Bayesian inference in particle physics

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1 Bayesian Higgs

We got our hands on some fresh LHC data. It seems to be invariant mass measurements of the channel $H \to \gamma\gamma$ in the region $m \in [100, 160]$ GeV.

Excitedly, you tell all your friends. One of them tells you that you should analyse the data to obtain a measurement of the signal strength μ. She also tells you, that for this channel the background is expected to have an exponential mass distribution of

$$p(m|m_b, \lambda_b) = \frac{1}{\lambda_b} \exp\left(\frac{m - m_b}{\lambda_b}\right),$$

where $m_b = 100$ GeV and $\lambda_b = 20$ GeV. The signal has an expected mass distribution given by

$$p(m|m_b, \lambda_b) = \mathcal{N}(m_s, \sigma_s),$$

where $m_s = 125$ GeV and $\sigma_s = 2$ GeV. You expect 10000 background events and 1000 signal events

Hint: Always work with twice negative logarithmic probabilities throughout the exercise.

1. Generate MC events for signal and background and load the data.
2. Construct a binned negative log-likelihood function

$$p(n|\mu, \mu_{bkg}) = \prod_{\text{bins } b} \text{Poisson}(n_b|\nu_b(\mu, \mu_{bkg}))$$

   where n_b are your measured data yields per bin b. The expected yields per bin are

$$\nu_b(\mu, \mu_{bkg}) = \mu n_{sig,b} + \mu_{bkg} n_{bkg,b},$$

   where $n_{sig,b}$ and $n_{bkg,b}$ are the expected signal and background yields per bin, respectively. Our model parameters are the normalization factors μ and μ_{bkg}. The goal of this analysis will be to infer μ.
3. Think about possible prior choices here. We are relatively confident in the background modeling, but are not so confident in the signal normalization. Think about possible prior choices for μ and μ_{bkg}.
4. Combine (twice negative log) likelihood and (twice negative log) prior into a (twice negative log) posterior.
5. What is the posterior mode? What is the impact of your priors here?
6. Advanced: Code up your own implementation of the Metropolis-Hastings algorithm.

   Hint: Implement it such that it works with twice negative logarithmic probabilities, for numerical stability.

   Alternative: Copy my implementation from here.
7. Generate samples from the posterior.

   Bonus: Make trace plots.
8. Use the samples to obtain estimates for the 1-and 2-dimensional marginal posteriors of μ and μ_{bkg}.

   Hint: Corner plots are useful here.
9. What is the 95% upper credible interval on μ?
10. What are the 68% and 95% central intervals of μ and μ_{bkg}?