**Umeå University**
Department of Computing Science

# Language and Computation 7.5 p
# 5DV162

## Assignment 1

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## Problem 1

Assume that $L$ is a regular language. Strings $w$ of language $L$ are constructed according to the specification where zero, one or several $a$'s and zero, one or several $b$s are mixed freely as long as there are more $b$'s than $a$'s in it. Let's have a look at such a generic string and how it can be split into three parts $xyz$ such that $xy$ is shorter than $m$, $x$ is 1 or larger and all $xy^iz$ are part of $L$ for all $i \in \mathbb{N}$. Independent of how $m$ is chosen, if $y$ contains more $a$'s than $b$'s, which according to the language definition is possible, the resulting strings for $xy^iz$ $i \in \mathbb{N}$ are for large enough $i$ no longer part of $L$. Hence, proven by contradiction, $L$ can not be a regular language.

## Problem 2

Assume that $L$ is a regular language. Strings $w$ of language $L$ are constructed according to the specification as $a^n$ with $n$ being a prime number equal or larger than 2. For every prime $n$, there has to be a number $m$ which is smaller than $n$, and that multiplied with any natural number plus the difference $n-m$ will be a prime. Such a number does not exist. Hence, L can not be a regular language.

## Problem 3

Conversion of context free grammar into Chomsky normal form:

First Rule 1 is applied;

$$
\begin{aligned}
&S \rightarrow aAB \\
&A \rightarrow aAa \\
&A \rightarrow bb \\
&B \rightarrow a
\end{aligned}
$$

$$
\begin{aligned}
&S_0 \rightarrow aAB \\
&S \rightarrow aAB \\
&A \rightarrow aAa|bb \\
&B \rightarrow a
\end{aligned}
$$

No application for the epsilon and unit rule. The last shown steps are according rule four:

$$
\begin{aligned}
&S_0 \rightarrow BAB \\
&S \rightarrow BAB \\
&A \rightarrow BAB|bb \\
&B \rightarrow a
\end{aligned}
$$

$$
\begin{aligned}
&S_0 \to BU \\
&S \to BU \\
&A \to BU|bb \\
&U \to AB
\end{aligned}
$$

$$
\begin{aligned}
&S_0 \to BU \\
&S \to BU \\
&A \to BU|bb \\
&U \to AB \\
&V \to b
\end{aligned}
$$

## Problem 4

Proving that the resulting Language $Ls$ from shuffeling the regular languages $L1$ and $L2$ is also regular was attempted here by construction.

The idea is to construct a NFA from NFA/DFA's that accept $L1$ and $L2$. Let's assume The wordlength of $L1$ and $L2$ is $|w1|$ and $|w2|$, hence the NFA/DFA to accept $L1$ or $L2$ will have $u_1 \dots u_{|w1|}$ respectively $v_1 \dots v_{|w2|}$ states. Now, for example state u1 is split into $|w1|$ substates to 'remember' how many states of NFA 2 already have been passed. The same is done for all states of the NFA 1 and 2. Hence the new NFA will in this case have 18 + 2 states (18 + start and accepted end state). All inbound transitions are the same as in the initial NFA's. Hence it has been proven by construction of a new NFA that the shuffle of $L1$ and $L2$ is also a regular language.

The solution is shown in figure 1 produced with graphviz. Figure 2 shows a hand draft of the same figure, but with a more clear grouping.

