Umeå University
Department of Computing Science

Language and Computation 7.5 p
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Assignment 4

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Problem 1, BLUE-RED-GREEN

Problem 2, Reduction ALL_{CFG} to EQ_{CFG}

If we can decide wether $L(E) \subseteq L(G)$ we could choose $L(G) = \Sigma^*$ and decide it's universality. But we know that ALL_{CFG} is not decidable. Hence EQ_{CFG} is also not decidable.

Problem 3, Proof closure under compliment for reduction

This can be concluded from the logical equivalence: $p \leftrightarrow q$ is the same as $\neg p \leftrightarrow \neg q$. So $p = q \in A$ and $q = f(w) \in B$. $A/leq_m B$ means there is a total computable f such that for all w, $w \in A \leftrightarrow f(w) \in B$. By the argument above, this is the same as $w \notin A \leftrightarrow f(w) \notin B$, or equivalently $w \in \bar{A} \leftrightarrow (w) \in \bar{B}$.

Problem 4, Asymptotic Problems

a)

Find c:
$\lim_{n\to\infty} \frac{f(n)}{f(g)} + 1$ where
$\lim_{n\to\infty}(\frac{5n^4+3n^2+4}{n^4}) + 1 = \lim_{n\to\infty}(5 + \frac{3}{n^2} + \frac{4}{n^4}) + 1 \implies c = 6$
find n_0 by trial: $n_0 = 2$

b)

First find by trial a n where $2^n < n!$. Then proof by induction that this holds for all $n > n_0$.
Choose $n_0 = 4$
Suppose thatn when $n = k(k \geq 4)$, we have that $k! > 2^k$.
Now, prove that $(k+1)! > 2^{k+1}$ when $n = (k+1)(k \geq 4)$.
$(k+1)! = (k+1)k! > (k+1)2^k$.
This implies $(k+1)! > 2^k \cdot 2$
Therefore, $(k+1)! > 2^{k+1}$
Hence, $n! > 2^n$ for all integers $n \geq 4$

c)

Assume $2^{2n} = O(2^n)$
then $lim_{n\to\infty} \frac{f(n)}{g(n)} + 1$
but $lim_{n\to\infty} \frac{2^{2n}}{2^n} + 1 = \infty$ hence, $2^{2n} \neq O(2^n)$

d)

For every constant a there is a constant c such that $(n+a)^3 = O(n^3)$