GV300 - Quantitative Political Analysis

University of Essex - Department of Government

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Week 18 – 27 January, 2020

What are we doing today?

Today's session:

1. Robust estimation and Generalized Least Squares (GLS)
2. Non-parametric tests

Robust estimation and Generalized Least Squares (GLS)

The importance of constant variance for OLS

Remember vcov matrix, OLS estimates of parameters and S.E.:

$$\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$$

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{N - K - 1}$$

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What if we have heteroscedasticity and/or serial correlation or autocorrelation?

Some matrix algebra

Let's work on the vcov matrix:

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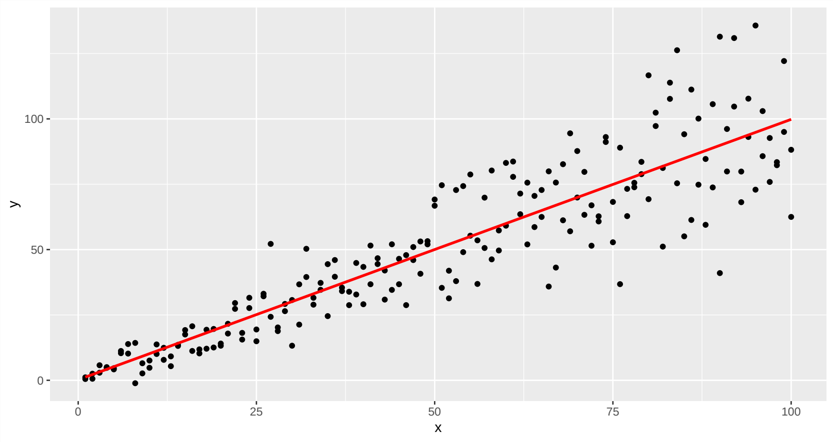
Problem with heteroscedasticity: no constant error variance!

Heteroscedasticity: visualization



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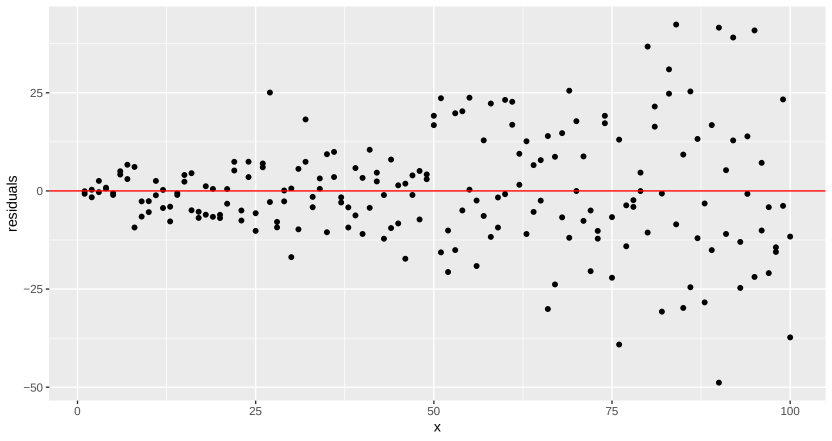

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- We can use **robust standard errors** to allow for such violation (while still assuming **no** serial correlation/autocorrelation)
- We **estimate** the covariance matrix $\Sigma = E(\hat{\mathbf{u}}\hat{\mathbf{u}}'|\mathbf{X})$ using $\hat{\Sigma}$

$$\hat{\Sigma} = \begin{bmatrix} \hat{u}^2 & 0 & \dots & 0 \\ 0 & \hat{u}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{u}^2 \end{bmatrix}$$

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- Your statistical software will multiply the squared residuals of each i by $n/df = n/(n-k-1)$ as a degree of freedom correction to have unbiased estimates
- The estimate of the variance of the OLS estimator becomes

$$\widehat{Var}(\hat{\beta}) = \frac{n}{df}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{\Sigma}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$$

- This robust variance estimator is called **Sandwich estimator** giving us **White-Huber standard errors**

