

Natural Language Processing - 2nd Semester (2024-2025)  
1038141

## 1.9 - Syntax



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\*\*credits are reported in the last slide

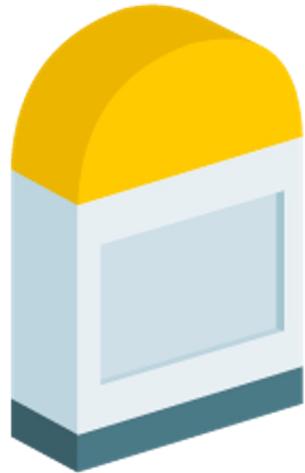


## 1.9 - Syntax

- Syntax
- Formal Grammars, Context-Free Grammars, Chomsky Normal Form, Constituent Grammars vs. Dependency Grammars, Ambiguity, Treebanks Syntactic Parsing, Cocke-Kasami-Younger Algorithm, Early Algorithm, Probabilistic CFG, Probabilistic CKY Algorithm
- Exercises
- Q&A

# Milestones in NLP

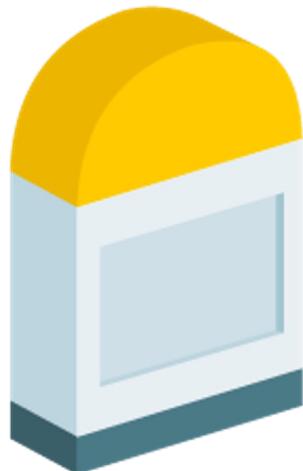
today topic is fundamental for the three areas.



rule-based systems



statistical classical machine learning  
models



deep learning models

# Syntax

## **definition:**

Syntax is the branch of linguistics that studies the way words are arranged together in sentences.

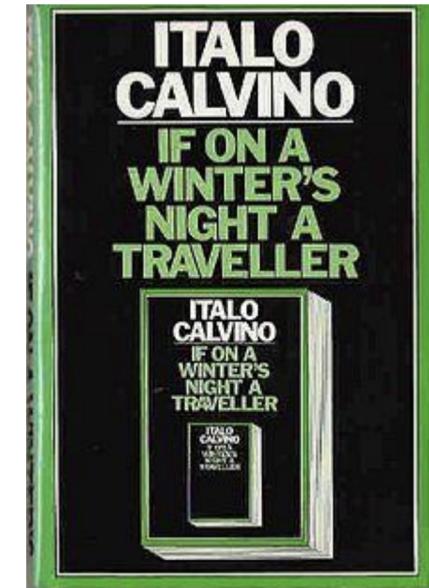
# Syntax

Consider the following book title:

*"If on a winter's night a traveler"*

and the following translations:

- *Se una notte d'inverno un viaggiatore*
- *\*Se notte una d'inverno un viaggiatore*
- *Una notte se d'inverno un viaggiatore*
- *\*Se un notte d'inverno una viaggiatore*
- *Se una notte un viaggiatore d'inverno*
- *Se un viaggiatore d'inverno una notte*
- *\*Se un una notte viaggiatore d'inverno*
- *\*Se un una d'notte viaggiatore inverno*
- *~Se un inverno d'notte un viaggiatore*
- *Se d'inverno un viaggiatore una notte*
- *Se d'inverno una notte un viaggiatore*



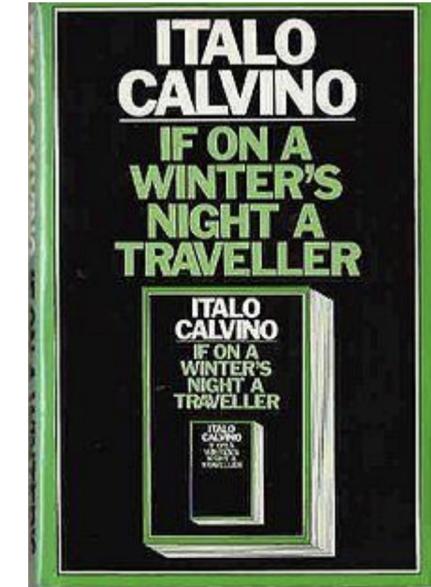
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- *\*Se un una d'notte viaggiatore inverno*
- *~Se un inverno d'notte un viaggiatore*
- *Se d'inverno un viaggiatore una notte*
- *Se d'inverno una notte un viaggiatore*



- some are good translations
- others are not syntactically correct
- part of the problem is related to the **order** of words

# Syntax

The “word order” problem has been partially addressed with:

- Regular languages and regular expressions
- N-gram models: Computing probabilities for word sequences
- Part-of-speech classes: equivalence classes for words



# We move to ...

We move to ...

formal grammars!



## Formal Grammars

Grammars are able to determine the correct syntax of a language.

We like Grammars but we love Formal Grammars.



# Context-free Grammars

A **context-free grammar (CFG)** or **phrase-structure grammar** is a formal grammar defined as a 4-tuple:

$$G = (N, T, P, S)$$

where:

- **N** is the set of **nonterminal symbols (phrases or clauses)**
- **T** is the set of **terminal symbols (lexicon)**
- **P** is the set of **productions (rules)**,  $P \subseteq N \times (N \cup T)^*$
- **S** is the **start symbol** such that  $S \in N, \exists (S, \beta) \in P$

# Example of CFG

$$G = (N, T, P, S)$$

where:

- $N$  is the set of nonterminal symbols (phrases or clauses)
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- $S$  is the start symbol such that  $|S \in N, \exists(S, \beta) \in P$

$$N = \{ S, NP, Nom, Det, Noun \}$$

$$T = \{ a, the, winter, night \}$$

$$\begin{aligned} P = \{ & S \rightarrow NP, \\ & NP \rightarrow Det\ Nom, \\ & Nom \rightarrow Noun \mid Nom\ Noun, \\ & Det \rightarrow a \mid the, \\ & Noun \rightarrow winter, \\ & Noun \rightarrow night \} \end{aligned}$$

S

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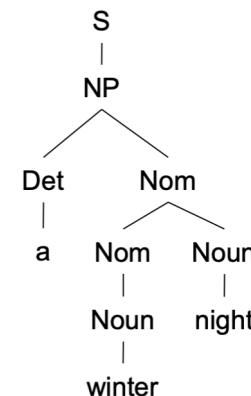
S

# CFGs

$N = \{ S, NP, Nom, Det, Noun \}$   
 $T = \{ a, the, winter, night \}$   
 $P = \{ S \rightarrow NP,$   
 $NP \rightarrow Det\ Nom,$   
 $Nom \rightarrow Noun \mid Noun$   
 $Noun,$   
 $Det \rightarrow a \mid the,$   
 $Noun \rightarrow winter,$   
 $Noun \rightarrow night \}$   
**S**

**"a winter night"**

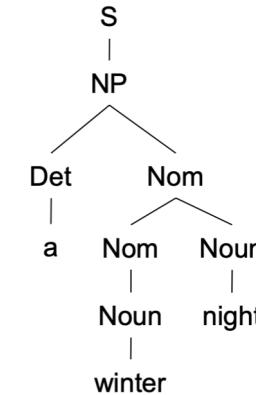
**Generating sentences**



**derivations**

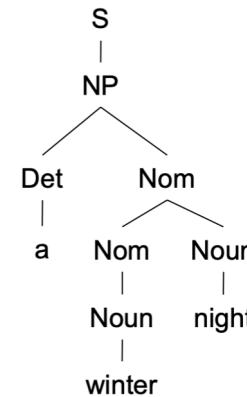
**Analysing sentence structures**

**all possible derivations**



**"a winter night"**

**Parse tree**



# $L(G)$

**definition:**

$L(G)$  is the set of strings of terminals  $w$  for which there is a sequence of productions (derivation) that yield  $w$  from  $S$ .

**formally:**

$$L(G) = \{ w \in T^* : S \Rightarrow^* w \}$$

$L(G)$  is the language of the grammar  $G$

$L(G)$  is a context-free language if  $G$  is context-free

$G$  is a CFG if is defined as:

$$G = (N, T, P, S)$$

where:

- $N$  is the set of nonterminal symbols (phrases or clauses)
- $T$  is the set of terminal symbols (lexicon)
- $P$  is the set of productions (rules),  $P \subseteq N \times (N \cup T)^*$
- $S$  is the start symbol such that  $S \in N, \exists (S, \beta) \in P$

# CFG in Normal Forms

a CFG is in Chomsky Normal Form (CNF) if:

- each production is in one of the following form:  
 $A \rightarrow B C$   
 $A \rightarrow a$
- the production of  $\epsilon$  is not allowed ( $\epsilon$ -free)

CNF grammars generate **binary branching**

Any CFG grammar can be converted into a **weakly equivalent** CNF grammar:

Convert rules of the kind:  $A \rightarrow B C D$  into two productions:

- $A \rightarrow B X$
- $X \rightarrow C D$

# Dependency Grammars

instead of defining CFGs where N as a set of constituents (e.g. NP etc...) which enable the definition of phrase-structure rules;

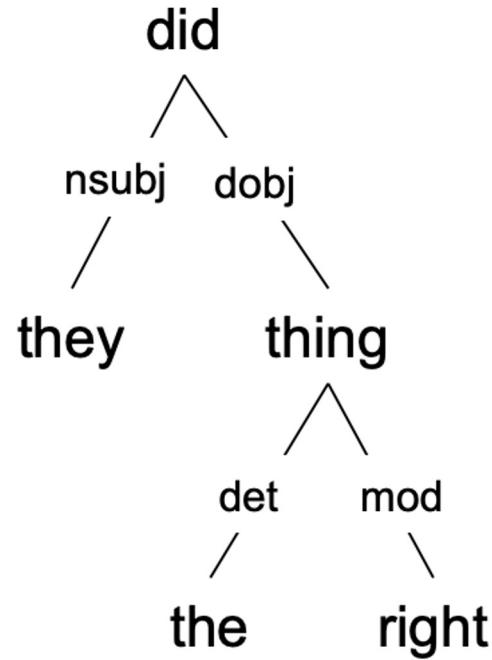
we define CFGs, where:

productions express lexical, syntactic, or semantic relations between words, called dependencies

each production is labelled with a dependency type

# Example of a typed dependency parse tree

"They did the right thing"



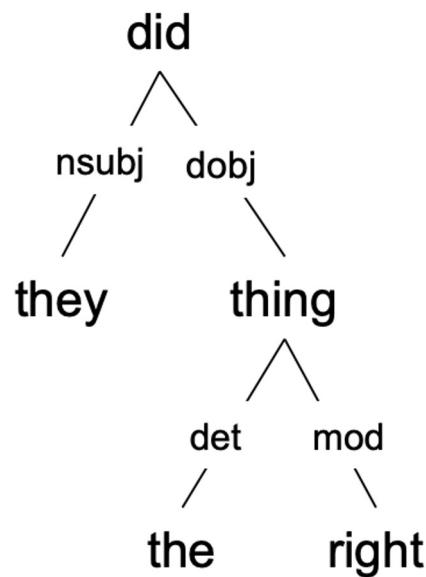
- No non-terminal or phrasal nodes!
- derivation links are labeled (a.k.a. typed dependency parse)

Argument Dependencies	Description
nsubj	They → did
dobj	did → thing
iobj	gave → me
pobj	to → me
Modifier Dependencies	Description
tmod	studied → yesterday
det	the → thing
prep	on → shelf

nsubj: nominal subject  
dobj: direct object  
iobj: indirect object  
pobj: object of preposition  
tmod: temporal modifier  
det: determiner  
prep: prepositional modifier  
...

