Variational approach to multimode nonlinear optical fibers

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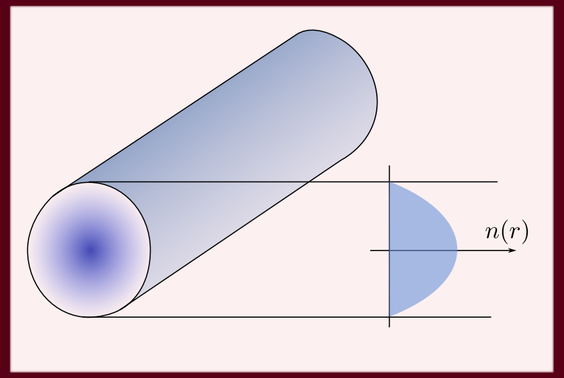
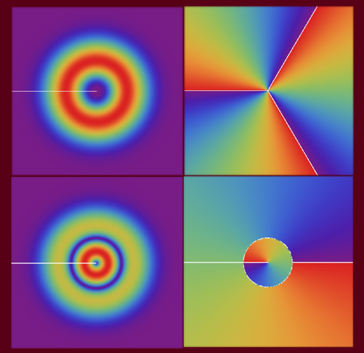




$$+\frac{g}{2}|\Phi|^4$$

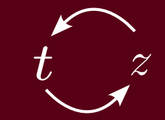
Helmholtz equation

$$\left[\nabla^2 + \beta^2(\omega)\right]\tilde{\mathbf{E}}(\mathbf{r},\omega) = \mathbf{0}$$

$$i\partial_z\Phi(x,y,z,t) = \Big[-\frac{1}{2}(\partial_x^2+\partial_y^2+\partial_t^2) + W(x,y) + g|\Phi(x,y,z,t)|^2\Big]\Phi(x,y,z,t)$$

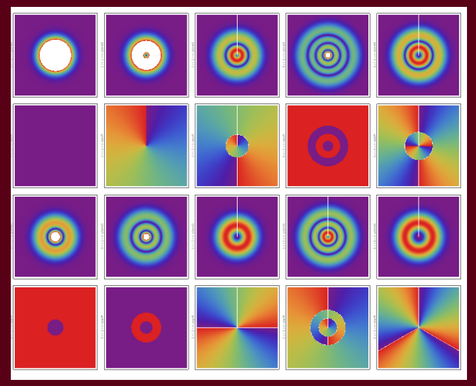
$$\mathscr{L} = \tfrac{i}{2}(\Phi^*\partial_z\Phi - \Phi\partial_z\Phi^*) - \tfrac{1}{2}(|\partial_x\Phi|^2 + |\partial_y\Phi|^2 + \underbrace{|\partial_t\Phi|^2}_{\text{anomalous dispersion}}) - \underbrace{W(x,y)}_{\text{harmonic external potential}}|\Phi|^2 - \underbrace{\tfrac{g}{2}}_{\text{Kerr nonlinearity}}|\Phi|^4$$

Formally analogous to GPE within



variational ansatz

$$\Phi_{nm}(r,\theta,z,t) = A_{nS}(z,t)\,T_{nS}(r;\sigma_{nS}(z,t))\;e^{im\theta}$$



Laguerre-Gauss modes

$A_{nS}(z,t)$
$\sigma_{nS}(z,t)$

variational parameters

(3+1)-D

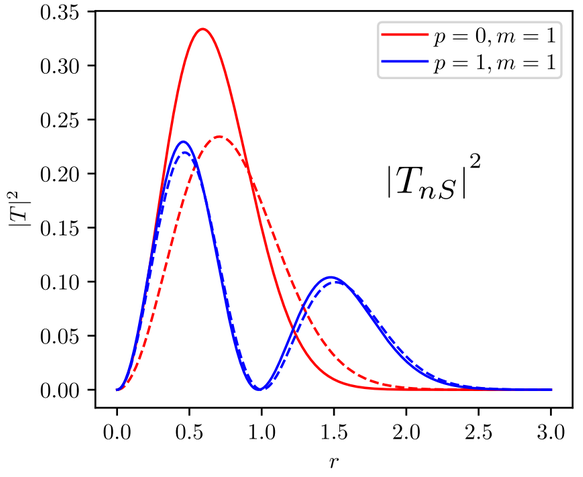
Dimensional reduction

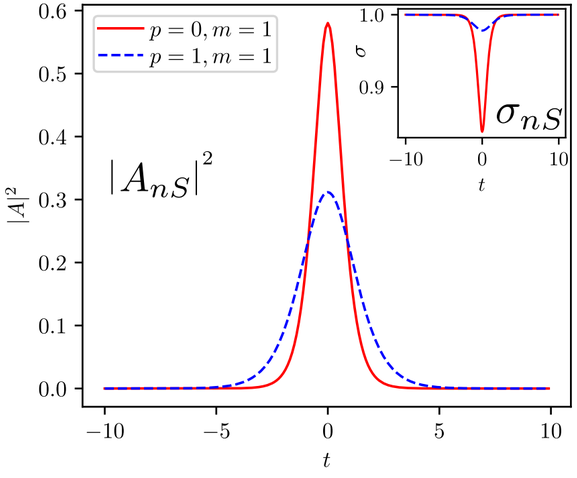
quasi (1+1)-D

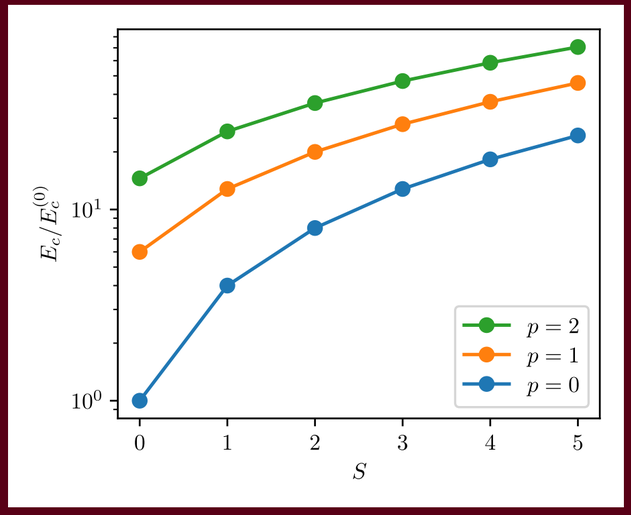
$$i\partial_z A = -\frac{1}{2}\partial_t^2 A + \xi_{pS}\left(\frac{1+(3/2)g_{pS}|A|^2}{\sqrt{1+g_{pS}|A|^2}}\right)A$$

$$\sigma^4 = 1 + g_{pS}|A|^2$$

Nonpolynomial Schrödinger equation





analytical (implicit) **solitonic** solutions

stability estimate through Vakhitov Kolokolov criterion