GV300 - Quantitative Political Analysis

University of Essex - Department of Government

Lorenzo Crippa

Week 24 – 9 March, 2020

2020 Department of Government Student Conference

Teaching evaluation. Be mindful of your unconscious biases!

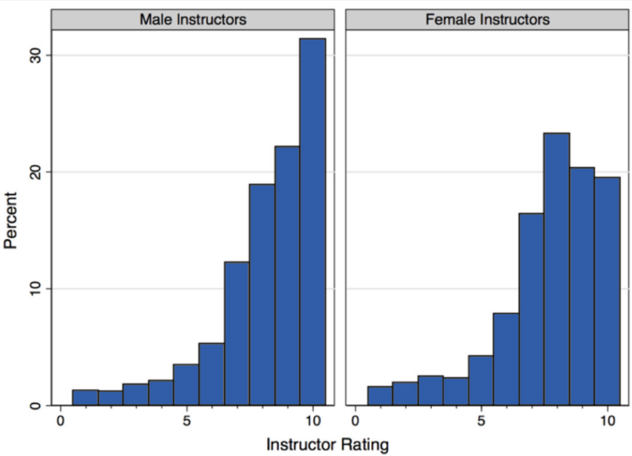

Figure 1: Source: Rivera and Tilcsik 2019

Today's class

1. Prep for exam: From Mac (or Linux) to PC
2. Correction of Problem Set 7
3. Maximum Likelihood Estimation

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- In a UNIX system (Mac or Linux) the / is different from a \. you have: `/Users/Your_name/Documents/your_folder`

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- In a UNIX system (Mac or Linux) the / is different from a \. you have: `/Users/Your_name/Documents/your_folder`
- In Microsoft Windows the / is identical to a \. So you can have both: `C:/Users/Your_name/Documents/your_folder` and `C:\Users\Your_name\Documents\your_folder`

Dealing with two different systems

If you need to work on both systems in R or Stata you can include both paths and comment out the one you don't need (depending on the machine you're working on).

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```
# Lorenzo Macbook
setwd("/Users/Lorenzo/Dropbox/Shared_Essex/GTA/GV300")

# Lorenzo Essex
#setwd("C:/Users/lc19059/Dropbox/Shared_Essex/GTA/
    GV300")
```

```
* Lorenzo Macbook
* cd "/Users/Lorenzo/Dropbox/Shared_Essex/GTA/GV300"

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- R works only if you turn each \into a / while Stata works both ways
- In any case it's always better to change each \into a /

Correction of Problem Set 7

Question 1 – (a)

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Countries in the treatment group: Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovak Republic, Slovenia.

Countries in the control group: Albania, Armenia, Bulgaria, Croatia, Georgia, Kosovo,Macedonia, FYR Moldova, Montenegro, Serbia, Ukraine

Question 1 – (b)

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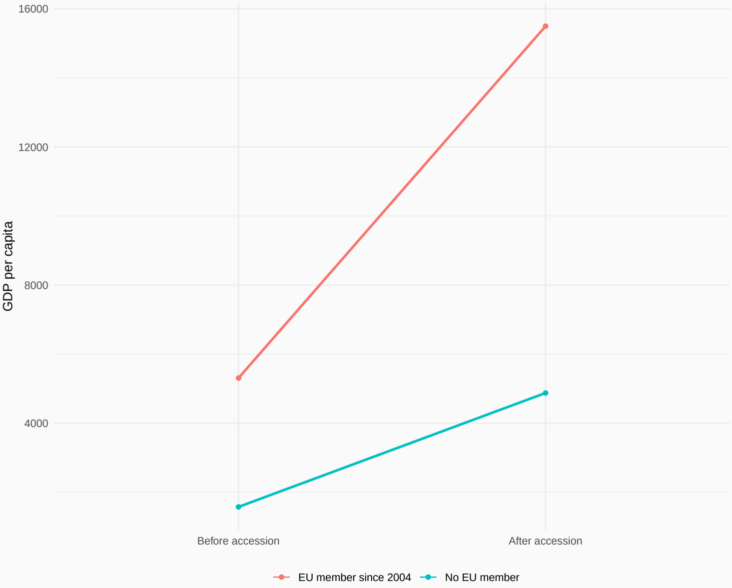
The difference in differences is
$(15495.61 - 5306.678) - (4873.54 - 1574.61) = 6890.301$

Question 1 – (b)


16000
12000
8000
4000
GDP per capita
Before accession
After accession
EU member since 2004
No EU member

Question 1 – (c)

Plot the mean of `GDPPerCapita` for treatment and control group over `year`. Add a line indicating the intervention year. Add the counterfactual `GDPPerCapita`. Evaluate whether the common trend assumption is met pre-intervention. Are the parts of SUTVA met that are relevant to the goodness of the differences-in-differences estimator? Why or why not?