[https://downloads.cs.stanford.edu/nlp/software/dependencies\\_manual.pdf](https://downloads.cs.stanford.edu/nlp/software/dependencies_manual.pdf)

# Advantages of Dependency Grammars



the structures are **word order-free**:

- better for languages like the Italian or Czech in which the order of words is much more flexible;
- a phrase-structure grammar would need separate rules for each possible position in the parse tree.

# Treebanks

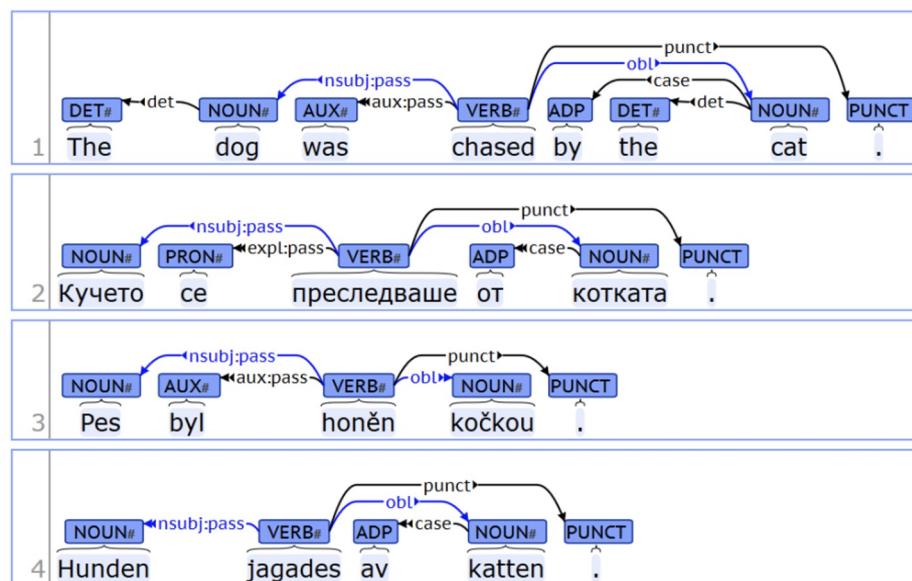
- CFGs can be used to assign a parse tree to any valid sentence
- We can build a corpus, called treebank, whose sentences are annotated with parse trees
- The most popular project of this kind is the Penn Treebank
  - From the Brown, Switchboard, ATIS and Wall Street Journal corpora of English
    - Wall Street Journal: 1.3 million words
    - Brown Corpus: 1 million words
    - Switchboard: 1 million words
  - All tagged with Part-of-Speech & syntactic structure
  - Developed 1988-1994

# Universal dependencies

(<https://universaldependencies.org>)

A project that provides treebank annotations that:

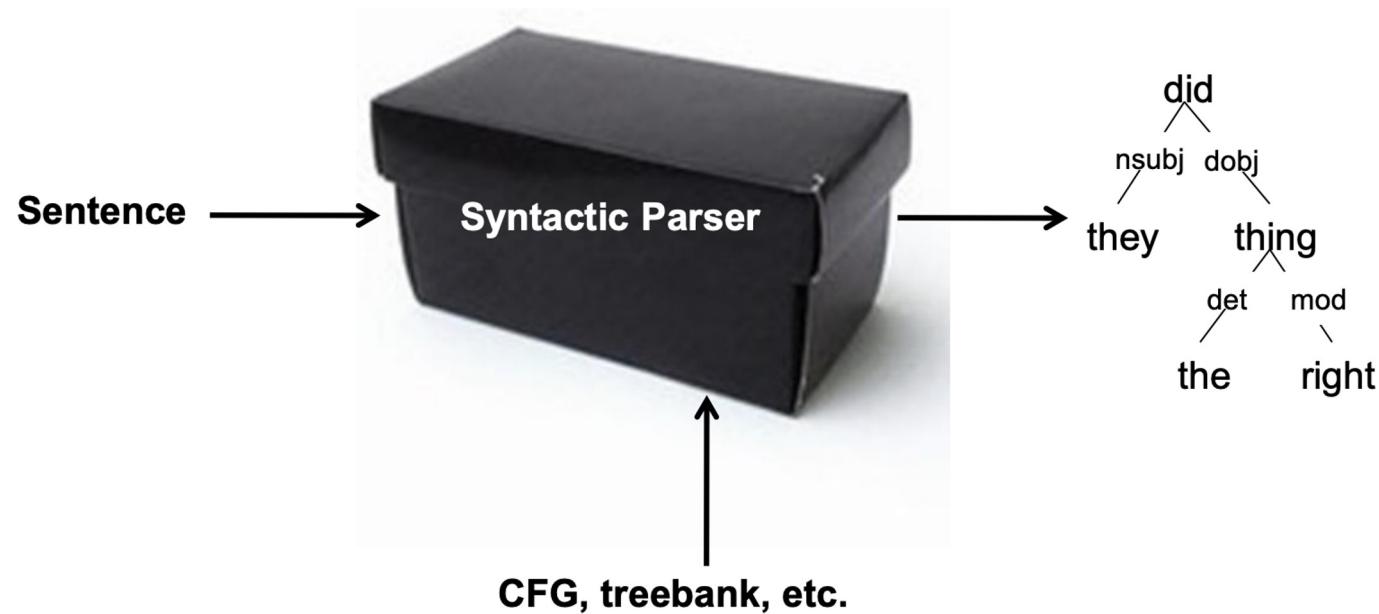
- are cross-linguistically consistent for many languages
- facilitate:
  - multilingual parser development
  - cross-lingual learning
  - parsing research also across languages



# Syntactic Parsing

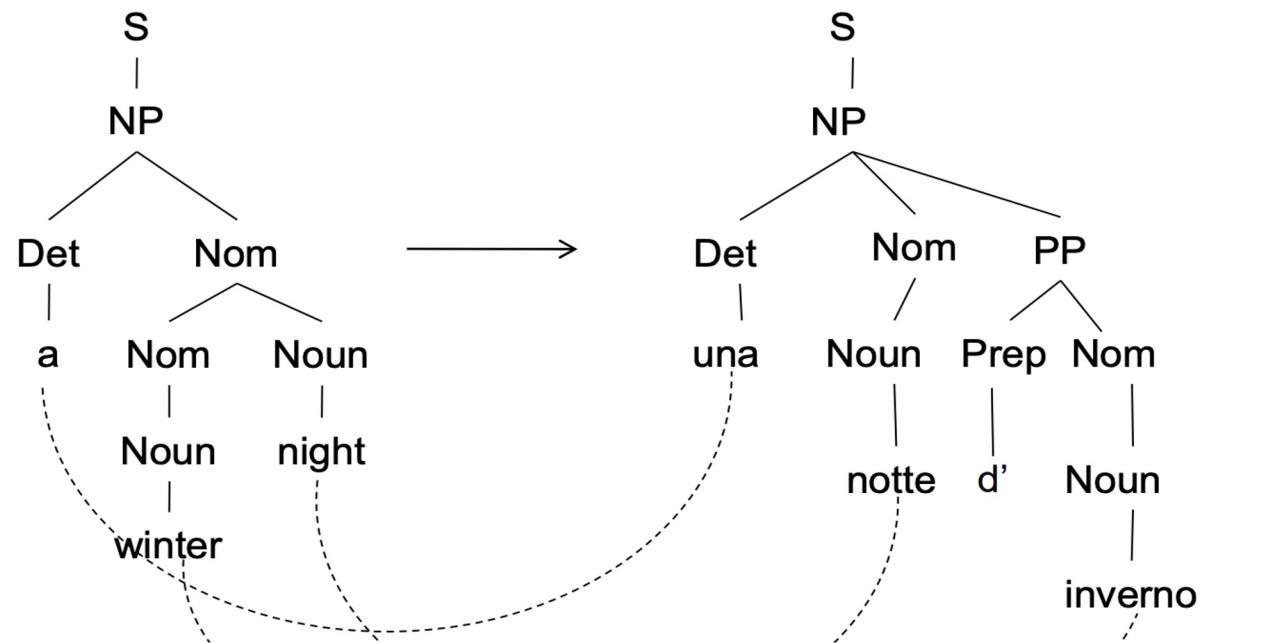
**definition:**

Syntactic Parsing is the task of recognizing a sentence and assigning a syntactic structure to it.



