

Algorithms Lab HS23
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Exercise – Schneewittchen

"Spieglein, Spieglein, an der Wand, wer ist die Schönste im ganzen Land?"

"Looking-glass upon the wall, who is fairest of us all?"

Kinder- und Hausmärchen by Jacob Grimm (1785-1863) and Wilhelm Grimm (1786-1859)

Once upon a time, there was a beautiful princess named Snow-white. But alas, poor Snow-white was in grave danger. The wicked Queen, who was jealous of Snow-white's beauty, had a magic mirror that told her where the child was. The Queen set out to kill Snow-white. Fortunately, Snow-white had the seven dwarfs, who were determined to keep her safe. And so, they hatched a plan. They would build a new magic mirror for the Queen, one that would not divulge the whereabouts of Snow-white, and secretly exchange the mirrors the next night. Thus, the Queen would no longer be able to find Snow-white, and the child would be safe once more.

So they went about to acquire the raw materials for the mirror. Perchance, these could be mined from the nearby mountain. The dwarfs knew this mountain like the back of their hands. There were n mines, one of which was the *entry* of the mountain. For each mine the dwarfs knew the exact amount of each of the m minerals available there. The mines were connected by railway tracks. Each *track* connected two mines, and from each mine there was a unique sequence of tracks to reach the entry. To collect the minerals they transported them to the entry using the railways. But some of the mines—luckily, only very few—were *dangerous*. Whenever a railcar passed through a dangerous mine, half of its contents (that is, half of the amount of every mineral) got lost. Furthermore, a dangerous mine would collapse if the total amount of minerals that enter it throughout the whole process was strictly larger than a certain *danger threshold*. A collapse had to be avoided at all costs.

Finally, there was a mineral shop in their village. The shop had a limited supply s_j for a price per unit p_j of the mineral j, for $j \in \{0, ..., m-1\}$. The dwarfs would try to get enough minerals combining the mines and the shop. They minimized the price they had to pay at the shop by carefully picking the minerals collected from the mountain. Could they obtain all the minerals they needed? If yes, how much did they have to pay?

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line containing two integers n m, separated by a space. They denote
 - n, the number of mines $(1 \le n \le 10^3)$ and
 - m, the number of minerals $(1 \le m \le 5)$.
- The following n lines describe the mines. The first line corresponds to the entry. Each line contains m+1 integers d $r_0 \ldots r_{m-1}$, separated by a space. They denote d, the danger threshold $(-1 \le d < 2^{28})$, and r_j , the quantity of mineral j available $(0 \le r_j < 2^{20})$. If $d \ge 0$, then the corresponding mine is dangerous. In a dangerous mine, no minerals are available $(r_0 = \ldots = r_{m-1} = 0)$. You may assume that at most 20 mines are dangerous and that the entry is not.

- The following n-1 lines describe the railway tracks. Each line contains two integers u v, separated by a space, indicating a railway track between mine u and mine v ($0 \le u, v < n$). You may assume that v lies on the (unique) path from u to the entry.
- The following m lines describe the minerals. Each line contains 3 integers c s p, separated by a space. They denote c, the amount of the mineral required for the mirror; s, the supply of the mineral at the shop; and p, its price per unit at the shop $(0 \le c, s, p < 2^{28})$.

Output For each test case the corresponding output appears on a separate line. If the dwarfs can obtain all the minerals required, the output is the price they had to pay, rounded down to the next integer. Otherwise, the output is Impossible!

Points There are five groups of test sets. Overall, you can achieve 100 points.

- 1. For the first group of test sets, worth 20 points, you may assume that (1) the railway tracks form a path that starts at the entry; (2) there exists at most one dangerous mine; and (3) the market is sold out $(s_i = 0, \text{ for all } j)$.
- 2. For the second group of test sets, worth 15 points, and for the corresponding hidden group, worth 5 points, you may assume that (1) for every mine there is at most one dangerous mine on its (unique) sequence of tracks to the entry; and (2) the market is sold out $(s_i = 0, \text{ for all } j)$.
- 3. For the third group of test sets, worth 20 points, and for the corresponding hidden group, worth 5 points, you may assume that the number of mines is small ($n \le 30$) and that the market is sold out ($s_i = 0$, for all j).
- 4. For the fourth group of test sets, worth 15 points, and for the corresponding hidden group, worth 5 points, you may assume that the market is sold out $(s_i = 0, \text{ for all } j)$.
- 5. For the fifth group of test sets, worth 10 points, and for the corresponding hidden group, worth 5 points, there are no additional assumptions.

Sample Input

Sample Output

Impossible! 2