# Scattering of matter-wave soliton from a narrow barrier: transmission coefficient and induced collapse

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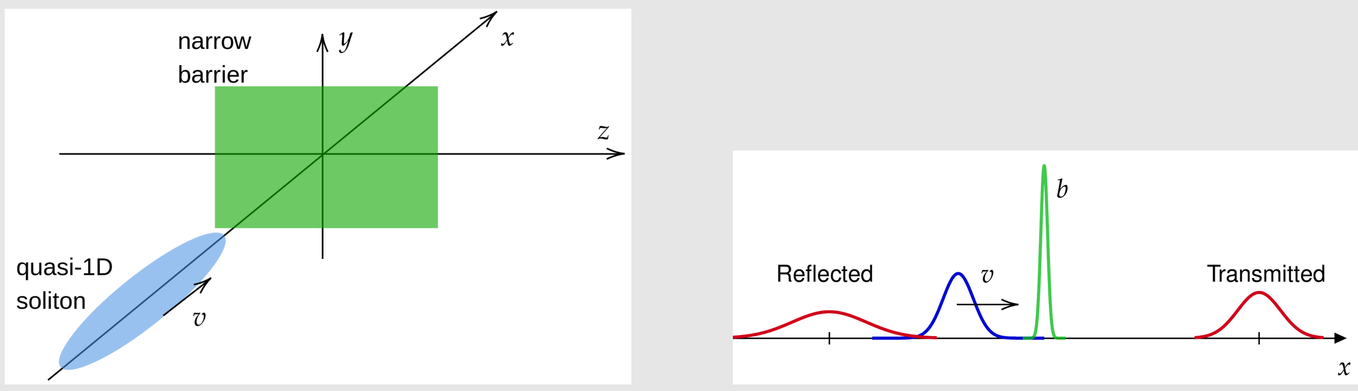
## Introduction

We consider a ultracold Bose gas trapped in a **quasi one-dimensional** setting.

- By using the three-dimensional Gross-Pitaevskii equation, we numerically obtain the dynamics of the collision of a matter-wave soliton with a narrow potential barrier.
- We determine how the **transmission coefficient** depends on the soliton impact velocity and the barrier height.
- We also obtain the regions of parameters where there is the **collapse** of the bright soliton induced by the collision.
- We compare the three-dimensional results with the ones obtained by **three different** one-dimensional nonlinear Schrödinger equations.

$$i\hbar\frac{\partial}{\partial t}\psi = \left[-\frac{\hbar^2}{2m}\nabla^2 + U + g(N-1)|\psi|^2\right]\psi. \quad (1)$$

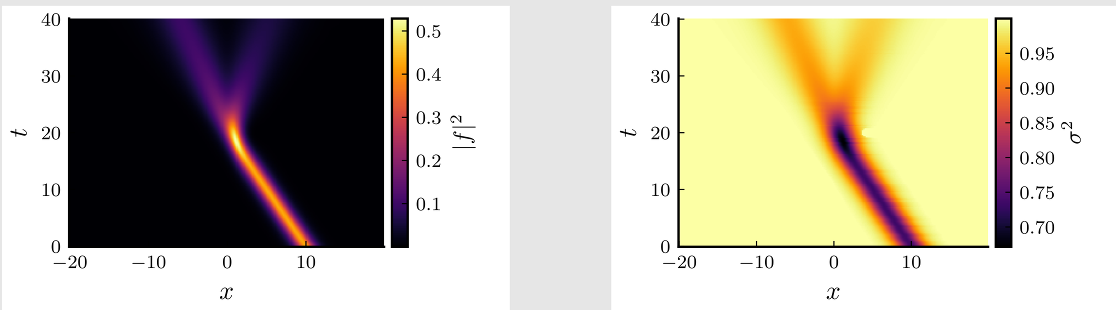
$$U(x,y,z) = \frac{1}{2}m\omega_\perp^2(y^2+z^2) + V(x), \quad (2)$$




(a) $v = 0.5$ (b) $v = 1$

## Dynamics at the collision

When a one-dimensional soliton impinge on a narrow barrier, the transmission coefficient $T$ is expected to be a **discontinuous function** of $v$, the impinging velocity, and $b$, the barrier energy, for sufficiently low values of the parameters. As shown in a previous work, collapse can be initiated [MALOMED] by the collision event, so it is important to distinguish between static and dynamic collapse.