# Why Parsing a Sentence?

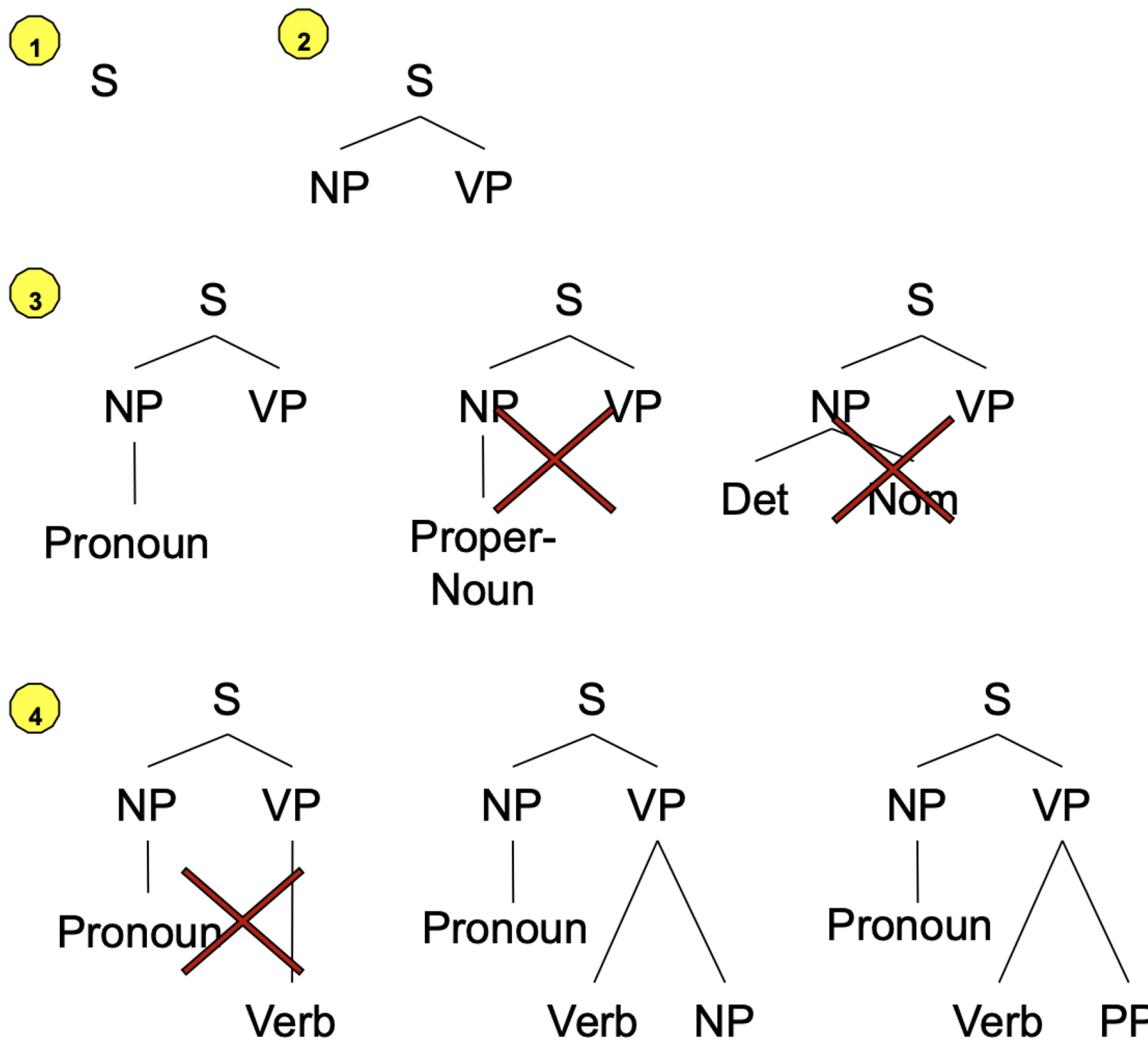
- **Grammar Checking:** dozens of real applications ...;
- **Semantic analysis:** to provide a ground for further analysis (e.g., the semantic one);
- **Question Answering:** a Question Answering system needs (at least...): the subject of the sentence (what books), the main verb (write), the by-adjunct (Raymond Queneau), etc;
- **Machine Translation:** to provide a syntactic-coherent ground alignment.



## Constituency Parsing is a Search Task

- We search through the space of possible parse trees for the best parse tree for a given sentence
- We have two constraints:
  1. Data constraint: given an input sentence of  $k$  words, the parse tree **must have  $k$  leaves**
  2. Grammar constraint: the final parse tree **must have one root**
- Depending on the constraint we start from, we have two search strategies:
  - Top-down or goal-directed search
  - Bottom-up or data-directed search

# Top-down Parsing: “He is a student in Rome”



$S \rightarrow NP\ VP$   
 $NP \rightarrow \text{Pronoun} \mid \text{Proper-Noun} \mid \text{Det Nom}$   
 $\text{Nom} \rightarrow \text{Nom Noun} \mid \text{Noun}$   
 $VP \rightarrow \text{Verb}$   
 |  $\text{Verb NP}$   
 |  $\text{Verb NP PP}$   
 |  $\text{Verb PP}$   
 $PP \rightarrow \text{Preposition NP}$   
 $\text{Noun} \rightarrow \text{student} \mid \text{researcher} \mid \text{research}$   
 $\text{Verb} \rightarrow \text{am} \mid \text{is}$   
 $\text{Adjective} \rightarrow \text{bright}$   
 $\text{Pronoun} \rightarrow \text{me} \mid \text{I} \mid \text{he} \mid \text{you} \mid \text{it} \mid \text{him} \mid \text{her}$   
 $\text{Proper-Noun} \rightarrow \text{Rome} \mid \text{Sapienza}$   
 $\text{Det} \rightarrow \text{the} \mid \text{a} \mid \text{an}$   
 $\text{Preposition} \rightarrow \text{from} \mid \text{to} \mid \text{on} \mid \text{in} \mid \text{near} \mid \text{at}$   
 $\text{Conjunction} \rightarrow \text{and} \mid \text{or} \mid \text{but}$

# Bottom-up Parsing: “He is a student in Rome”

1

He is a student in Rome

$S \rightarrow NP VP$

$NP \rightarrow Pronoun \mid Proper-Noun \mid Det\ Nom$

$Nom \rightarrow Nom\ Noun \mid Noun$

$VP \rightarrow Verb$

$\mid Verb\ NP$

$\mid Verb\ NP\ PP$

$\mid Verb\ PP$

$PP \rightarrow Preposition\ NP$

$Noun \rightarrow student \mid researcher \mid research$

$Verb \rightarrow am \mid is$

$Adjective \rightarrow bright$

$Pronoun \rightarrow me \mid I \mid he \mid you \mid it \mid him \mid her$

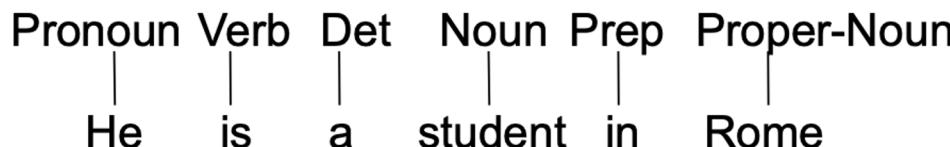
$Proper-Noun \rightarrow Rome \mid Sapienza$

$Det \rightarrow the \mid a \mid an$

$Preposition \rightarrow from \mid to \mid on \mid in \mid near \mid at$

$Conjunction \rightarrow and \mid or \mid but$

2



$S \rightarrow NP VP$

$NP \rightarrow Pronoun \mid Proper-Noun \mid Det\ Nom$

$Nom \rightarrow Nom\ Noun \mid Noun$

$VP \rightarrow Verb$

$\mid Verb\ NP$

$\mid Verb\ NP\ PP$

$\mid Verb\ PP$

$PP \rightarrow Preposition\ NP$

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$Verb \rightarrow am \mid is$

$Adjective \rightarrow bright$

$Pronoun \rightarrow me \mid I \mid he \mid you \mid it \mid him \mid her$

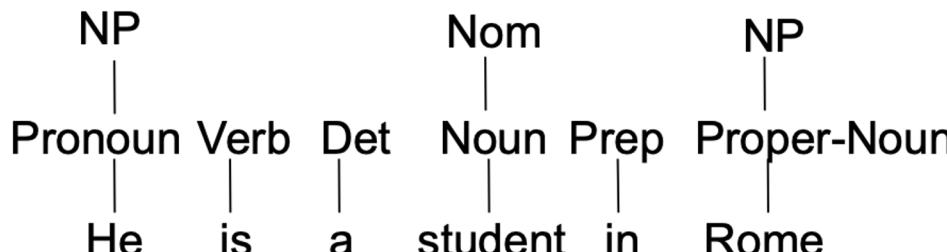
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$VP \rightarrow Verb$

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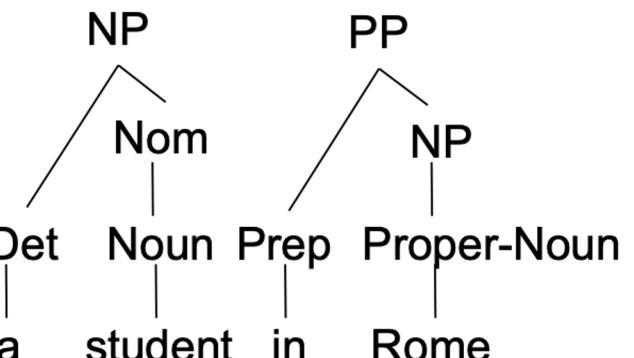
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$Det \rightarrow the \mid a \mid an$

$Preposition \rightarrow from \mid to \mid on \mid in \mid near \mid at$

$Conjunction \rightarrow and \mid or \mid but$

4



$S \rightarrow NP VP$

$NP \rightarrow Pronoun \mid Proper-Noun \mid Det\ Nom$

$Nom \rightarrow Nom\ Noun \mid Noun$

$VP \rightarrow Verb$

$\mid Verb\ NP$

$\mid Verb\ NP\ PP$

$\mid Verb\ PP$

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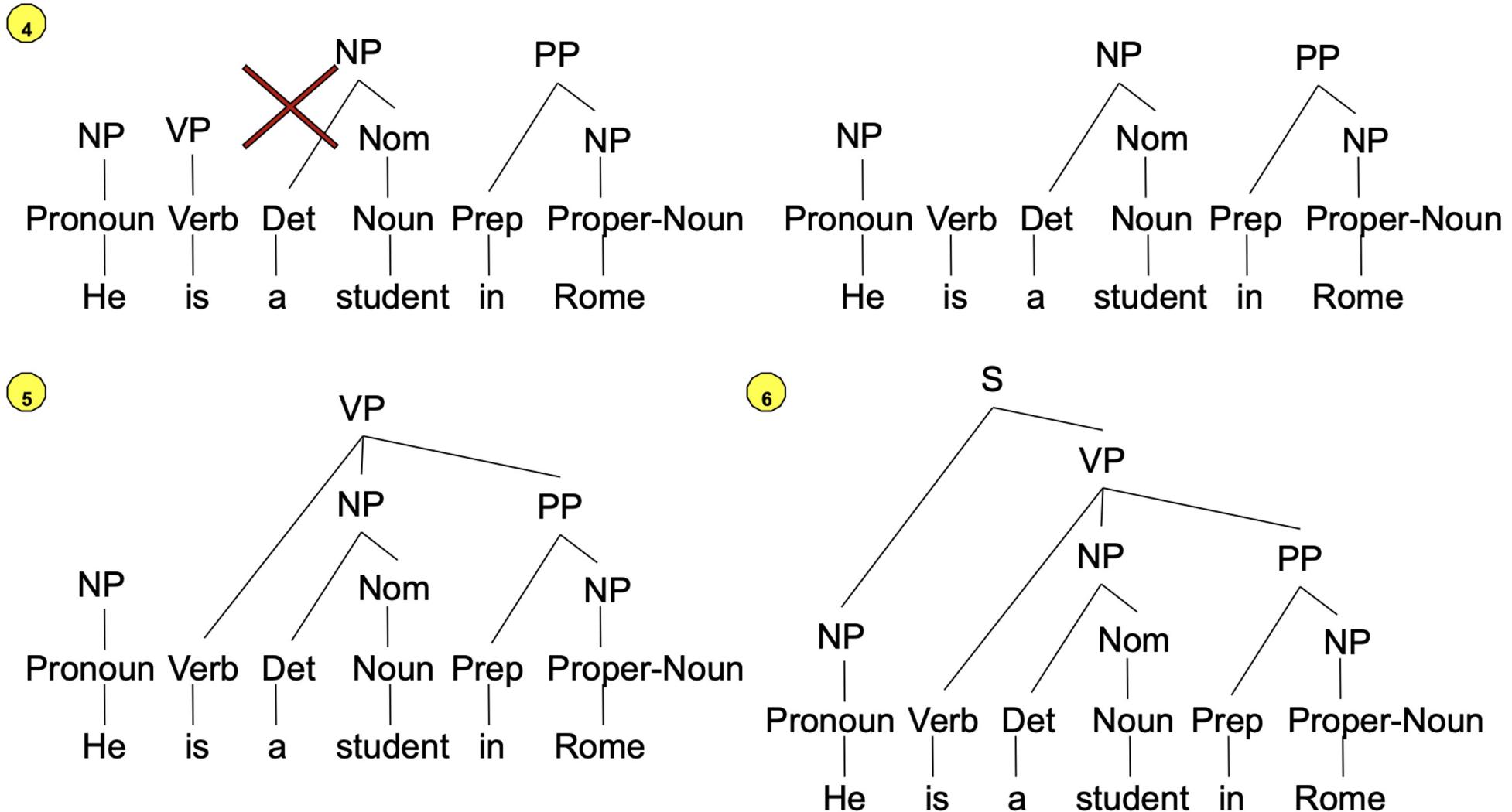
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$Det \rightarrow the \mid a \mid an$

