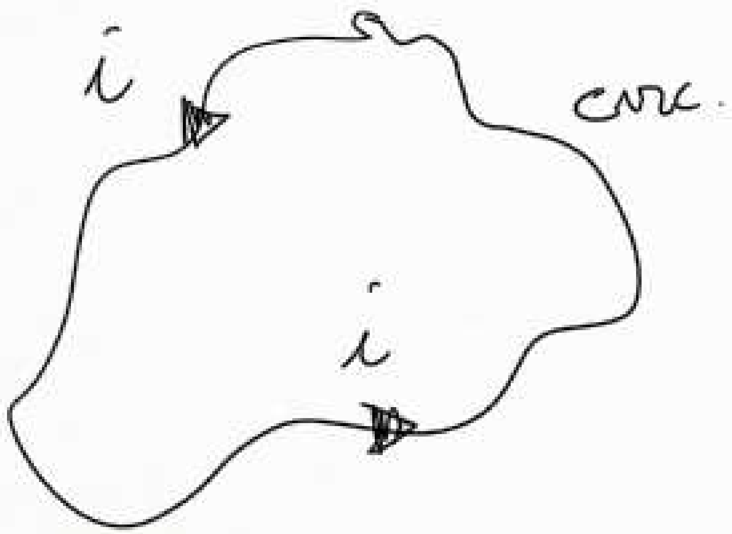
AUTOINDUZIONE



$$\vec{B} = \frac{\mu_0 i}{4\pi}\oint_{circ} \frac{d\vec{l}\times\hat{r}}{r^2}$$

AUTOFLUSSO

$$\Phi_{circ}(\vec{B}) = \int_{\Sigma(circ)} \vec{B}\cdot\hat{n}\,d\Sigma = \int_{\Sigma(circ)}\left[\frac{\mu_0 i}{4\pi}\oint_{circ}\frac{d\vec{l}\times\hat{r}}{r^2}\right]\cdot\hat{n}\,d\Sigma ,$$

$$\Phi_{circ}(\vec{B}) = L i$$

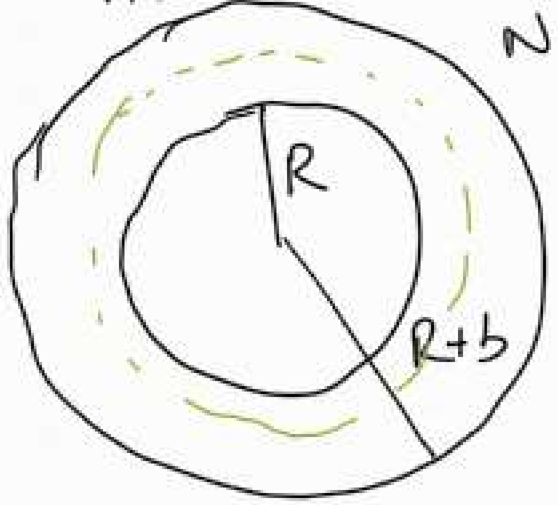
materiale

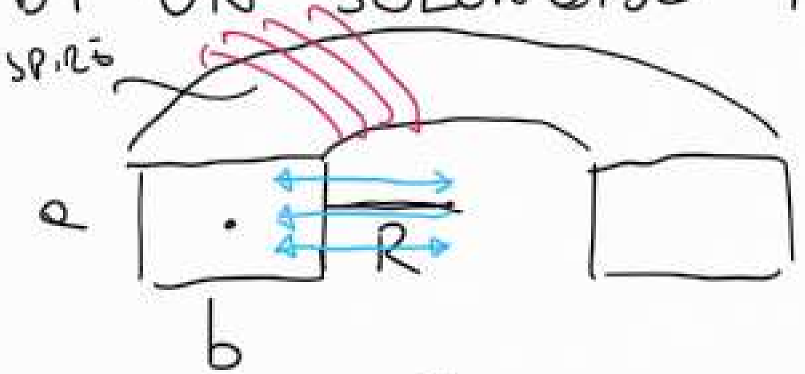
$$L \equiv \int_{\Sigma(circ)} \left(\frac{\mu_0}{4\pi}\oint_{circ}\frac{d\vec{l}\times\hat{r}}{r^2}\right)\cdot\hat{n}\, d\Sigma$$

geometria

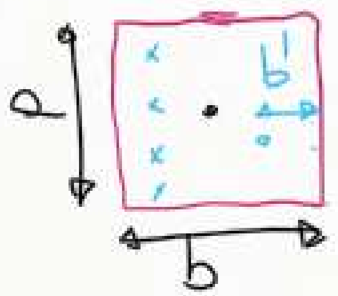
induttanza (coefficiente di induzione)

