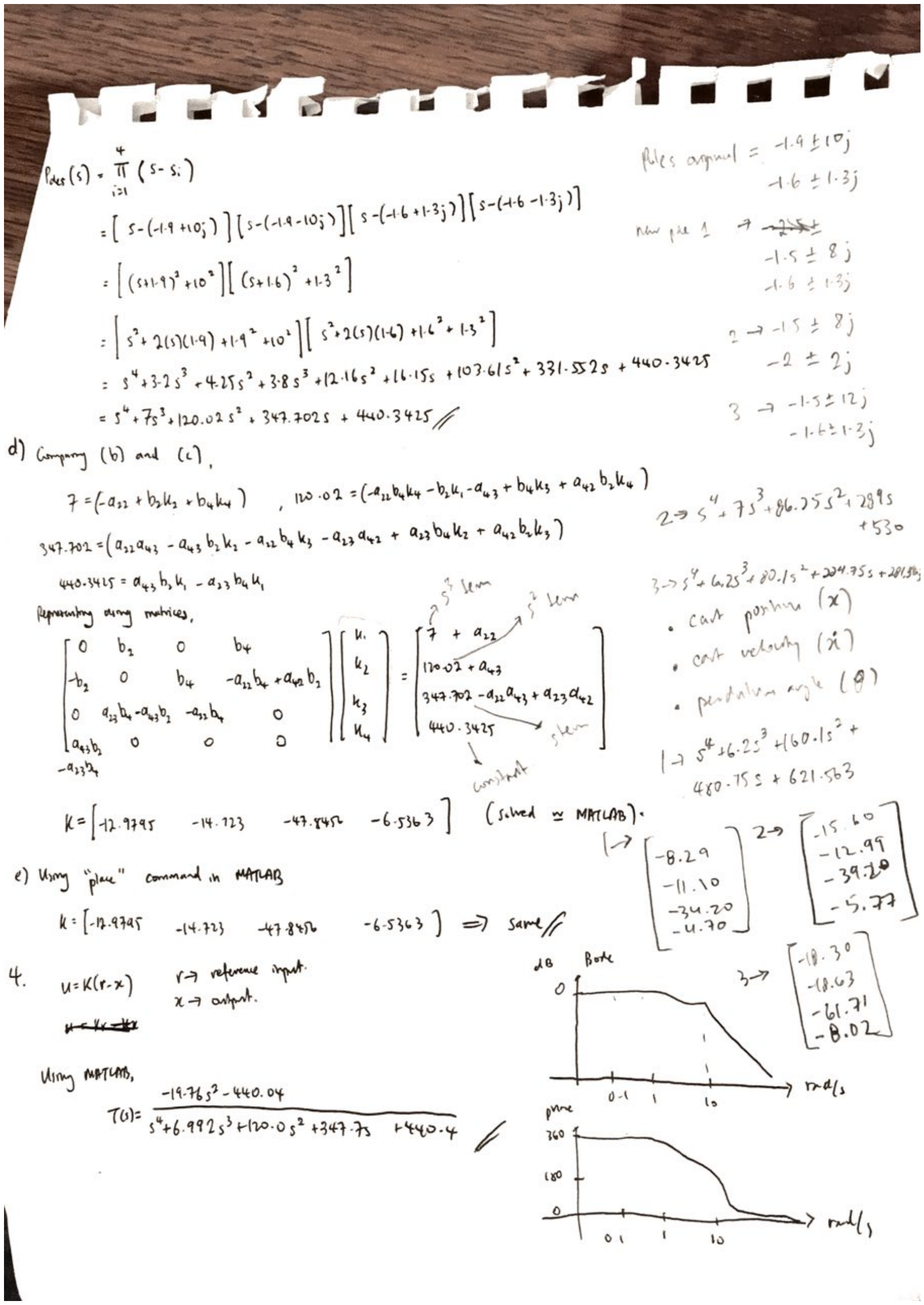
b)

$$sI - A_k = \begin{bmatrix} s & -1 & 0 & 0 \\ +b_2k_1 & s-a_{22}+b_2k_1 & b_2k_3-a_{23} & b_2b_4 \\ 0 & 0 & s & -1 \\ b_4k_1 & b_4k_2-a_{42} & b_4k_3-a_{43} & s+b_4k_4 \end{bmatrix}$$

$$\begin{aligned} \det(sI - A_k) &= s(s-a_{22}+b_2k_1)(s^2+sb_4k_4+b_4k_3-a_{43}) \\ &\quad + s(-a_{42}+b_4k_2)(a_{23}-b_2k_3-sb_2b_4) \\ &\quad - (b_2k_1)(s^2+sb_4k_4-a_{43}+b_4k_3) - (b_4k_1)(a_{23}-b_2k_3-sb_2k_4) \end{aligned}$$

$$\begin{aligned} p(k;s) &= (s^2-sa_{22}+sb_2k_1)(s^2+sb_4k_4+b_4k_3-a_{43}) \\ &\quad + (sb_4k_2-sa_{42})(a_{23}-b_2k_3-sb_2b_4) \\ &\quad - (b_2k_1)(s^2+sb_4k_4-a_{43}+b_4k_3) \\ &\quad - (b_4k_1)(a_{23}-b_2k_3-sb_2k_4) \\ &= \ldots \\ &= s^4+s^3(-a_{22}+b_2k_2+b_4k_4) \\ &\quad + s^2(-a_{22}b_4k_4-b_2k_1-a_{43}+b_4k_3+a_{42}b_2k_4) \\ &\quad + s(a_{22}a_{43}-a_{43}b_2k_2-a_{22}b_4k_3-a_{23}a_{42}+a_{23}b_4k_2+a_{42}b_2 \\ &\quad + (a_{43}b_2k_1-a_{23}b_4k_1) // \end{aligned}$$

Figure 2



$$P_{des}(s) = \prod_{i=1}^{4} (s - s_i)$$

$$= \left[s-(-1.9+10j) \right]\left[s-(-1.9-10j) \right]\left[s-(-1.6+1.3j) \right]\left[s-(-1.6-1.3j) \right]$$

$$= \left[(s+1.9)^2+10^2 \right]\left[(s+1.6)^2+1.3^2 \right]$$

$$= \left[s^2+2(s)(1.9)+1.9^2+10^2 \right]\left[s^2+2(s)(1.6)+1.6^2+1.3^2 \right]$$

$$= s^4+3.2s^3+4.25s^2+3.8s^3+12.16s^2+16.15s+103.61s^2+331.552s+440.3425$$

$$= s^4+7s^3+120.02s^2+347.702s+440.3425$$

Poles original $= -1.9 \pm 10j$, $-1.6 \pm 1.3j$

new pole 1 $\rightarrow$ ~~-2.5±~~ $-1.5 \pm 8j$, $-1.6 \pm 1.3j$

$2 \rightarrow -1.5 \pm 8j$, $-2 \pm 2j$

$3 \rightarrow -1.5 \pm 12j$, $-1.6 \pm 1.3j$

d) Comparing (b) and (c),

$$7 = (-a_{22} + b_2 k_2 + b_4 k_4), \quad 120.02 = (-a_{22} b_4 k_4 - b_2 k_1 - a_{43} + b_4 k_3 + a_{42} b_2 k_4)$$

$$347.702 = (a_{22} a_{43} - a_{43} b_2 k_2 - a_{22} b_4 k_3 - a_{23} a_{42} + a_{23} b_4 k_2 + a_{42} b_2 k_3)$$

$$440.3425 = a_{43} b_2 k_1 - a_{23} b_4 k_1$$

$2 \rightarrow s^4 + 7s^3 + 86.25s^2 + 289s + 530$

$3 \rightarrow s^4 + 6.2s^3 + 80.1s^2 + 204.75s + 281.36$

Representing using matrices,

$$\begin{bmatrix} 0 & b_2 & 0 & b_4 \\ -b_2 & 0 & b_4 & -a_{22}b_4 + a_{42}b_2 \\ 0 & a_{23}b_4 - a_{43}b_2 & -a_{22}b_4 & 0 \\ a_{43}b_2 - a_{23}b_4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 7 + a_{22} \\ 120.02 + a_{43} \\ 347.702 - a_{22}a_{43} + a_{23}a_{42} \\ 440.3425 \end{bmatrix}$$

s^3 term, s^2 term, s term, constant

- cart position (x)
- cart velocity ($\dot{x}$)
- pendulum angle (θ)

$1 \rightarrow s^4 + 6.2s^3 + 160.1s^2 + 480.75s + 621.563$

$$K = \begin{bmatrix} -12.9795 & -14.723 & -47.8452 & -6.5363 \end{bmatrix} \quad \text{(solved w MATLAB)}.$$

$$1 \rightarrow \begin{bmatrix} -8.29 \\ -11.10 \\ -34.20 \\ -4.70 \end{bmatrix} \quad 2 \rightarrow \begin{bmatrix} -15.60 \\ -12.99 \\ -39.20 \\ -5.77 \end{bmatrix} \quad 3 \rightarrow \begin{bmatrix} -18.30 \\ -18.63 \\ -61.71 \\ -8.02 \end{bmatrix}$$

e) Using "place" command in MATLAB

$$k = \begin{bmatrix} -12.9795 & -14.723 & -47.8456 & -6.5363 \end{bmatrix} \Rightarrow \text{same}$$

4. $u = K(r - x)$ $\quad r \rightarrow$ reference input. $\quad x \rightarrow$ output.

~~u = Kr - Kx~~

Using MATLAB,

$$T(s) = \frac{-19.76s^2 - 440.04}{s^4 + 6.992s^3 + 120.0s^2 + 347.7s + 440.4}$$

Bode

dB: 0; rad/s: 0.1, 1, 10

phase: 360, 180, 0; rad/s: 0.1, 1, 10

Figure 3

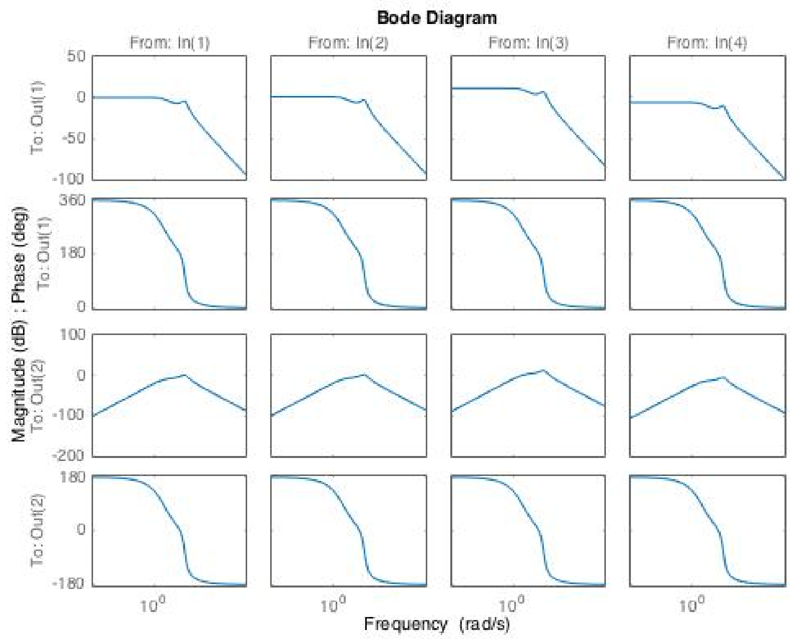


Figure 4

3 Lab Results

3.1 Implementing the Controller in Simulink

We implemented the state-feedback controller with the Quanser I/O blocks in Simulink as shown in Figure 5. In order to prevent the gear from slipping, we put a saturation block before the Analog Output block, set to -6V to 6V. The top gain block was set to a value of $\frac{1}{4.396}$ m/counts. The bottom gain block was set to a value of $\frac{1}{666.7}$ rad/counts.