Robust estimation in R

Using R today we will:

1. Compute OLS and estimates of the S.E. using `lm()` and matrix algebra
2. Represent heteroscedasticity
3. Compute White-robust estimators for S.E. using `vcovHC()`, matrix algebra and `lm_robust()`
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We are not doing it in Stata today, cause matrix algebra is definitely less handy... Equivalent commands exist, yet:

- To estimate robust standard errors: `reg ..., robust`
- To perform a Breusch-Pagan test: `estat hettest`
- To visualise heteroscedasticity after a regression: `rvfplot`

Serial correlation (time, space, both)

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- With $i \neq j$,

$$E[u_{(i)g}, u_{(j)g'}] = \begin{cases} 0 & \text{if } g = g' \\ \sigma_{(ij)g} & \text{if } g \neq g' \end{cases}$$

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$$Var(\mathbf{b}|\mathbf{X}) = E\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\Sigma\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\right]$$

- with Σ given by

$$\begin{bmatrix}
\sigma^2_{(11)1} & \dots & \sigma^2_{(1N_1)1} & 0 & \dots & 0 & 0 & \dots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\sigma^2_{(N_11)1} & \dots & \sigma^2_{(N_1N_1)1} & 0 & \dots & 0 & 0 & \dots & 0 \\
0 & \dots & 0 & \sigma^2_{(11)2} & \dots & \sigma^2_{(1N_2)2} & 0 & \dots & 0 \\
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0 & \dots & 0 & \sigma^2_{(N_21)2} & \dots & \sigma^2_{(N_2N_2)1} & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & \dots & 0 & \sigma^2_{(11)G} & \dots & \sigma^2_{(1N_G)G} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \dots & 0 & 0 & \dots & 0 & \sigma^2_{(N_G1)G} & \dots & \sigma^2_{(N_GN_G)G}
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What to do? We need an estimator for the variance that accounts for clustering:

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or $\hat{\mathbf{u}}_\mathbf{g}' \hat{\mathbf{u}}_\mathbf{g} / N_g$ where the numerator is SSR for all observations within each cluster.

Robust estimation and GLS

We can do even better:

- **Robust standard errors** use a robust covariance matrix. Keep the coefficient estimate but adjust **standard errors** to account for the violation of the assumption of i.i.d. $\rightarrow$ we get a conservative estimate!
- **Generalized least squares** proceed in two stages. First estimate the covariance matrix (stage 1) and then adjust coefficients (stage 2) $\rightarrow$ GLS may be more efficient

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- See an example in `R`

Take away points

1. **Always use robust standard errors!!!**
2. Consider whether your observations are clustered in a meaningful way, if yes, use clustered standard errors
3. Consider GLS as further robustness check on your estimation but (1) and (2) are mostly good enough.
4. R and Stata feature plenty of ways to implement robust and clustered standard errors; know how they work.

Non-parametric tests

Introduction

- There are alternatives to the t-test and F-test
- Choose your test based on your problem, and not the other way around
- Do not stick to parametric tests if your problem requires you to do otherwise

Pick an appropriate (non-parametric) test

1. Intro to non-parametric tests
2. Success probability: binomial test
3. Tests of differences between groups – paired from one-sample and independent from two-sample
    - Fisher sign test and Wilcoxon sign-rank test
    - Comparing success probabilities: Fisher's exact test

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- Are applicable for interval and ordinal data – some even for nominal data
- Have test statistics that are distributed normally when N is large

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- Often easier to compute manually than their parametric alternatives
- We can often derive the exact distribution of the test statistic

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- Need more observations to draw a conclusion with same certainty i.e., less powerful as parametric alternative when assumptions for parametric tests are met
- **Yet** often differences are small, and parametric alternatives perform vastly worse when assumptions are not met

Binomial test: Basics

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- Under assumptions 1-3, it is a distribution-free test of H_0 because the probability distribution of B is determined without further assumptions on the distribution of the underlying population

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- Where b_{α_2} is the upper α_2 percentile point and b_{α_1} is the lower α_1 percentile point with $\alpha = \alpha_1 + \alpha_2$

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  - leaves 2.5% of the observations to the left of 1
  - leaves 2.5% of the observations to the right of 6
- We reject $H_0 : p = .4$ if 6 or more successes are observed as well as if 1 or less successes are observed

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- Use when **direction** of difference between two measurements on same unit can be determined

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- n^+ is our test statistic. What is its distribution? It's binomial if we consider "being positive" as a Bernoulli random variable!

