# **GV300** - Quantitative Political Analysis

University of Essex - Department of Government

Lorenzo Crippa Week 10 – 2 December, 2019

# **Regression Analysis**

Today we'll work on regression analysis and apply what you have learnt in class. We'll use the dataset on sales of Monet paintings that we have started to explore in week 8.

# Explaining prices of paintings – Part 1

Load your data of Monet paintings and perform the following steps:

- 1. Explore how variables relate to each other (bivariate *descriptive* analysis)
  - Use appropriate statistics to talk about these relations
  - Use appropriate plots to show the relations
- 2. Propose a theory about what explains the price of a painting
  - What is your theory?
  - What is the underlining population of your theory?
  - What is the sample you are using?
- 3. Build an appropriate linear model to test your theory
  - What variables do you include?
  - Think and justify whether you meet Gauss-Markov assumptions
  - What could violate them?
  - What could improve your model?

# Explaining prices of paintings - Part 2

- 4. Run your model using your preferred statistical software
  - Interpret the coefficients
  - What test is performed on each variable?
  - What variables are significant? Is your theory rejected?
  - Do you see any problem in your model?
- 5. How good is your model, jointly considered?
  - What test is performed on the overall model?
  - Interpret the F-test
  - Interpret the *R*<sup>2</sup>
  - Which one of the two would you use to evaluate your model?
- 6. Based on your model, predict what would be the price of a painting with arbitrary values for the dependent variables

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### In R:

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#### In R:

## Outputs:

```
1 [,1] [,2] [,3]
2 [1,] 1.0000000 0.3145808 0.3468806
3 [2,] 0.3145808 1.0000000 0.5032801
4 [3,] 0.3468806 0.5032801 1.0000000
```

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### In Stata:

1 pwcorr price height width

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```

# Outputs:

1		1	price	height	width
2		-+-			
3	price	1	1.0000		
4	height	1	0.3146	1.0000	
5	width	1	0.3469	0.5033	1.0000

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#### In R:

```
# scatterplots (for continuous variables)
2 ggplot(Greene, aes(x = HEIGHT, y = PRICE)) + geom_
     point() + xlab("height") + ylab("price")
3
4 # boxplots (for ordinal variables)
5 ggplot(Greene, aes(x = SIGNED, y = PRICE)) + geom_
     boxplot() + xlab("signed") + ylab("price")
6
7 # multivariate relations
8 ggplot(Greene, aes(x = WIDTH, y = PRICE, col = SIGNED)
     ) + geom_point() +
    xlab("width") + ylab("price") +
g
  scale_color_discrete("signed", breaks = c(0,1),
10
                         labels = c("no", "yes"))
```

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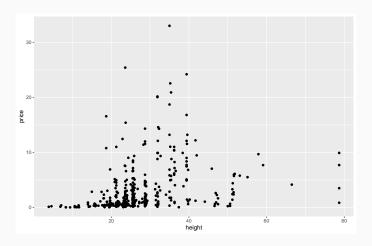
#### In Stata:

```
* * scatterplots
twoway scatter price height

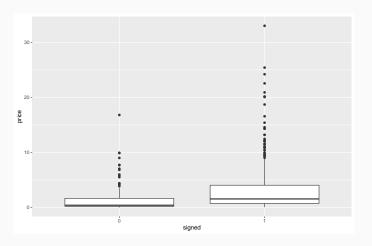
* boxplots
graph box price, over(signed)

* * multivariate
twoway (scatter price height if signed == 0) ///
(scatter price height if signed == 1), ///
legend(label(1 not signed) label(2 signed))
```

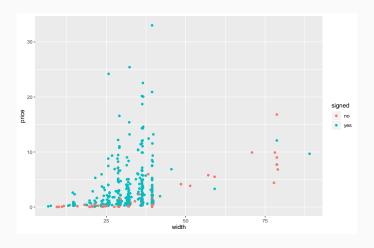
# Scatterplots:



# Boxplots:



# Multivariate:



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# **Theory**

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# Sample

The sample observations are sales of paintings from Monet from houses number 1, 2, 3.

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$$Price_i = \beta_0 + \beta_1 Height_i + \beta_2 Width_i + \beta_3 Signed_i + \beta_4 House_i + u_i$$

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Gauss-Markov assumptions make OLS BLUE (Greene, 2003):

1. Linearity

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<sup>\*</sup>not necessary

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#### In R:

```
1 model <- lm(data = Greene, PRICE ~ HEIGHT + WIDTH +
         SIGNED + HOUSE)
2 summary(model)</pre>
```

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### In R:

#### In Stata:

1 reg price height width signed house

OLS can also be computed using matrix algebra:

• **X** is the  $n \times k$  matrix of k-1 independent variables (the intercept is the +1 variable)

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$$\hat{\boldsymbol{\beta}} = \left( \mathbf{X}' \mathbf{X} \right)^{-1} \mathbf{X}' \mathbf{y} \tag{1}$$

Computing OLS using matrix algebra (see R script):

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Outputs:

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### Outputs:

```
[,1]
2 [1,] -5.52043300
3 [2,] 0.09096657
4 [3,] 0.11169822
5 [4,] 2.29231727
6 [5,] 0.38896084
```

Results from the stargazer package in R (Hlavac, 2018):

	Dependent variable:	
	PRICE	
HEIGHT	0.091***	
	(0.022)	
WIDTH	0.112***	
	(0.021)	
SIGNED	2.292***	
	(0.503)	
HOUSE	0.389	
	(0.328)	
Constant	-5.520***	
	(1.092)	
Observations	430	
Adjusted R <sup>2</sup>	0.179	
F Statistic	24.395*** (df = 4; 425)	
Note:	*p<0.1; **p<0.05; ***p<0.01	

17

$$Price = -5.520 + 0.091 Height + 0.112 Width + 2.292 Signed + 0.389 House + u$$
 (2)

Interpretation of the model:

$$Price = -5.520 + 0.091 Height + 0.112 Width + 2.292 Signed + 0.389 House + u$$
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 Ceteris paribus, 1 inch increase in Height (Width) raises the price by 0.091 (0.112) units. Similar for the other variables.