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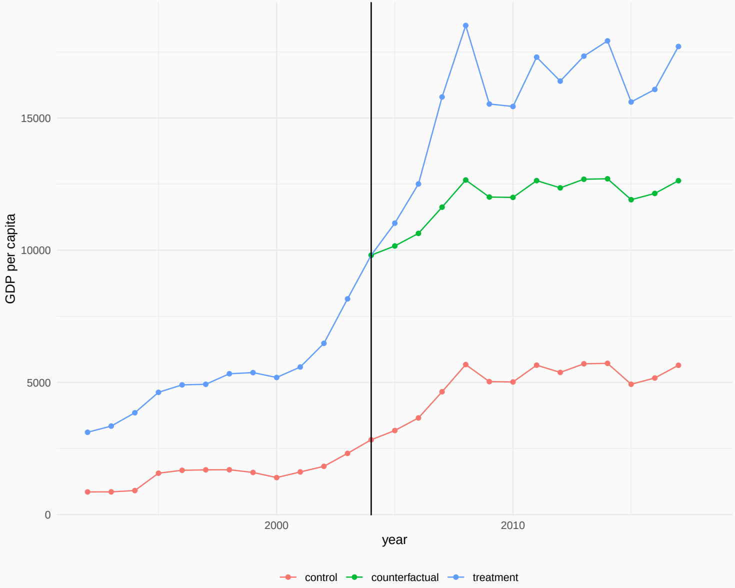

15000
10000
5000
0
GDP per capita
2000
2010
year
control
counterfactual
treatment

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Hence, SUTVA may be violated

Question 1 – (d), (e) and (f)

(d) Run a regression to compute the differences-in-differences estimator. Report and interpret the result. Speak to the three relevant coefficients.

(e) Improve your regression in (d) by computing clustered standard errors.

(f) Improve your estimate of the causal effect of joining the EU in (e) by including one relevant country-level covariate into the regression. Report and interpret your result. Was your estimate in (e) an over- or underestimate of the causal effect? Speculate why failing to include this covariate led to bias in your estimate in (e).

Question 1 – (d), (e) and (f)

	(d)	(e)	(f)	(f)
(Intercept)	1574.91^{***}	1574.91^{***}	-497.75	6153.33^{***}
	(299.71)	(136.46)	(314.89)	(588.39)
intervention	3298.63^{***}	3298.63^{***}	2950.32^{***}	3361.34^{***}
	(390.18)	(286.10)	(297.74)	(225.31)
treatment	3731.77^{***}	3731.77^{***}	2961.21^{***}	241.61
	(451.94)	(315.27)	(349.38)	(547.94)
intervention:treatment	6890.30^{***}	6890.30^{***}	5751.81^{***}	6827.59^{***}
	(593.70)	(575.16)	(608.09)	(547.98)
exportsShareGDP			64.37^{***}	
			(9.00)	
yearJoinEU				-0.54^{***}
				(0.06)
R^2	0.74	0.74	0.76	0.78
Adj. R^2	0.73	0.73	0.75	0.78
Num. obs.	457	457	451	457

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Question 2 – (a)

Panel data exercise. Provide summary statistics and plots of the 6 variables in the data set that are not the panel identifiers year and state. Plot `FTM` by state over time.

