

Umeå University
Department of Computing Science

Parallel Programming 7.5 p
5DV152

Exercises, Chapter/Topic 3

Submitted 2017-02-16
Author: Lorenz Gerber (dv15lgr@cs.umu.se lozger03@student.umu.se)
Instructor: Lars Karlsson / Mikael Ränner

Contents

1	Introduction	1
2	3.2 - Generalization of algorithm for trapezoidal rule	1
3	3.6 - Array distributions	2
4	3.8 - Tree-structured algorithms for scatter and gather	2
5	3.9 - Vector scaling and dot product	2
6	3.11 - Prefix sums	6
7	3.13 - Generalization of vector scaling and dot product	8
8	3.16 - Diagram for a butterfly implementation of allgather	11
9	3.18 - Derived data types	11
10	3.20 - Pack and unpack	12
11	3.21 - Matrix-vector multiplication	12
12	3.22 - Timing the trapezoidal rule	12
13	3.27 - Speedup and efficiency of odd-even sort	12
	References	12

1 Introduction

This report is part of the mandatory coursework. It describes the solutions for several chosen exercises from the course book [3].

2 3.2 - Generalization of algorithm for trapezoidal rule

Two functions to adapt the *trapezoidal rule* for `calc_local_a` and `calc_local_b` were written and tested with the source code from the book (*mpi_trap.c*).

```
double calc_local_a(int my_rank, double a, double b, int n, int comm_sz){
    double local_a = 0;
    double h = 0;
    int local_n = 0;
    int rest_n = 0;

    h = (b-a)/n;
    local_n = n/comm_sz;

    rest_n = n%comm_sz;

    if(my_rank < rest_n){
        local_a = a + my_rank*local_n*h + my_rank*h;
    } else {
        local_a = a + my_rank*local_n*h + rest_n*h;
        local_a += (my_rank-rest_n) * h;
    }

    return local_a;
}

double calc_local_b(int my_rank, double a, double b, int n, int comm_sz){
    double h;
    int local_n;

    h = (b-a)/n;
    local_n = n/comm_sz;

    if (my_rank == (comm_sz-1)){
        return a + my_rank+1*local_n*h;
    } else {
        return calc_local_a(my_rank+1, a, b, n, comm_sz);
    }
}
```

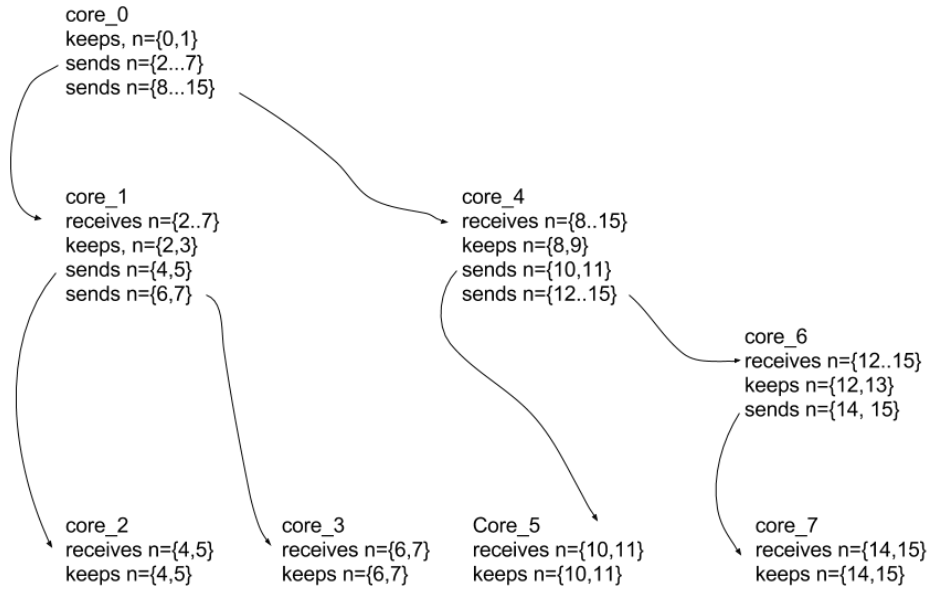


Figure 1: This graph shows a tree based implementation of scatter for $\text{comm_sz} = 8$ and $n = 16$.