INDUTTANZA DI UN SOLENOIDE TOROIDALE (BASE RETTANGOLARE)





$$\vec{B} = \frac{\mu_0 N i}{2\pi r}\hat{\varphi} \quad , \quad \Sigma \equiv ab$$

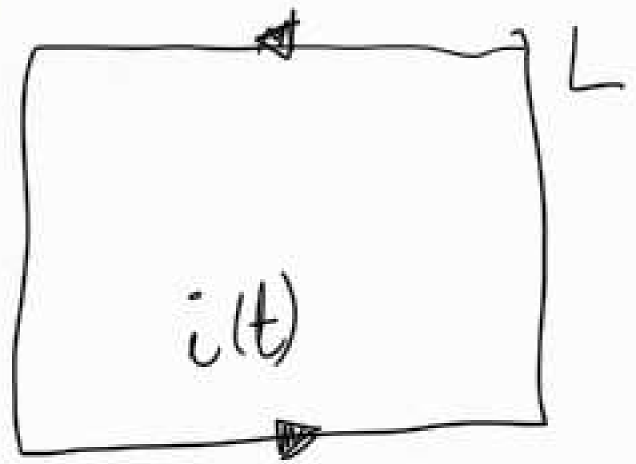


$$\Phi_s(\vec{B}) = \int_\Sigma \vec{B}\cdot\hat{n}\, d\Sigma = \frac{\mu_0 N i}{2\pi}\int_0^a da \int_0^b \frac{db'}{R+b'} = \frac{\mu_0 N i a}{2\pi}\int_0^b \frac{db'}{R+b'} =$$

$$= \frac{\mu_0 N i a}{2\pi}\log(R+b')\Big|_0^b = \frac{\mu_0 N i a}{2\pi}\log\left(\frac{R+b}{R}\right) \Rightarrow$$

$$\Phi(\vec{B}) = N\,\Phi_s(\vec{B}) = \frac{\mu_0 N^2 i a}{2\pi}\log\left(\frac{R+b}{R}\right) = L i$$

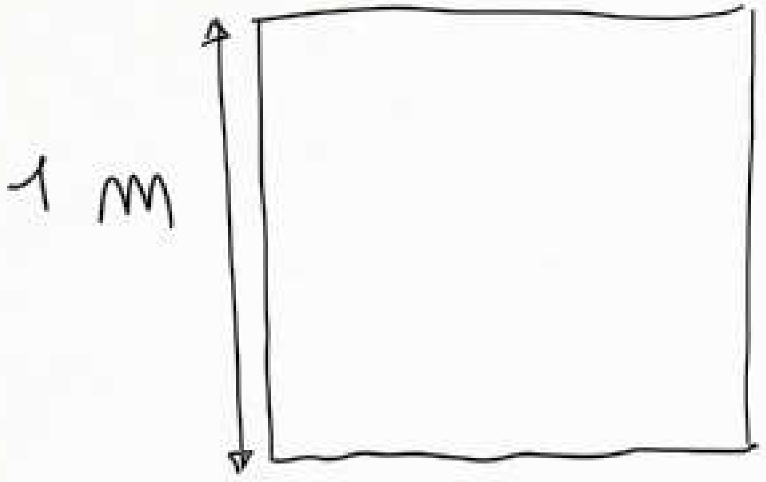
$$L = \frac{\mu_0 N^2 a}{2\pi}\log\left(\frac{R+b}{R}\right)$$



$$\boxed{\mathcal{E}_L = -\frac{d\Phi(\vec{B})}{dt}} = -\frac{d}{dt}(L\,i) = -L\frac{di}{dt} \Rightarrow$$