$Preposition \rightarrow from \mid to \mid on \mid in \mid near \mid at$

$Conjunction \rightarrow and \mid or \mid but$

## Bottom-up Parsing: “He is a student in Rome”

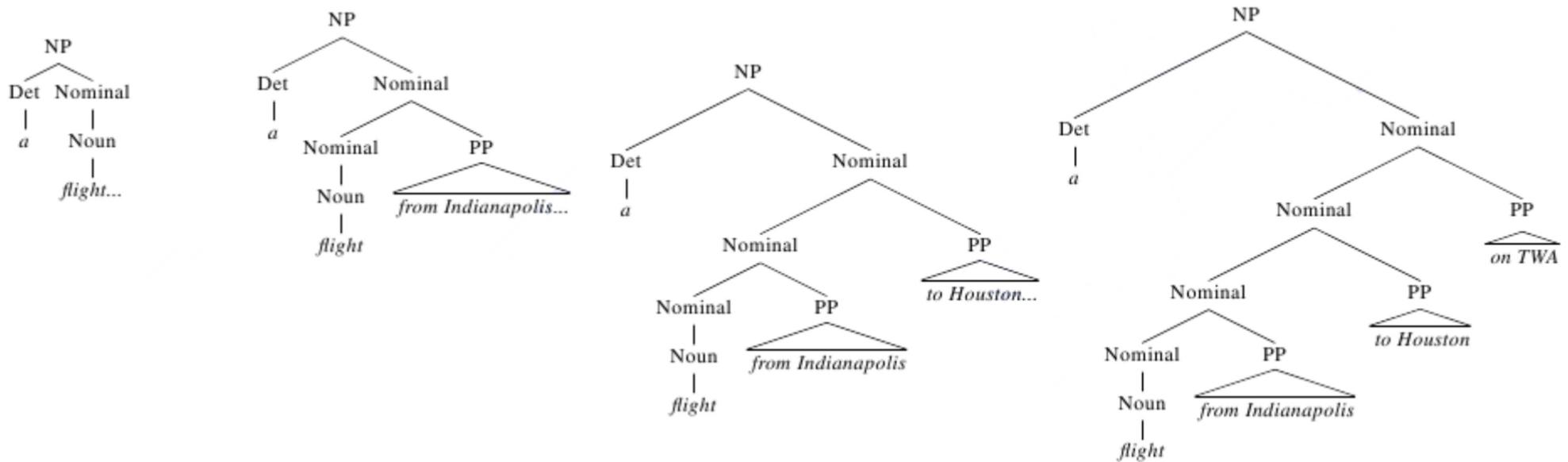


# Structural Ambiguity

- A kind of ambiguity which arises in parse trees
- Structural ambiguity is due to the existence of more parse trees for the same sentence
- Choosing the correct parse tree is a matter of syntactic disambiguation
- Even if a sentence is not ambiguous, it can be inefficient to parse because of local ambiguity
  - The sentence is not ambiguous globally, like in “Book that flight”
  - When the parser sees “book” it does not know its POS

# Using Backtracking to Parse

- Is a top-down, depth-first, left-to-right backtracking strategy efficient?



- Much parsing work is repeated many times

# Dynamic Programming

- Dynamic programming uses memory tables to save solutions to sub-problems
- Tables save subtrees for each constituent of the input sentence
  - No repeated parsing of the same phrase
  - Subtrees are looked up, not parsed again and again!



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Can you tell when we already discussed of Dynamic Programming algorithms in this course?



# The Cocke-Kasami-Younger (CKY) Algorithm

- A bottom-up dynamic programming parsing approach
- Takes as input a CFG in Chomsky Normal Form
- Given a sentence of  $n$  words, we need an  $(n+1) \times (n+1)$  matrix
- Cell  $(i,j)$  contains the set of non-terminals that produce all the constituents spanning positions from  $i$  to  $j$  of the input sentence
- The cell that represents the entire sentence is  $(0,n)$
- Main idea: if a non-terminal  $A$  is in  $(i,j)$ , there is a production  $A \rightarrow B C$ , so there must be an intermediate position  $k$  with  $B$  in  $(i,k)$  and  $C$  in  $(k,j)$

## How to fill in the table?

- Bottom-up
- Given a cell  $(i,j)$  all the cells containing its parts are already filled
  - Cells to the left and below

```
function CKY-PARSE(words, grammar) returns table
    for  $j \leftarrow$  from 1 to LENGTH(words) do
        for all  $\{A \mid A \rightarrow words[j] \in grammar\}$ 
            table[ $j - 1, j$ ]  $\leftarrow$  table[ $j - 1, j$ ]  $\cup$  A
        for  $i \leftarrow$  from  $j - 2$  down to 0 do
            for  $k \leftarrow i + 1$  to  $j - 1$  do
                for all  $\{A \mid A \rightarrow BC \in grammar \text{ and } B \in table[i, k] \text{ and } C \in table[k, j]\}$ 
                    table[ $i, j$ ]  $\leftarrow$  table[ $i, j$ ]  $\cup$  A
```

- An input sentence is **recognized** if  $(0,n)$  contains S

# Exercise

*Book the flight through Houston*

```

function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
  
```

words	Book	the	flight	through	Houston
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

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        for k ← i + 1 to j - 1 do
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	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

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    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book   include   prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I   she   me$
$NP \rightarrow TWA   Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book   flight   meal   money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book   include   prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$
<b>Lexicon</b>
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```

function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
    
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	<b>Det</b>				
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**

$Det \rightarrow that \mid this \mid the \mid a$

$Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	<b>Det</b>				
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
[0,1]	S, VP, Nominal Noun, Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det				
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	



no derivations  
of the form  
\* Det  
on the right  
side

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det				
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$

*Nominal → book | flight | meal | money*

$Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

<i>Det → that   this   the   a</i>
<i>Noun → book   flight   meal   money</i>
<i>Verb → book   include   prefer</i>
<i>Pronoun → I   she   me</i>
<i>Proper-Noun → Houston   NWA</i>
<i>Aux → does</i>
<i>Preposition → from   to   on   near   through</i>

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det				
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det ← NP [1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$

$Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
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            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=3
i= 0

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det ← NP [1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$	
$S \rightarrow X1 VP$	
$X1 \rightarrow Aux NP$	
$S \rightarrow book   include   prefer$	
$S \rightarrow Verb NP$	
$S \rightarrow X2 PP$	
$S \rightarrow Verb PP$	
$S \rightarrow VP PP$	
$NP \rightarrow I   she   me$	
$NP \rightarrow TWA   Houston$	
$NP \rightarrow Det Nominal$	
$Nominal \rightarrow book   flight   meal   money$	
$Nominal \rightarrow Nominal Noun$	
$Nominal \rightarrow Nominal PP$	
$VP \rightarrow book   include   prefer$	
$VP \rightarrow Verb NP$	
$VP \rightarrow X2 PP$	
$X2 \rightarrow Verb NP$	
$VP \rightarrow Verb PP$	
$VP \rightarrow VP PP$	
$PP \rightarrow Preposition NP$	
<b>Lexicon</b>	
$Det \rightarrow that   this   the   a$	
$Noun \rightarrow book   flight   meal   money$	
$Verb \rightarrow book   include   prefer$	
$Pronoun \rightarrow I   she   me$	
$Proper-Noun \rightarrow Houston   NWA$	
$Aux \rightarrow does$	
$Preposition \rightarrow from   to   on   near   through$	

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book   include   prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I   she   me$
$NP \rightarrow TWA   Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book   flight   meal   money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book   include   prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$
<b>Lexicon</b>
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

no symbols in  
[0,2] we skip

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=3
i= 0

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	[3,5]	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that \mid this \mid the \mid a$
$Noun \rightarrow book \mid flight \mid meal \mid money$
$Verb \rightarrow book \mid include \mid prefer$
$Pronoun \rightarrow I \mid she \mid me$
$Proper-Noun \rightarrow Houston \mid NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=4  
i= 2

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	[3,5]	
					[4,5]

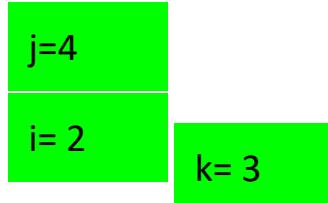
$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
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                table[i, j] ← table[i, j] ∪ A
```



words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

no derivations  
of the form  
B Preposition  
on the right  
side

$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book   include   prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I   she   me$
$NP \rightarrow TWA   Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book   flight   meal   money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book   include   prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$
<b>Lexicon</b>
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=4
i= 1

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

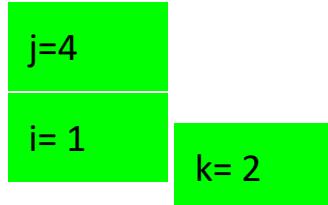
Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
```

```

for j ← from 1 to LENGTH(words) do
    for all {A | A → words[j] ∈ grammar}
        table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
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                table[i, j] ← table[i, j] ∪ A

