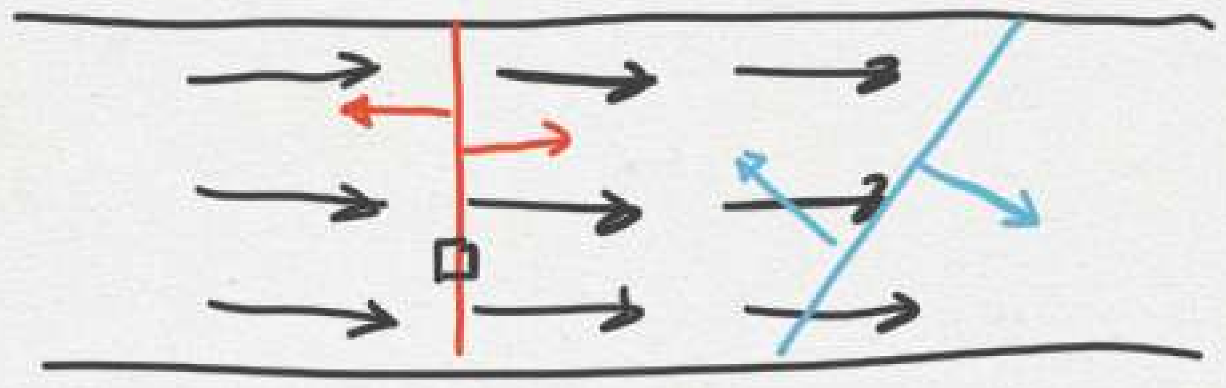
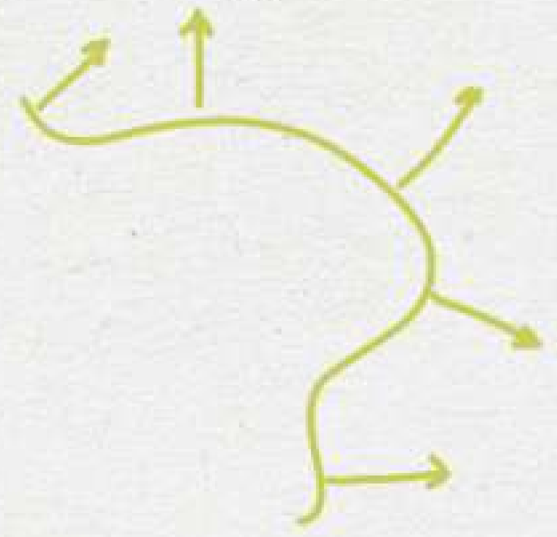
$$\vec{\nabla} \times \vec{E} = 0 \longleftrightarrow \oint \vec{E} \cdot d\vec{s} = 0$$