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## Dimensional reduction strategies for the Gross-Pitaevskii equation

Starting from the GP Lagrangian in 3D:

$$\mathcal{L} = \int d^3\mathbf{r}\;\psi^*\left[i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\nabla^2 - U - \frac{g}{2}(N-1)|\psi|^2\right]\psi, \quad (3)$$

with

$$g = \frac{4\pi\hbar^2 a_s}{m}, \quad (4)$$

one can compute the corresponding Euler-Lagrange equation:

**ciao**

$$V(x;\ b) = b\ \exp[-\frac{x^2}{2w^2}]. \quad (5)$$

A better approach is to assume a **variable transverse width**

$$\phi(y,z,\sigma(x,t)) = \frac{1}{\sqrt{\pi}\sigma(x,t)}\exp\left[-\frac{y^2+z^2}{2\sigma(x,t)^2}\right] \quad (6)$$

Writing the 3D GP Lagrangian, we may hope to integrate along the transverse variables
Integrating the term proportional to $\frac{\partial}{\partial t}\sigma$ gives $0$, as one may realize by looking at the symmetry of its prefactor. However, the term proportional to $\frac{\partial^2}{\partial x^2}\phi$ gives a non-null contribution to the 1D Lagrangian.
Let us now consider the corresponding Euler-Lagrange equations. These will be computed for $f$ and $\sigma$, thus $\sigma$ is a proper variational parameter, and constitute a set of coupled PDE and ODE.

## One dimensional effective equations

The simplest dimensional reduction

$$\text{GPE:}\quad i\hbar\frac{\partial}{\partial t}f = \left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}^2 + V + \hbar\omega_\perp + \frac{N-1}{2\pi\sigma^2}g|f|^2\right] \quad (7)$$

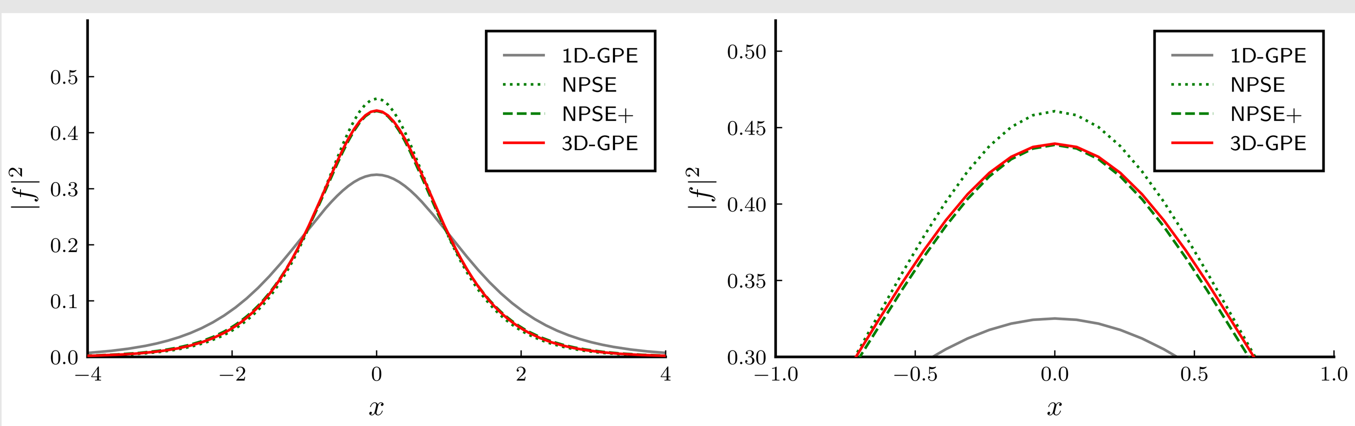
The effective equations that are compared are NPSE