```



words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

no symbols in  
[2,4] we skip

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

Book the flight through Houston

function CKY-PARSE(*words*, *grammar*) returns *table*

```

for j ← from 1 to LENGTH(words) do
    for all {A | A → words[j] ∈ grammar}
        table[j − 1, j] ← table[j − 1, j] ∪ A
for i ← from j − 2 down to 0 do
    for k ← i + 1 to j − 1 do
        for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
            table[i, j] ← table[i, j] ∪ A

```



<i>words</i>	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
<i>S,VP, Nominal Noun,Verb</i> [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

no derivation  
of the form \*  
Preposition on  
the right side

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
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            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
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                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
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**Lexicon**  
 $Det \rightarrow that | this | the | a$   
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# Exercise

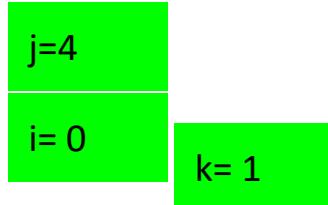
Book the flight through Houston

function CKY-PARSE(*words*, *grammar*) returns *table*

```

for j ← from 1 to LENGTH(words) do
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```



<i>words</i>	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

no symbols in  
[1,4] we skip

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
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 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
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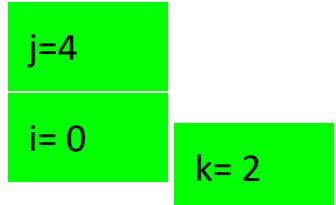
## Lexicon

$Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
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 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
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words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

no symbols in [1,2] and in [2,4] we skip

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
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 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
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 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

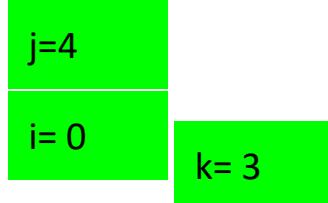
## Lexicon

$Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
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# Exercise

Book the flight through Houston

```
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    for j ← from 1 to LENGTH(words) do
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            table[j - 1, j] ← table[j - 1, j] ∪ A
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        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```



words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
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 $S \rightarrow Verb NP$   
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 $S \rightarrow Verb PP$   
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 $VP \rightarrow Verb NP$   
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 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
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 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that \mid this \mid the \mid a$
$Noun \rightarrow book \mid flight \mid meal \mid money$
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$Pronoun \rightarrow I \mid she \mid me$
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$Aux \rightarrow does$
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# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=4
i= 0

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	[3,5]	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun			
		[2,3]	[2,4]	[2,5]	
			Preposition		
			[3,4]	[3,5]	
				NP, Proper- Noun	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$

$Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	[3,5]	
				NP, Proper- Noun [4,5]	

$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book   include   prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I   she   me$
$NP \rightarrow TWA   Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book   flight   meal   money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book   include   prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$
<b>Lexicon</b>
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=3

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]		S,VP,X2 [0,3]		[0,4]
		Det [1,2]	NP [1,3]		[0,5]
				[1,4]	[1,5]
			Nominal Noun [2,3]		[2,5]
				Preposition [3,4]	[3,5]
					NP, Proper- Noun [4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=3  
k=4

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	PP [3,5]	
				NP, Proper- Noun [4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=2

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that \mid this \mid the \mid a$
$Noun \rightarrow book \mid flight \mid meal \mid money$
$Verb \rightarrow book \mid include \mid prefer$
$Pronoun \rightarrow I \mid she \mid me$
$Proper-Noun \rightarrow Houston \mid NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=2  
k=3

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	[2,4]	[2,5]	Nominal
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]

$S \rightarrow NP VP$
$S \rightarrow X1 VP$
$X1 \rightarrow Aux NP$
$S \rightarrow book   include   prefer$
$S \rightarrow Verb NP$
$S \rightarrow X2 PP$
$S \rightarrow Verb PP$
$S \rightarrow VP PP$
$NP \rightarrow I   she   me$
$NP \rightarrow TWA   Houston$
$NP \rightarrow Det Nominal$
$Nominal \rightarrow book   flight   meal   money$
$Nominal \rightarrow Nominal Noun$
$Nominal \rightarrow Nominal PP$
$VP \rightarrow book   include   prefer$
$VP \rightarrow Verb NP$
$VP \rightarrow X2 PP$
$X2 \rightarrow Verb NP$
$VP \rightarrow Verb PP$
$VP \rightarrow VP PP$
$PP \rightarrow Preposition NP$
<b>Lexicon</b>
$Det \rightarrow that   this   the   a$
$Noun \rightarrow book   flight   meal   money$
$Verb \rightarrow book   include   prefer$
$Pronoun \rightarrow I   she   me$
$Proper-Noun \rightarrow Houston   NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from   to   on   near   through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=2  
k=4

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb [0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]	
	Det [1,2]	NP [1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]	Nominal [2,4]	[2,5]	
			Preposition [3,4]	PP [3,5]	
				NP, Proper- Noun [4,5]	

no symbols in  
[2,4] we skip

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=1

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=1  
k=2

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$

$Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
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 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP	NP		
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=1  
k=4

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston \mid NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=0

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP	NP		
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
$Det \rightarrow that \mid this \mid the \mid a$
$Noun \rightarrow book \mid flight \mid meal \mid money$
$Verb \rightarrow book \mid include \mid prefer$
$Pronoun \rightarrow I \mid she \mid me$
$Proper-Noun \rightarrow Houston \mid NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=0  
k=1

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	S,VP,X2			S1,VP, X2
	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$	
$S \rightarrow X1 VP$	
$X1 \rightarrow Aux NP$	
$S \rightarrow book   include   prefer$	
$S \rightarrow Verb NP$	
$S \rightarrow X2 PP$	
$S \rightarrow Verb PP$	
$S \rightarrow VP PP$	
$NP \rightarrow I   she   me$	
$NP \rightarrow TWA   Houston$	
$NP \rightarrow Det Nominal$	
$Nominal \rightarrow book   flight   meal   money$	
$Nominal \rightarrow Nominal Noun$	
$Nominal \rightarrow Nominal PP$	
$VP \rightarrow book   include   prefer$	
$VP \rightarrow Verb NP$	
$VP \rightarrow X2 PP$	
$X2 \rightarrow Verb NP$	
$VP \rightarrow Verb PP$	
$VP \rightarrow VP PP$	
$PP \rightarrow Preposition NP$	
<b>Lexicon</b>	
$Det \rightarrow that   this   the   a$	
$Noun \rightarrow book   flight   meal   money$	
$Verb \rightarrow book   include   prefer$	
$Pronoun \rightarrow I   she   me$	
$Proper-Noun \rightarrow Houston   NWA$	
$Aux \rightarrow does$	
$Preposition \rightarrow from   to   on   near   through$	

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=0  
k=2

words	Book	the	flight	trough	Houston
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]		Nominal [2,5]	
			Preposition [3,4]	PP	
					NP, Proper- Noun [4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

Book the flight through Houston

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=0  
k=3

words	Book	the	flight	trough	Houston
[0,1]	S,VP, Nominal Noun,Verb		S,VP,X2		S1,VP, X2 S2,VP S3
	[0,2]	[0,3]	[0,4]		
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book | include | prefer$   
 $S \rightarrow Verb NP$

$S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$

$NP \rightarrow I | she | me$   
 $NP \rightarrow TWA | Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book | flight | meal | money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book | include | prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

## Lexicon

$Det \rightarrow that | this | the | a$   
 $Noun \rightarrow book | flight | meal | money$   
 $Verb \rightarrow book | include | prefer$   
 $Pronoun \rightarrow I | she | me$   
 $Proper-Noun \rightarrow Houston | NWA$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from | to | on | near | through$

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

j=5  
i=0  
k=4

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb	[0,2]	[0,3]	[0,4]	[0,5]
	Det	NP			NP
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
					[4,5]

$S \rightarrow NP VP$   
 $S \rightarrow X1 VP$   
 $X1 \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>

# Exercise

*Book the flight through Houston*

```
function CKY-PARSE(words, grammar) returns table
    for j ← from 1 to LENGTH(words) do
        for all {A | A → words[j] ∈ grammar}
            table[j - 1, j] ← table[j - 1, j] ∪ A
    for i ← from j - 2 down to 0 do
        for k ← i + 1 to j - 1 do
            for all {A | A → BC ∈ grammar and B ∈ table[i, k] and C ∈ table[k, j]}
                table[i, j] ← table[i, j] ∪ A
```

words	Book	the	flight	through	Houston
[0,1]	S,VP, Nominal Noun,Verb		S,VP,X2		S1,VP, X2 S2,VP S3
	[0,2]	[0,3]	[0,4]	[0,5]	
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun		Nominal	
		[2,3]	[2,4]	[2,5]	
			Preposition	PP	
			[3,4]	[3,5]	
				NP, Proper- Noun	
				[4,5]	

$S \rightarrow NP VP$   
 $S \rightarrow XI VP$   
 $XI \rightarrow Aux NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb NP$   
 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal Noun$   
 $Nominal \rightarrow Nominal PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

**Lexicon**  
 $Det \rightarrow that \mid this \mid the \mid a$   
 $Noun \rightarrow book \mid flight \mid meal \mid money$   
 $Verb \rightarrow book \mid include \mid prefer$   
 $Pronoun \rightarrow I \mid she \mid me$   
 $Proper-Noun \rightarrow Houston$   
 $Aux \rightarrow does$   
 $Preposition \rightarrow from \mid to \mid in \mid near \mid through$

RECOGNIZED since [0,5]  
contains **S**



# Exercise

starting from **S1**

words	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
	S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5] $\downarrow$ S1,VP, X2 S2,VP S3
	Det	NP			NP
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]			Nominal [2,5]
			Preposition	PP	
			[3,4]	[3,5] $\downarrow$ NP, Proper- Noun	
					[4,5]

Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>

$S \rightarrow NP\ VP$   
 $S \rightarrow XI\ VP$   
 $XI \rightarrow Aux\ NP$   
 $S \rightarrow book\mid include\mid prefer$   
 $S \rightarrow Verb\ NP$   
 $S \rightarrow X2\ PP$   
 $S \rightarrow Verb\ PP$   
 $S \rightarrow VP\ PP$   
 $NP \rightarrow I\mid she\mid me$   
 $NP \rightarrow TWA\mid Houston$   
 $NP \rightarrow Det\ Nominal$   
 $Nominal \rightarrow book\mid flight\mid meal\mid money$   
 $Nominal \rightarrow Nominal\ Noun$   
 $Nominal \rightarrow Nominal\ PP$   
 $VP \rightarrow book\mid include\mid prefer$   
 $VP \rightarrow Verb\ NP$   
 $VP \rightarrow X2\ PP$   
 $X2 \rightarrow Verb\ NP$   
 $VP \rightarrow Verb\ PP$   
 $VP \rightarrow VP\ PP$   
 $PP \rightarrow Preposition\ NP$