LOCALE

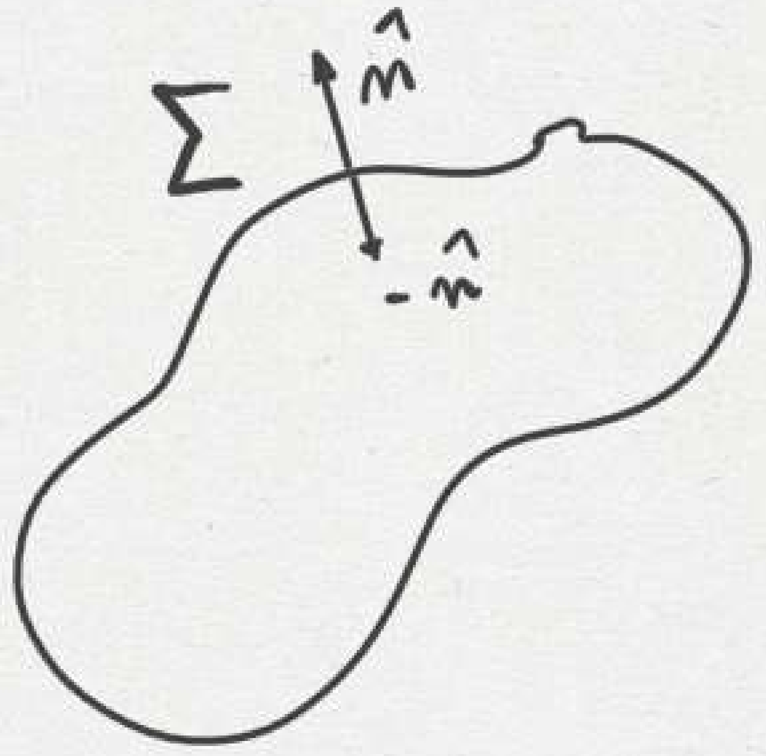
INTEGRALE

TEOREMA DI GAUSS



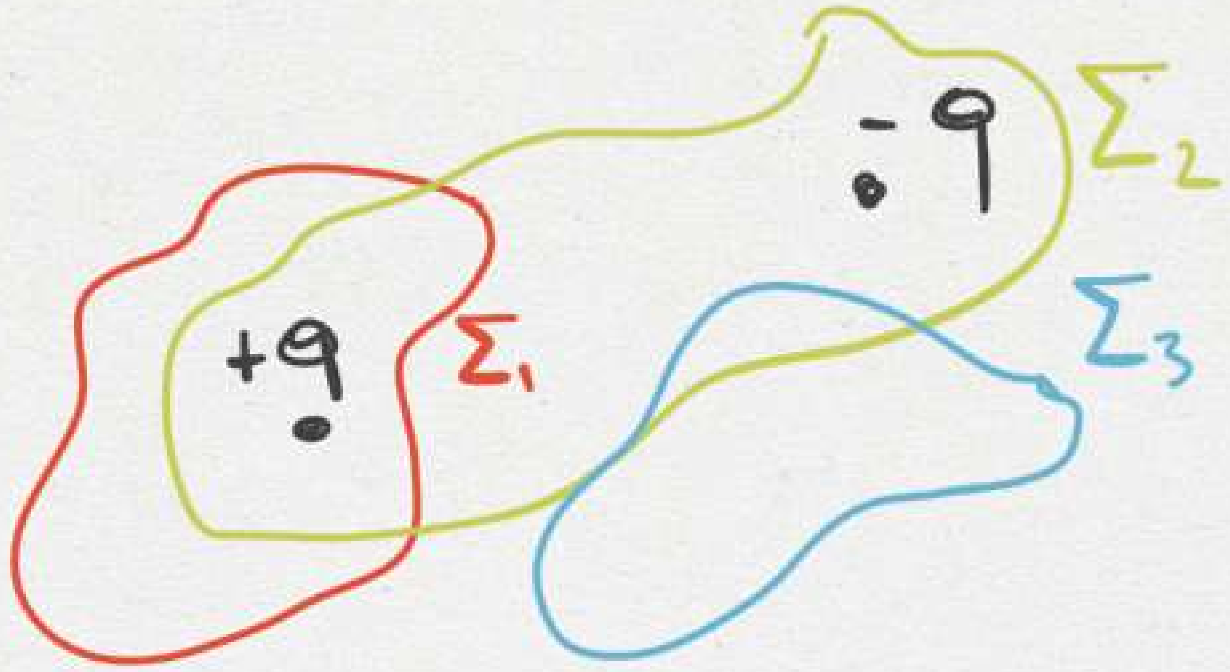


$\vec{E}$, $\quad d\Phi(\vec{E}) \equiv \vec{E}\cdot\boxed{\hat{n}}\,d\Sigma \Rightarrow \Phi_\Sigma(\vec{E}) = \int_\Sigma \vec{E}\cdot\hat{n}\,d\Sigma$



$$\Phi_\Sigma(\vec{E}) = \oint_\Sigma \vec{E}\cdot\hat{n}\,d\Sigma \overset{\text{TEOREMA}}{=} \frac{Q_\Sigma}{\varepsilon_0}$$