$$\text{NPSE:}\begin{cases} i\hbar\frac{\partial}{\partial t}f = \left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}^2 + V + \frac{\hbar^2}{2m}\frac{1}{\sigma^2} + \frac{m\omega_\perp^2}{2}\sigma^2 + \frac{N-1}{2\pi\sigma^2}g|f|^2\right] \\ -m\omega_\perp^2\sigma + \left[\frac{\hbar^2}{m} + \frac{N-1}{2\pi}g|f|^2\right]\sigma^{-3} + \frac{\hbar^2}{m}\sigma^{-3}\left(\sigma\frac{\partial}{\partial x}^2\sigma - \left(\frac{\partial}{\partial x}\sigma\right)^2\right) = 0. \end{cases} \quad (8)$$

NPSE+

$$\text{NPSE+:}\begin{cases} i\hbar\frac{\partial}{\partial t}f = \left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial x}^2 + V + \frac{\hbar^2}{2m}\frac{1}{\sigma^2} + \frac{m\omega_\perp^2}{2}\sigma^2 + \frac{N-1}{2\pi\sigma^2}g|f|^2\right] \\ -m\omega_\perp^2\sigma + \left[\frac{\hbar^2}{m} + \frac{N-1}{2\pi}g|f|^2\right]\sigma^{-3} + \frac{\hbar^2}{m}\sigma^{-3}\left(\sigma\frac{\partial}{\partial x}^2\sigma - \left(\frac{\partial}{\partial x}\sigma\right)^2\right) = 0. \end{cases} \quad (9)$$

## Soliton-like solutions

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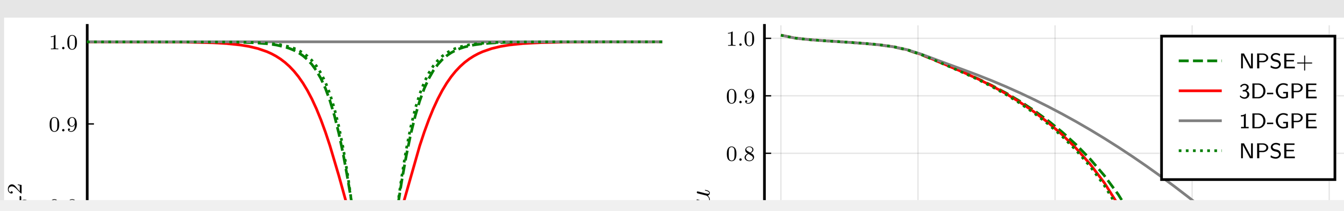