3 3.6 - Array distributions

Block distribution

Block distribution can be obtained by $b = \lfloor i \div p \rfloor$ where b is the block number, i the index of n and p is the number of processes. This solution is however not fair. An improved, fair expression can be devised using a ternary operator:

$$i < n \bmod p \times \lceil n \div p \rceil ? \lfloor i \div \lceil n \div p \rceil \rfloor : n \bmod p + \lfloor (i - n \bmod p \times \lceil n \div p \rceil) \div \lfloor n \bmod p \rfloor \rfloor$$

Cyclic distribution

Cyclic distribution is described by $b = i \bmod p$ with b as block number i as index of n and p as number of processes.

Block cyclic distribution

Block cyclic distribution can be expressed as $b = \lfloor i \div l \rfloor \bmod p$ where b is block index, i index of n , l block length and p number of processes.

4 3.8 - Tree-structured algorithms for scatter and gather

5 3.9 - Vector scaling and dot product

```
#include <stdio.h>
```

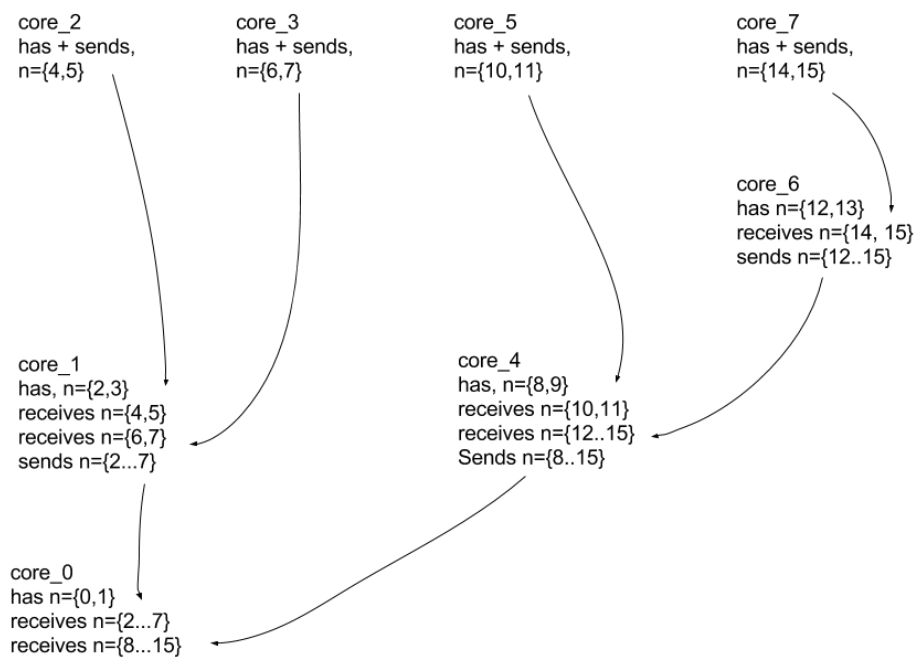


Figure 2: This graph shows a tree based implementation of gather for $comm_sz = 8$ and $n = 16$.

4(12)

```
#include <mpi.h>
#include <string.h>
#include <stdlib.h>
#include <time.h>

int main(int argc, char *argv[]) {
    int my_rank, comm_sz;
    int n, local_n, local_dotp_sum = 0, scalar, result_dot;
    int* local_vec1;
    int* local_vec2;
    int* vector1;
    int* vector2;

    /* Initializing */
    MPI_Init(NULL, NULL);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
    srand(time(NULL));

    /* Obtaining Data */
    if(my_rank==0 && argc > 1){

        if(strcmp(argv[1], "r") == 0){
            printf("using random data, vector length = %d\n", 100*comm_sz);
            n = 100*comm_sz;
            vector1 = (int *) malloc(100*comm_sz * sizeof(int));
            vector2 = (int *) malloc(100*comm_sz * sizeof(int));

            for(int i = 0; i < n;i++){
vector1[i] = rand() % 1000;
vector2[i] = rand() % 1000;
            }
            scalar = rand() % 1000;
        }

        } else if (my_rank==0){

            printf("enter vector length\n");
            scanf("%d", &n);
            printf("enter integer vector 1\n");

            vector1 = (int *) malloc(n * sizeof(int));
            vector2 = (int *) malloc(n * sizeof(int));

            for(int i = 0; i < n; i++){
                scanf("%d", &vector1[i]);
            }
        }
    }
}
```

```

printf("enter integer vector 2\n");

for(int i = 0; i < n; i++){
    scanf("%d", &vector2[i]);
}

printf("enter integer scalar\n");
scanf("%d", &scalar);
}

/* Distribute Data */
MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);

local_n = n/comm_sz;
local_vec1 = (int*) malloc(local_n * sizeof(int));
local_vec2 = (int*) malloc(local_n * sizeof(int));

MPI_Bcast(&scalar, 1, MPI_INT, 0, MPI_COMM_WORLD);
MPI_Scatter(vector1, local_n, MPI_INT, local_vec1, local_n, MPI_INT, 0, MPI_COMM_WORLD);
MPI_Scatter(vector2, local_n, MPI_INT, local_vec2, local_n, MPI_INT, 0, MPI_COMM_WORLD);

/* Calculations */
/* Calculate Dot Product */
for(int i = 0; i < local_n; i++){
    local_vec2[i]*=local_vec1[i];
}

/* Calculate vector-scalar product */
for(int i = 0; i < local_n; i++){
    local_vec1[i]*=scalar;
}

/* Summing for dot product */
for(int i = 0; i < local_n; i++){
    local_dotp_sum += local_vec2[i];
}

/* Collect Data */
MPI_Gather(local_vec1, local_n, MPI_INT, vector1, local_n, MPI_INT, 0, MPI_COMM_WORLD);
MPI_Reduce(&local_dotp_sum, &result_dot, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);

/* Results */
if(my_rank == 0){
    printf("dot product = %d\n", result_dot);

    printf("vector-scalar product = ");
    for(int i = 0; i < n; i++){