Q_Σ é la somma algebrica delle cariche all'interno di Σ

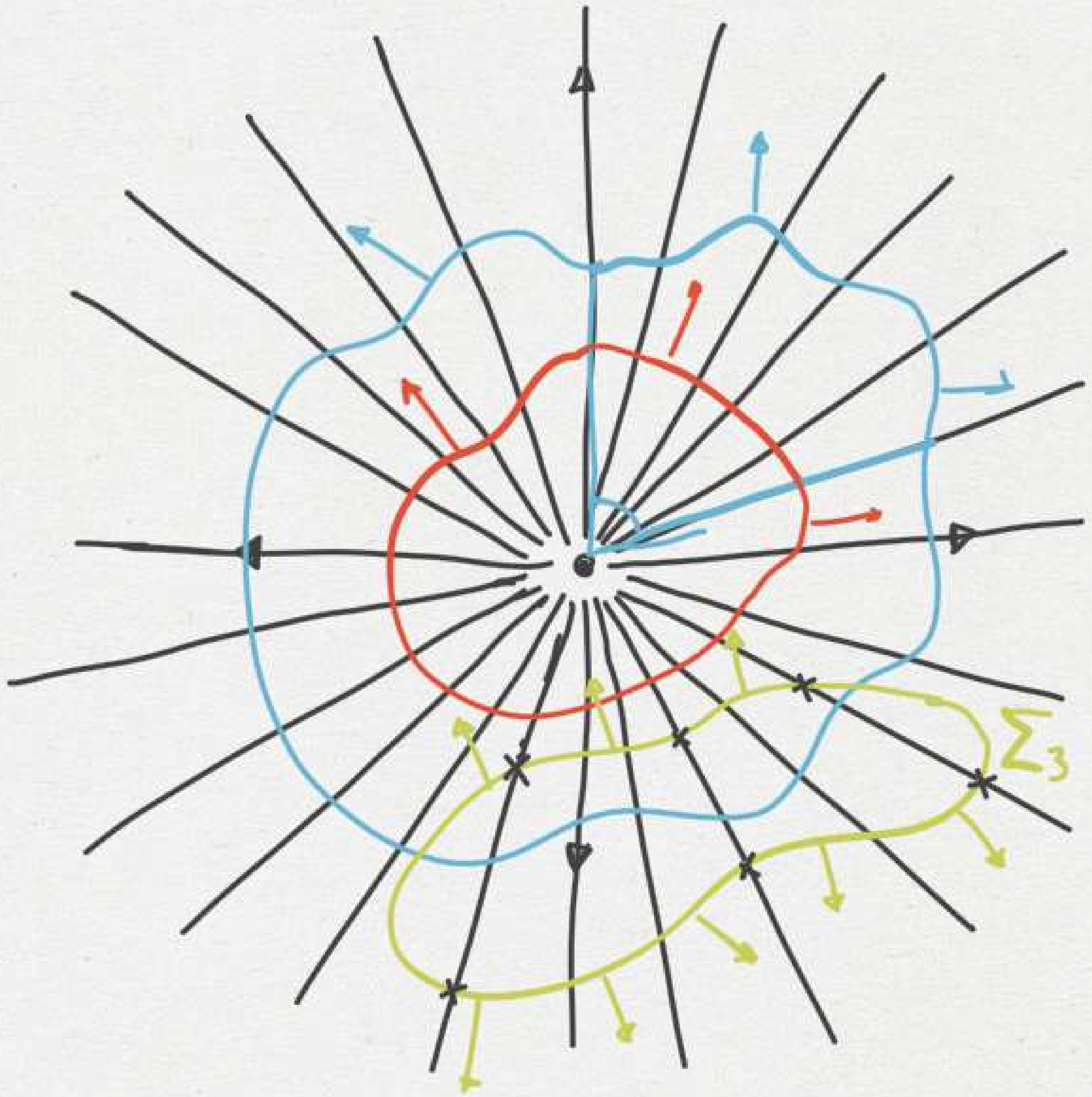


$$\underbrace{\Phi_{\Sigma_1}(\vec{E}) = \frac{q}{\varepsilon_0}}$$

$$\Phi_{\Sigma_2}(\vec{E}) = 0$$

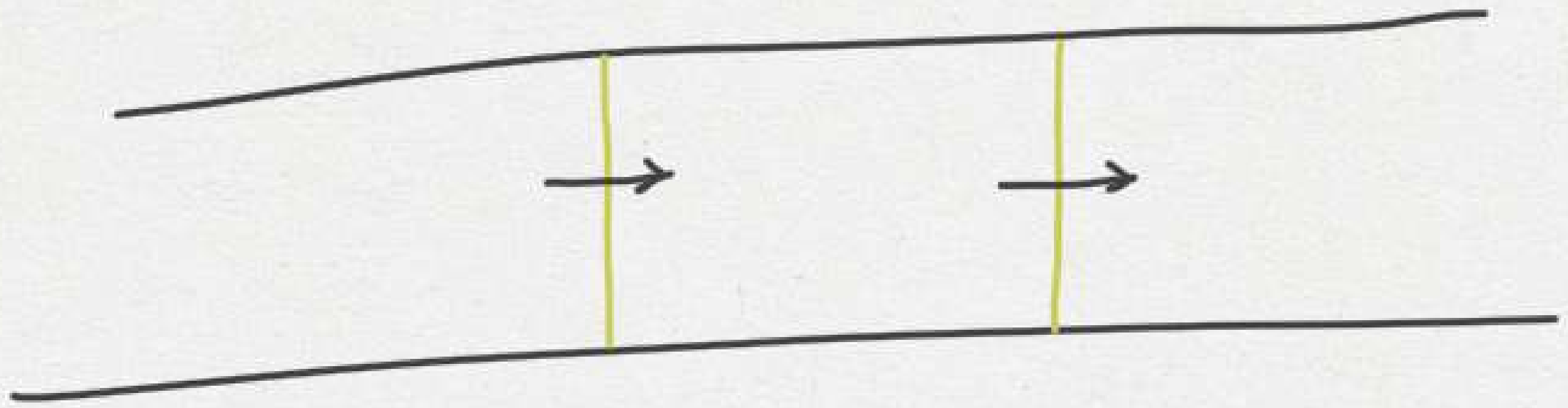
$$\underbrace{\Phi_{\Sigma_3}(\vec{E})} = 0$$

$$\int_\Sigma \vec{E}\cdot\hat{n}\,d\Sigma = \int_\Sigma \sum_i \vec{E}_i\cdot\hat{n}\,d\Sigma = \sum_i \underbrace{\int_\Sigma \vec{E}_i\cdot\hat{n}\,d\Sigma}$$