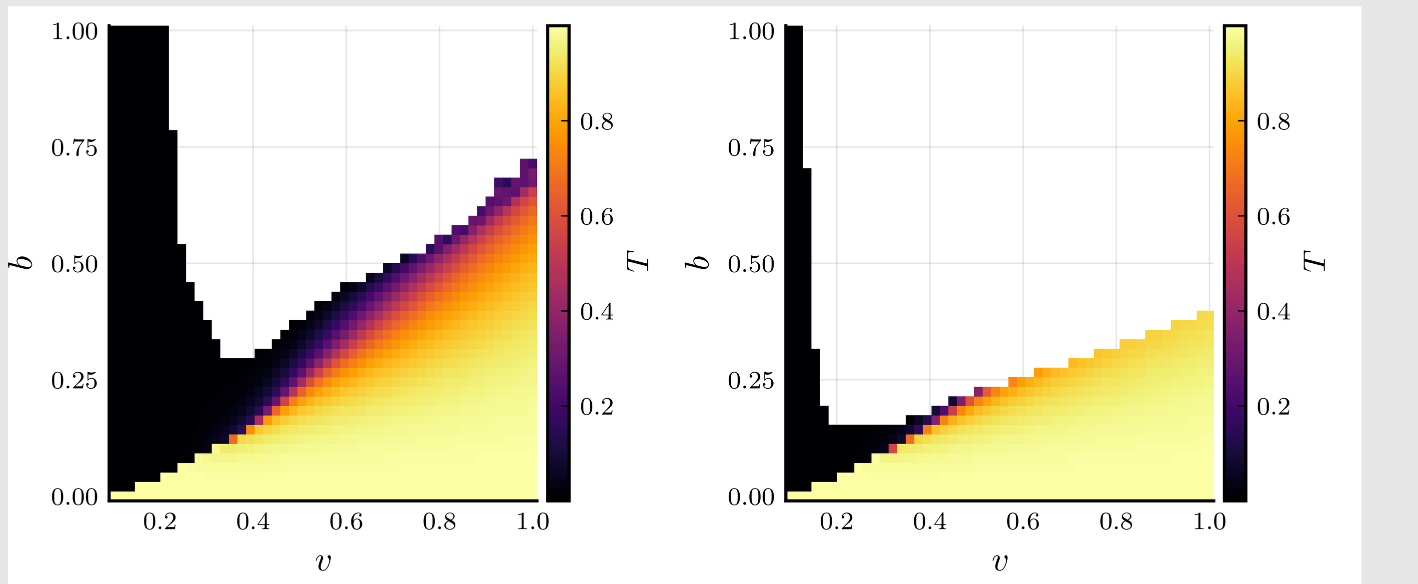

## Numerical methods

We assume the field to be localized away from the boundary in order to neglect this problem. The time step in both setups is chosen to be $h_t = 0.01$. These parameters have been proven to give a total truncation error in the $L_\infty$ norm of the order of $10^{-4}$, and allow for a reasonable computation time of all the calculations.
The ground state solutions are computed using an imaginary-time propagation method. We point out that some modifications of this method are available under the name of normalized gradient-flow methods.
We have an isotropic confinement, in which we have units: **energy** $\longrightarrow \hbar\omega_\perp$, **time** $\rightarrow \omega_\perp^{-1}$, **length** $\longrightarrow l_\perp$.

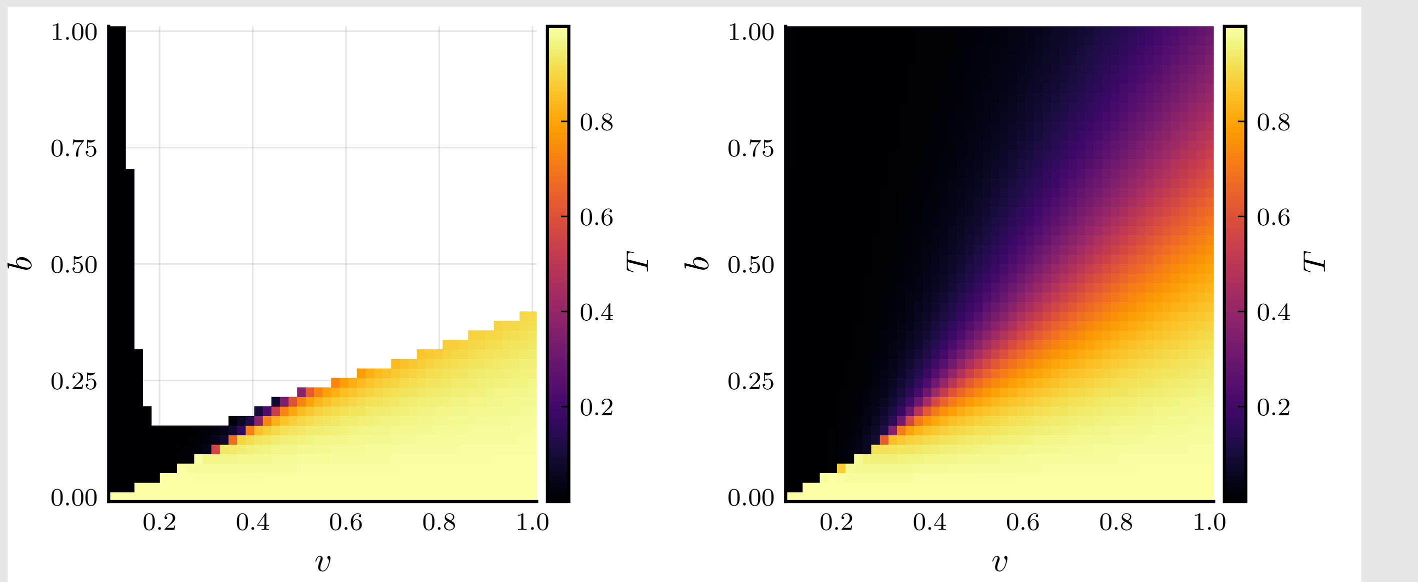
- 1D simulations a total length of $L = 40$, with a grid of $N = 512$ points.
- 3D simulations, we use a grid of $(N_x, N_y, N_z) = (512, 40, 40)$ points, with total lengths of $(L_x, L_y, L_z) = (40, 10, 10)$.