```

6(12)

```
        printf("%d ", vector1[i]);
    }
    printf("\n");
}

/* Clean up */
if(my_rank==0){
    free(vector1);
    free(vector2);
}

free(local_vec1);
free(local_vec2);

MPI_Finalize();

return 0;
}
```

6 3.11 - Prefix sums

Serial Algorithm

Assume vector x of length n and prefix sums vector y to be calculated.

```
y_0 = x_0
for (i = 1, i < n , i++)
    y_i = y_{i-1} + x_i
```

Parallel Algorithm

After Blelloch [1], parallel prefix-sum, or ‘Sum Scan Algorithm’ can be calculated in two steps, the ‘Up-sweep’ (Reduce) and ‘Down-sweep’. Pseudo code from M. Valdez [4] where x is the input data, n the size of the input and d the number of processeors. If $n \neq 2^k$, then n has to be extended with *zero*’s. The ‘Up-Sweep’:

```
for d = 0 to log2(n) - 1 do
    for all k = 0 to n - 1 by 2^(d+1) in parallel do
        x[k + 2^(d+1) - 1] = x[k + 2^d - 1] + x[k + 2^(d+1) - 1]
```

The ‘Down-Sweep’:

```
x[n - 1] = 0
for d = log2(n) - 1 down to 0 do
    for all k = 0 to n - 1 by 2^(d+1) in parallel do
        t = x[k + 2^d - 1]
        x[k + 2^d - 1] = x[k + 2^(d+1) - 1]
        x[k + 2^(d+1) - 1] = t + x[k + 2^(d+1) - 1]
```