$$\boxed{\mathcal{E}_L = -L\frac{di}{dt}}$$

$$[L] = \frac{[\Phi]}{[i]} = \frac{Wb}{A} \equiv H$$ (Henry), mH, μH



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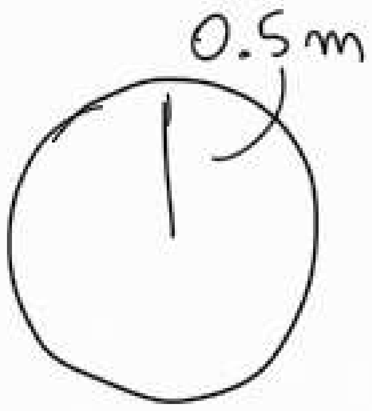
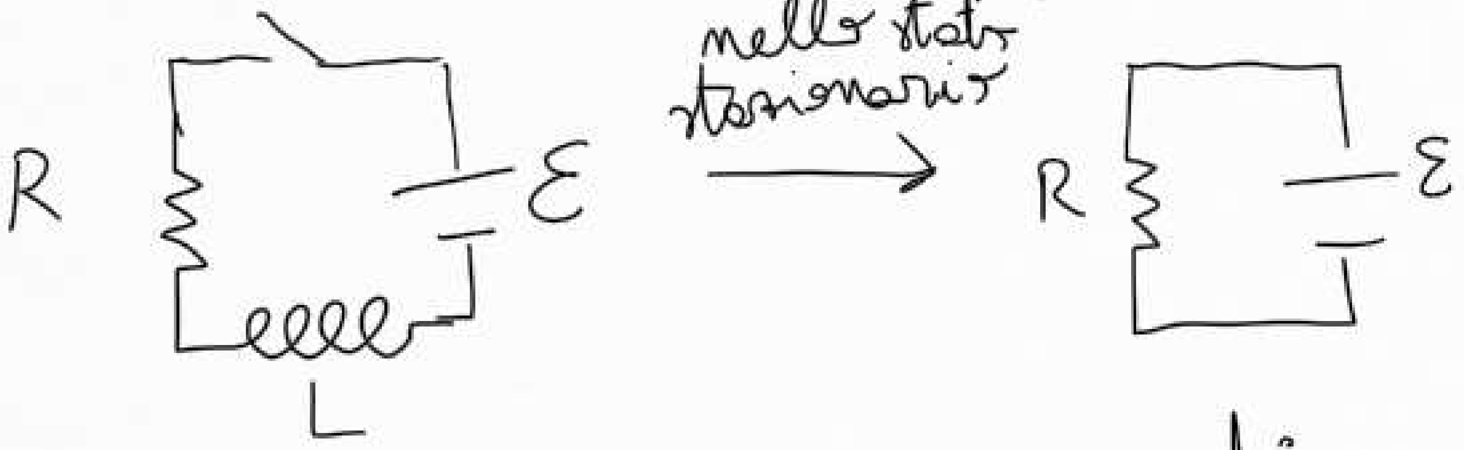


entrambe hanno $L \approx 4\cdot 10^{-6}$ H

quindi

INDUTTORE

CIRCUITO RL



a $t=0$ chiudiamo l'interruttore $\Rightarrow$

$$i(0)=0$$

$$Ri(t)=\mathcal{E}+\mathcal{E}_L=\mathcal{E}-L\frac{di}{dt} \Rightarrow \frac{di}{i-\mathcal{E}/R}=-\frac{R}{L}\,dt \Rightarrow$$

$$\int_{i(0)}^{i(t)}\frac{di}{i-\mathcal{E}/R}=-\frac{R}{L}\int_0^t dt' \Rightarrow \log\left(i-\mathcal{E}/R\right)\Big|_0^{i(t)}=-\frac{R}{L}t \Rightarrow$$

$$\log\left(\frac{i-\mathcal{E}/R}{-\mathcal{E}/R}\right)=\log\left(\frac{\mathcal{E}/R-i(t)}{\mathcal{E}/R}\right)=-\frac{R}{L}t \Rightarrow \frac{\mathcal{E}/R-i(t)}{\mathcal{E}/R}=e^{-\frac{R}{L}t} \Rightarrow$$

