

Analytic single and double scattering

$$f_2(\omega_i, \omega_\sigma) \langle \omega_\sigma, \omega_g \rangle =$$

$$(1) \quad \lambda_p(\omega_i) f_p(\omega_i, \omega_\sigma, \omega_p) \langle \omega_\sigma, \omega_p \rangle g_1(\omega_\sigma, \omega_p)$$

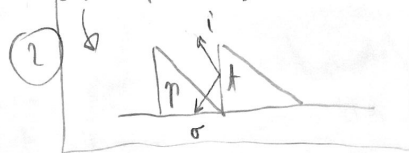
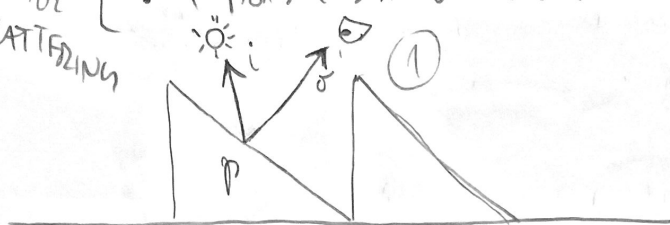
$$(2) \quad + \lambda_p(\omega_i) f_p(\omega_i, \omega_\sigma', \omega_p) \langle \omega_\sigma', \omega_p \rangle (1 - g_1(\omega_\sigma', \omega_p)) g_1(\omega_\sigma, \omega_p)$$

$$(3) \quad + \lambda_t(\omega_i) f_p(\omega_i, \omega_\sigma, \omega_p) \langle \omega_\sigma, \omega_p \rangle g_1(\omega_\sigma, \omega_p) (1 - g_1(\omega_i', \omega_t))$$

$$(1) \quad \lambda_p(\omega_i) f_p(\omega_i, \omega_\sigma, \omega_p) \langle \omega_\sigma, \omega_p \rangle g_1(\omega_\sigma, \omega_p)$$

Single Scattering {

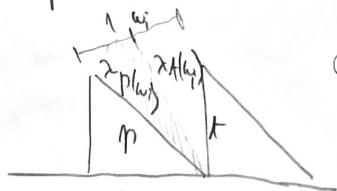
- paths $i \rightarrow p \rightarrow \sigma$
- (paths $i \rightarrow t \rightarrow \sigma$ excludes to \emptyset because reflect $(\omega_\sigma, \omega_t)$ is in lower hemisphere)



$\lambda_p(\omega_i)$ - interaction probability of incoming ray with perturbed facet

$\lambda_p(\omega_i) = \frac{a_p(\omega_i)}{a_p(\omega_i) + a_t(\omega_i)}$, $a_p(\omega_i) = \frac{\langle \omega_i, \omega_g \rangle}{\langle \omega_p, \omega_g \rangle}$

$a_t(\omega_i) = \frac{\langle \omega_i, \omega_t \rangle \sqrt{1 - \langle \omega_p, \omega_g \rangle^2}}{\langle \omega_p, \omega_g \rangle}$



$$f_p(\omega_i, \omega_\sigma, \omega_p)$$

↳ evaluation of perturbed facet RDP.

• ω_i, ω_σ are used in local space transformed using basis around ω_p

$$\langle \omega_\sigma, \omega_p \rangle$$

↳ cosine weight for perturbed facet

$$g_1(\omega_\sigma, \omega_p) - \text{masking of } \omega_\sigma \text{ by } \omega_p$$

$$g_1(\omega_i, \omega_m) = H(\langle \omega_i, \omega_m \rangle) \min \left[1, \frac{\langle \omega_i, \omega_g \rangle}{\langle a_p(\omega_i) + a_t(\omega_i) \rangle} \right]$$

cur dir ω_i trans dir ω_p or ω_t