Sign test: Procedure

- Compute differences $D_i = X_i^1 - X_i^2$ between N pairs of matched observations.
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- n^+ is our test statistic. What is its distribution? It's binomial if we consider "being positive" as a Bernoulli random variable!
- Calculate the binomial coefficient $B = \binom{N}{n^+}$. $B/2^N$ is the probability of getting **exactly** the amount of n^+ and n^- under the null.
- To get a p-value, sum all binomial coefficients that are $\leq B$ and divide by 2^N

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```
    +-----------------------------+
    | income08   income12    D_i |
    |-----------------------------|
 1. |        3         19    -16 |
 2. |        4          9     -5 |
 3. |       17         20     -3 |
 4. |       15         17     -2 |
 5. |       14         16     -2 |
...
```

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- Statistical packages will then compute these for you

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- Information taken from signs in difference between paired observations (pre- vs post-treatment)
- When actual difference pre- vs post-treatment is greater than 0, tendency to larger proportion of positive differences

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- Test of null hypothesis of zero shift in location (no treatment effect), $H_0 : \theta = 0$ – null hypothesis states that each of the distributions for the differences is symmetrically distributed around the 0
- Use when direction of difference **and magnitude** between two measurements on same unit can be determined

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- Order **absolute values** of differences from smallest to largest
- Let S_i denote the rank of D_i in the joint ordering (assign average rank to ties)
- "Wilcoxon signed rank statistic", W^+, is sum of positive signed ranks
- Under $H_0 : \theta = 0$, W^+ is distributed according to the distribution derived by Wilcoxon (1954)
- Reject H_0 if $W^+ \geq w_{\alpha/2}$ or $W^+ \leq \frac{N(N+2)}{2} - w_{\alpha/2}$
- distribution of W is based on permutations of all possible rankings

Wilcoxon sign-rank test: Example

$N = 15$