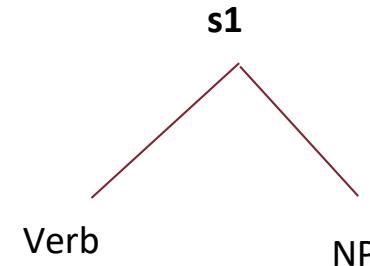
# Exercise

words	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5]	$S_1, VP, X_2$ $S_2, VP$ $S^3$
	Det	NP		NP	
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]		Nominal [2,5]	
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]

Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>

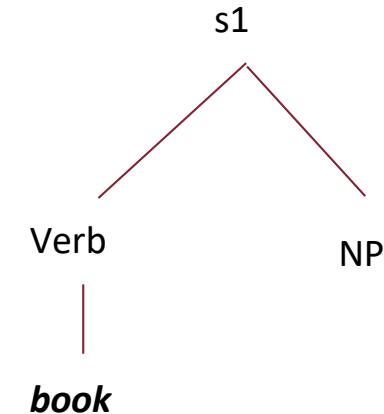
$S \rightarrow NP\ VP$   
 $S \rightarrow XI\ VP$   
 $XI \rightarrow Aux\ NP$   
 $S \rightarrow book\mid include\mid prefer$   
 $S \rightarrow Verb\ NP$  (highlighted)  
 $S \rightarrow X2\ PP$   
 $S \rightarrow Verb\ PP$   
 $S \rightarrow VP\ PP$   
 $NP \rightarrow I\mid she\mid me$   
 $NP \rightarrow TWA\mid Houston$   
 $NP \rightarrow Det\ Nominal$   
 $Nominal \rightarrow book\mid flight\mid meal\mid money$   
 $Nominal \rightarrow Nominal\ Noun$   
 $Nominal \rightarrow Nominal\ PP$   
 $VP \rightarrow book\mid include\mid prefer$   
 $VP \rightarrow Verb\ NP$   
 $VP \rightarrow X2\ PP$   
 $X2 \rightarrow Verb\ NP$   
 $VP \rightarrow Verb\ PP$   
 $VP \rightarrow VP\ PP$   
 $PP \rightarrow Preposition\ NP$

starting from **S1**



# Exercise

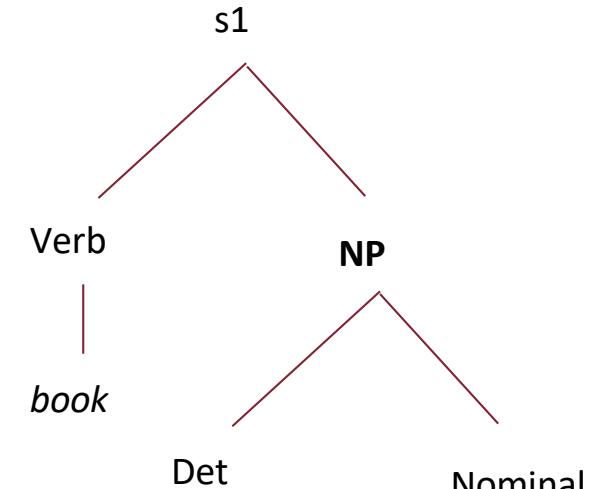
words	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
	S,VP, Nominal Noun,Verb [0,1]	[0,2]	[0,3]	[0,4]	[0,5] S1,VP, X2 S2,VP S3
	Det	NP [1,2]	NP [1,3]	NP [1,4]	NP [1,5]
		Nominal Noun [2,3]		Nominal [2,4]	Nominal [2,5]
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]



Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>
<i>S</i> → <i>NP VP</i>
<i>S</i> → <i>XI VP</i>
<i>XI</i> → <i>Aux NP</i>
<i>S</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>S</i> → <i>Verb NP</i>
<i>S</i> → <i>X2 PP</i>
<i>S</i> → <i>Verb PP</i>
<i>S</i> → <i>VP PP</i>
<i>NP</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>NP</i> → <i>TWA</i>   <i>Houston</i>
<i>NP</i> → <i>Det Nominal</i>
<i>Nominal</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Nominal</i> → <i>Nominal Noun</i>
<i>Nominal</i> → <i>Nominal PP</i>
<i>VP</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>VP</i> → <i>Verb NP</i>
<i>VP</i> → <i>X2 PP</i>
<i>X2</i> → <i>Verb NP</i>
<i>VP</i> → <i>Verb PP</i>
<i>VP</i> → <i>VP PP</i>
<i>PP</i> → <i>Preposition NP</i>

# Exercise

words	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]			NP [1,5]
		Nominal Noun [2,3]			Nominal [2,5]
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]



Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
<i>Preposition</i> → <i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>   <i>through</i>

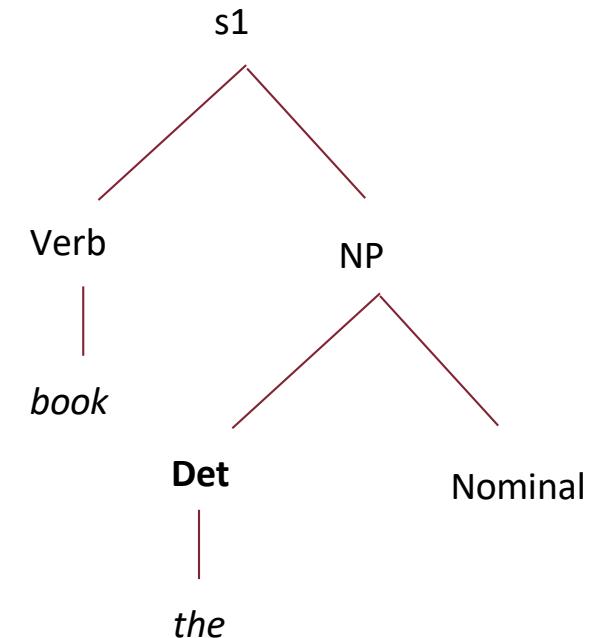
$S \rightarrow NP\ VP$   
 $S \rightarrow XI\ VP$   
 $XI \rightarrow Aux\ NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb\ NP$   
 $S \rightarrow X2\ PP$   
 $S \rightarrow Verb\ PP$   
 $S \rightarrow VP\ PP$   
 $NP \rightarrow I \mid she \mid me$   
 $NP \rightarrow TWA \mid Houston$   
 $NP \rightarrow Det\ Nominal$   
 $Nominal \rightarrow book \mid flight \mid meal \mid money$   
 $Nominal \rightarrow Nominal\ Noun$   
 $Nominal \rightarrow Nominal\ PP$   
 $VP \rightarrow book \mid include \mid prefer$   
 $VP \rightarrow Verb\ NP$   
 $VP \rightarrow X2\ PP$   
 $X2 \rightarrow Verb\ NP$   
 $VP \rightarrow Verb\ PP$   
 $VP \rightarrow VP\ PP$   
 $PP \rightarrow Preposition\ NP$

# Exercise

words	<i>Book</i>	<i>the</i>	<i>flight</i>	<i>trough</i>	<i>Houston</i>
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	<b>Det</b>	<b>NP</b>			<b>NP</b>
	[1,2]	[1,3]	[1,4]	[1,5]	
		Nominal Noun [2,3]			Nominal [2,5]
			Preposition [2,4]	PP [3,5]	
					NP, Proper- Noun [4,5]

Lexicon
<i>Det</i> → <i>that</i>   <i>this</i>   <i>the</i>   <i>a</i>
<i>Noun</i> → <i>book</i>   <i>flight</i>   <i>meal</i>   <i>money</i>
<i>Verb</i> → <i>book</i>   <i>include</i>   <i>prefer</i>
<i>Pronoun</i> → <i>I</i>   <i>she</i>   <i>me</i>
<i>Proper-Noun</i> → <i>Houston</i>   <i>NWA</i>
<i>Aux</i> → <i>does</i>
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 $S \rightarrow XI\ VP$   
 $XI \rightarrow Aux\ NP$   
 $S \rightarrow book \mid include \mid prefer$   
 $S \rightarrow Verb\ NP$   
 $S \rightarrow X2\ PP$   
 $S \rightarrow Verb\ PP$   
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 $VP \rightarrow Verb\ NP$   
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 $VP \rightarrow Verb\ PP$   
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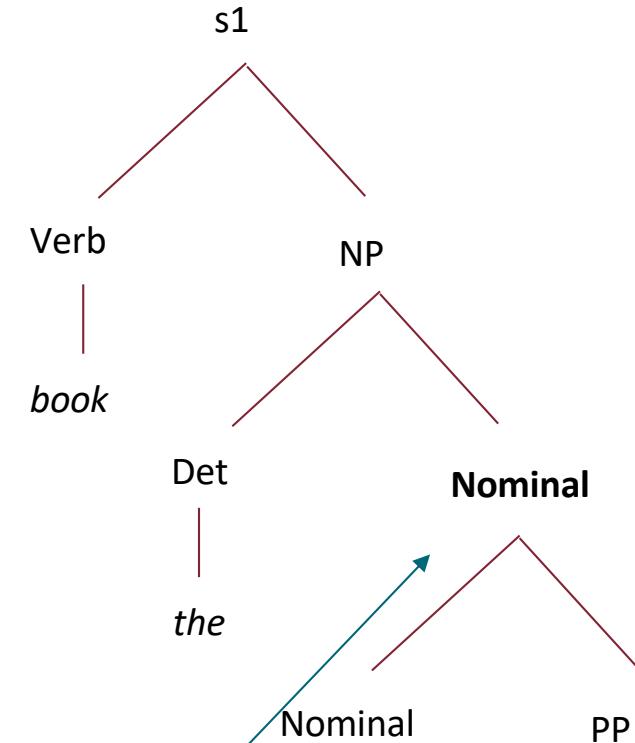


# Exercise

words	Book	the	flight	trough	Houston
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]		NP [1,4]	
		Nominal Noun [2,3]		Nominal [2,5]	
			Preposition [3,4]	PP [3,5]	
					NP, Proper- Noun [4,5]

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$Det \rightarrow that \mid this \mid the \mid a$
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 $XI \rightarrow Aux NP$   
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 $S \rightarrow X2 PP$   
 $S \rightarrow Verb PP$   
 $S \rightarrow VP PP$   
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 $NP \rightarrow TWA \mid Houston$   
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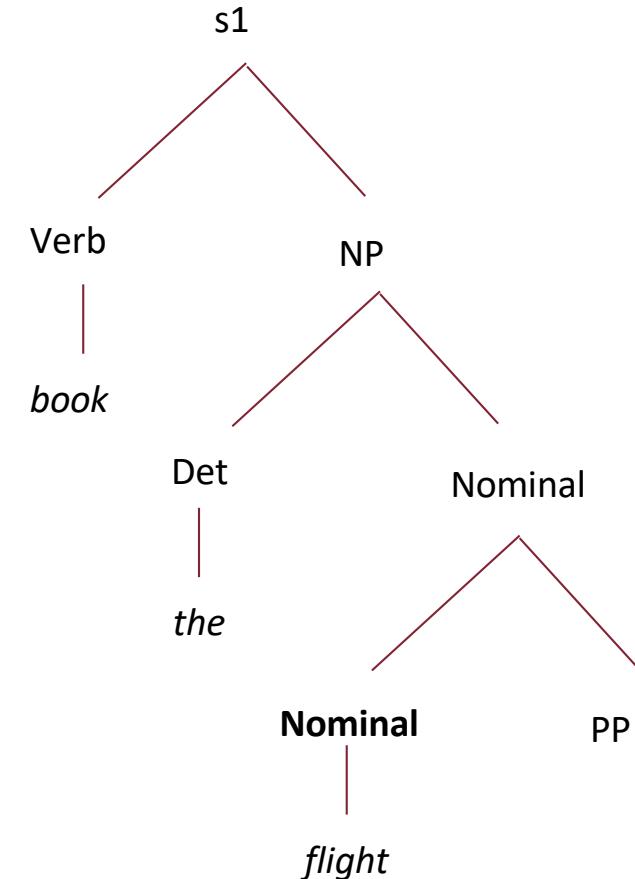
2 possible derivations here!