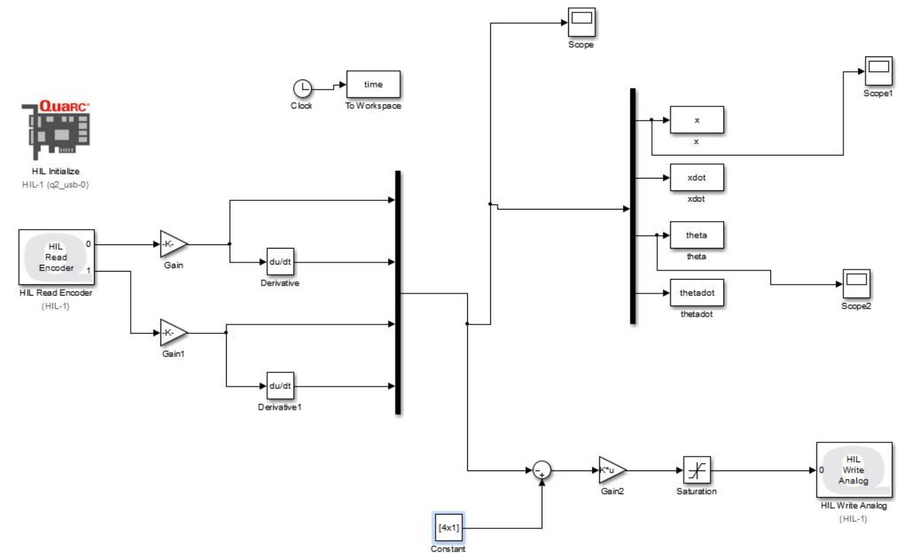


Figure 5: Simulink block diagram.

3.2 Running the Controller on the Hardware

3.2.1 Initially balancing the pendulum

We ran the controller on the hardware (with reference r set to $r = [0\ 0\ 0\ 0]^T$) and made sure it balanced. We held the pendulum exactly vertical before connecting to the real time target (our controller stabilized the unstable equilibrium $\theta = 0°$). If we try to connect the hardware when the pendulum is in its stable equilibrium position ($\theta = 180°$), nothing should happen since the pendulum will always be in its desired position.

3.2.2 System response to small perturbations

With the pendulum balancing, we manually applied small perturbations to the pendulum and checked the response.

When the pendulum was perturbed in one direction, the cart responded by moving the same direction in order to balance the torque applied by the perturbation.

We then plotted the variation of the cart position (Figure 6) and pendulum angle (Figure 7) with time for small perturbations (manually induced) about the equilibrium value. The spikes in the plots represent the manual perturbations. Overall, the response time to correct the perturbations is decent, but we noticed that the hardware continued to oscillate about the equilibrium point even when undisturbed. This is because when the pendulum will always be slightly left or right of the equilibrium point. When the hardware attempts to correct the pendulum angle, there will be overshoot, and this will repeat in an oscillatory fashion.

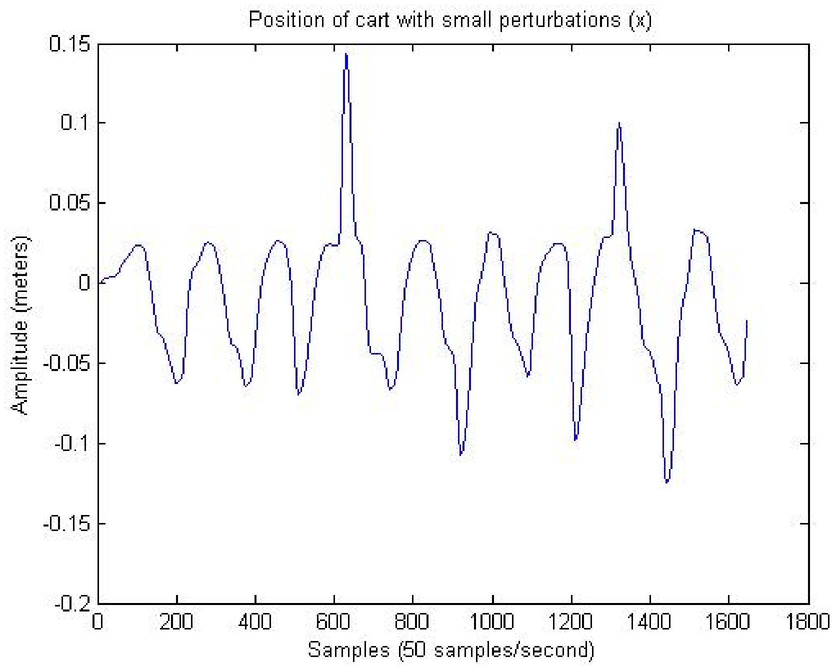


Figure 6: Cart position with small perturbations.

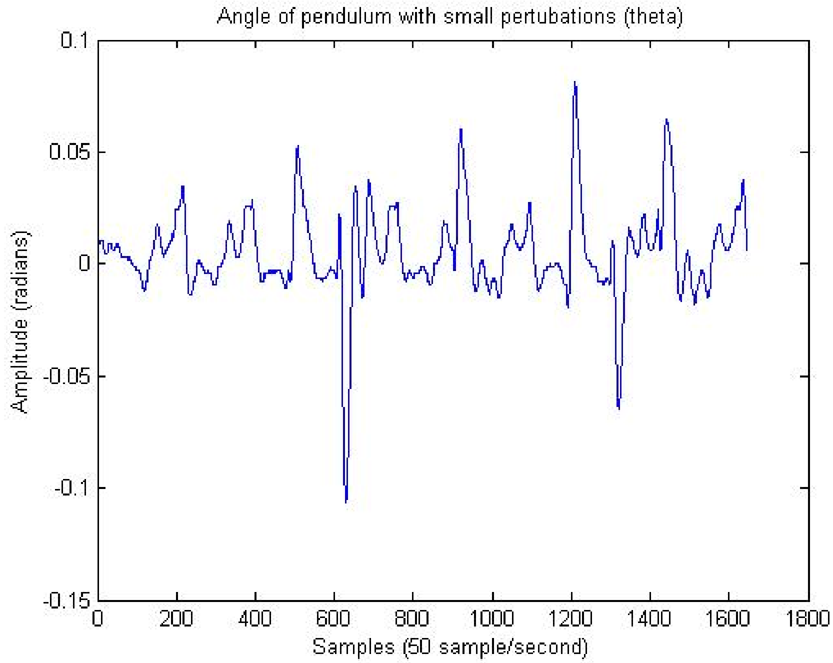


Figure 7: Pendulum angle with small perturbations.

3.2.3 Introducing a sine wave reference signal

We introduced a sine wave reference signal by changing the constant block in Figure 1 to be of the form $[Msin\omega t, 0, 0, 0]^T$, with $M = 0.1$m and ω being varied at $\omega = 1, 2, 5 rad/s$. We then ran the simulation and analyzed the results.
The plots for $M = 0.1$m and $\omega = 1$ rad/s are shown in Figure 8, 9, 10 below:

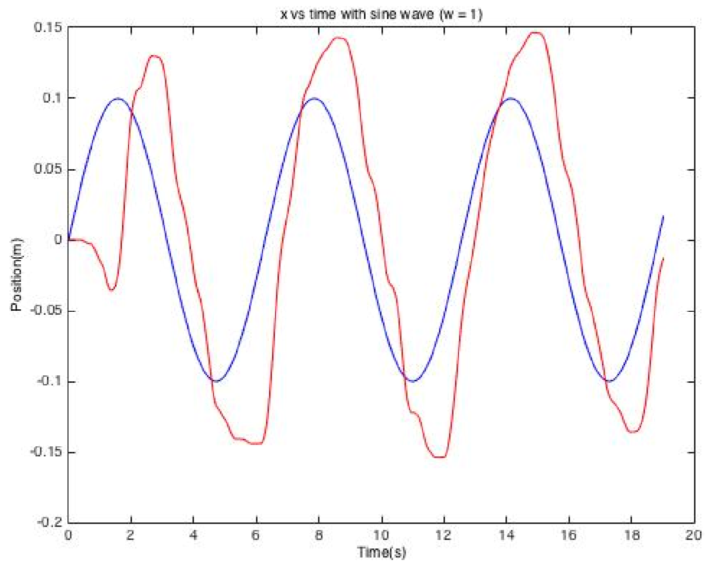


Figure 8: Position vs time graph

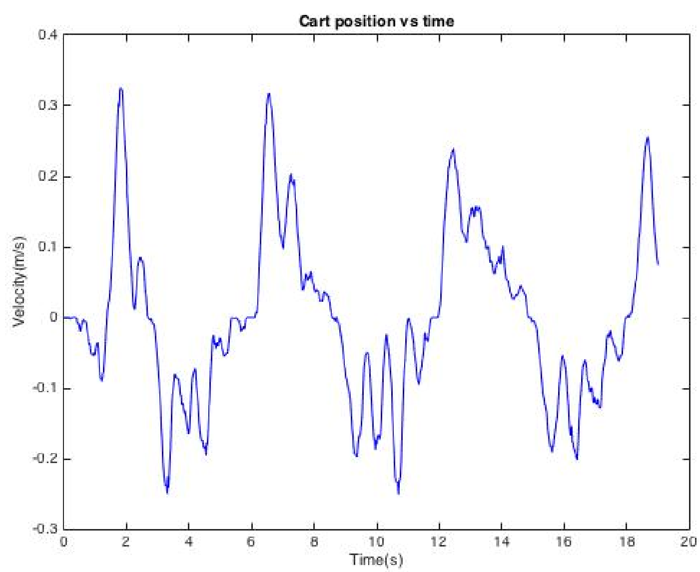


Figure 9: Velocity vs time graph

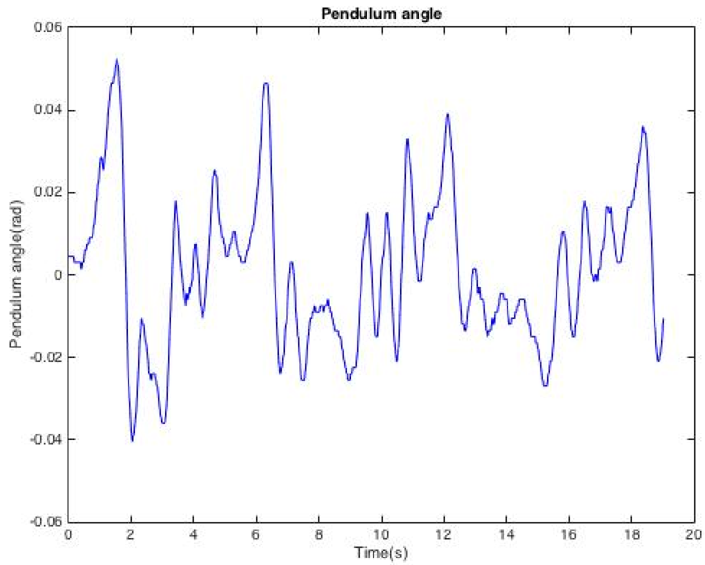


Figure 10: Pendulum angle vs time graph

The plots for $M = 0.1$m and $\omega = 2$rad/s are shown in Figure 11, 12, 13 below:

