The spring elastic constant evaluation

1. Purpose

The objective of the experiment is to determine the spring constant of a spiral spring using Hooke's law and the period of oscillatory motion in response to a weight.

Apparatus: A spiral spring, a set of weights, a weight hanger, a stop watch, and a lab scale.

2. Theory

A. Static method

We use a spiral spring with elastic constant k and undeformed length l_o and bodies with different mass.

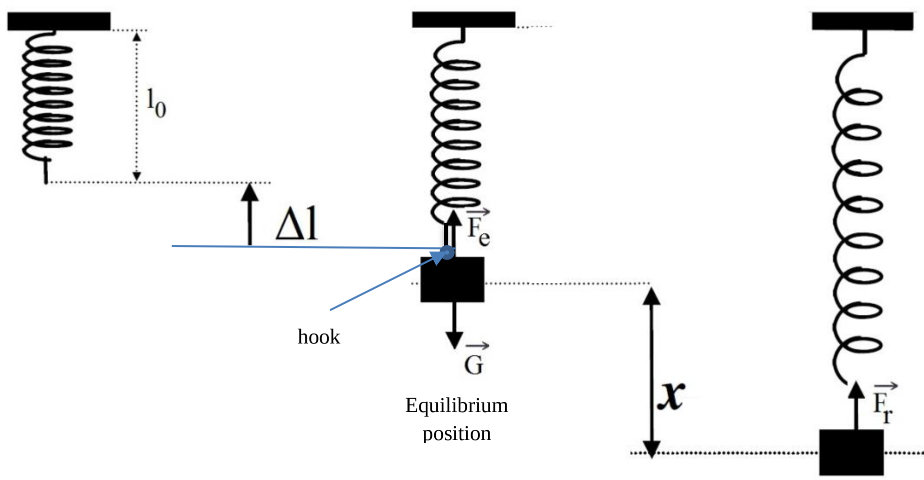

Fig. 1.

When a spring is stretched, according to Hooke's law, a restoring force F proportional to its elongation, x (or $\Delta l = l - l_o$) appears. Every spring obeys the Hooke's law if the deformation is not too great.

$$\vec{F_e} = -k\overrightarrow{\Delta l} \tag{1}$$

For the equilibrium

$$\Rightarrow \; mg = k\Delta l \tag{2}$$

$$k = \frac{mg}{\Delta l} \tag{3}$$

B. Dynamic method

When we move the body connected to a spring from its equilibrium position it starts to oscillate around the equilibrium position under the action of restoring (elastic) force. With x the distance from equilibrium position the Newton's second law is:

$$\mathrm{ma} = -\mathrm{kx} \tag{4}$$

$$\frac{\mathrm{d}^2\mathrm{x}}{\mathrm{dt}^2} + \frac{\mathrm{k}}{\mathrm{m}}\mathrm{x} = 0 \tag{5}$$

We note $\boldsymbol{\omega} = \sqrt{\frac{\mathbf{k}}{\mathbf{m}}}$ and we call it the natural angular frequency. The second Newton law for the spring (5) is a differential equation having the solution

$$x(t) = A\sin(\omega t + \varphi) \tag{7}$$

The period for the harmonic oscillation is connected to the natural angular frequency throught:

$$\boldsymbol{\omega} = \frac{\mathbf{2\pi}}{\mathbf{T}} \tag{8}$$

$$k = m\omega^2 = 4\pi^2\frac{m}{T^2} \tag{9}$$

3. What to do

Static method

1) We measure the undeformed spring length l_o.
2) We hang the weight hanger with mass m_1 on the spring and we measure the deformed spring length l_1. We calculate the spring elongation Δl_1.
3) Successively we hang weights (m_i) and we calculate corresponding spring elongations Δl_i.
4) Use the Table 1. for experimental data and calculate elastic constant using relation (3).
5) Plot the graph of force (the deformative force) produced by different masses ($F=m\cdot g$) as a function the displacement from equilibrium Δl: $\mathbf{F(\Delta l)}$. The data should be linear. Hence, the slope of the line will be equal to the spring constant k according to the relation 2.
6) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$$\Delta k_i = k_i - \bar{k} \qquad \overline{\Delta k} = \frac{\sum|\Delta k_i|}{n} \qquad \sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n(\Delta k_i)^2}{n(n-1)}}$$

Table 1.

$l_0 = \qquad cm$

Nr. crt	**m [kg]**	**l** [cm]	$\Delta \boldsymbol{l}$ **[m]**	***F*** [N]	$\boldsymbol{k}$ **[N/m]**	$\bar{\boldsymbol{k}}$ **[N/m]**	$\Delta \boldsymbol{k}$ **[N/m]**	$\sigma_{\bar{k}}$ **[N/m]**	$\boldsymbol{k_{true}}$ **[N/m]**
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									

Dynamic method

1) Hang the weight hanger with several weights (mass m_1) on the spring and set the equilibrium position of the system.
2) Pull the system out of its equilibrium position to make oscillations with 1-2cm amplitude.
3) Record the time for n=20 oscilations and find the period: $T = t/n$.
4) Repeat 1), 2) and 3) for different masses.
5) Complete the Table 2 using relation (9) for elastic constant.
6) Plot the graph of $T^2(s^2)$ as function of m(kg). The data should be linear. Find elastic constant from the slope (i.e. $T^2 = \frac{4\pi^2}{k}m \Leftrightarrow y = \text{slope}\cdot x$).
7) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$$\Delta k_i = k_i - \bar{k} \qquad \overline{\Delta k} = \frac{\sum|\Delta k_i|}{n} \qquad \sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n(\Delta k_i)^2}{n(n-1)}}$$

Table 2.

Nr. crt.	m **[kg]**	t **[s]**	n	T **[s]**	T^2 (s^2)	k **[N/m]**	$\bar{k}$ **[N/m]**	Δk **[N/m]**	$\sigma_{\bar{k}}$ **[N/m]**	k_{true} **[N/m]**
1										
2										
3										
4										
5										
6										
7										
8										

Compare the results obtained by the 2 methods, respectively by arithmetic and graphic mediation !!