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Summary statistics (pooled):

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
FTM	201	58.249	9.516	26.296	53.364	63.511	91.833
white	261	0.809	0.162	0	0.7	0.9	1
poor	261	0.181	0.117	0.000	0.109	0.227	0.692
turnout	260	1.654	0.166	1.000	1.559	1.779	2.000
voteDem	261	0.164	0.171	0.000	0.000	0.295	0.667
dem	261	0.282	0.195	0.000	0.000	0.417	0.737

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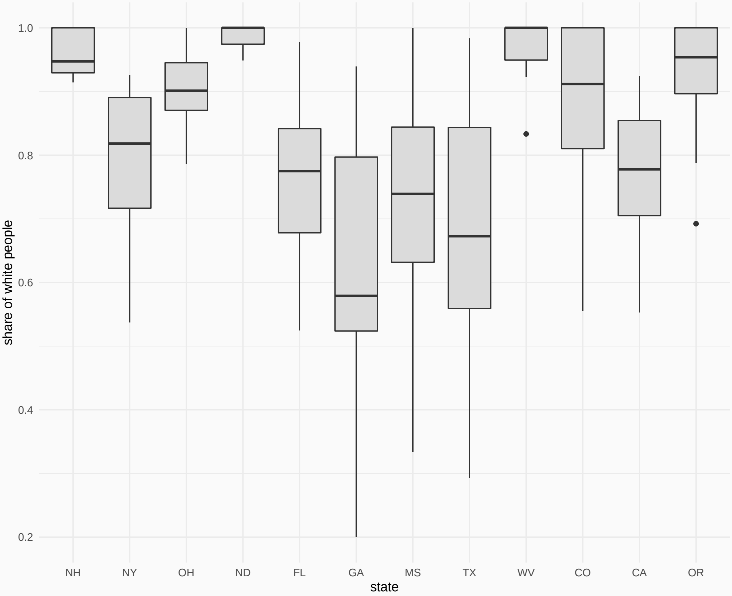

1.0
0.8
0.6
0.4
0.2
share of white people
NH
NY
OH
ND
FL
GA
MS
TX
WV
CO
CA
OR
state

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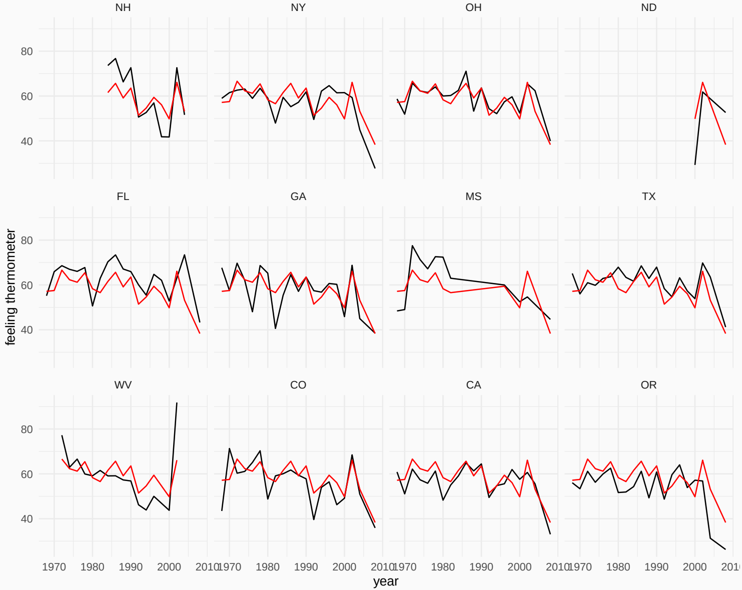
NH
NY
OH
ND
FL
GA
MS
TX
WV
CO
CA
OR
feeling thermometer
year
40
60
80
1970
1980
1990
2000
2010

Question 2 – (b)

Get overall, within, and between variation (standard deviation) of the variables in the data set from R or Stata.

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```
overall.variation
    year      FTM     white      poor
15.66012 9.516032 0.1615002 0.1174961

within.variation
var.year  var.FTM var.white  var.poor
15.06844 9.070157 0.1257591 0.1014254

