Scattering of matter-wave soliton from a narrow barrier: transmission coefficient and induced collapse

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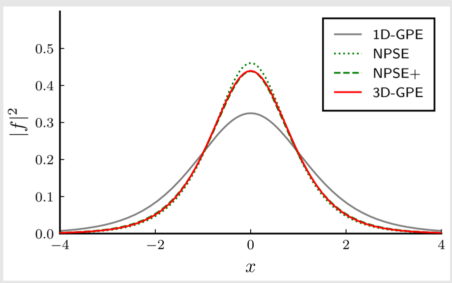
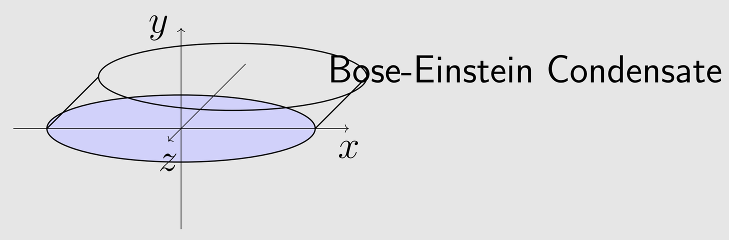
Introduction

We consider a ultracold Bose gas trapped in a **quasi one-dimensional** setting. By using the three-dimensional Gross-Pitaevskii equation, we numerically obtain the dynamics of the collision of a matter-wave soliton with a narrow potential barrier. In this way, we determine how the **transmission coefficient** depends on the soliton impact velocity and the barrier height. Quite remarkably, we also obtain the regions of parameters where there is the **collapse** of the bright soliton induced by the collision. We compare these three-dimensional results with the ones obtained by three different one-dimensional nonlinear Schrödinger equations. We find that a specifically modified nonpolynomial Schrödinger equation is able to capture the main features of the three-dimensional findings.

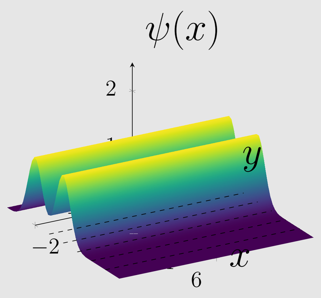




$$U(x,y,z) = \frac{1}{2}m\omega_\perp^2(y^2+z^2)+V(x), \quad (1)$$

Phenomenology of the collision event

When a one-dimensional soliton impinge on a narrow barrier, the transmission coefficient T is expected to be a **discontinuous function** of v, the impinging velocity, and b, the barrier energy, for sufficiently low values of the parameters. As shown in a previous work, collapse can be initiated [MALOMED] by the collision event, so it is important to distinguish between static and dynamic collapse.



Dimensional reduction strategies for the Gross-Pitaevskii equation

Starting from the GP Lagrangian in 3D:

$$\mathcal{L} = \int d^3\mathbf{r}\, \psi^* \left[i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2 - U - \frac{g}{2}(N-1)|\psi|^2\right]\psi, \quad (2)$$

with

$$g = \frac{4\pi\hbar^2 a_s}{m}, \quad (3)$$

one can compute the corresponding Euler-Lagrange equation:

$$i\hbar\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U + g(N-1)|\psi|^2\right]\psi. \quad (4)$$

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$$V(x;\ b) = b\ \exp[-\frac{x^2}{2w^2}]. \quad (5)$$

A better approach is to assume a **variable transverse width**

$$\phi(y,z,\sigma(x,t)) = \frac{1}{\sqrt{\pi}\sigma(x,t)}\exp\left[-\frac{y^2+z^2}{2\sigma(x,t)^2}\right] \quad (6)$$

Writing the 3D GP Lagrangian, we may hope to integrate along the transverse variables

$$\mathcal{L} = \int dx \int dy\, dz\ f^*\phi^*\left[i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2 - U - \frac{1}{2}g(N-1)|f\phi|^2\right]f\phi. \quad (7)$$

Computing the derivatives, we have

$$\mathcal{L} = \int dx \int dy\, dz\ f^*\phi^*\left[i\hbar\phi\frac{\partial}{\partial t}f + i\hbar f\phi\left(\frac{y^2+z^2}{\sigma^3}-\frac{1}{\sigma}\right)\frac{\partial}{\partial t}\sigma + \frac{\hbar^2}{2m}\left(f\nabla_\perp^2\phi + f\frac{\partial^2}{\partial x^2}\phi + \phi\frac{\partial^2}{\partial x^2}f\right) - Uf\phi - \frac{1}{2}g(N-1)|f\phi|^2 f\phi\right]. \quad (8)$$

Integrating the term proportional to $\frac{\partial}{\partial t}\sigma$ gives 0, as one may realize by looking at the symmetry of its prefactor. However, the term proportional to $\frac{\partial^2}{\partial x^2}\phi$ gives a non-null contribution to the 1D Lagrangian. The integration gives

$$\mathcal{L} = \int dx\, f^*\left[i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}^2 - V - \frac{\hbar^2}{2m\sigma^2}\left(1+\left(\frac{\partial}{\partial x}\sigma\right)^2\right) - \frac{m\omega_\perp^2}{2}\sigma^2 - \frac{1}{2}\frac{N-1}{2\pi\sigma^2}g|f|^2\right]f. \quad (9)$$