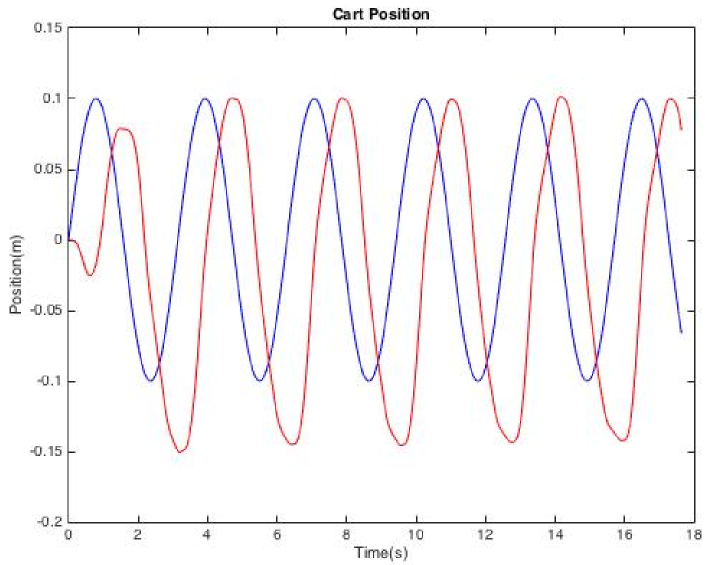


Figure 11: Position vs time graph

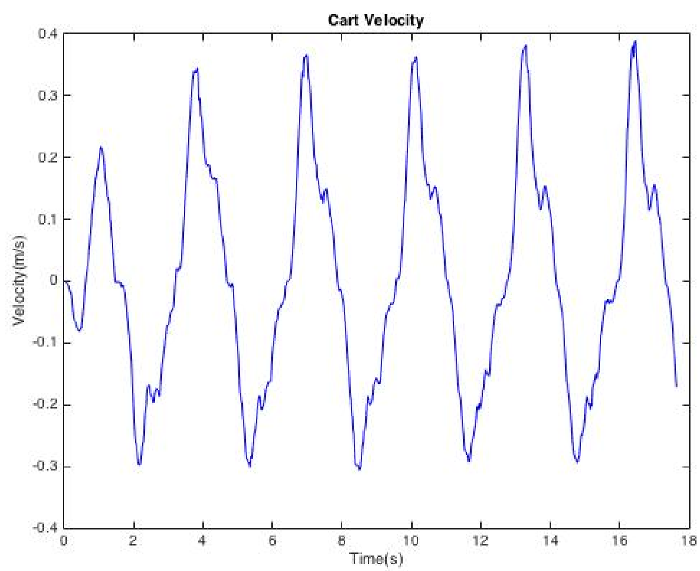


Figure 12: Velocity vs time graph

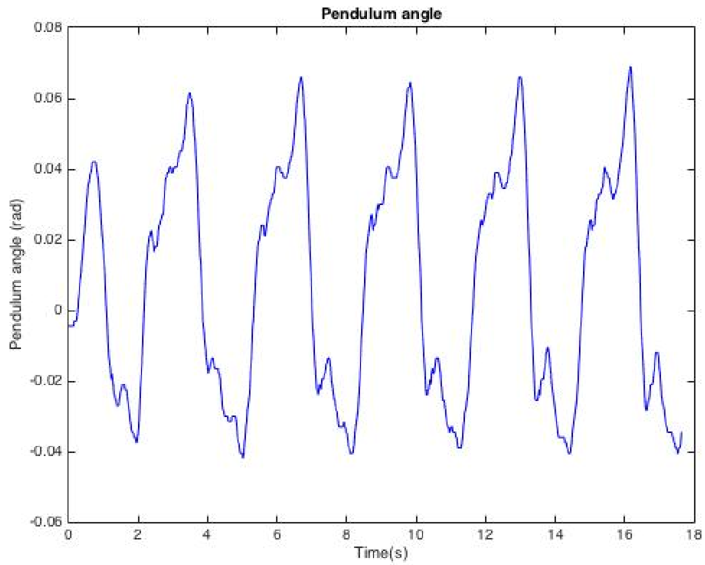


Figure 13: Pendulum angle vs time graph

The plots for $M = 0.1$m and $\omega = 5$rad/s are shown in Figure 14, 15, 16 below:

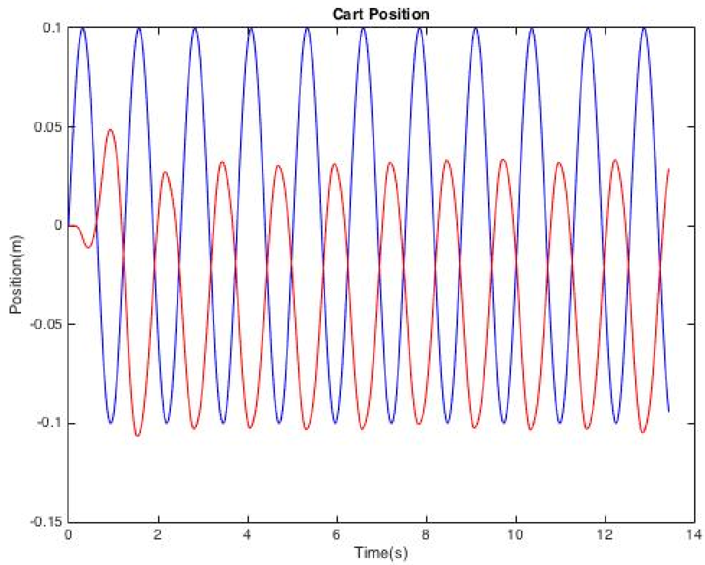


Figure 14: Position vs time graph

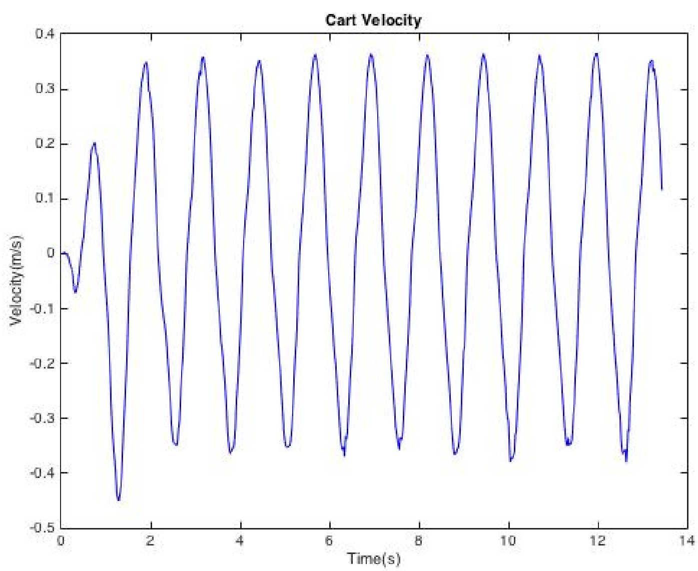


Figure 15: Velocity vs time graph

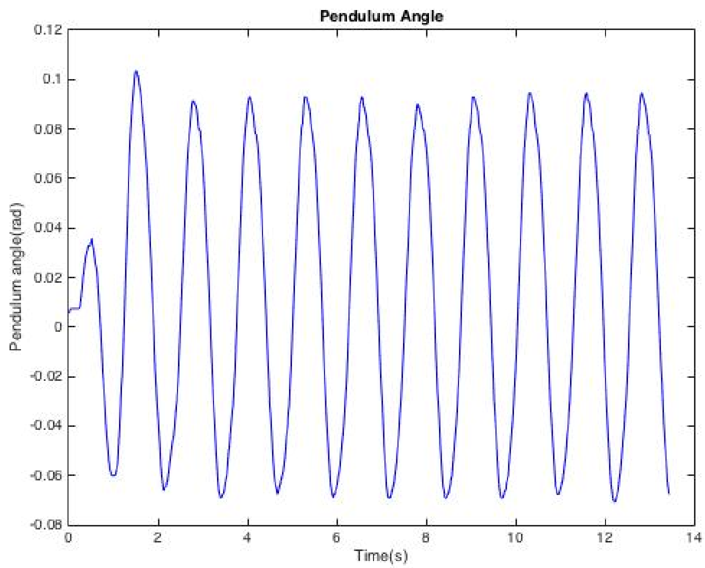


Figure 16: Pendulum angle vs time graph

3.2.4 Calculate the gain and phase for each of the frequencies in your frequency response (ignoring the offset from the hardware response). Locate these frequencies on the Bode plot from Part 3.3.4 of your Pre-Lab and compare the results. Do your values match for each frequency? If not, explain possible causes for the difference.

Frequency (rad/s)	Actual Gain	Actual Phase(rad)	Bode Gain	Bode Phase(rad)
1	1.400	1.13	0.980	0.820
2	1.125	1.76	0.806	1.61
5	0.675	2.14	0.445	2.74

Table 1: Table of actual gain and phase and bode gain and phase

The values do not match. The modelled system does not factor in the non linearities, which are definitely present in the actual system. Also, the model ignores friction and internal motor resistance.

3.2.5 Slightly change the position of the desired closed-loop poles. Try a couple of different values and run the resulting controllers on the hardware. Again include plots of cart position, cart velocity and pendulum angle in your report. Discuss how the changes in the position of the poles affect the behavior of the system. You do not need to repeat part 3a.

The position of the closed loop poles were changed as follows:

Pole Values	K Values
-1.5 $\pm j8.0, -1.6 \pm j1.3$	[-8.29 -11.10 -34.20 -4.70]
-1.5 $\pm j8.0, -2.0 \pm j2.0$	[-15.60 -12.99 -39.20 -5.77]
-1.5 $\pm j12.0, -1.6 \pm j1.3$	[-18.30 -18.63 -61.71 -8.02]

Table 2: Table of actual gain and phase and bode gain and phase

The resulting plots of the first pole change are shown below:

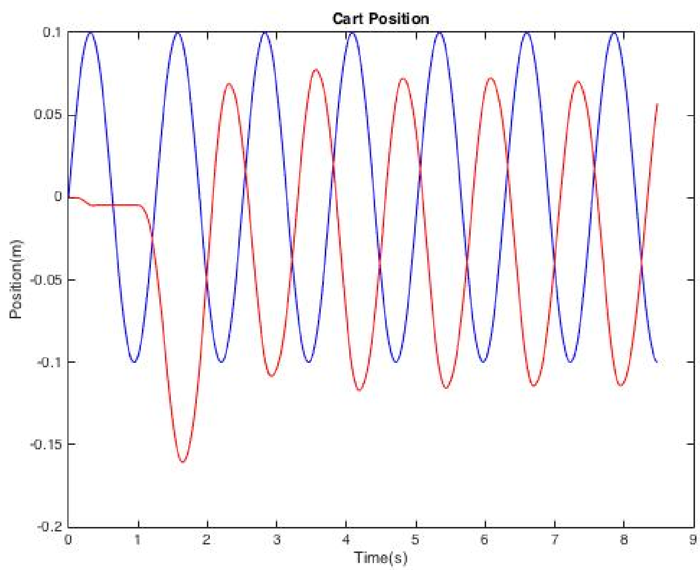


Figure 17: Position vs time graph

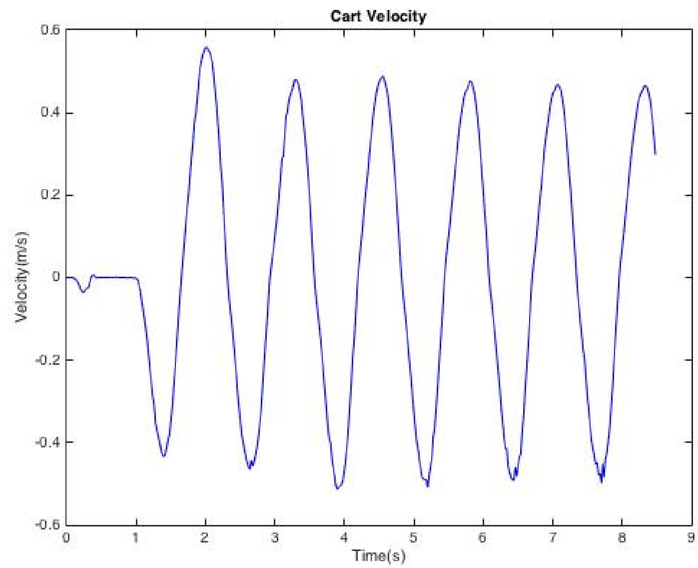


Figure 18: Velocity vs time graph

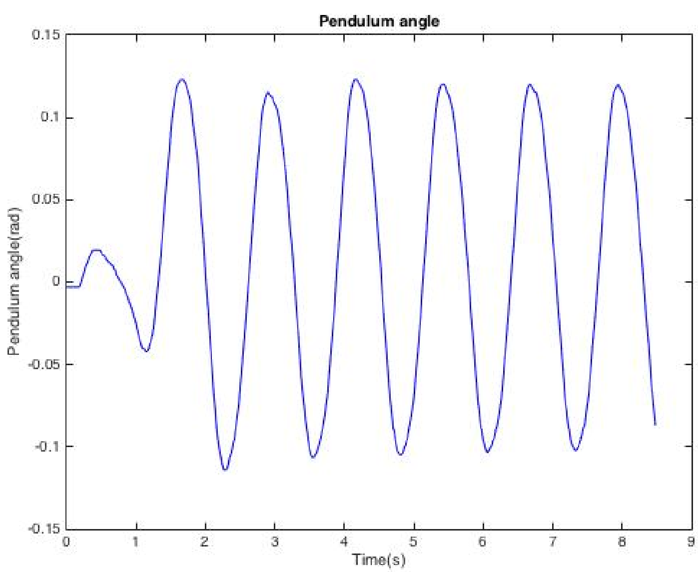


Figure 19: Pendulum angle vs time graph

The resulting plots of the second pole change are shown below:

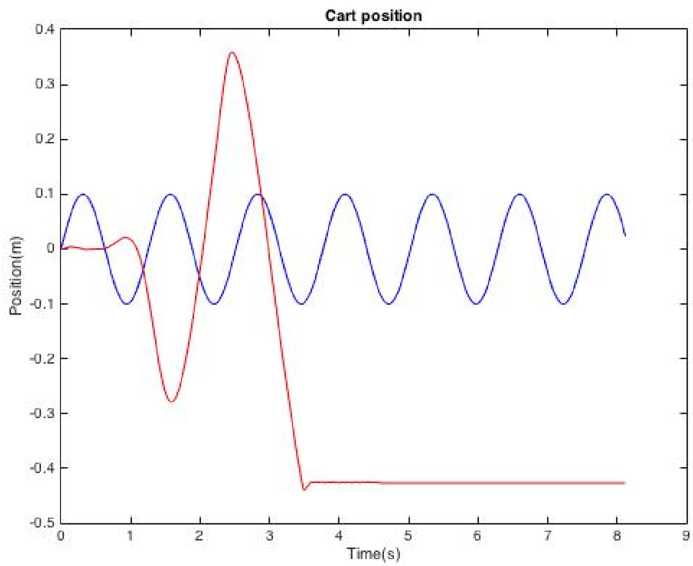


Figure 20: Position vs time graph

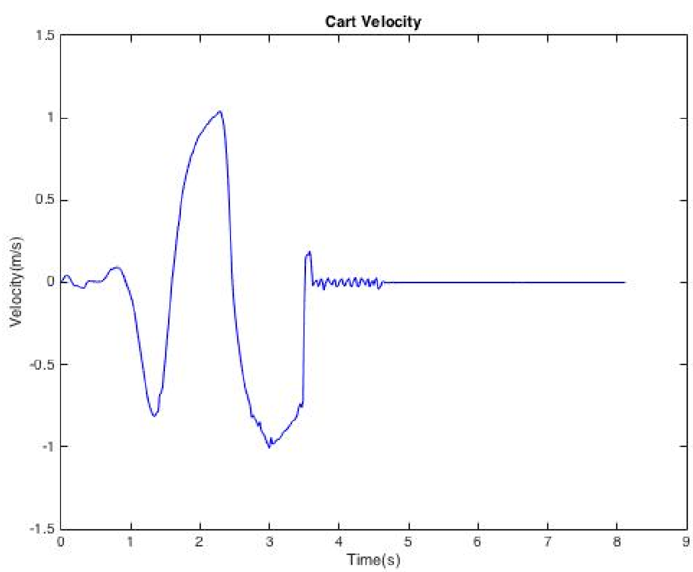


Figure 21: Velocity vs time graph

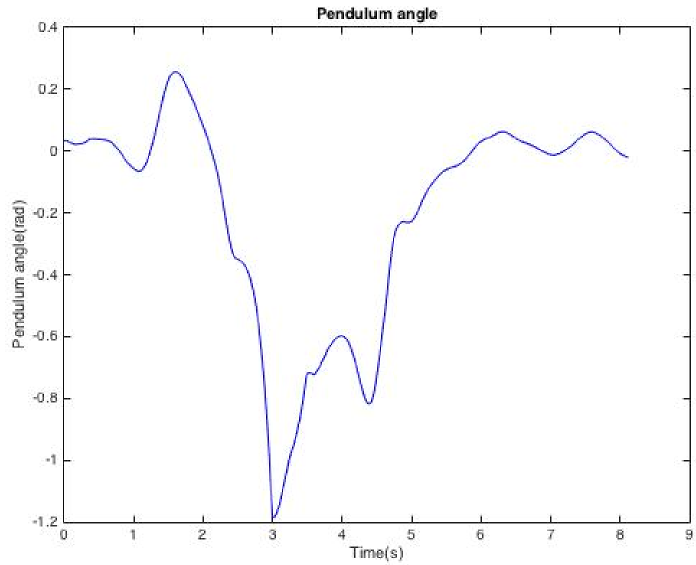


Figure 22: Pendulum angle vs time graph

The resulting plots of the third pole change are shown below:

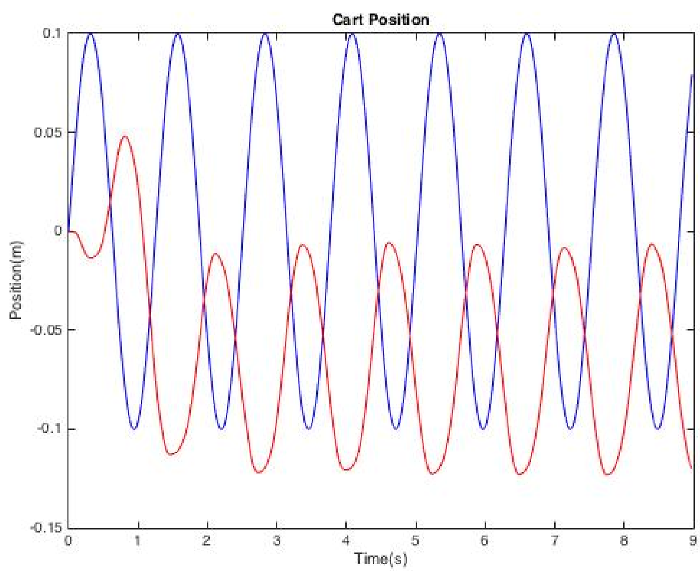


Figure 23: Position vs time graph

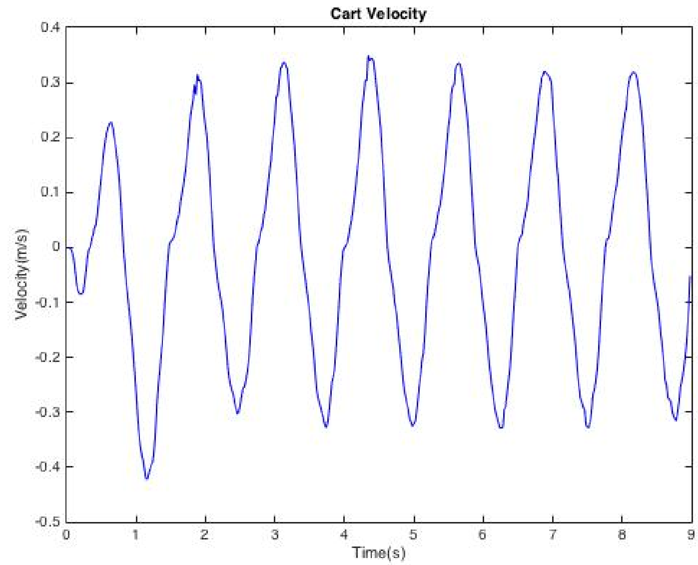


Figure 24: Velocity vs time graph

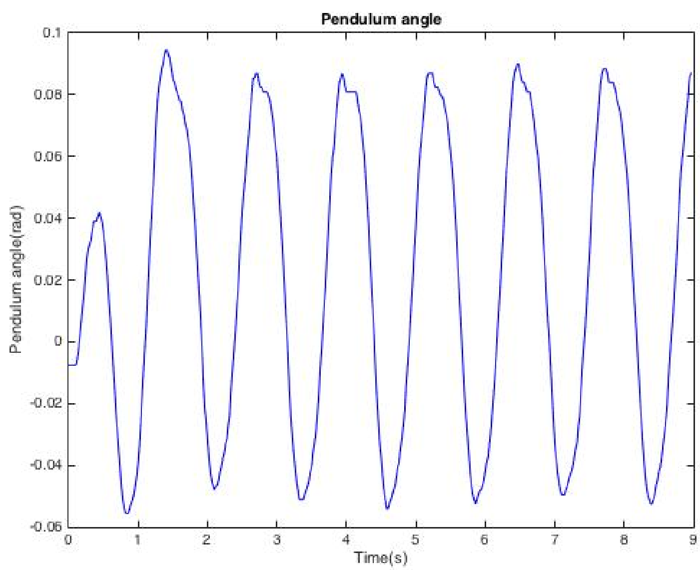


Figure 25: Pendulum angle vs time graph

The system will be stable as long as no right hand poles are present. As we changed the poles to be closer to the imaginary axis, the system turned out to be less stable, as seen from the larger fluctuations in position, velocity and pendulum angle. For the second pole pairing that we tried, the fluctuations got too large and the system failed (the pendulum crashed).

3.3 Plot the cart velocity and the pendulums angular velocity , which are obtained by numerically differentiating the signals x and θ, respectively. Comment on the quality of the obtained signals.

The plots were derived from numerically differentiating the signals.

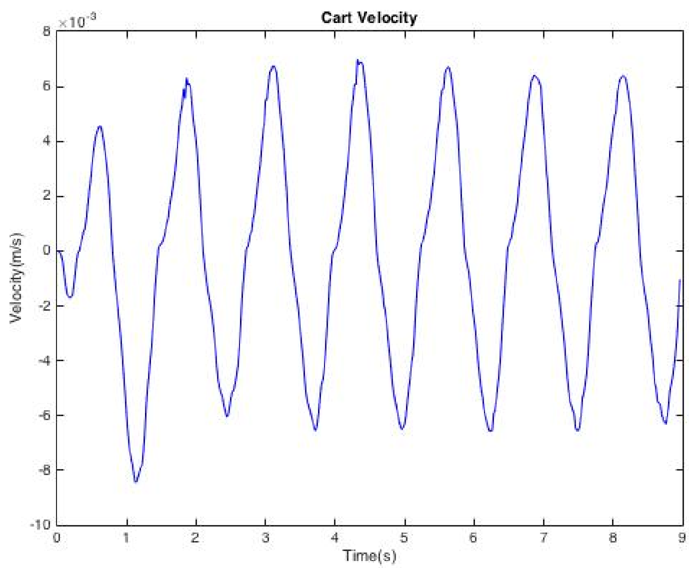


Figure 26: Derivative graph of cart position

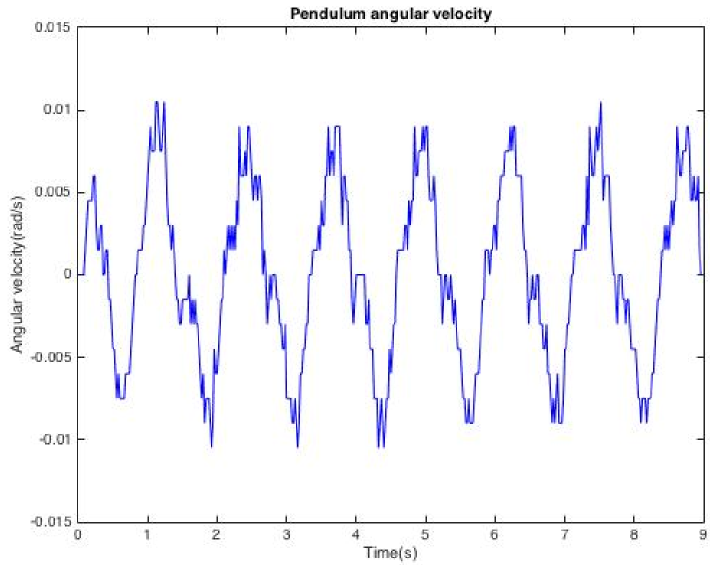


Figure 27: Derivative graph of pendulum angle

A lot of noise was observed from the derivative plots. This is possibly because the read encoder samples the outputs x and theta at discrete intervals. The derivatives for the time period inbetween these discrete intervals would fluctuate between infinity and zero, hence resulting in the noise we see from the plots.

Lab 6b
Luenberger Observer Design for Inverted Pendulum
ELENG128

Clarkson CHANG (SID: 3032524418)
Loren JIANG (SID: 25403779)

April 17, 2017

Date Performed: April 10, 2017
Lab Instructor: Eric CHOI

1 Objective

The objective of the lab was to design a Luenberger observer that enables state estimation of the inverted pendulum, given the position of the cart and pendulum. The estimates will be used as a form of feedback control.

