$$\lim_{N\to\infty}\frac{\sum_{i=2}^{\log_2 N}\frac{N}{2^i}*\left(2^i-2\right)}{N*\log N}=C$$

The upper part can be written as:

$$\sum_{i=2}^{\log_2 N}\frac{N}{2^i}*\left(2^i-2\right)=N*\sum_{i=2}^{\log_2 N}\left(1-\frac{1}{2^{i-1}}\right)=N*\left(\sum_{i=2}^{\log_2 N}1-\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}\right)$$
$$=N*\left(\log_2 N-2-\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}\right)$$

Going back to the limit calculation:

$$\lim_{N\to\infty}\frac{N*\left(\log_2 N-2-\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}\right)}{N*\log N}=\lim_{N\to\infty}\frac{\left(\log_2 N-2-\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}\right)}{\log N}$$
$$=\lim_{N\to\infty}\frac{\log_2 N}{\log N}-\lim_{N\to\infty}\frac{2}{\log N}-\lim_{N\to\infty}\frac{\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}}{\log N}=\lim_{N\to\infty}\frac{\frac{\log N}{\log 2}}{\log N}-\lim_{N\to\infty}\frac{\sum_{i=2}^{\log_2 N}\frac{1}{2^{i-1}}}{\log N}$$
$$=\frac{1}{\log 2}-2\lim_{N\to\infty}\frac{\sum_{i=2}^{\log_2 N}\left(\frac{1}{2}\right)^i}{\log N}=\frac{1}{\log 2}-2\lim_{N\to\infty}\frac{1-\left(\frac{1}{2}\right)^{1+\log_2 N}}{\left(1-\frac{1}{2}\right)*\log N}$$
$$=\frac{1}{\log 2}-2\lim_{N\to\infty}\frac{1-\left(\frac{1}{2N}\right)}{\left(1-\frac{1}{2}\right)*\log N}=\frac{1}{\log 2}$$

Therefore we can conclude that the sum is of the same order of complexity as O(N*log N)