Let us now consider the corresponding Euler-Lagrange equations. These will be computed for f and σ, thus σ is a proper variational parameter, and constitute a set of coupled PDE and ODE.

$$i\hbar\frac{\partial}{\partial t}f = \left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}^2 + V + \frac{\hbar^2}{2m}\frac{1}{\sigma^2} + \frac{m\omega_\perp^2}{2}\sigma^2 + \frac{N-1}{2\pi\sigma^2}g|f|^2\right]f \quad (10)$$

$$-m\omega_\perp^2\sigma + \left[\frac{\hbar^2}{m} + \frac{N-1}{2\pi}g|f|^2\right]\sigma^{-3} + \frac{\hbar^2}{m}\sigma^{-3}\left(\sigma\frac{\partial}{\partial x}^2\sigma - \left(\frac{\partial}{\partial x}\sigma\right)^2\right) = 0. \quad (11)$$

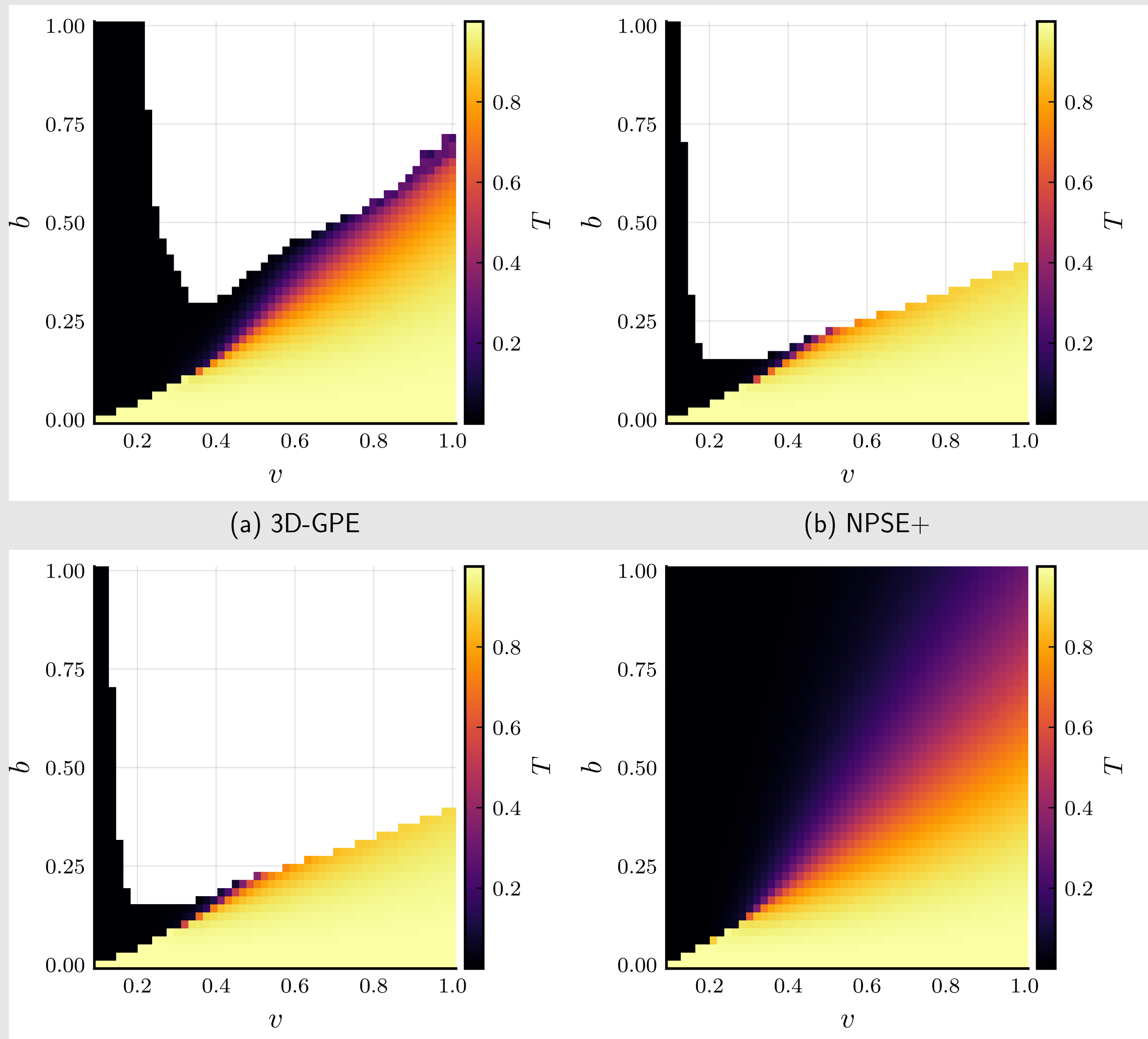
Numerical methods

We assume the field to be localized away from the boundary in order to neglect this problem. The time step in both setups is chosen to be $h_t = 0.01$. These parameters have been proven to give a total truncation error in the L_∞ norm of the order of 10^{-4}, and allow for a reasonable computation time of all the calculations. The ground state solutions are computed using an imaginary-time propagation method. We point out that some modifications of this method are available under the name of normalized gradient-flow methods.
We have an isotropic confinement, in which we have units:

- energy $\longrightarrow \hbar\omega_\perp$,
- time $\longrightarrow \omega_\perp^{-1}$,
- length $\longrightarrow l_\perp$.
- 1D simulations a total length of $L = 40$, with a grid of $N = 512$ points.
- 3D simulations, we use a grid of $(N_x, N_y, N_z) = (512, 40, 40)$ points, with total lengths of $(L_x, L_y, L_z) = (40, 10, 10)$.

Collapse threshold is set to a probability per point of 0.3. The time step in both setups is chosen to be $h_t = 0.01$. These parameters have been proven to give a total truncation error in the L_∞ norm of the order of 10^{-4}, and allow for a reasonable computation time of all the calculations.

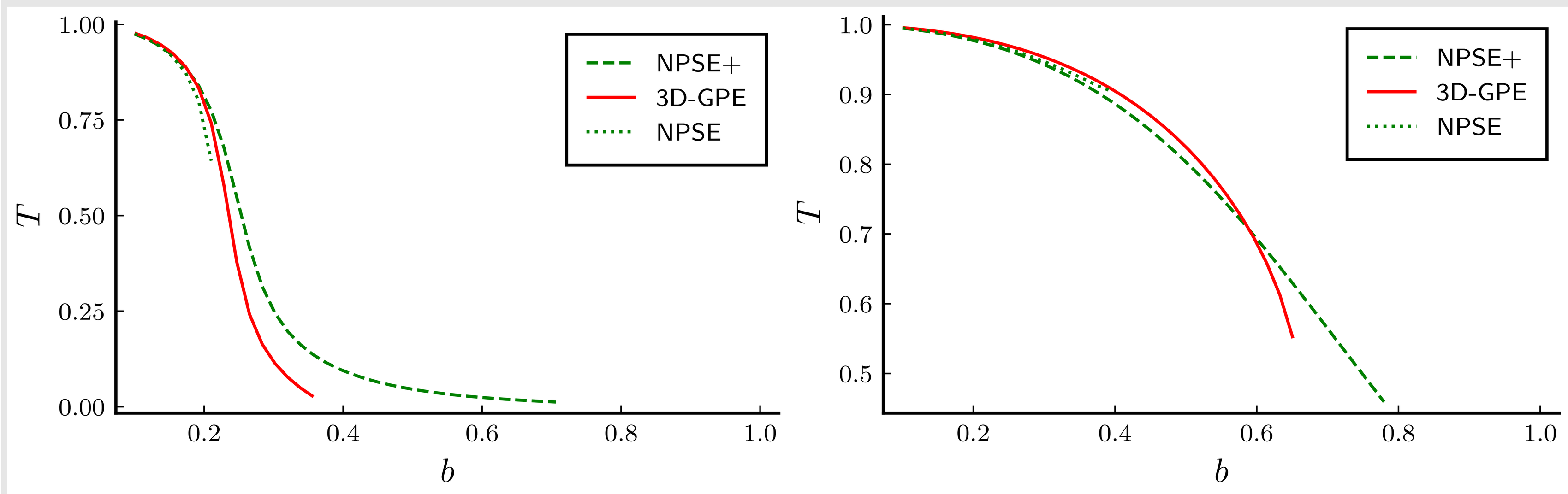
Results



(a) 3D-GPE (b) NPSE+

(c) NPSE (d) 1D-GPE

Figure 1. Comparison of the transmission coefficient and the collapse regions for the various equations. v is in units of $\sqrt{\hbar\omega_\perp/m}$, b is in units of $\hbar\omega_\perp$.



(a) $v = 0.5$ (b) $v = 1$

Conclusions and perspectives

We investigated how the choice of dimensional reduction impacts the description of some features of the process, namely the transmission coefficient and the onset of barrier-induced collapse, also using the familiar one-dimensional Gross-Pitaevskii. We first reviewed the ground state properties given by all the schemes, highlighting the role of the variational transverse width. Then we compared the scattering properties: our results show that by using the NPSE in a regime of high barrier energy and high velocity it fails to describe the 3D dynamics due to the the vanishing of the transverse width of the solution. In such cases the collapse phenomena in the 3D solutions are absent, and the NPSE is not capturing the correct dynamics. Our main result is that by adopting a slight modification of the transverse width using the true variational solution with the NPSE ansatz, the efficacy of the method is restored, showing good agreement with the 3D-GPE.

References

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