2 Pre-lab assignments

Pre-lab 6b

Clarkson Chung
3032524418
M 2-5PM

6

3.1 From last week's lab,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6.8312 & -1.4967 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 15.5161 & 25.6815 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1.5244 \\ 0 \\ -3.4625 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Using matlab "ctrb" on the system, the controllability matrix was found to be

$$C_{control} = (1 \times 10^3) \begin{bmatrix} 0 & 0.0015 & -0.0104 & 0.0763 \\ 0.0015 & -0.0104 & 0.0763 & -0.5568 \\ 0 & -0.0035 & 0.0237 & -0.2505 \\ -0.0035 & 0.0237 & -0.2505 & 1.7916 \end{bmatrix}$$

Calling on matlab "rank" on the $C_{control}$ matrix,

$$\text{rank}(C_{control}) = 4$$

Hence we can say that the system is controllable. //

3.2

1. LC is a 4x4 matrix.

Since C is ~~also~~ a 2x4 matrix,

L must be a 4x2 matrix ~~as well~~

$\therefore$ L is 4x2 matrix. //

2. $A - LC \Rightarrow A^T - L^T C^T$

$$L^T = \text{place}(A^T, C^T, (-10+j15, -10-j15, -12+j17, -12-j17))$$

$$L = \begin{bmatrix} 16.1384 & -2.3771 \\ 254.6804 & -5.4468 \\ 15.7454 & 21.0304 \\ 180.9431 & 378.2680 \end{bmatrix}$$ //

3.3

1. $K = [-12.9796 \quad -14.7230 \quad -47.8456 \quad -6.5363]$

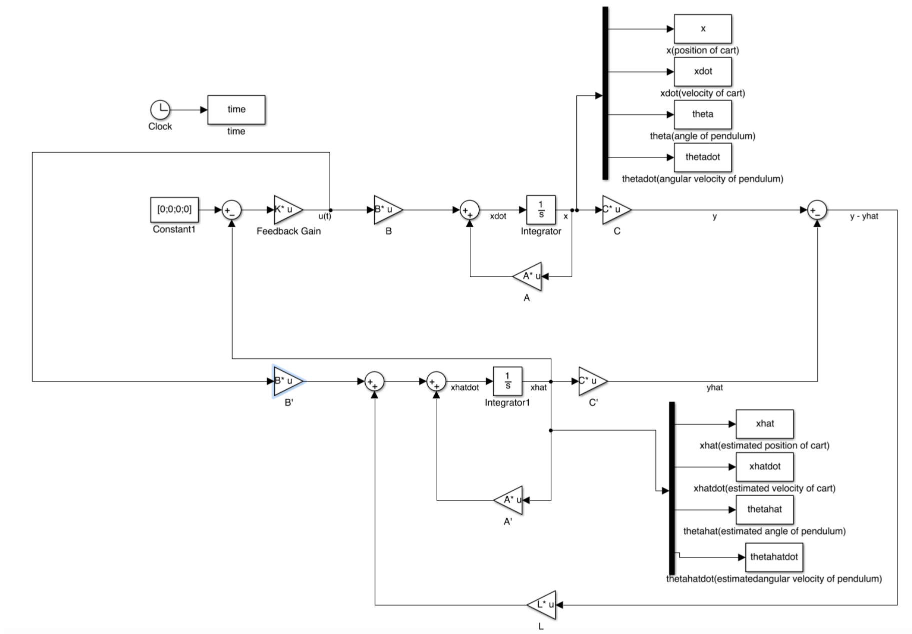


Figure 1: Block diagram for observer implementation

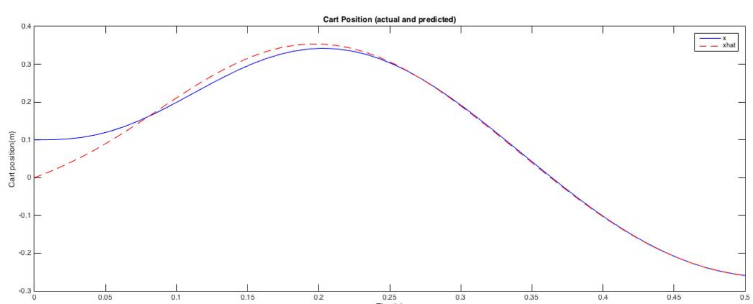


Figure 2

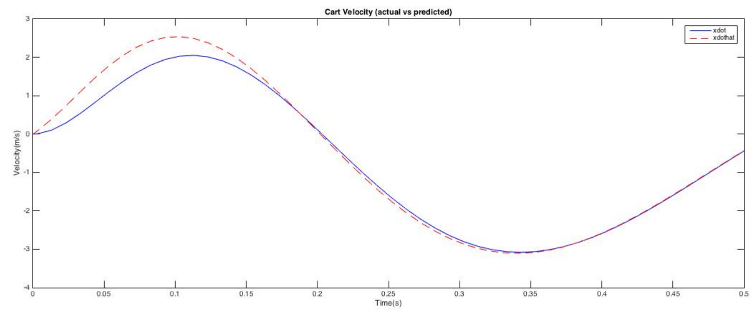


Figure 3

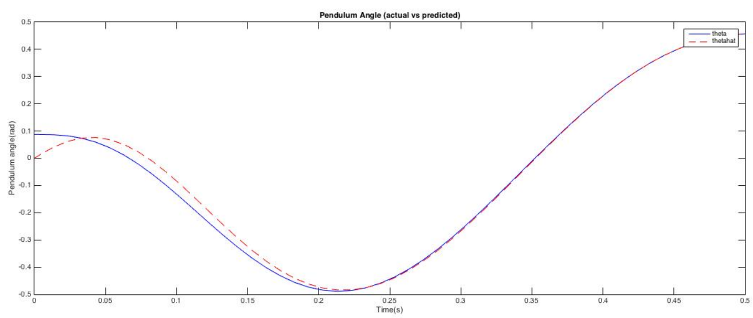


Figure 4

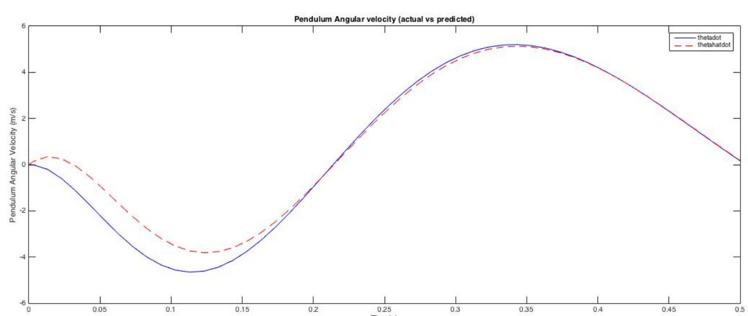


Figure 5

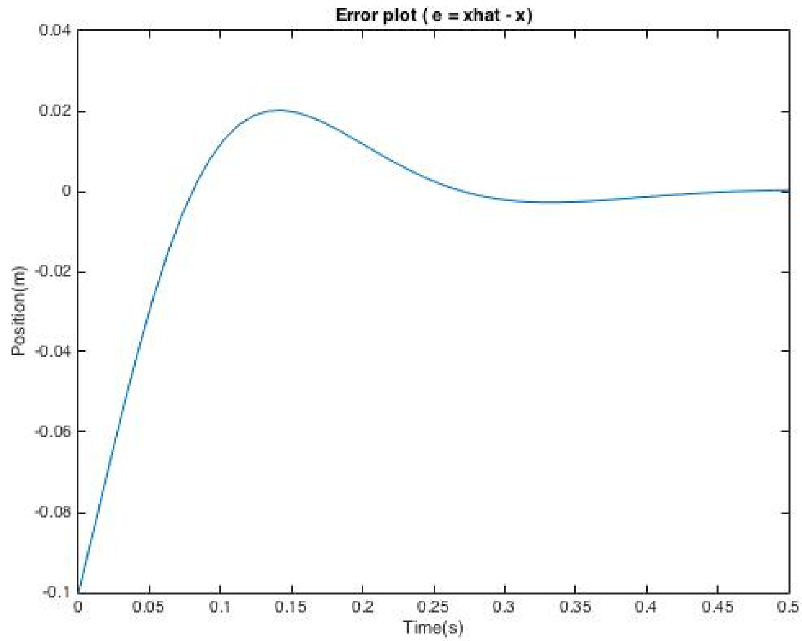


Figure 6

When t = 0s, the error is -0.1m. This is because we set the initial conditions of the integrator block to be of a 10cm disturbance. Over a time span of about 0.4s, the observer was able to correct the error, hence the convergence to zero after a certain time.

3 Lab Results

The system was implemented in Simulink with the Quanser hardware as shown in the figure below.

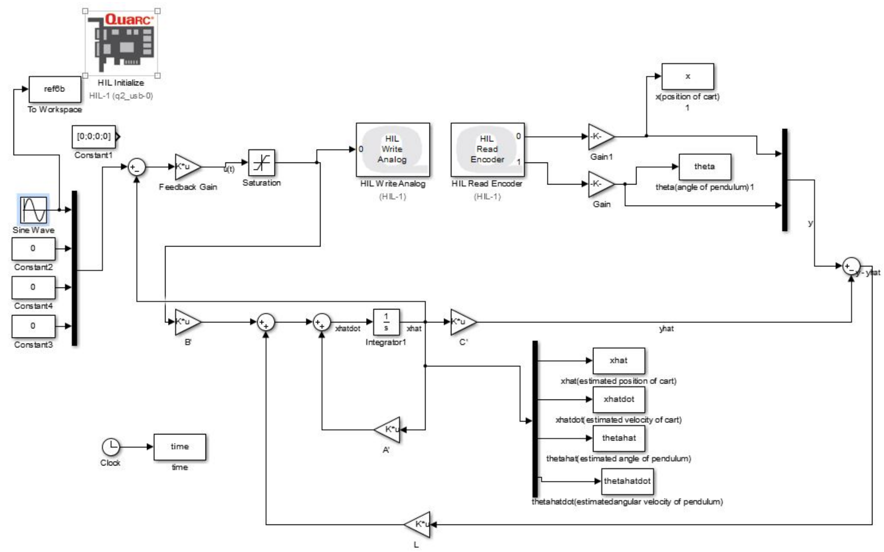


Figure 7: Simulink model for hardware system.

3.1 Hardware Response with Luenberger Observer

For a zero reference signal, observe and record the output y of the observer and the actual measurement y when manually applying small perturbations. That is, plot both the estimated and actual signals on the same graph for the position of the cart and the pendulum. The difference between these two signals indicates how well the observer estimates the state of the system.
Setting the reference input to zero, the graphs in Figure 8 and 9 were obtained for cart position and pendulum angle respectively, when small perturbations were applied. The red lines are for the state estimates from the Observer, while the blue lines are from the actual system itself.

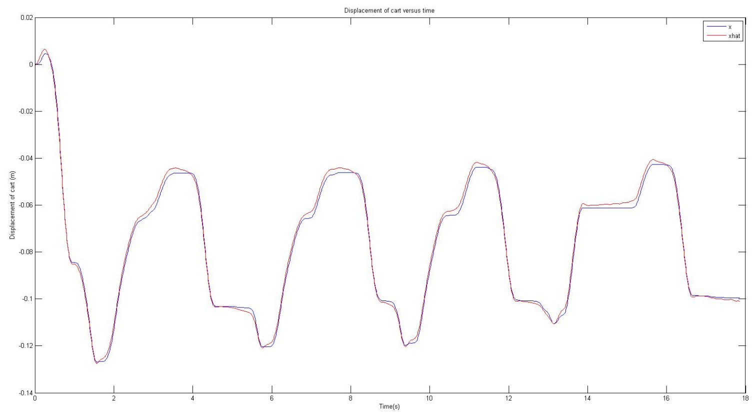


Figure 8: Cart position against time, zero reference input with small perturbations

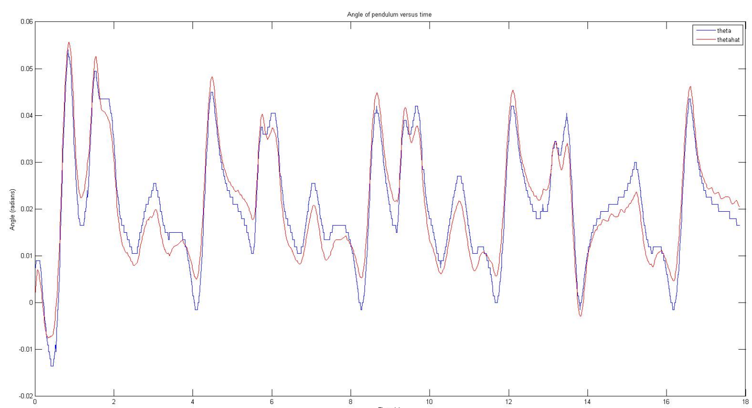


Figure 9: Pendulum angle against time, zero reference input with small perturbations

3.2 Comparing with controller from Lab 6a

For each of the following reference signals, qualitatively describe any noticeable differences in performance, and plot the cart position and the angular position of the rod for both controllers on top of each other and compare their tracking abilities.

