GV300 - Quantitative Political Analysis

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Math refresher (part 1)

1. Notation
   - 1.1 Variables and constants
   - 1.2 Sets
   - 1.3 Operators
2. Linear algebra
   - 2.1 Scalars
   - 2.2 Vectors
   - 2.3 Matrices
3. Functions
   - 3.1 Basic definitions
   - 3.2 Properties
   - 3.3 Important functions

Math refresher (part 2)

4. Calculus
    - 4.1 Basics
    - 4.2 Limits
    - 4.3 Derivative
    - 4.4 Rules of differentiation
    - 4.5 Extrema
5. Integrals
    - 5.1 Definite integral
    - 5.2 Indefinite integral
    - 5.3 Fundamental theorem of calculus
    - 5.4 Rules of operation

Math Refresher - Exercises in R and Stata

Compute the following (using either R/Stata or just pen and paper):

1.
$$\begin{bmatrix} 2 & 0 & -4 \\ 1 & 3 & 5 \\ -3 & 1 & 4 \\ 1 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -4 \\ -1 & 2 \\ 3 & 5 \end{bmatrix}$$

2.
$$\begin{bmatrix} 2 & 0 & -4 \\ 1 & 3 & 5 \\ -3 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -4 & 1 \\ -1 & 2 & 3 \end{bmatrix}$$

3.
$$\begin{bmatrix} 2 & 0 & -4 \\ 1 & 3 & 5 \\ -3 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & -3 \\ 0 & 3 & 1 \\ -4 & 5 & 4 \end{bmatrix}$$

Math Refresher - Exercises in R and Stata

Generate a vector of 1000 random numbers (it should contain at least a 0) and call it x. Obtain the functions below and plot them in twoway graphs. Then calculate their derivatives and plot them on a twoway graph with the original functions.

1. Linear

$$y = 3x - 4$$

2. Quadratic

$$y = -x^2 + 3x - 4$$

3. Cubic

$$y = x^3 - x^2 + 3x - 4$$

4. Logarithm

$$y = ln(x)$$

5. Exponential

$$y = e^{(x)}$$

6. Trigonometric

$$y = sin(x) + 1.3$$

Exercises for R users:

1. A function for the median and the standard deviation
   a. re-program a function for median and population standard deviation and call them "median2" and "sd2"
   b. generate two vectors (of even and uneven length), apply the new functions to them and compare them with those obtained by applying base R's functions.
2. Imagine you're tossing a coin n times. (X: number of heads)
   a. write a function that returns a data frame with all X, all possible ways to get X, and the probability of each X
   b. apply the function to n $= 10$, p $= 0.5$. Generate a twoway plot (with X on the x-axis and p on the y-axis).
   c. Draw 1000 observations for the number of heads obtained by tossing 10 times a fair coin. Obtain a histogram reporting the results and compare it with the plot from b.

Exercises for Stata users:

1. Program a function that takes as arguments:
   a. the number of observations to be drawn from a random binomial distribution
   b. the number of trials
   c. the probability of success for a trial
2. Simulate the following:
   a. toss a fair coin 2, 20, 200, 2000, 4000 and 8000 times and compute the mean
   b. store a boxplot and a histogram relative to the means obtained for each of the six iterations and export a pdf showing them side by side
   c. export a graph showing all iterations and all plots together (a total of 12 plots, arranged in 3 rows)

Median, mean and population standard deviation

Median: "it is the point such that as many cases are greater as are less" (Gill 2006, 362). What if the variable is even in length?

Mean:

$$\mu_X = \frac{1}{N} \sum_{i=1}^{N} X_i$$

Population standard deviation:

$$\sigma_X = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_i - \mu_X)^2}$$

The binomial distribution

Number of possible ways each outcome can occur:

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

Probability that a certain outcome occurs:

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Conclusion

All clear? Questions?

Thanks and see you next week!

References

Gill, Jeff (2006). *Essential Mathematics for Political and Social Research*. Cambridge University Press.