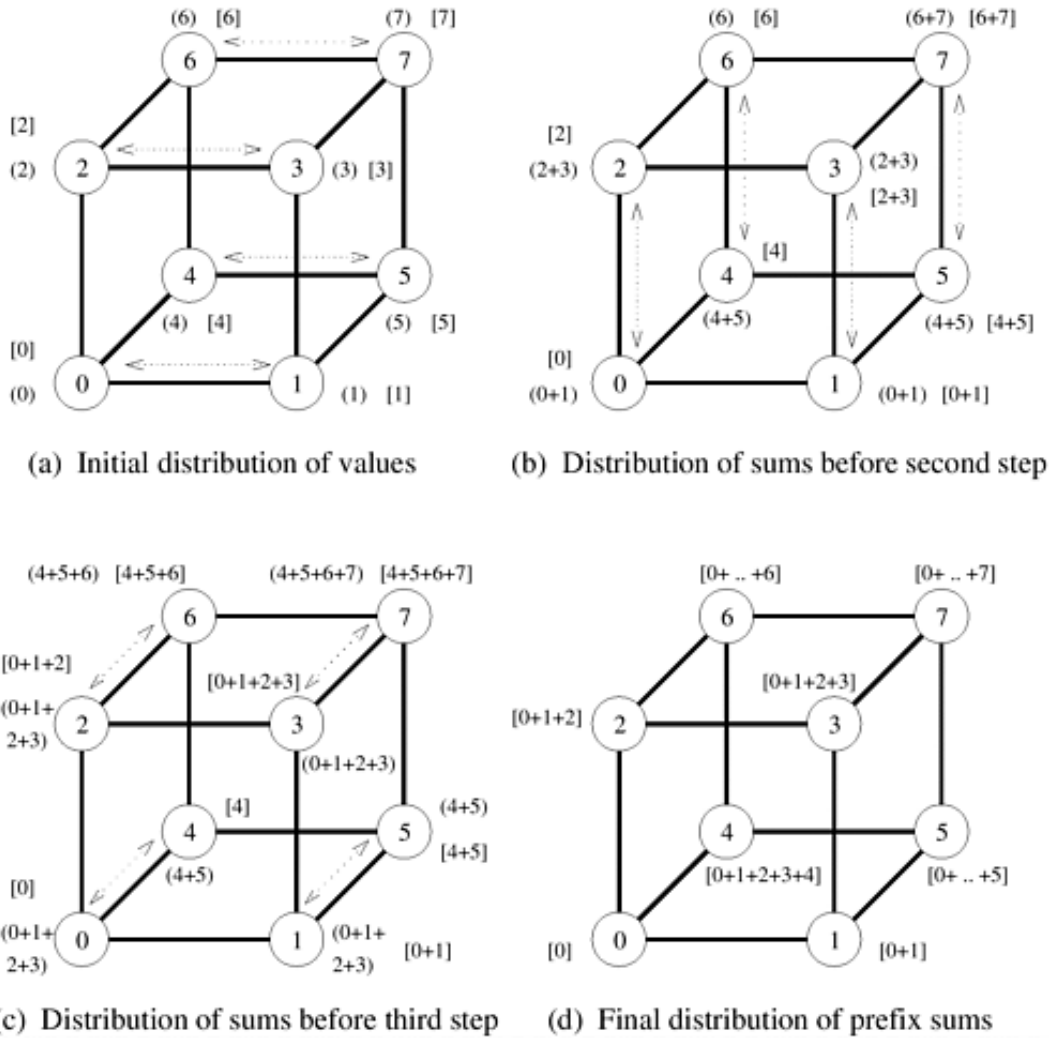



Figure 3: This figure shows the calculation of prefix sums with minimal communication phases. The figure is taken from [2, chap 4.3]

Minimum Communication

An algorithm that uses for the prefix-sum of length $n = 2^k$ only k communication steps can be derived from the communication model in a hypercube as shown in figure 3.

Below follows pseudo code for parallel hypercube prefix sum (from [2]):

```

procedure PREFIX_SUMS_HCUBE(my_id, my_number, d, result)
begin
  result := my_number;
  msg := result;
  for i := 0 to d - 1 do
    partner := my_id XOR 2i;
    send msg to partner;
    receive number from partner;
    msg := msg + number;
    if (partner < my_id) then result := result + number;
  endfor;

```

```
end PREFIX_SUMS_HCUBE
```

MPI Program using MPI_Scan

A minimal working example using MPI_Scan.

```
#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>
#include <time.h>

int main(void){

    int my_rank, comm_sz;
    int* initial_vector;
    int* prefix_sums;

    MPI_Init(NULL, NULL);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
    srand(time(NULL));

    initial_vector = (int *) malloc(10*sizeof(int));
    prefix_sums = (int *) calloc(10, sizeof(int));

    for(int i = 0; i < 10;i++){
        initial_vector[i] = rand() % 1000;
    }

    MPI_Scan(initial_vector, prefix_sums, 10,MPI_INT, MPI_SUM, MPI_COMM_WORLD);

    for(int i = 0; i< comm_sz;i++){
        if (i == my_rank){
            for(int j = 0;j < 10;j++){
                printf("%d ", prefix_sums[j]);
            }
        }
    }
    printf("\n");

    MPI_Finalize();
    return 0;
}
```