1. zero reference

2. zero reference with small perturbations (try to be consistent in how you apply

the perturbations)
3. sinusoidal reference position with amplitude 5 cm and frequency 1 rad/s
The reference velocity, angle, and angular velocity should be set to 0.
In this section, we compare the performance of the hardware system from last week's lab with this week's implementation.

3.2.1 Performance comparison for zero reference

We ran the experiment with zero reference on both systems and obtained the graphs below. From Figure 10, both the controller from Lab 6a and 6b performed about the same. However, there was an audible difference in performance. The controller from 6a made grinding noise, and this is partially evident in Figure 11 where the angle of the pendulum spikes up and down very quickly (the darker red sections of the plot).

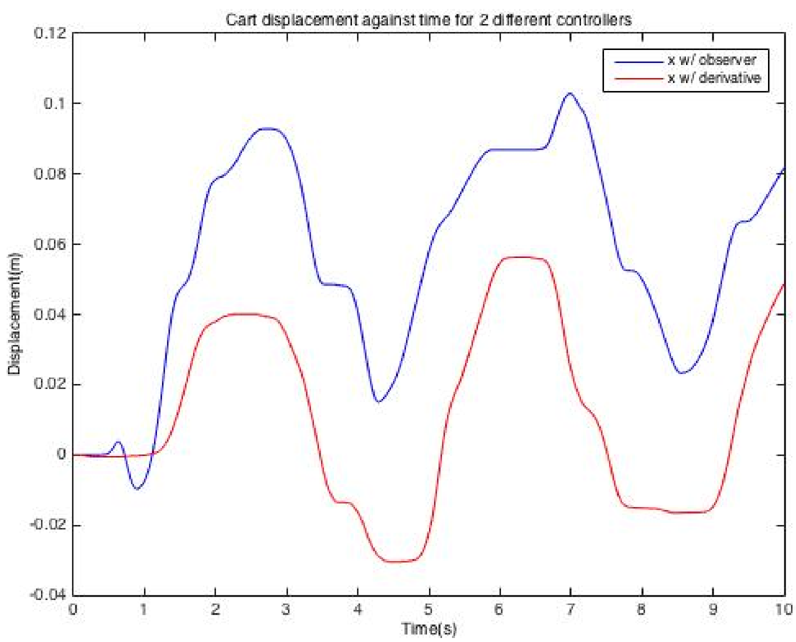


Figure 10: Comparison of cart position for different systems, zero reference input

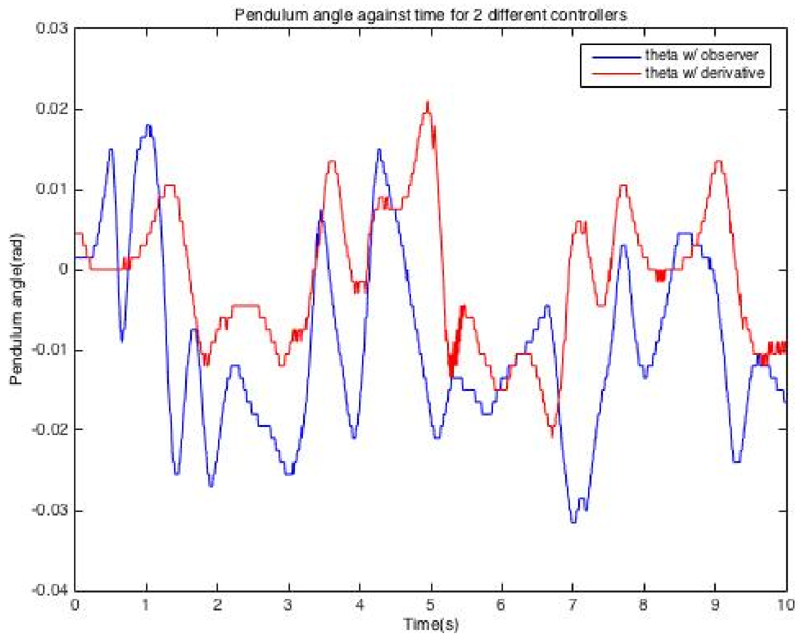


Figure 11: Comparison of pendulum angle for different systems, zero reference input

3.2.2 Performance comparison for zero reference with small perturbations

We ran the experiment with zero reference and small perturbations on both systems and obtained the graphs below. In both systems, small perturbations were applied every two seconds. From Figure 12, the derivative controller seemed to slightly outperform the observer controller, but just barely. In Figure 13, this is more apparent. The observer controller seems to have a slower response time meaning the angle sweeps would be slightly bigger and, subsequently, the displacement of the cart would also be bigger to correct for this.

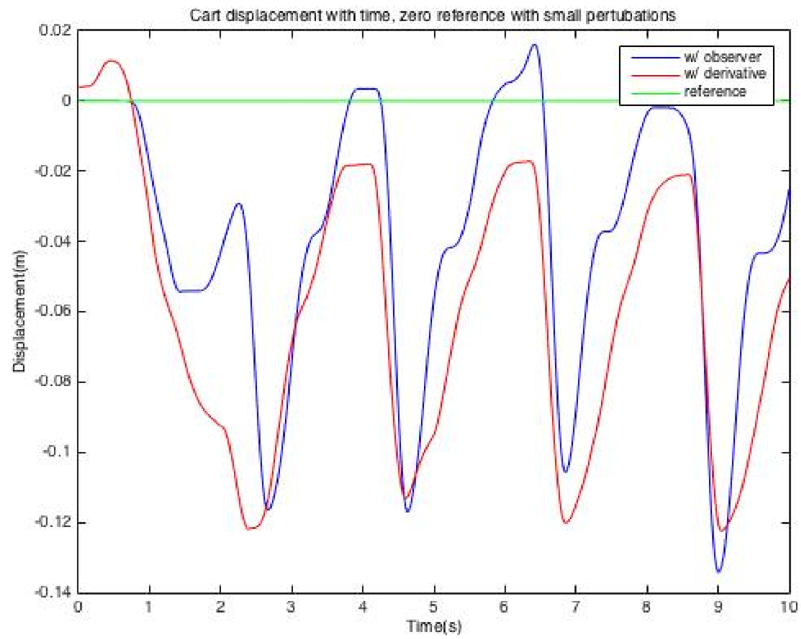


Figure 12: Comparison of cart position for different systems, zero reference input with small perturbations

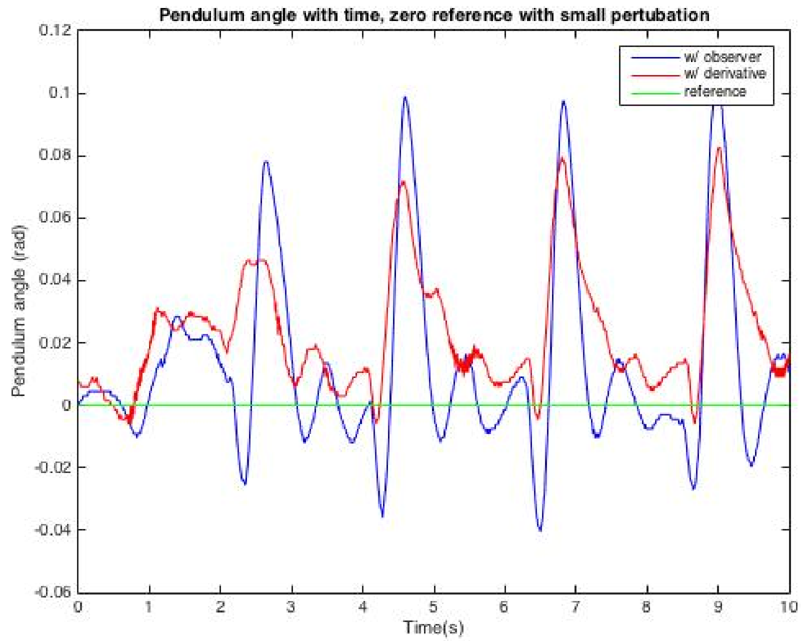


Figure 13: Comparison of pendulum angle for different systems, zero reference input with small perturbations

3.2.3 Performance comparison sinusoidal reference

We ran the experiment with sinusoidal reference position of amplitude 5cm and frequency of 1 rad/s on both systems and obtained the graphs below. From Figure 14, we can see that the observer system tracks a sinusoidal reference better since there is less resulting gain. Figure 15 does not really show any distinguishable features between the two controllers; however, there are still the sections of where the angle of the pendulum is spiking up and down rapidly caused by the grinding noise heard previously.

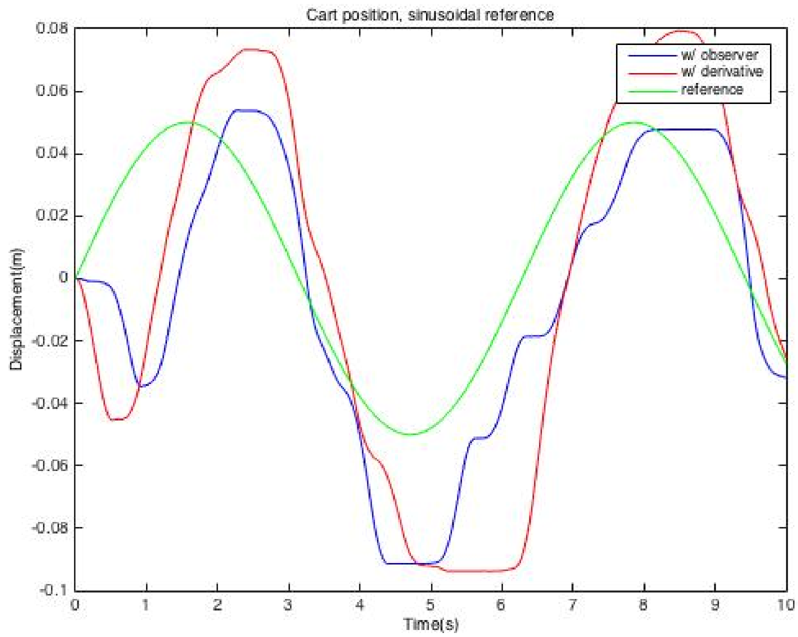


Figure 14: Comparison of cart position for different systems, sinusoidal reference input

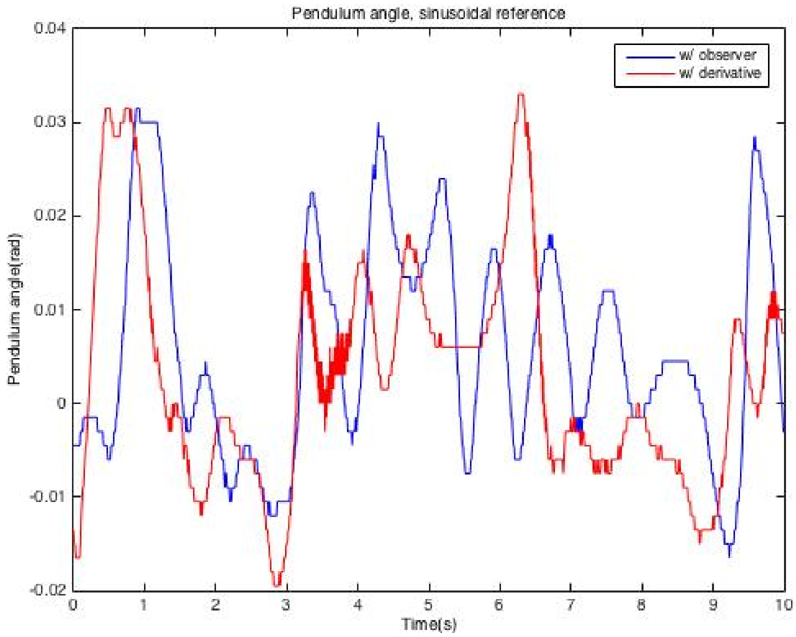


Figure 15: Comparison of pendulum angle for different systems, sinusoidal reference input

3.3 Comparing the estimates of the cart and pendulum velocities

It is suspected that the system with the derivative controller will be more susceptible to noise. That is, the effects of noise will be amplified because we are taking the derivative of potential noise that may be greatly changing. The observer system will be therefore be more robust to noise because the observer estimates the states and their derivatives, not the actual measurements themselves. However, from Figure 16 and Figure 17 there isn't an appreciable difference in noise minimization. The amplitudes of both plots are within a similar range of relatively small values.

