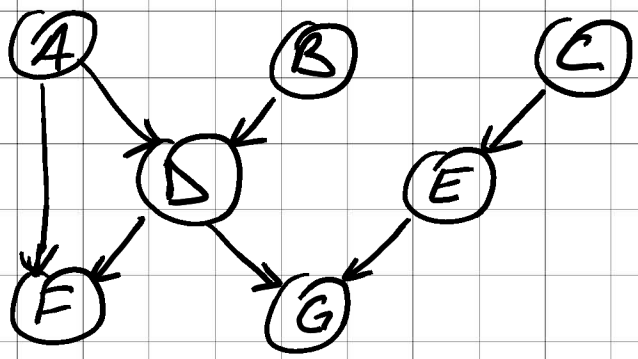
ELIMINAREA VARIABILELOR
~ EXEMPLU ~

- Fie rețeaua Bayesiană



definită de următoarele tabele de probabilități:

$p(A) = 0.3$

ϕ_A:

A	
0	0.7
1	0.3

$p(B) = 0.5$

ϕ_B:

B	
0	0.5
1	0.5

$p(C) = 0.6$

ϕ_C :

C	
0	0.4
1	0.6

A	B	P(D | A, B)
0	0	.25
0	1	.3
1	0	.4
1	1	.8

ϕ_D :

A	B	D	
0	0	0	.75
0	0	1	.25
0	1	0	.7
0	1	1	.3
1	0	0	.6
1	0	1	.4
1	1	0	.2
1	1	1	.8

C	P(E | C)
0	0.75
1	0.25

ϕ_E :

C	E	
0	0	0.25
0	1	0.75
1	0	0.75
1	1	0.25

A	Δ	P(F | A, Δ)
0	0	.4
0	1	.6
1	0	.3
1	1	.2

ϕ_F :

A	Δ	F	
0	0	0	.6
0	0	1	.4
0	1	0	.4
0	1	1	.6
1	0	0	.7
1	0	1	.3
1	1	0	.8
1	1	1	.2

Δ	E	P(G | Δ, E)
0	0	.9
0	1	.8
1	0	.5
1	1	.6

ϕ_G :

Δ	E	G	
0	0	0	.1
0	0	1	.9
0	1	0	.2
0	1	1	.8
1	0	0	.5
1	0	1	.5
1	1	0	.4
1	1	1	.6

- Ne propunem să calculăm

$$P(C, F \mid G = 0)$$

- Vom aplica eliminarea succesivă a variabilelor pentru a calcula:

ϕ_{CFG} astfel:

1. "Conditionăm" factorii cu $G = 0$
2. Eliminăm pe rând variabilele

$$E, B, A, D$$

3. Înmulțim factorii rămași
4. $$P(C, F \mid G = 0) = \frac{\phi_{CFG}(C, F, G = 0)}{\sum_{C,F} \phi_{CFG}}$$

1. Un singur factor contine variabila G: ϕ_G

$\phi_G \rightarrow \phi_{G0}$

ϕ_{G0}:

D	E	G	
0	0	0	0.1
0	1	0	0.2
1	0	0	0.5
1	1	0	0.4

2 Eliminăm variabile

(2.1) Eliminăm E

$$w_E = \phi_E \cdot \phi_{G0}$$

C	D	E	G					
0	0	0	0	0.25	·	0.1	=	0.025 *
0	0	1	0	0.75	·	0.2	=	0.15 *
0	1	0	0	0.25	·	0.5	=	0.125
0	1	1	0	0.75	·	0.4	=	0.3
1	0	0	0	0.75	·	0.1	=	0.075
1	0	1	0	0.25	·	0.2	=	0.05
1	1	0	0	0.75	·	0.5	=	0.375
1	1	1	0	0.25	·	0.4	=	0.1

$$\phi_{CDG} = \sum_E w_E =$$

C	D	G	
0	0	0	0.175
0	1	0	0.425
1	0	0	0.125
1	1	0	0.475

(2.2) Eliminate B

$$\omega_B = \phi_B \cdot \phi_D$$

A	B	D			
0	0	0	.5 ·	.75	= 0.375
0	0	1	.5 ·	.25	= 0.125
0	1	0	.5 ·	.7	= 0.350
0	1	1	.5 ·	.3	= 0.15
1	0	0	.5 ·	.6	= 0.3
1	0	1	.5 ·	.4	= 0.2
1	1	0	.5 ·	.2	= 0.1
1	1	1	.5 ·	.8	= 0.4

$$\phi_{AD} = \sum_B \omega_B :$$

A	D	
0	0	0.725
0	1	0.275
1	0	0.4
1	1	0.6

(2.3) Eliminăm A

$$\omega_A = \phi_A \cdot \phi_{AD} \cdot \phi_F$$

A	D	F				
0	0	0	0.7	× 0.725	× 0.6	= 0.3045
0	0	1	0.7	× 0.725	× 0.4	= 0.203
0	1	0	0.7	× 0.275	× 0.4	= 0.077
0	1	1	0.7	× 0.275	× 0.6	= 0.1155 ←
1	0	0	0.3	× 0.4	× 0.7	= 0.084
1	0	1	0.3	× 0.4	× 0.3	= 0.036
1	1	0	0.3	× 0.6	× 0.8	= 0.144
1	1	1	0.3	× 0.6	× 0.2	= 0.036 ←

$$\phi_{DF} = \sum_A \omega_A :$$

D	F	
0	0	0.3885
0	1	0.239
1	0	0.221
1	1	0.1515 ←

Factori rămași $\phi_C, \phi_{DF}, \phi_{CDG}$

(2.4) Eliminar variable Δ

$$\omega_\Delta = \phi_{C\Delta G} \cdot \phi_{\Delta F}$$

C	Δ	F	G	
0	0	0	0	0.175 · 0.3885
0	0	1	0	0.175 · 0.239
0	1	0	0	0.425 · 0.221
0	1	1	0	0.425 · 0.1515
1	0	0	0	0.125 · 0.3885
1	0	1	0	0.125 · 0.239
1	1	0	0	0.475 · 0.221
1	1	1	0	0.475 · 0.1515

$$\phi'_{CFG} = \sum_\Delta \omega_\Delta =$$

C	F	G	
0	0	0	0.1619125
0	1	0	0.1062125
1	0	0	0.1535375
1	1	0	0.1018375

3 Înmulțim cei doi factori

rămași: $\phi_{CFG} = \phi_C \cdot \phi'_{CFG}$

C F G	
0 0 0	$0.161925 \cdot 0.4 = 0.06477$
0 1 0	$0.1062125 \cdot 0.4 = 0.042485$
1 0 0	$0.1535375 \cdot 0.6 = 0.0921225$
1 1 0	$0.1018375 \cdot 0.6 = 0.0611025$

4 $P(C=1, F=0 \mid G=0)$

$$= \frac{0.0921225}{0.26048} \simeq 0.3537$$

TB, 2017