7 3.13 - Generalization of vector scaling and dot product

Below follows an implementation of the ‘Vector Scaling and Dot Product’ using ‘MPI_Scatterv’/‘MPI_Gatherv’.

```
#include <stdio.h>
```

```

#include <mpi.h>
#include <string.h>
#include <stdlib.h>
#include <time.h>

int main(int argc, char *argv[]) {
    int my_rank, comm_sz;
    int n, local_n, local_dotp_sum = 0, scalar, result_dot;
    int* sendcounts;
    int* displs;
    int* local_vec1;
    int* local_vec2;
    int* vector1;
    int* vector2;

    /* Initializing */
    MPI_Init(NULL, NULL);
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
    srand(time(NULL));

    /* Obtaining Data */
    if(my_rank==0 && argc > 1){

        if(strcmp(argv[1], "r") == 0){
            printf("using random data, vector length = %d\n", 100*comm_sz);
            n = 100*comm_sz;
            vector1 = (int *) malloc(100*comm_sz * sizeof(int));
            vector2 = (int *) malloc(100*comm_sz * sizeof(int));

            for(int i = 0; i < n;i++){
vector1[i] = rand() % 1000;
vector2[i] = rand() % 1000;
            }
            scalar = rand() % 1000;
        }

        } else if (my_rank==0){

            printf("enter vector length\n");
            scanf("%d", &n);
            printf("enter integer vector 1\n");

            vector1 = (int *) malloc(n * sizeof(int));
            vector2 = (int *) malloc(n * sizeof(int));

            for(int i = 0; i < n; i++){

```

10(12)

```
        scanf("%d", &vector1[i]);
    }

    printf("enter integer vector 2\n");

    for(int i = 0; i < n; i++){
        scanf("%d", &vector2[i]);
    }

    printf("enter integer scalar\n");
    scanf("%d", &scalar);
}

/* Distribute Data */
MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);

/* Fixing sendcounts for general n */
sendcounts = (int *) malloc(comm_sz * sizeof(int));
displs = (int *) calloc(comm_sz, sizeof(int));

for(int i = 0; i < comm_sz; i++){
    if(n % comm_sz > i){
        sendcounts[i] = (n/comm_sz) + 1;
    } else {
        sendcounts[i] = (n/comm_sz);
    }
}

for(int i = 1; i < comm_sz; i++){
    displs[i] = displs[i-1] + sendcounts[i];
}

local_n = sendcounts[my_rank];

local_vec1 = (int*) malloc(local_n * sizeof(int));
local_vec2 = (int*) malloc(local_n * sizeof(int));

MPI_Bcast(&scalar, 1, MPI_INT, 0, MPI_COMM_WORLD);
MPI_Scatterv(vector1, sendcounts, displs, MPI_INT, local_vec1, sendcounts[my_rank], MPI_COMM_WORLD);
MPI_Scatterv(vector2, sendcounts, displs, MPI_INT, local_vec2, sendcounts[my_rank], MPI_COMM_WORLD);

/* Calculations */
/* Calculate Dot Product */
for(int i = 0; i < local_n; i++){
    local_vec2[i] *= local_vec1[i];
}
```

```

    }

    /* Calculate vector-scalar product */
    for(int i = 0; i < local_n; i++){
        local_vec1[i]*=scalar;
    }

    /* Summing for dot product */
    for(int i = 0; i < local_n; i++){
        local_dotp_sum += local_vec2[i];
    }

    /* Collect Data */
    MPI_Gatherv(local_vec1, sendcounts[my_rank], MPI_INT, vector1, sendcounts, displs, MPI_
    MPI_Reduce(&local_dotp_sum, &result_dot, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);

    /* Results */
    if(my_rank == 0){
        printf("dot product = %d\n", result_dot);

        printf("vector-scalar product = ");
        for(int i = 0; i < n; i++){
            printf("%d ", vector1[i]);
        }
        printf("\n");
    }

    /* Clean up */
    if(my_rank==0){
        free(vector1);
        free(vector2);
    }

    free(local_vec1);
    free(local_vec2);

    MPI_Finalize();

    return 0;
}

```

8 3.16 - Diagram for a butterfly implementation of allgather

9 3.18 - Derived data types

takes a while to solve requires programming

10 3.20 - Pack and unpack

requires programming

11 3.21 - Matrix-vector multiplication

takes a while to solve requires programming requires testing

12 3.22 - Timing the trapezoidal rule

takes a while to solve Requires programming requires testing

13 3.27 - Speedup and efficienci of odd-even sort

References

- [1] G.E. Blelloch. *Vector Models for Data-Parallel Computing*. MIT Press, Cambridge MA, USA, 1990.
- [2] A Grama, A Gupta, G Karypis, and V Kumar. *Introduction to Parallel Computing*. Pearson, Essex, England, 2003.
- [3] P.S. Pacheco. *An Introduction to Parallel Programming*. Morgan Kaufman, 2011.
- [4] Stackoverflow, parallel prefix sum - fastest implementation. <http://stackoverflow.com/questions/10053629/parallel-prefix-sum-fastest-implementation>, 2012. accessed: 2017-02-04.