Collapse threshold is set to a probability per point of $0.3$. The time step in both setups is chosen to be $h_t = 0.01$. These parameters have been proven to give a total truncation error in the $L_\infty$ norm of the order of $10^{-4}$, and allow for a reasonable computation time of all the calculations.

## Results

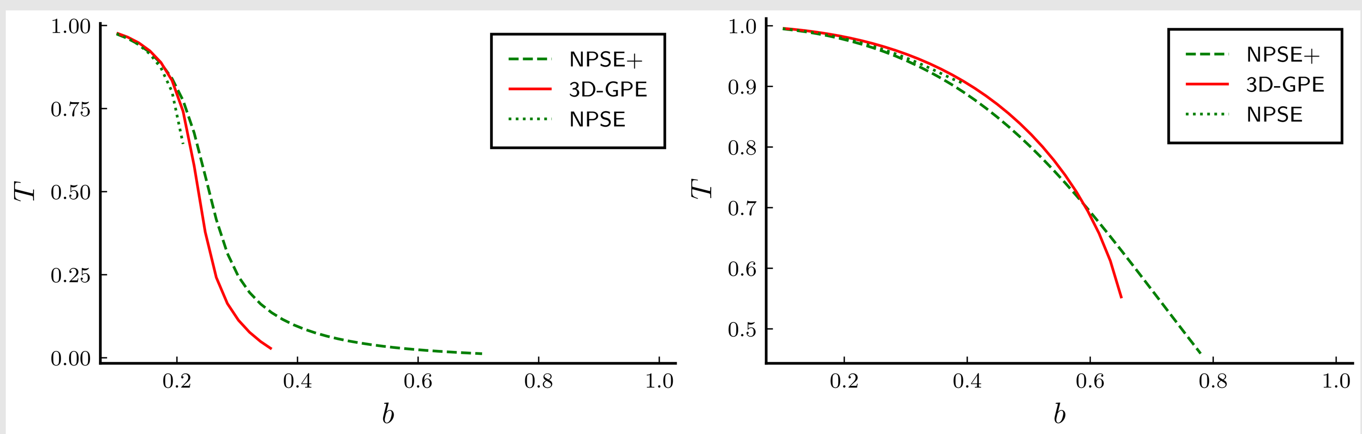



(a) 3D-GPE (b) NPSE+




(c) NPSE (d) 1D-GPE

Figure 5. Comparison of the transmission coefficient and the collapse regions for the various equations. $v$ is in units of $\sqrt{\hbar\omega_\perp/m}$, $b$ is in units of $\hbar\omega_\perp$.

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## Conclusions and perspectives

We investigated how the choice of **dimensional reduction** impacts the description of some features of the process, namely the transmission coefficient and the onset of barrier-induced collapse, also using the familiar one-dimensional Gross-Pitaevskii. We first reviewed the **ground state properties** given by all the schemes, highlighting the role of the variational transverse width. Then we compared the scattering properties: our results show that by using the NPSE in a regime of high barrier energy and high velocity it **fails to describe the 3D dynamics** due to the vanishing of the transverse width of the solution. Our main result is that by adopting a slight modification of the transverse width using the **true variational** solution with the NPSE ansatz, the method has a good agreement with the 3D-GPE.

## References

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