

## Exercise – *Rumpelstilzchen*

“Heute back ich, Morgen brau ich, Übermorgen hol ich der Königin ihr Kind; Ach, wie gut ist, daß niemand weiß, daß ich Rumpelstilzchen hei!”

“Today I bake, tomorrow I brew, The day after that the queen’s child comes in; And oh! I am glad that nobody knew that the name I am called is Rumpelstiltskin!”

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*Kinder- und Hausmrchen by Jacob Grimm (1785–1863) and Wilhelm Grimm (1786–1859)*

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After many years, Rumpelstilzchen is living on a group of islands. One day he decides to spin as much gold as he can in one night at the islands. He discovers that he can spin one yarn of superior gold if he uses one sack of wheat straw and one sack of barley straw. However, wheat straw is only available in the castle of the kingdom to the west of the islands and barley straw is only available in the castle of the kingdom to the east of the islands.

The queen’s father, the miller, wants to repay Rumpelstilzchen for helping his daughter and agrees to transport the straw from both kingdoms to the islands for free. The miller transports the straw during the day, and Rumpelstilzchen can spin gold during the night. The miller has two independent transportation networks. The transportation networks consist of roads between cities, and shipping routes connecting cities of the two kingdoms to the islands. The two transportation networks do not have any cities in common, and there are no roads connecting cities of one transportation network with cities of the other transportation network. Each road and shipping route has a maximum capacity associated with it, which denotes the maximum number of sacks that can be transported through it. Both kingdoms have strict traffic rules, and all roads and shipping routes are one-way, i.e. they can only be traversed in a prespecified direction. The castle is situated in the city with index zero in both kingdoms.

On each island is one spinning wheel that Rumpelstilzchen can use to spin gold. However, the spinning wheels are not well maintained, and they will break down after spinning exactly one yarn of gold, i.e. each island can produce at most one yarn of gold. Like all gold spinners who take themselves seriously, Rumpelstilzchen prefers some spinning wheels over others. More specifically, he has a *preference score* for each spinning wheel, indicating how much he enjoys spinning on that particular spinning wheel. Rumpelstilzchen would like to maximize the number of yarns of superior gold spinned that night. If there exist multiple strategies leading to the same number of yarns of superior gold, he chooses a combination of spinning wheels with the highest total sum of preference scores. You can assume that Rumpelstilzchen has enough time during the night to visit all islands, if needed.

**Input** The first line of the input contains the number  $t \leq 30$  of test cases. Each of the  $t$  test cases is described as follows:

- It starts with a single line containing nine integers  $n \ p_G \ p_H \ e_G \ e_H \ f_G \ f_H \ s_G \ s_H$ , separated by a space. For  $X \in \{G, H\}$ , they denote
  - $n$ , the number of islands ( $1 \leq n \leq 500$ );
  - $p_X$ , the number of cities in the network  $X$  ( $1 \leq p_X \leq 10^3$ );

- $e_X$ , the number of edges of  $X$  ( $1 \leq e_X \leq 5 \cdot 10^3$ );
- $f_X$ , the number of edges from  $X$  to the islands ( $1 \leq f_X \leq 2 \cdot 10^3$ ); and
- $s_X$ , the number of sacks of straw available in the castle of  $X$  ( $1 \leq s_X \leq 10^3$ ).
- The following line contains  $n$  integers  $a_0 \dots a_{n-1}$ , separated by a space. They denote the preference score of the spinning wheel on each of the  $n$  islands ( $0 \leq a_i \leq 2^{10}$ ).
- The following  $e_G + e_H$  lines define the edges of  $G$  (first) and  $H$  (second). Each line contains three integers  $u \ v \ c$ , separated by a space, that define a one-way road of capacity  $c$  from city  $u$  to city  $v$  in the respective network ( $0 \leq u < p_X$ ,  $0 \leq v < p_X$ ,  $u \neq v$ ,  $0 \leq c \leq 2^{24}$ ).
- The next  $f_G + f_H$  lines define the edges between the networks  $G$  (first) and  $H$  (second) on one side and the islands on the other side. Each line contains three integers  $u \ v \ c$ , separated by a space, that define a one-way shipping route of capacity  $c$  from city  $u$  in the respective network to an island  $v$  ( $0 \leq u < p_X$ ,  $0 \leq v < n$ ,  $0 \leq c \leq 2^{24}$ ).

**Output** The output for each test case consists of a separate line containing two integers  $y \ s$ , separated by a space. They denote  $y$ , the maximum number of yarns of superior gold that Rumpelstilzchen can spin during the night; and  $s$ , the corresponding maximum total preference score of the used spinning wheels for that number of yarns of superior gold.

**Points** There are four groups of test sets. For each group there is also a corresponding hidden test set, each worth 5 points. Overall, you can achieve 100 points.

1. For the first group of test sets, worth 25 points, you may assume that all preference scores  $a_i$  are equal to zero, that the second transportation network  $H$  contains a direct shipping route from the castle (index 0) to each island with capacity equal to 1, and that the number of sacks of barley straw available in the second transportation network  $H$  is equal to the total number of islands  $n$ .
2. For the second group of test sets, worth 25 points, you may assume the same as for the first group of test sets, except that the preference scores are not necessarily equal to zero. Moreover, you may assume  $n \leq 100$ ,  $p_G + p_H \leq 200$ ,  $e_G + e_H + f_G + f_H \leq 1200$ .
3. For the third group of test sets, worth 15 points, you may assume that all preference scores  $a_i$  are equal to zero.
4. For the fourth group of test sets, worth 15 points, there are no additional assumptions.

Corresponding sample test sets are contained in `testi.in/out`, for  $i \in \{1, 2, 3, 4\}$ .

**Sample Input**

```
2
2 2 2 1 1 1 2 2 2
10 5
0 1 3
0 1 2
0 1 2
1 0 1
1 1 3
3 2 3 1 3 3 2 1 2
5 2 1
0 1 1
0 1 2
0 2 1
2 1 1
1 0 5
1 1 3
0 2 2
1 0 2
1 2 3
```

**Sample Output**

```
1 5
1 5
```