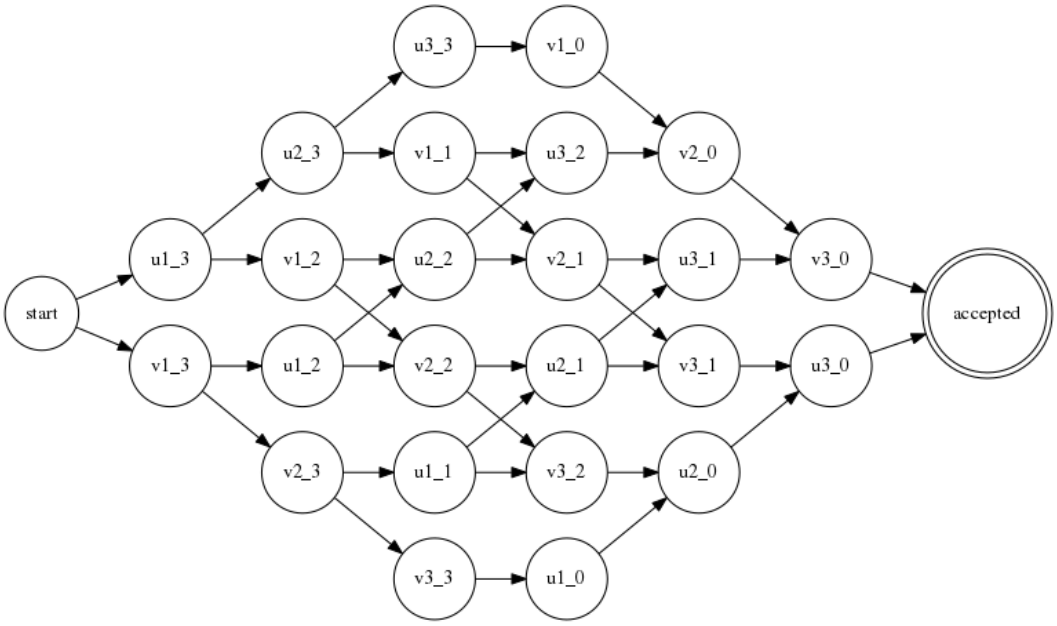


**Figure 1:** *Non-Deterministic Finite Automaton of shuffle between L1 and L2, generated with graphviz.*

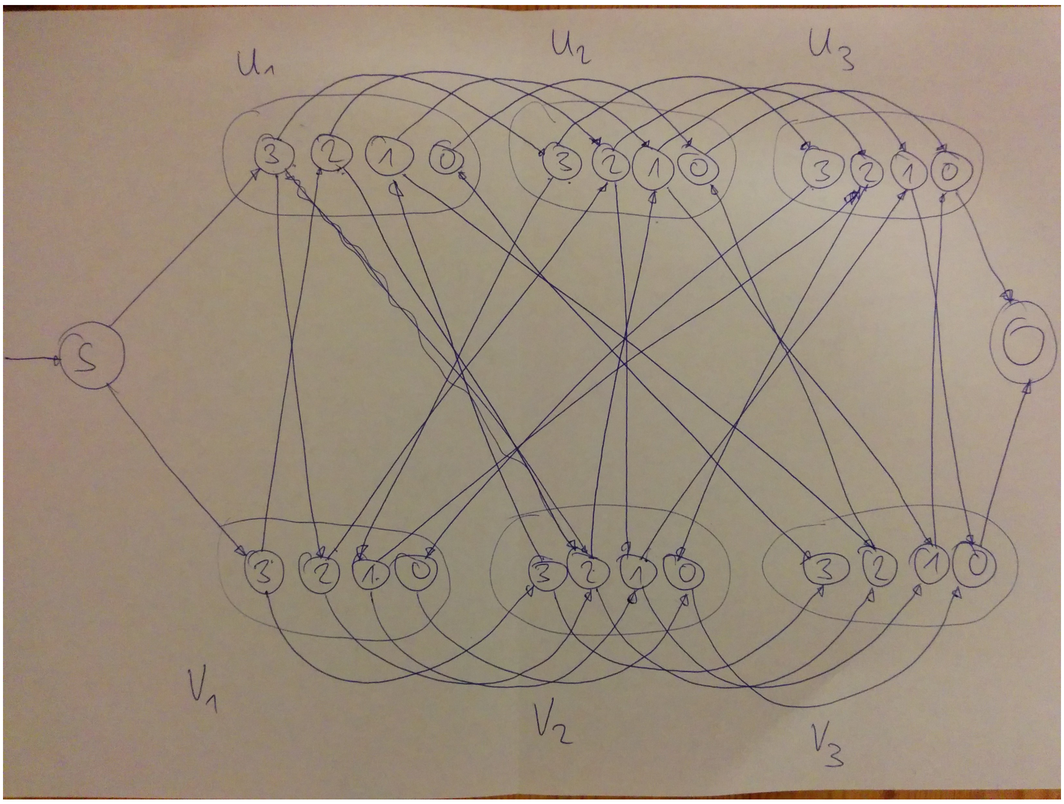


**Figure 2:** *Non-Deterministic Finite Automaton of shuffle between L1 and L2, hand draft.*