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- The theory is not rejected, although our null-hypotheses are  $H_0: \beta_i = 0$ .

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- The theory is not rejected, although our null-hypotheses are  $H_0: \beta_i = 0$ . One-tailed test would be more appropriate (see script).

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Check the following. In R:

```
1 qplot(x = model$fitted.values, y = model$residuals) +
2    geom_point() + xlab("fitted values") +
3    ylab("residuals") +
4    geom_hline(yintercept = 0, color = c("red"))
```

Do you see any problem in your model?

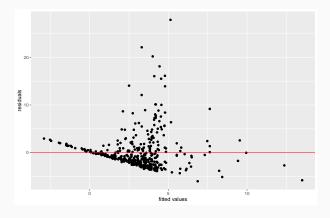
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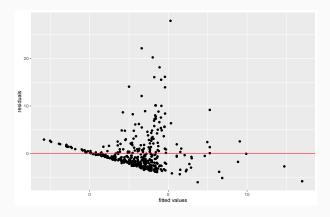
### In Stata:

```
1 rvfplot, yline(0)
```

What does the following picture suggest to you?

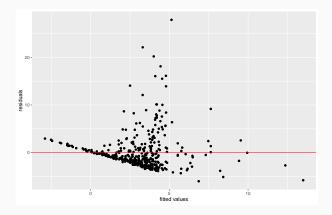


What does the following picture suggest to you?



It is very likely that our errors (not residuals) are heteroskedastic.

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It is very likely that our *errors* (**not** residuals) are heteroskedastic. We cannot trust the standard errors of our model.

Procedure to have heteroskedasticity-robust estimators for the standard errors (and trustworthy t-tests):

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#### In R:

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library(sandwich)
library(lmtest)
robust <- vcovHC(model, type = "HC1")
coeftest(model, vcov. = robust)</pre>
```

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## In Stata:

```
1 reg price height width signed house, robust
2 ereturn list r2_a
```

Heteroskedasticity-robust estimators for the standard errors:

Dependent variable: PRICE	
(1)	(2)
0.091***	0.091***
(0.022)	(0.022)
0.112***	0.112***
(0.021)	(0.019)
2.292***	2.292***
(0.503)	(0.349)
0.389	0.389
(0.328)	(0.289)
-5.520***	-5.520***
(1.092)	(0.939)
430	430
0.179	0.179
24.395***	24.395***
	PI non-robust (1) (2) 0.091*** (0.022) (0.022) (0.112*** (0.021) (0.503) (0.503) (0.328) (1.092) (1.092) (1.092)

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- We reject the null-hypothesis that the independent variables, jointly considered, are not significant in determining the dependent variable.
- The R<sup>2</sup> tells how much of the variance of the observations for the dependent variable is explained by the included regressors. (note: we use the adjusted R<sup>2</sup>)

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- The R<sup>2</sup> tells how much of the variance of the observations for the dependent variable is explained by the included regressors. (note: we use the adjusted R<sup>2</sup>)
- The F-test is a better statistic to evaluate the model, because
  it is a test which refers to the population. The R<sup>2</sup> has no
  population counterpart: it merely refers to the sample at
  hand. Thus its meaning is limited.

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$$R^{2} = \frac{ESS}{TSS} = \frac{\sum_{i=1}^{n} (\hat{y}_{i} - \bar{y})^{2}}{\sum_{i=1}^{n} (y_{i} - \bar{y})^{2}} = 1 - \frac{\sum_{i=1}^{n} \hat{u}_{i}^{2}}{\sum_{i=1}^{n} (y_{i} - \bar{y})^{2}}$$

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How do we perform the F test?

Start from an F distribution with k-1 and n-k degrees of freedom (k independent variables, intercept included, n observations).

The critical value (F statistic) from such F distribution is:

$$F[k-1, n-k] = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

6. Based on your model, predict what would be the price of a painting with arbitrary values for the dependent variables

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6. Based on your model, predict what would be the price of a painting with arbitrary values for the dependent variables

We can do it with pen and paper, we don't really need a software: If Height = 30, Width = 30, Signed = 0, House = 3 then:

$$Price = -5.520 + 0.091 Height + 0.112 Width + 2.292 Signed + 0.389 House$$
 (3)

$$P\hat{rice} = -5.520 + 0.091 \times 30 + 0.112 \times 30 + 2.292 \times 0 + 0.389 \times 3 = 1.737$$
(4)

## Conclusion

All clear? Questions? Thanks and see you next week!

## References

Greene, W. H. (2003). *Econometric analysis*. Pearson Education India.

Hlavac, M. (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables. Central European Labour Studies Institute (CELSI), Bratislava, Slovakia. R package version 5.2.2.