between.variation
year_g.mean FTM_g.mean white_g.mean poor_g.mean
7.81        3.98       0.115        0.0725
```

Question 2 – (c), (d), (e), (f), (g)

(c) Run a pooled OLS regression of `FTM` on `white`, `poor`, `dem`, and `turnout`. Explain why the estimated coefficients and standard errors may be biased.

(d) Re-run the regression above but allow errors to be clustered by `state`. How are the estimation results different? Explain why they are different.

(e) Re-run the regression above but include dummies for each state. How are the estimation results different? Explain why they are different.

(f) Re-run the regression above but use the fixed effects estimator (or within estimator).

(g) Re-run the regression above but use the random effects estimator.

Question 2 – (c), (d), (e), (f), (g)

	(c)	(d)	(e)	(f)	(g)
(Intercept)	83.91^{***}	83.91^{***}	50.52^{***}		58.84^{***}
	(8.24)	(11.07)	(7.86)		(5.87)
white	8.30^{**}	8.30^{*}	12.04^{*}	12.04^{*}	1.37
	(4.18)	(4.47)	(6.42)	(7.28)	(4.75)
poor	-5.66	-5.66	-1.83	-1.83	3.33
	(6.94)	(10.35)	(7.87)	(10.42)	(9.14)
dem	2.35	2.35	5.46	5.46	5.82^{*}
	(5.05)	(4.48)	(5.15)	(3.55)	(3.36)
turnout	-19.35^{***}	-19.35^{***}			
	(4.32)	(4.59)			
voteDem			-23.42^{***}	-23.42^{***}	-27.66^{***}
			(4.03)	(3.00)	(3.09)
R^2	0.10	0.10	0.29	0.22	0.22
Adj. R^2	0.08	0.08	0.23	0.16	0.20
Num. obs.	201	201	201	201	201

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- **Yet** a Hausman test only cares about the precision of the estimate, and it is not everything we need to consider
- In this case we know we have lots of within and not so much between variation: a RE model still makes sense
- In these cases you can estimate both and hope to get similar results

Maximum Likelihood Estimation

Introduction to MLE

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    - We are asking the same question, only starting from a different starting point: $p(y, \mathbf{X}|\theta) = \mathcal{L}(\theta|y, \mathbf{X})$
    - To perform MLE you need to have some prior idea of what functional form the distribution of your dependent variable can follow

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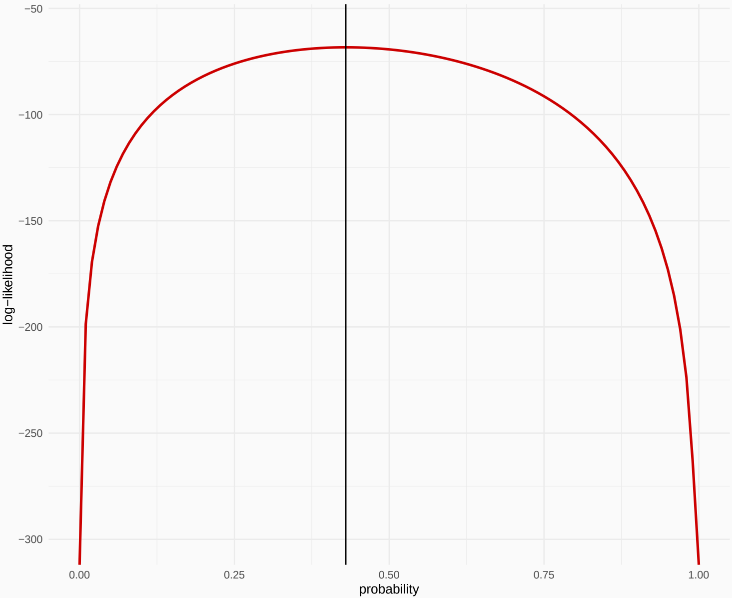
4. Take the second derivative and **verify** that it is smaller than 0 (second order condition) to verify it's a maximum (not a minimum) $\frac{\partial^2 log\mathcal{L}}{\partial\theta} < 0$

Log-likelihood function of a binomial distribution



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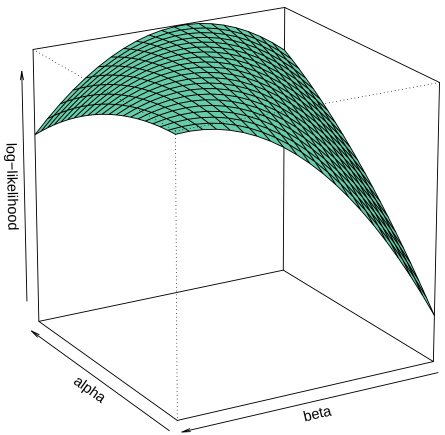
- Most often, though, we will use functions that rely on a particular log-likelihood specification (such as the functions of the GLM family). We won't need to write a $log\mathcal{L}$ function

Log-likelihood function of a linear model with two parameters



Conclusion

All clear? More questions?

Thanks and see you next week!