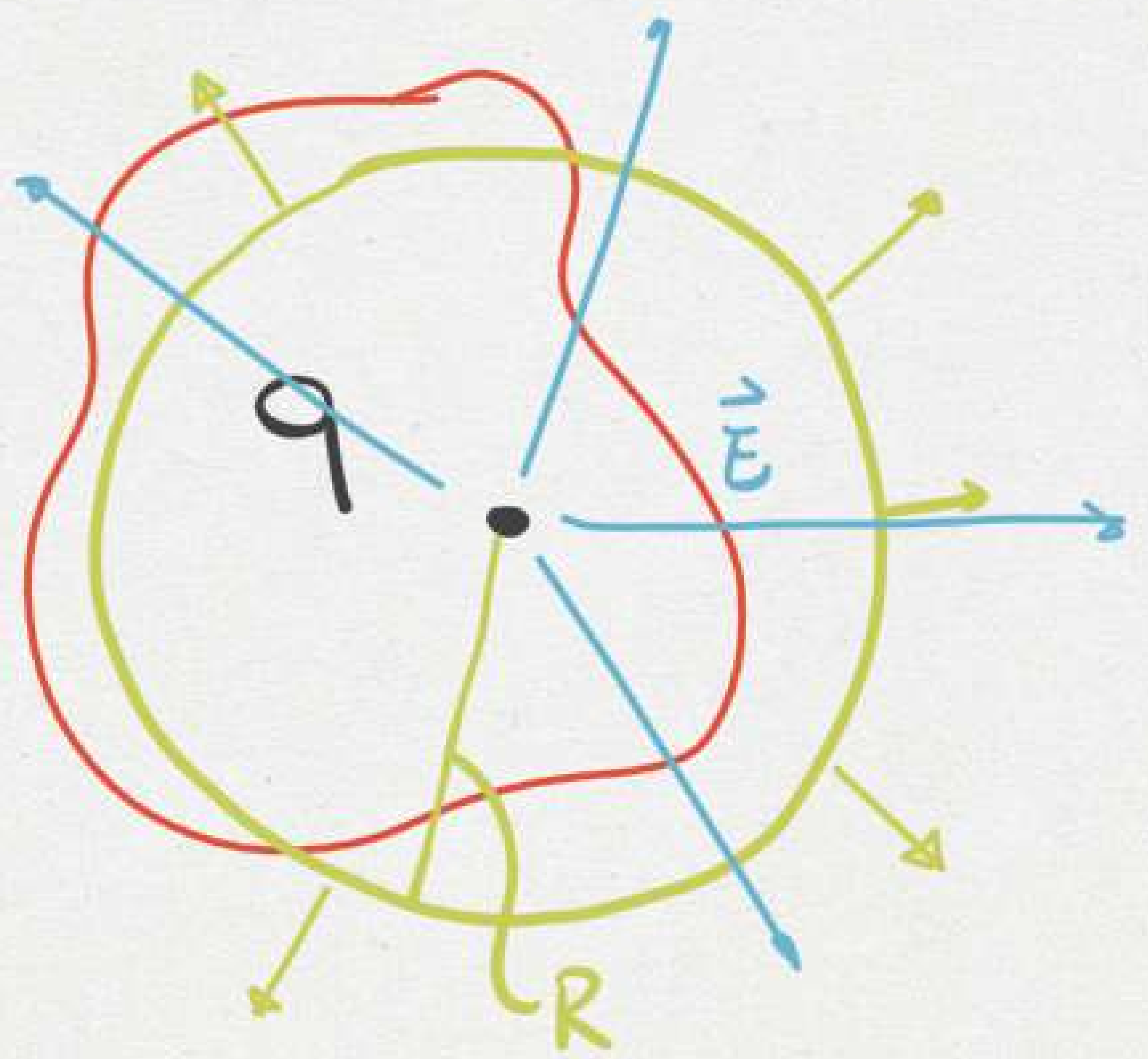


$\Phi_\Sigma > 0$ se $q > 0$

$\Phi < 0$ se $q < 0$



ESEMPIO



$$V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

$$d\Phi = \vec{E}\cdot\hat{n}\, d\Sigma = E(r)\, d\Sigma$$

$$\Rightarrow \Phi_\Sigma(\vec{E}) = \int_\Sigma E(r)\, d\Sigma = \int_\Sigma E(R)\, d\Sigma = E(R)\int_\Sigma d\Sigma = E(R)\, 4\pi R^2$$