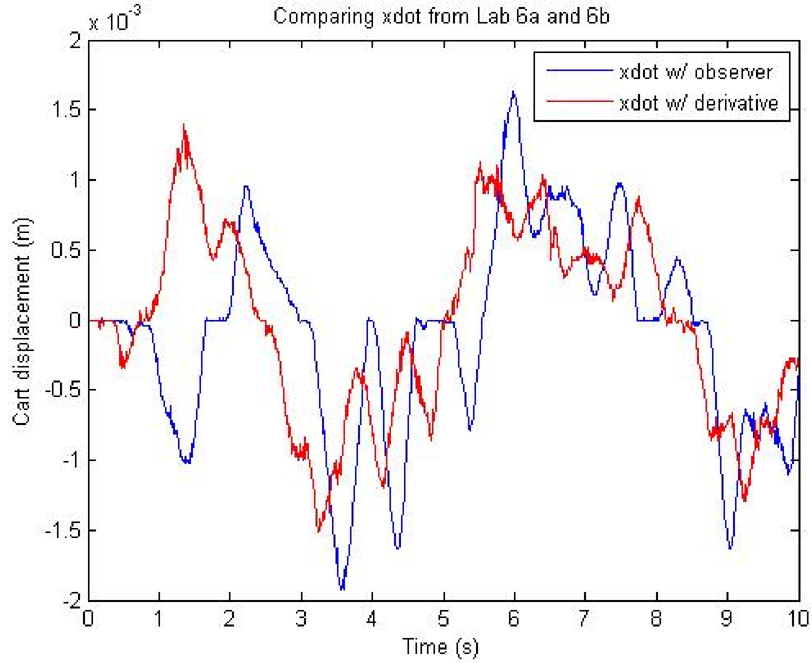


Figure 16: Comparison of the derivative of cart position for different systems, sinusoidal reference input

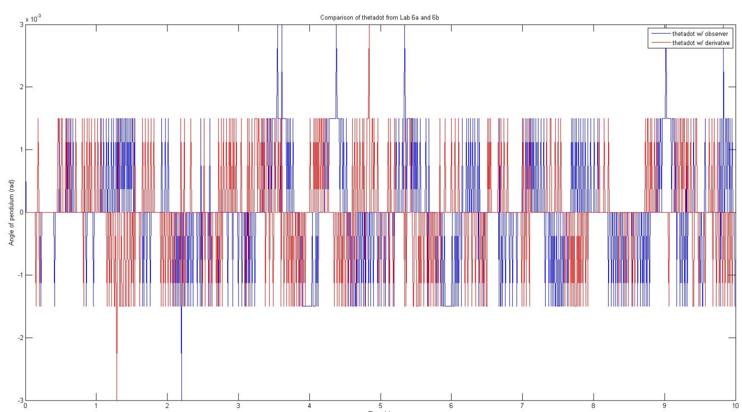


Figure 17: Comparison of the derivative pendulum angle for different systems, sinusoidal reference input

3.4 Comparison of schemes

The system with the observer should perform better due to the minimization of the effect of noise present in the measurements of and , resulting in smoother performance. However, the comparison of plots do not conclusively show that one system is significantly better than the other. Qualitatively, the observer system eliminated the grinding noise, but seemed a tiny bit slower in response time. Overall, we would choose to use the observer system since the noise might damage the equipment in the long term due to the abrupt and frequent changes in motor direction leading to wear and tear.

Lab 6c
LQR Controller Design for Inverted Pendulum
ELENG128

Clarkson CHANG (SID: 3032524418)

Loren JIANG (SID: 25403779)

April 17, 2017

Date Performed:	February 13, 2017
Lab Instructor:	Eric CHOI

1 Objectives

The objective of this lab was to design a Linear Quadratic Regulator, and to understand the effects of the penalty matrices P and Q on the performance of the closed loop system.

2 Pre-lab assignments

From the previous labs, the state space representation of the system is given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6.8123 & -1.4973 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 15.4731 & 25.6828 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 1.5226 \\ 0 \\ -3.4583 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

What will the dimensions of Q and R be?

Q will be a 4x4 matrix, while R is a 1x1 matrix.

Individually vary the weights from their nominal values and study the influence of the weights on how the system outputs and control effort varies with time. The weights are relative, so you may assume q1 = 1 in all cases, and vary only the other two. Choose your weights such that you can clearly see the effect in the system behavior (you can restrict your weights to the range 0 100). Consider the following five cases: (nominal, $q3 << 1$***,*** $q3 >> 1$***,*** $r << 1$***, and*** $r >> 1$***).***

The MATLAB script found in Figure 1 was used to get the K values for different values of q3 and r. The results obtained were recorded in the table below. The table also contains information regarding maximum deviations of x and θ, as well as the absolute maximum value of the input voltage u. The plots of output y and control action u can be found in the subsequent figures.

```
%Defining state space matrices
a22=-6.8123; a23=-1.4973; a42=15.4731;
    a43=25.6828;b2=1.5226;b4=-3.4583;
A = [0 1 0 0; 0 a22 a23 0; 0 0 0 1; 0 a42 a43 0]; B = [0;
    b2;0;b4]; C=[1 0 0 0; 0 0 1 0]; D =[0;0];

%Set q1,q3,r, changed for each iteration
Q1 = 1/(0.3*0.3); Q3=1/(0.05*0.05); R=1/(6*6);
sys= ss(A,B,C,D);

%Use the same L matrix each time
L = [16.1384 -2.3771;254.6804 -5.4468; 15.7454 21.0304;
    180.9431 378.2680;]
Q = [Q1 0 0 0; 0 0 0 0; 0 0 Q3 0; 0 0 0 0];

%Get the K matrix and eigenvalues
[K, S, e] = lqr(sys, Q, R);
AK = A-B*K;
eig(AK);
```

Figure 1: MATLAB Script for determining K, closed loop poles.

Case	q1,q3,r values	K	Closed Loop Poles
Nominal	1, 1, 1	[-20.0000 -29.3892 -154.7433 -20.3651]	$-15.2976 \pm 13.6130i$, $-0.9488 \pm 0.8473i$
$q3 << 1$	1, 0.05, 1	[-20.0000 -21.1577 -70.7772 -13.6477]	$-8.7946 \pm 5.5717i$, $-2.1030 \pm 1.3556i$
$q3 >> 1$	1, 50, 1	[-20.0000 -64.3096 -880.3236 -48.8805]	$-38.6262 \pm 37.9882i$, $-0.3427 \pm 0.3372i$
$r << 1$	1, 1, 0.05	[-89.4427 -110.1821 -619.0558 -64.9555]	$-30.9063 \pm 30.1071i$, $-0.9361 \pm 0.8682i$
$r >> 1$	1, 1, 50	[-2.8284 -10.8285 -42.7271 -8.0768]	$-7.8436 \pm 3.8312i$, -1.9106,-0.6591

Case	q1,q3,r values	Max x	Min x	Max θ	Min θ	Max u
Nominal	1, 1, 1	0.3455	-0.0104	0.0500	-0.0501	13.7372
$q3 << 1$	1, 0.05, 1	0.3672	-0.0034	0.0500	-0.1081	9.5389
$q3 >> 1$	1, 50, 1	0.3274	-0.0134	0.0500	-0.0106	50.0162
$r << 1$	1, 1, 0.05	0.3462	-0.0118	0.0500	-0.0544	57.7856
$r >> 1$	1, 1, 50	0.3481	0.001	0.0500	-0.0324	2.9849

Figure 2: Table of results for different combinations of q1, q3 and r.

2.1 Plots of output y and control action u

For the nominal case, the plots below were obtained.

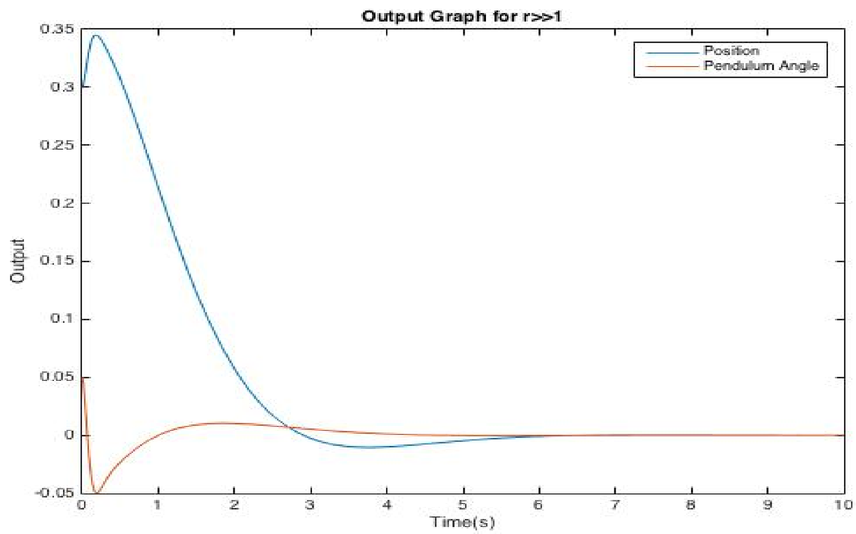


Figure 3: Variation of cart position and pendulum angle with time

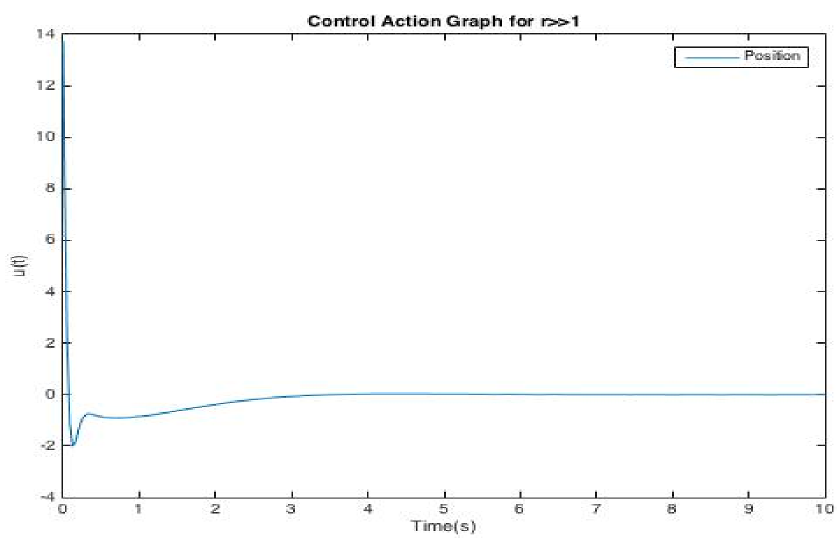


Figure 4: Variation of input motor voltage with time

For the $q3 << 1$ case, the plots below were obtained.