# Exercise

words	Book	the	flight	trough	Houston
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]		NP [1,4]	
		Nominal Noun [2,3]			Nominal [2,5]
			Preposition [3,4]	PP [3,5]	
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 $S \rightarrow Verb NP$   
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 $VP \rightarrow Verb NP$   
 $VP \rightarrow X2 PP$   
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 $VP \rightarrow Verb PP$   
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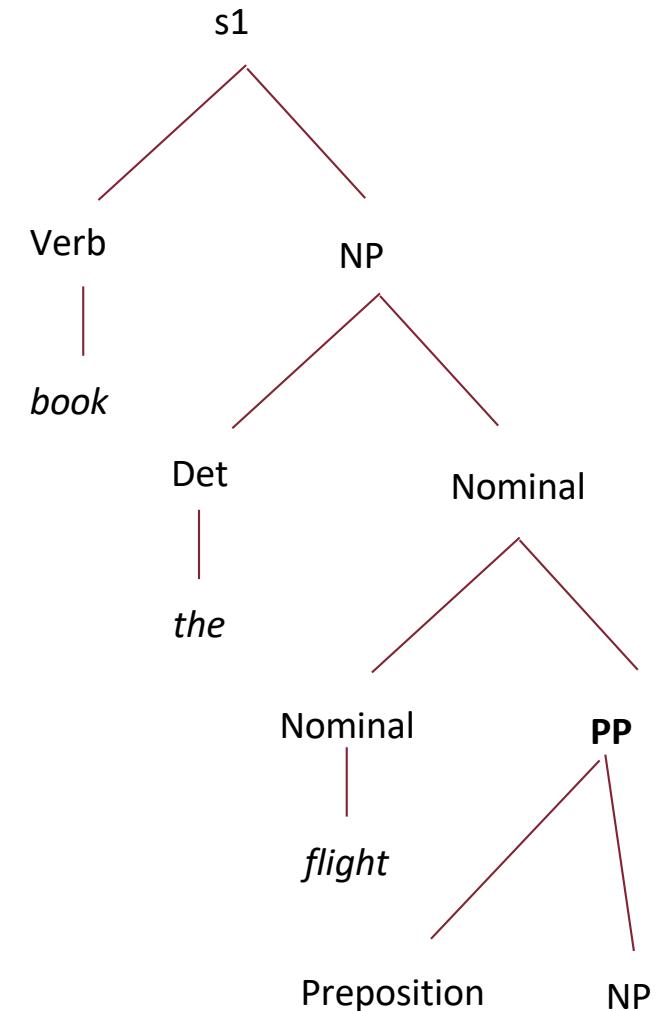


# Exercise

words	Book	the	flight	trough	Houston
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]		NP [1,4]	
		Nominal Noun [2,3]		Nominal [2,5]	
			Preposition [3,4]	PP [3,5]	
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 $VP \rightarrow X2 PP$   
 $X2 \rightarrow Verb NP$   
 $VP \rightarrow Verb PP$   
 $VP \rightarrow VP PP$   
 $PP \rightarrow Preposition NP$

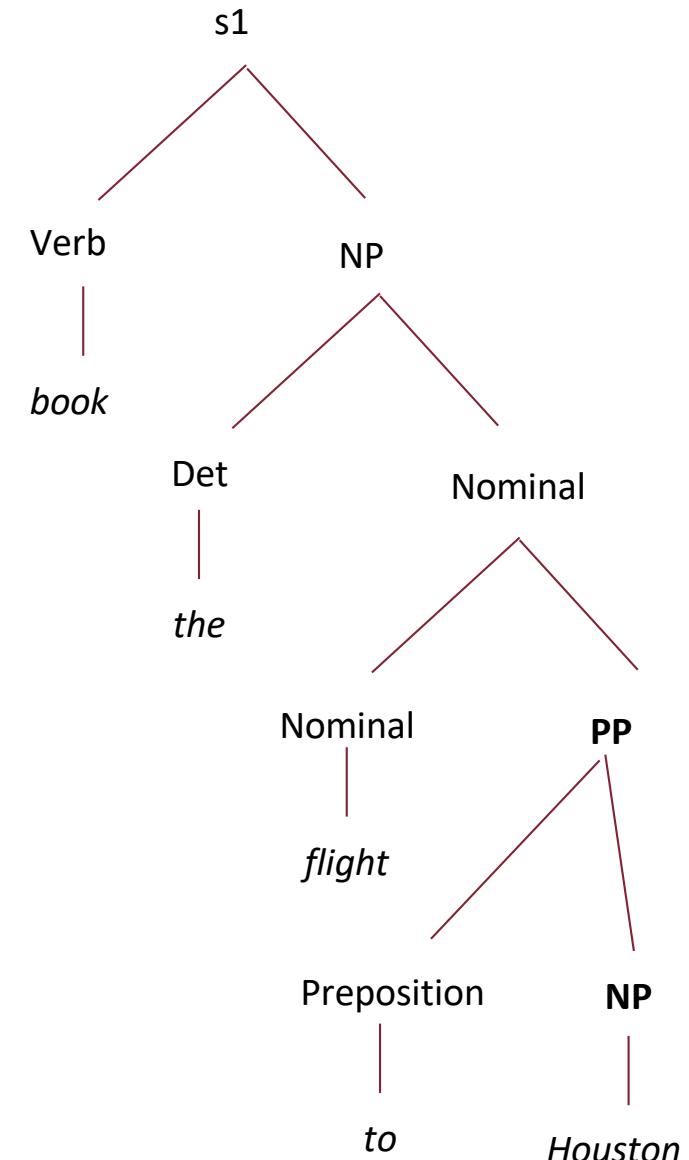


# Exercise

words	Book	the	flight	trough	Houston
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]		NP [1,4]	
		Nominal Noun [2,3]		Nominal [2,5]	
			Preposition [3,4]	PP [3,5]	
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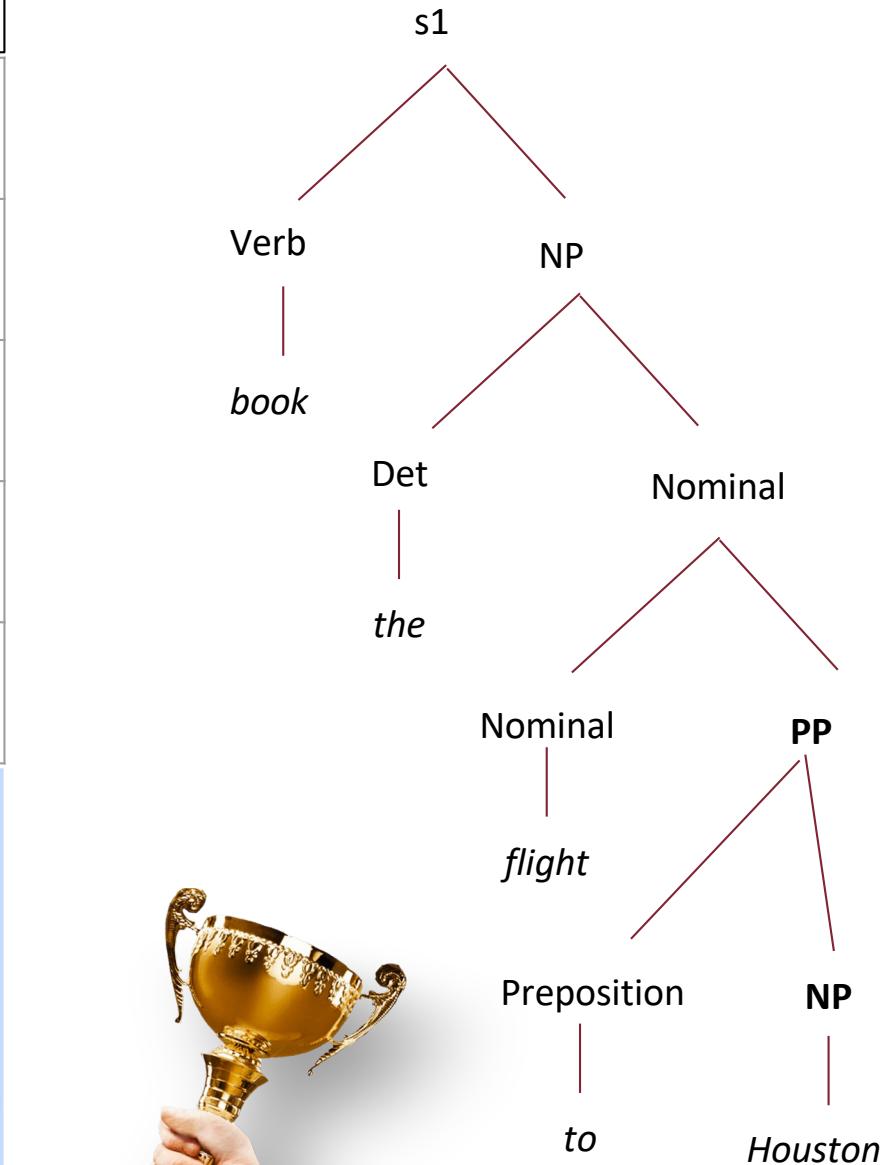