$$\Phi_\Sigma(\vec{E}) = \frac{q}{\varepsilon_0} = E(R)\; 4\pi R^2 \Rightarrow$$

$$E(R) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}$$

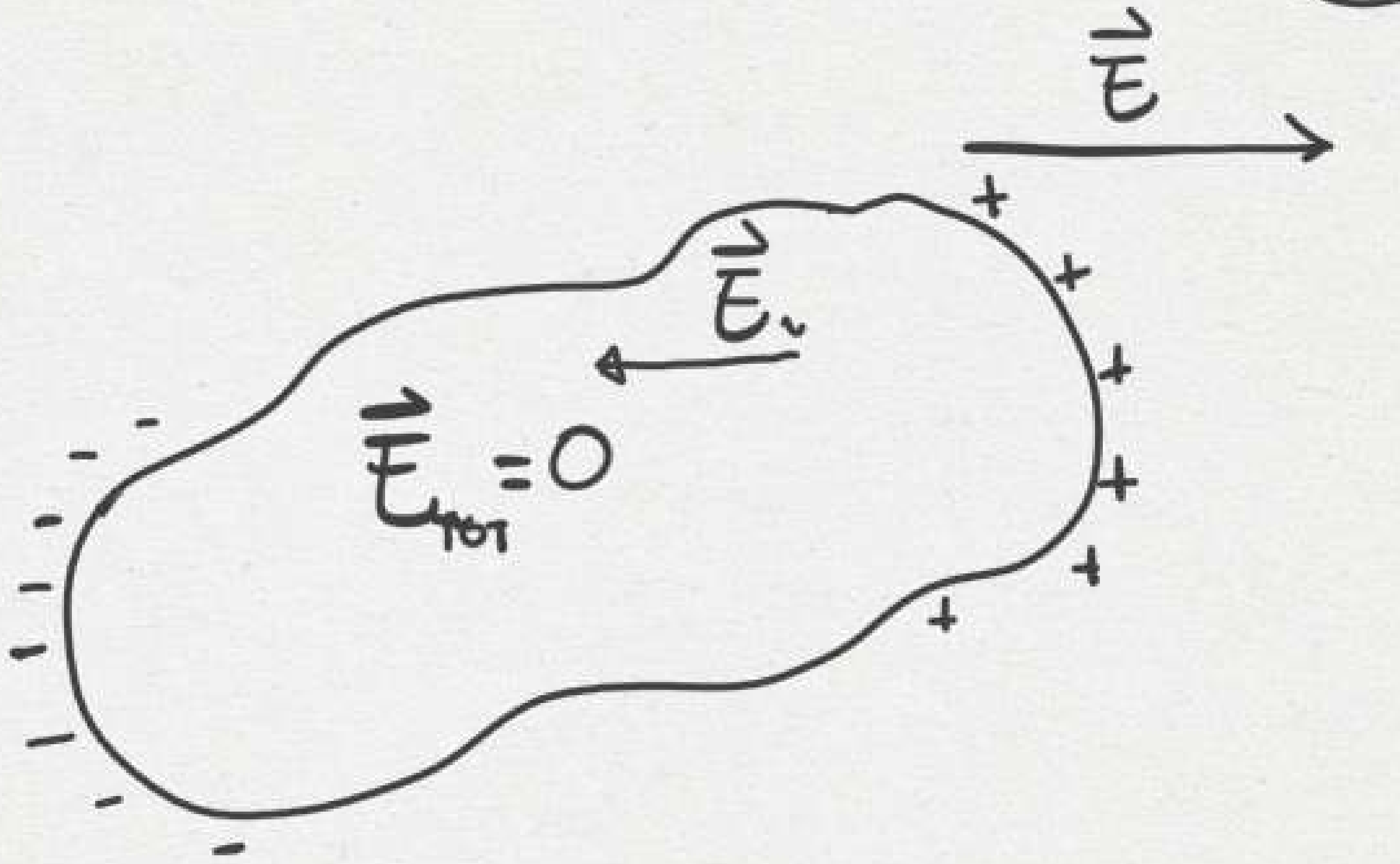
$$\int_{\Sigma} \vec{E}\cdot\hat{n}\, d\Sigma \underset{\text{della divergenza}}{=} \int_{\tau(\Sigma)} \underbrace{\vec{\nabla}\cdot\vec{E}}_{\text{divergenza}}\, d\tau = \frac{Q_{\Sigma}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_{\tau} \rho\, d\tau$$

$$\int_{\tau} \vec{\nabla}\cdot\vec{E}\, d\tau = \int_{\tau} \frac{\rho}{\varepsilon_0}\, d\tau \implies \boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