```
    +-------------------------------------------+
    | income08   income12   D_i   absD_i   rank |
    |-------------------------------------------|
 1. |       22         22     0        0      1 |
 2. |       20         21    -1        1    2.5 |
 3. |       20         21    -1        1    2.5 |
 4. |       14         16    -2        2    4.5 |
 5. |       15         17    -2        2    4.5 |
    |-------------------------------------------|
 6. |       16         13     3        3    6.5 |
 7. |       17         20    -3        3    6.5 |
 8. |       21         17     4        4    8.5 |
...
```

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- What is the smallest significance level at which these data lead to rejection of H_0?
- Consulting statistical tables of W^+ distribution, for our example we have $N = 14$ and $W^+ = 74.5$

.117	$n = 14$	66	.213		93
.102		67	.196		94
.088		68	.179		95
.076		69	.163		96
.065		70	.148		97
.055		71	.134		98
.046		72	.121		99
.039		73	.108		100
.032		74	.097		101
.026		75	.086		102
.021		76	.077		103
.017		77	.068		104
.013		78	.059		105
.010		79	.052		106
.008		80	.045		107
.006		81	.039		108
.005		82	.034		109
.003		83	.029		110
.002		84	.025		111
.002		85	.021		112
.001		86	.018		113
.001		87	.015		114
<.0005		88	.012	$n = 16$	93
.207		89	.010		94
.188		90	.008		95
.170		91	.007		96
.153		92	.005		97
.137		93	.004		98
.122		94	.003		99
.108		95	.003		100
.095		96	.002		101
.084		97	.002		102
.073		98	.001		103
.064		99	.001		104
.055		100	.001		105
.047		101	<.0005		106
					107

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  - With $n \to \infty$, $W^+ \sim N(0, 1)$
  - Adjustments for ties are needed
- Your statistical package will perform this test for you, but this is what happens under the surface

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Fisher's exact test – Intuition

When do we use it?

1. When we have two independent samples arranged in a contingency table
2. When in each sample we measure the frequency of success *vs* failure (Bernoulli)
3. When expected frequency of any success or failure is below 5. Otherwise use χ^2 test

Fisher's exact test – Three cases

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    - The hypothesis tested is $p_1 = p_2$
2. Both row totals and column totals are random
    - The hypothesis tested is bivariate independence
3. Both row totals and column totals are fixed
    - The hypothesis tested is independence

Fisher's exact test – Basics

1. indicate as O_{1S} the *S*uccessful outcome you observe from sample 1 (similarly you have O_{1F}, O_{2S}, O_{2F}).
2. Indicate as $n_{1\cdot}$ ($n_{2\cdot}$) the independent repeated Bernoulli trials from sample 1 (2), each one with success probability p_1 (p_2)

We will have:

	Successes	Failures	Totals
Sample 1	O_{1S}	O_{1F}	$n_{1\cdot}$
Sample 2	O_{2S}	O_{2F}	$n_{2\cdot}$
Totals	$n_{\cdot S}$	$n_{\cdot F}$	n

$$H_0 : p_1 = p_2 = p$$
$$H_1 : p_1 \neq p_2$$

Fisher's exact test – Procedure

- In effect the test asks "what is the probability of having a table as extreme as the one we observe, if the null hypothesis is true"?
- Hypergeometric distribution: $Prob(O_{1S} = x | n_{1\cdot}, n_{2\cdot}, n_{\cdot S}, n_{\cdot F}) = \frac{n_{1\cdot}! n_{2\cdot}! n_{\cdot S}! n_{\cdot F}!}{n! x! O_{1F}! O_{2S}! O_{2F}!}$
- Fisher's exact test rejects $H_0 : p_1 = p_2$ if your observed $O_{1S} \geq q_\alpha$
- Where q_α is chosen from the conditional distribution described above so that $Prob(O_{1S} = q_\alpha | n_{1\cdot}, n_{2\cdot}, n_{\cdot S}, n_{\cdot F}) = \alpha$ where α is our desired level of significance

Fisher's exact test – Examples

Suppose you are curing two independent samples of patients, one gets an innovative drug and the other gets the traditional one. You measure success as cure, failure as not cure

	S (cured)	F (not cured)	Totals
S1 (new drug)	5	4	9
S2 (old drug)	4	2	6
Totals	9	6	15

- Assume horizontal and vertical totals are fixed
- $H_0 : p_1 < p_2$
- Under the null-hypothesis, what are the probabilities of the tables that would give us a value as small as or smaller than the observed value of $O_{1S} = 5$?

Fisher's exact test – Example

Sample 1	3	6	4	5	5	4	6	3	7	2	8	1	9	0
Sample 2	6	0	5	1	4	2	3	3	2	4	1	5	0	6
p	.017		.151		.378		.336		.108		.011		.000	

- $H_0 : p_1 < p_2$
- Under the null-hypothesis, what are the probabilities of the tables that would give us a value as small as or smaller than the observed value of $O_{1S} = 5$?
- It will be $p = .017 + .151 + .378 = .546$

Fisher's exact test – Example

If (with the same horizontal and vertical totals) what we observe were:

	S (cured)	F (not cured)	Totals
S1 (new drug)	8	1	9
S2 (old drug)	1	5	6
Totals	9	6	15

The probability of having a table as extreme (or more) as the one you observe would be: $p = 0.0107 + 0.0002 = 0.0109$!

Conclusion

All clear? Questions?

Thanks and see you next week!