# Exercise

words	Book	the	flight	trough	Houston
	S,VP, Nominal Noun,Verb [0,1]		S,VP,X2 [0,3]		S1,VP, X2 S2,VP S3 [0,5]
	Det [1,2]	NP [1,3]		NP [1,5]	
		Nominal Noun [2,3]		Nominal [2,5]	
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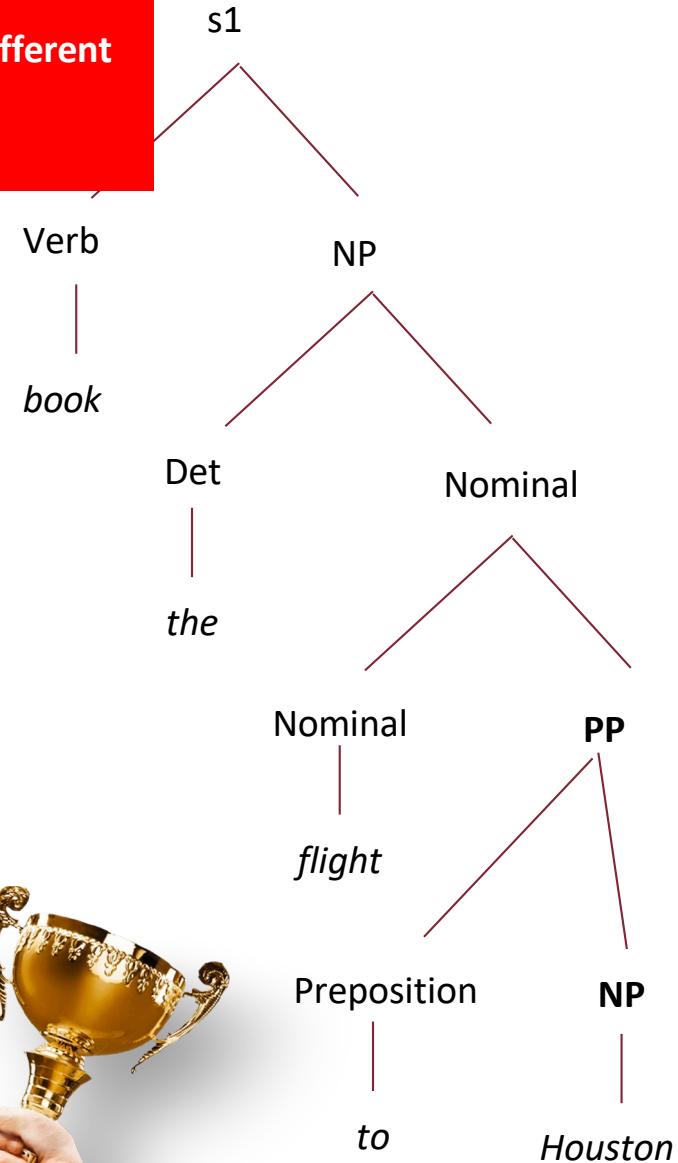


# Exercise

words	Book	the	
	S, VP, Nominal Noun, Verb [0,1]	S, [0,2]	[0, ]
	Det [1,2]	NP [1,3]	NP [1,4] [1,5]
		Nominal Noun [2,3]	Nominal [2,5]
		Preposition [3,4]	PP [3,5]
			NP, Proper- Noun [4,5]

Lexicon
$Det \rightarrow that \mid this \mid the \mid a$
$Noun \rightarrow book \mid flight \mid meal \mid money$
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$Pronoun \rightarrow I \mid she \mid me$
$Proper-Noun \rightarrow Houston \mid NWA$
$Aux \rightarrow does$
$Preposition \rightarrow from \mid to \mid on \mid near \mid through$

A sentence can be recognized as part of the language, correct syntax but the parser can generate all existing different valid parses



# Practical Problems with CKY

Chomsky Normal Form

- We need grammars to be in CNF
  - This complicates syntax-driven approaches to semantic analysis
- **Solution:** keep enough information to transform the parse trees back to the original version
  - Easy for transformation rules such as:  $A \rightarrow BCw \Rightarrow X \rightarrow BC; A \rightarrow Xw$
  - More complex for unit productions such as:  $A \rightarrow B$

# Exercise with CKY

- Given the following CFG grammar:
- $S \rightarrow NP\ V \mid V\ NP$
- $NP \rightarrow Det\ N$
- $N \rightarrow book \mid flight$
- $Det \rightarrow the \mid that$
- $V \rightarrow book \mid flies$
- Parse the sentence “book that flight”

# Probabilistic Parsing

(intuition)

- The most commonly used probabilistic grammar:  
probabilistic CFGs
  - Each rule is assigned a probability
  - Trained on a hand-labeled Treebank grammar
  - Parsing with a probabilistic version of CKY

## Probabilistic (or Stochastic) CFGs

- First proposed by Taylor Booth (1969)
- In a probabilistic CFG  $G = (N, T, P, S)$ , each production

$$A \rightarrow w [p]$$

- is assigned a probability  $p = P(w|A) = P(A \rightarrow w)$
- For each left-hand-side non-terminal  $A$ , it must hold:

$$\sum_w P(A \rightarrow w) = 1$$

# An Example of PCFG (from Jurafsky & Martin)

$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book [.10] \mid flight [.30]$
$S \rightarrow VP$	[.05]	$\mid meal [.15] \mid money [.05]$
$NP \rightarrow Pronoun$	[.35]	$\mid flights [.40] \mid dinner [.10]$
$NP \rightarrow Proper-Noun$	[.30]	$Verb \rightarrow book [.30] \mid include [.30]$
$NP \rightarrow Det Nominal$	[.20]	$\mid prefer; [.40]$
$NP \rightarrow Nominal$	[.15]	$Pronoun \rightarrow I [.40] \mid she [.05]$
$Nominal \rightarrow Noun$	[.75]	$\mid me [.15] \mid you [.40]$
$Nominal \rightarrow Nominal Noun$	[.20]	$Proper-Noun \rightarrow Houston [.60]$
$Nominal \rightarrow Nominal PP$	[.05]	$\mid TWA [.40]$
$VP \rightarrow Verb$	[.35]	$Aux \rightarrow does [.60] \mid can [.40]$
$VP \rightarrow Verb NP$	[.20]	$Preposition \rightarrow from [.30] \mid to [.30]$
$VP \rightarrow Verb NP PP$	[.10]	$\mid on [.20] \mid near [.15]$
$VP \rightarrow Verb PP$	[.15]	$\mid through [.05]$
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	

## PCFGs for Disambiguation

- So What Is the Probability of a Parse Tree?
- Defined as the product of the probabilities of all the productions used in the derivation

$$P(T, s) = \prod_{i=1}^n P(RHS_i \mid LHS_i)$$

where  $s$  is the sentence whose parse is  $T$  and  $LHS_i \rightarrow RHS_i$  is just the  $i$ -th production in the derivation

- The joint probability  $P(T, s) =$  probability of the parse tree  $P(T)$ :

$$P(T, s) = P(T)P(s \mid T) = P(T)$$

because  $T$  includes all the words in  $s$

# Probabilistic CKY Algorithm

- We just add probabilities to non-terminals in the bidimensional matrix cells
- If cell  $(i, j)$  contains non-terminal A, we specify its probability to span positions i through j of the input sentence

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
and its probability
for  $j \leftarrow$  from 1 to LENGTH(words) do
    for all  $\{ A \mid A \rightarrow words[j] \in grammar \}$ 
         $table[j-1, j, A] \leftarrow P(A \rightarrow words[j])$ 
    for  $i \leftarrow$  from  $j-2$  downto 0 do
        for  $k \leftarrow i+1$  to  $j-1$  do
            for all  $\{ A \mid A \rightarrow BC \in grammar,$ 
                    and  $table[i, k, B] > 0$  and  $table[k, j, C] > 0 \}$ 
                if ( $table[i, j, A] < P(A \rightarrow BC) \times table[i, k, B] \times table[k, j, C]$ ) then
                     $table[i, j, A] \leftarrow P(A \rightarrow BC) \times table[i, k, B] \times table[k, j, C]$ 
                     $back[i, j, A] \leftarrow \{k, B, C\}$ 
return BUILD_TREE(back[1, LENGTH(words), S]), table[1, LENGTH(words), S]
```

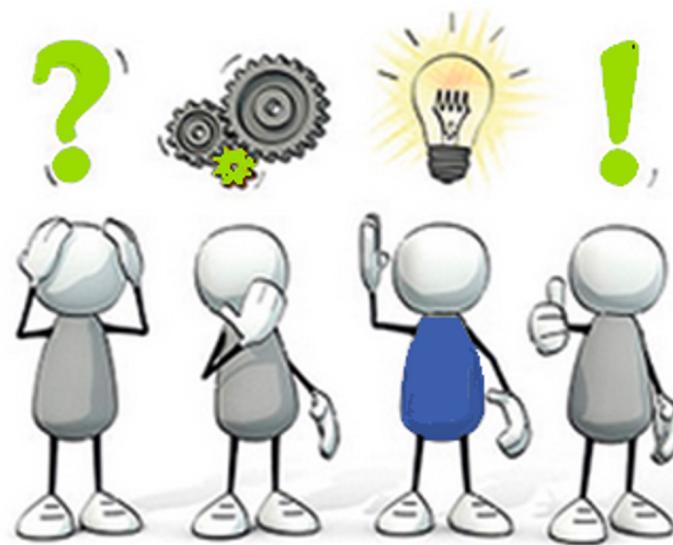
# Example of Probabilistic CKY Chart (intuition)

$S \rightarrow NP VP$	.80	$Det \rightarrow the$	.50
$NP \rightarrow Det N$	.30	$Det \rightarrow a$	.40
$VP \rightarrow V NP$	.20	$N \rightarrow meal$	.01
$V \rightarrow includes$	.05	$N \rightarrow flight$	.02

Det: .40 [0,1]	NP: .30 * .40 * .02 = .0024 [0,2]	[0,3]	[0,4]	[0,5]
	N: .02 [1,2]	[1,3]	[1,4]	[1,5]
		V: .05 [2,3]	[2,4]	[3,5]
			[3,4]	[3,5]
				[4,5]

To be continued...

# Q&A



# Resources and References

[Jurafsky&Martin, 2022] Jurafsky and Martin. Speech and Language Processing, Prentice Hall, third edition  
<https://web.stanford.edu/~jurafsky/slp3/ed3book.pdf>

## **\*\*Credits**

The slides of this part of the course are the result of a personal reworking of the slides and of the course material from different sources:

1. The NLP course of Prof. Roberto Navigli, Sapienza University of Rome
2. The NLP course of Prof. Simone Paolo Ponzetto, University of Mannheim, Germany
3. The NLP course of Prof. Chris Biemann, University of Hamburg, Germany
4. The NLP course of Prof. Dan Jurafsky, Stanford University, USA

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