I LEGGE DI MAXWELL

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

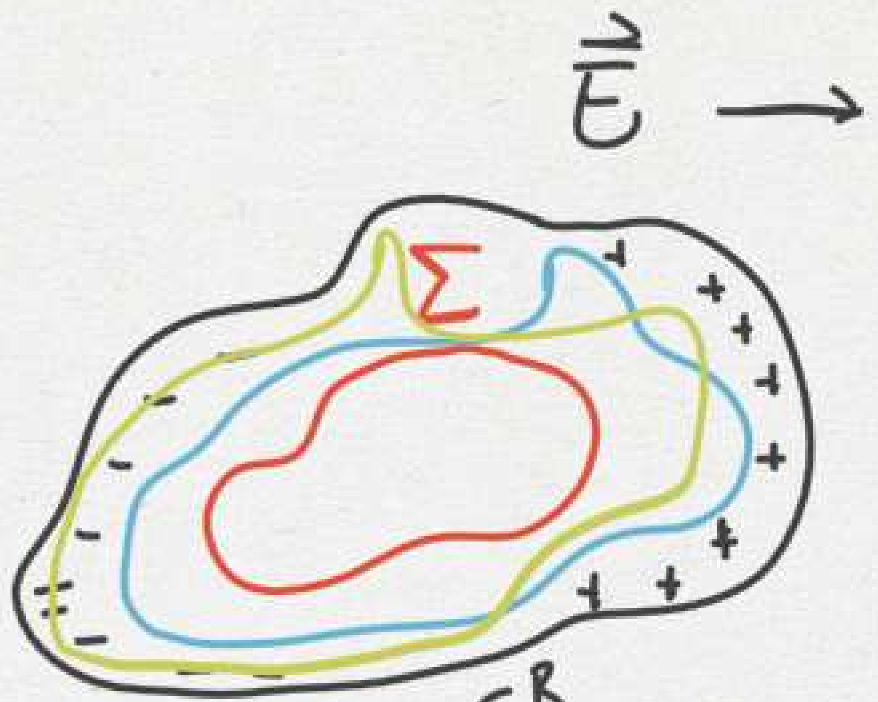
CONDUTTORI



MOTO TRANSIENTE

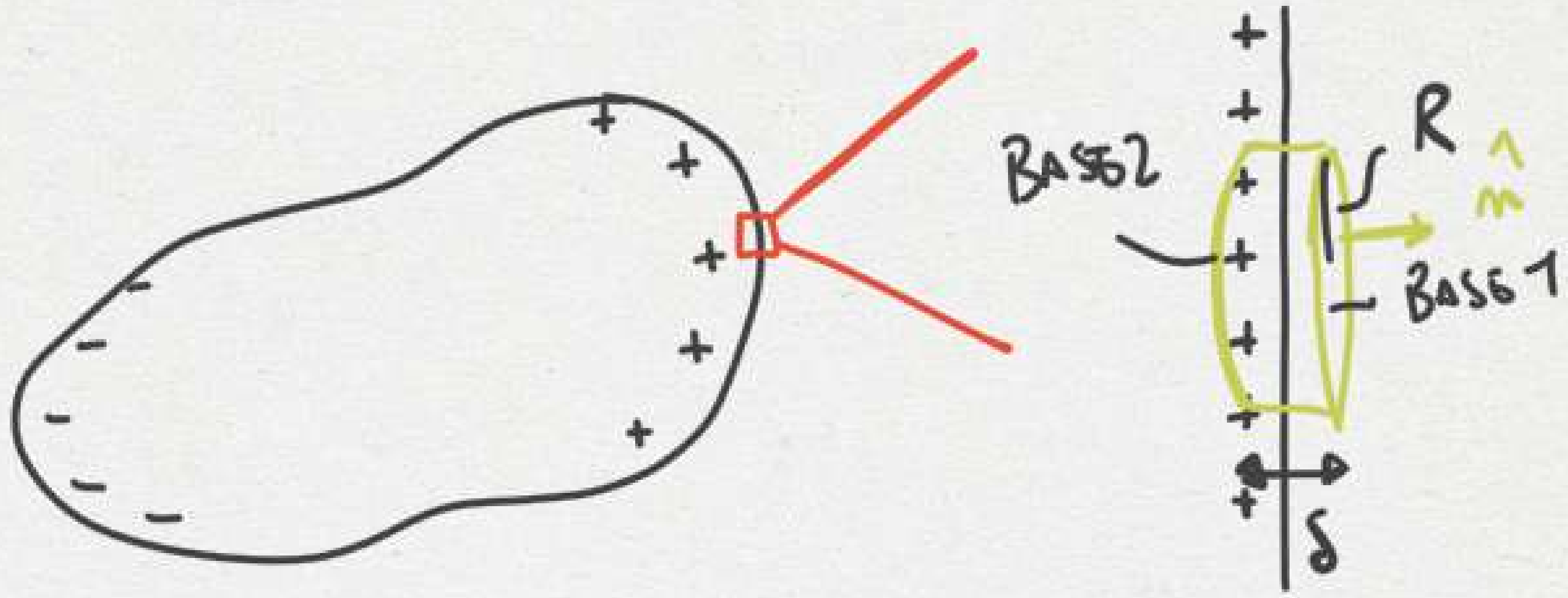
→ ① $\vec{E} = 0$ all'interno dei conduttori