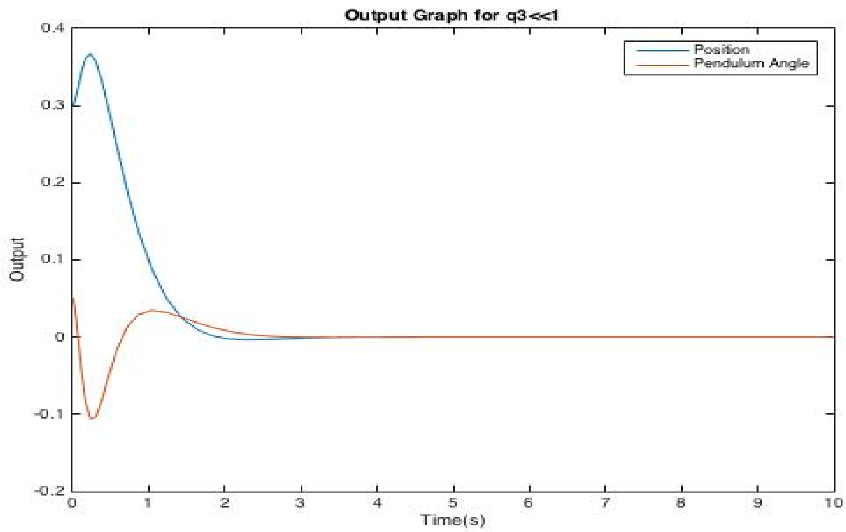


Figure 5: Variation of cart position and pendulum angle with time, $q3 << 1$

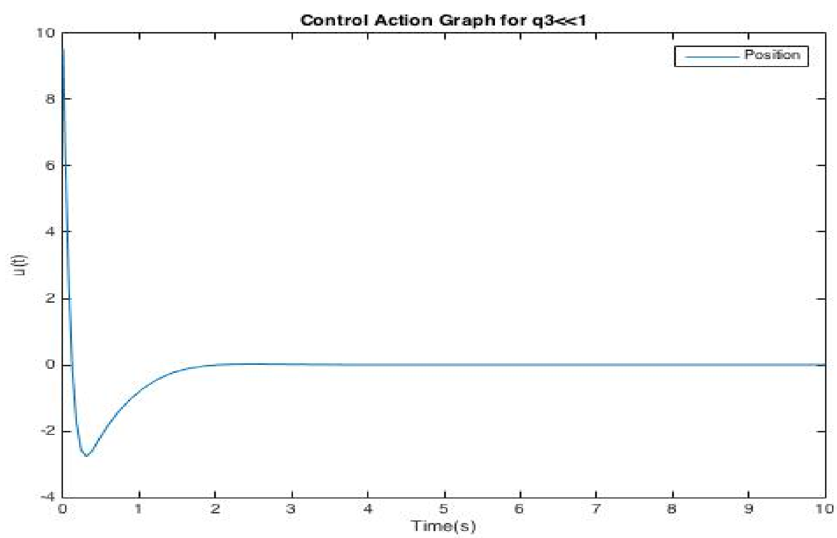


Figure 6: Variation of input motor voltage with time, $q3 << 1$

For the $q3 >> 1$ case, the plots below were obtained.

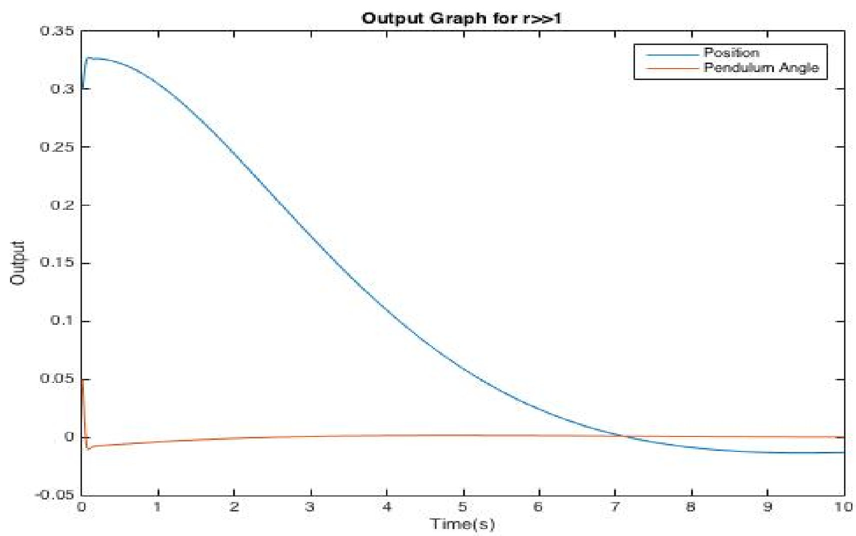


Figure 7: Variation of cart position and pendulum angle with time, $q3 >> 1$

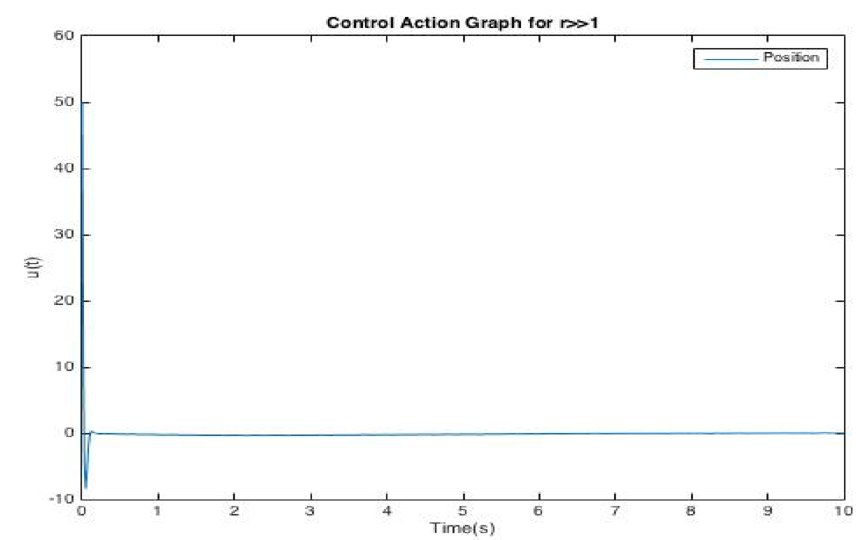


Figure 8: Variation of input motor voltage with time, $q3 >> 1$

For the $r << 1$ case, the plots below were obtained.

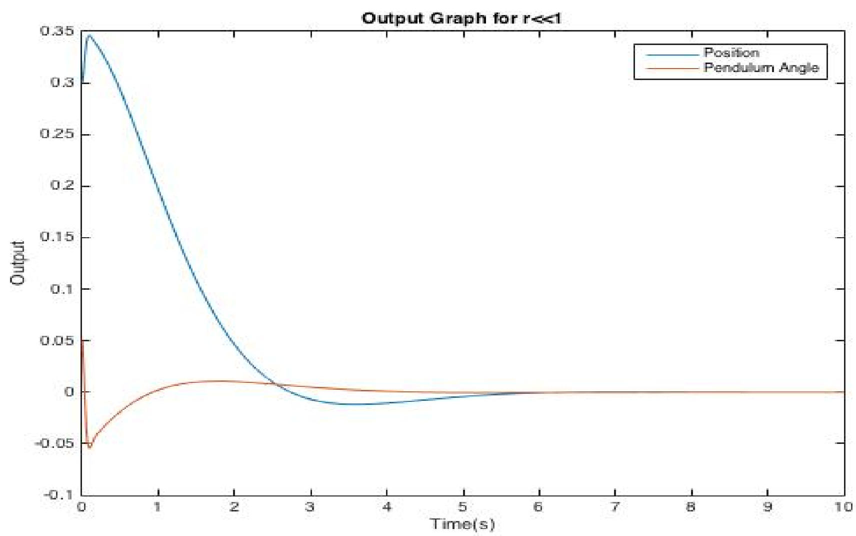


Figure 9: Variation of cart position and pendulum angle with time, $r << 1$

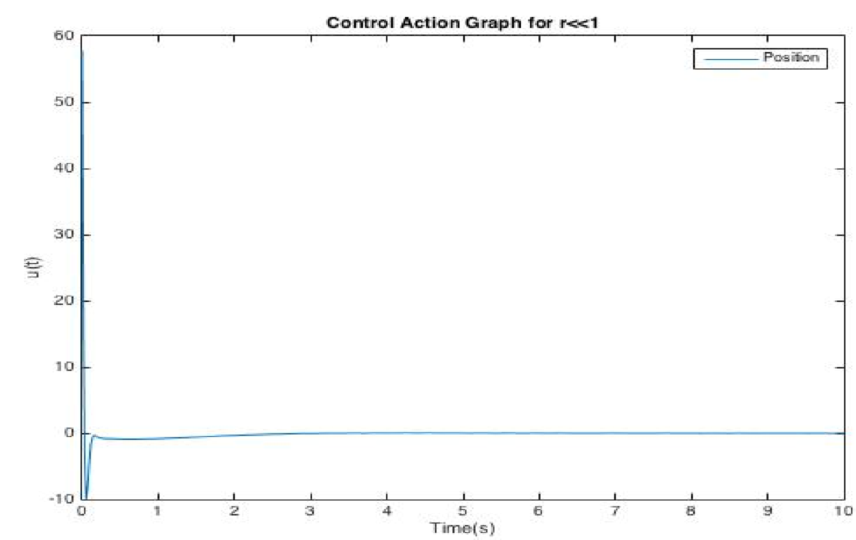


Figure 10: Variation of input motor voltage with time, $r << 1$

For the $r >> 1$ case, the plots below were obtained.

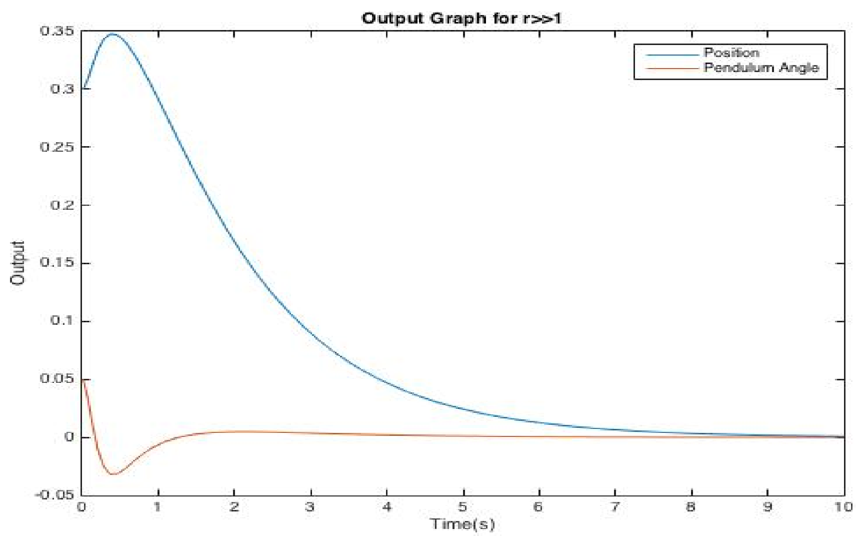


Figure 11: Variation of cart position and pendulum angle with time, $r >> 1$

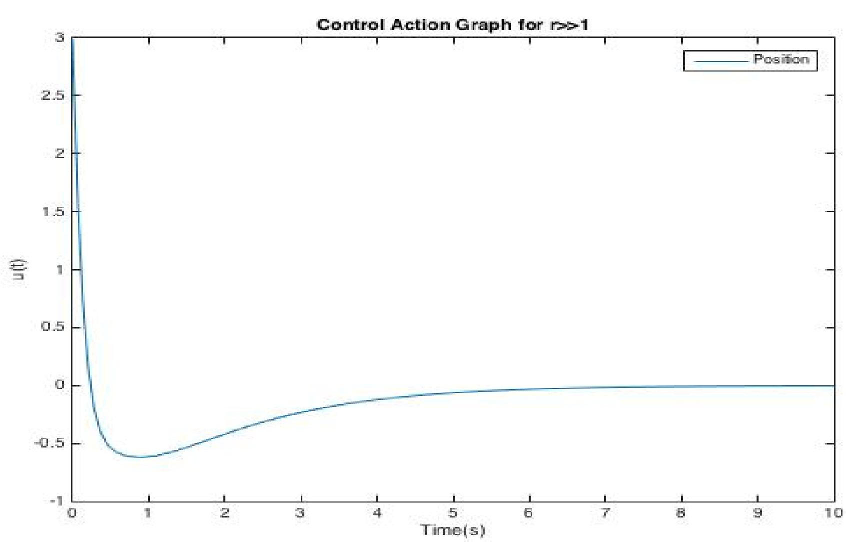


Figure 12: Variation of input motor voltage with time, $r >> 1$

Describe briefly the effect of changing the weights on the closed-loop system behavior

When the relative weight of a particular variable was increased, the performance of that variable improved, but at the expense of other variables. For the case $q3 >> 1$, the undershoot of the pendulum angle was reduced, but the cart position had a larger deviation. This led to an increase in input voltage required. When r was increased, the required input voltage decreases, but the performance of the variables suffers as noticed by the increased deviations for cart position and pendulum angle.

You will observe that the position x will first increase before converging to zero. What is the physical reason for this behavior?

To move the cart from the initial condition of 30cm to equilibrium position at 0cm, the pendulum has to first tilt in the direction of the intended motion of the cart. In order to provide the tilt of the pendulum, the cart has to first move in the opposite direction to create the desired tilt. The cart then attempts to move to the 0cm position after the pendulum tilt has been achieved. This explains why position x first increases before converging to zero.