② le cariche sono solo sulla superficie



$$\Phi_\Sigma(\vec{E}) = 0 \overset{\text{GAUSS}}{=} \frac{Q_\Sigma}{\varepsilon_0} \Rightarrow Q_\Sigma = 0$$

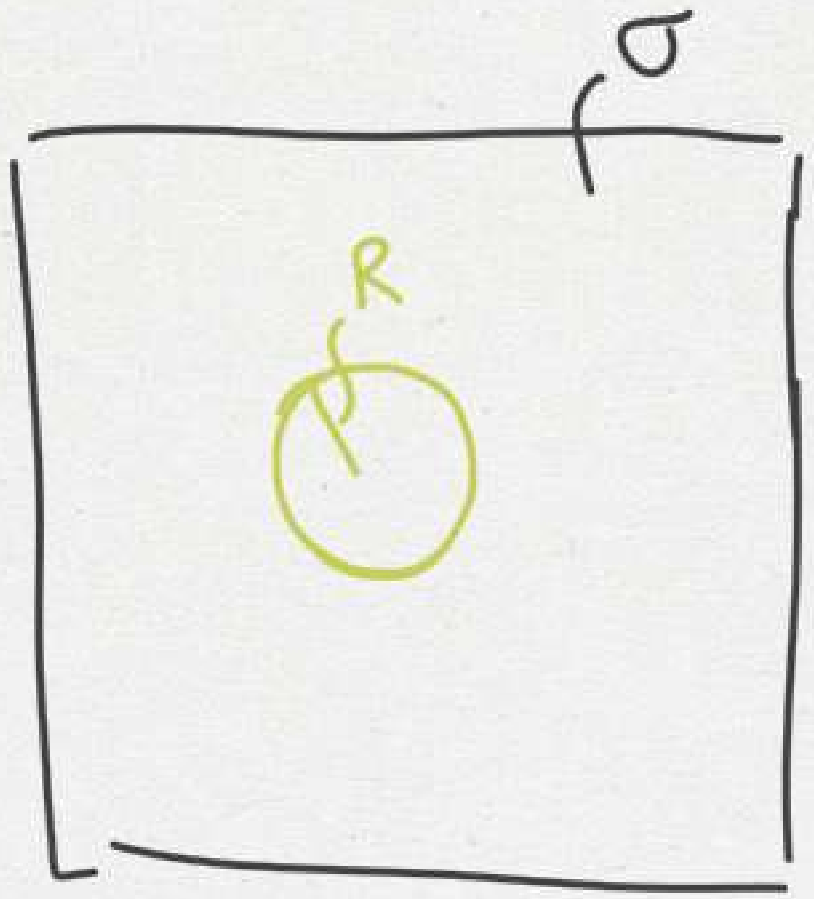
③ $\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = 0 = V(B) - V(A) \Rightarrow V = cost$ in un conduttore



$$\Phi_\Sigma(\vec{E}) = \Phi_\Sigma^{LAT} + \Phi^{BASE\,1} + \Phi^{BASE\,2} \underset{\delta \ll R}{\approx} \Phi^{BASE\,1} + \Phi^{BASE\,2} =$$

$$= \Phi^{BASE\,1} = \int_{BASE\,1} \vec{E} \cdot \hat{n}\, d\Sigma = \int_{BASE\,1} E\, d\Sigma = E \int_{BASE\,1} d\Sigma = E \pi R^2$$

$$\Phi_\Sigma(\vec{E}) = \frac{Q_\Sigma}{\varepsilon_0} = \frac{\sigma \pi R^2}{\varepsilon_0}$$



$$Q_\Sigma = \int_{BASE\,1} \sigma\, d\Sigma = \sigma \int_{BASE\,1} d\Sigma = \sigma \pi R^2 \Rightarrow$$

$$E \pi R^2 = \frac{\sigma \pi R^2}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

$\boxed{\text{teorema di Coulomb}}$