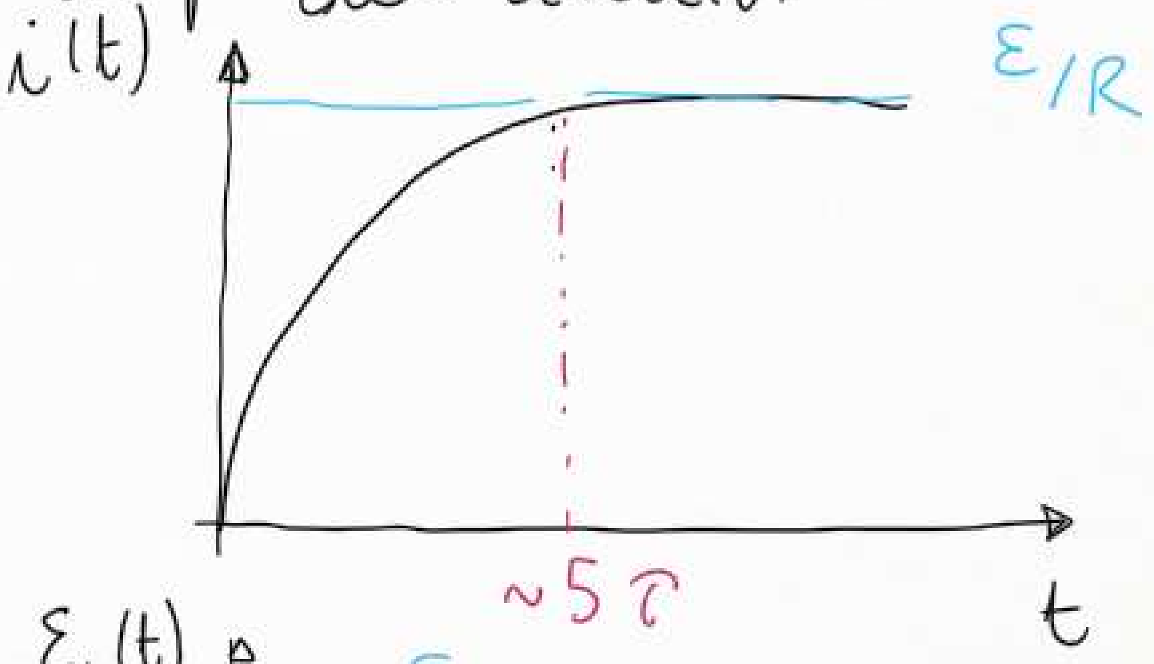
$$i(t)=\frac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right),\ \tau\equiv\frac{L}{R}$$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right), \quad \tau = \frac{L}{R} \text{ costante di tempo del circuito}$$

$$\mathcal{E}_L(t) = -L\frac{di(t)}{dt} = -\mathcal{E}\, e^{-t/\tau}$$

$$i_L(t) = \frac{\mathcal{E}_L(t)}{R} = -\frac{\mathcal{E}}{R}\, e^{-t/\tau}$$

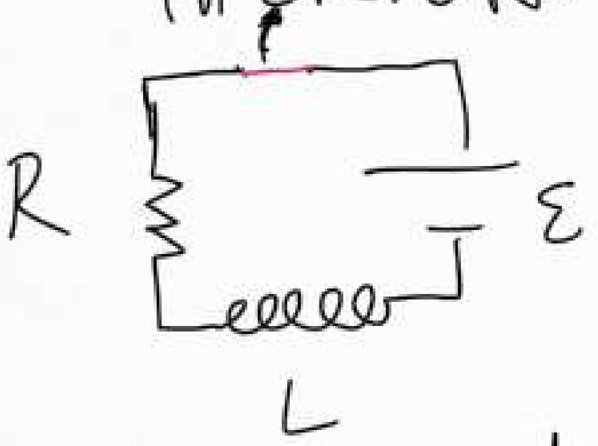
↳ verso opposto alla corrente dovuta al generatore



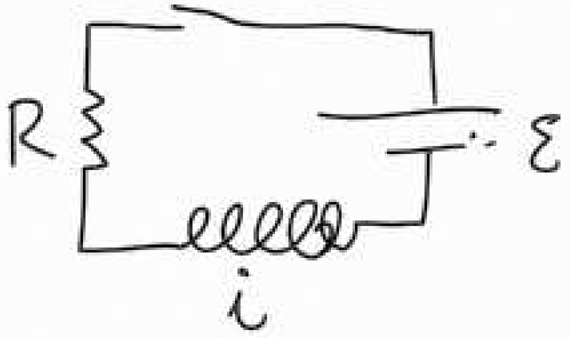
{ in assenza di induttori $\tau \sim 10^{-5} \div 10^{-8}$ s

{ se invece $L = 1$ H e $R = 1\,\Omega \Rightarrow \tau = 1$ s

APERTURA DI UN CIRCUITO INDUTTIVO



$$i(0) \equiv i_\infty = \frac{\mathcal{E}}{R}$$



$\approx$

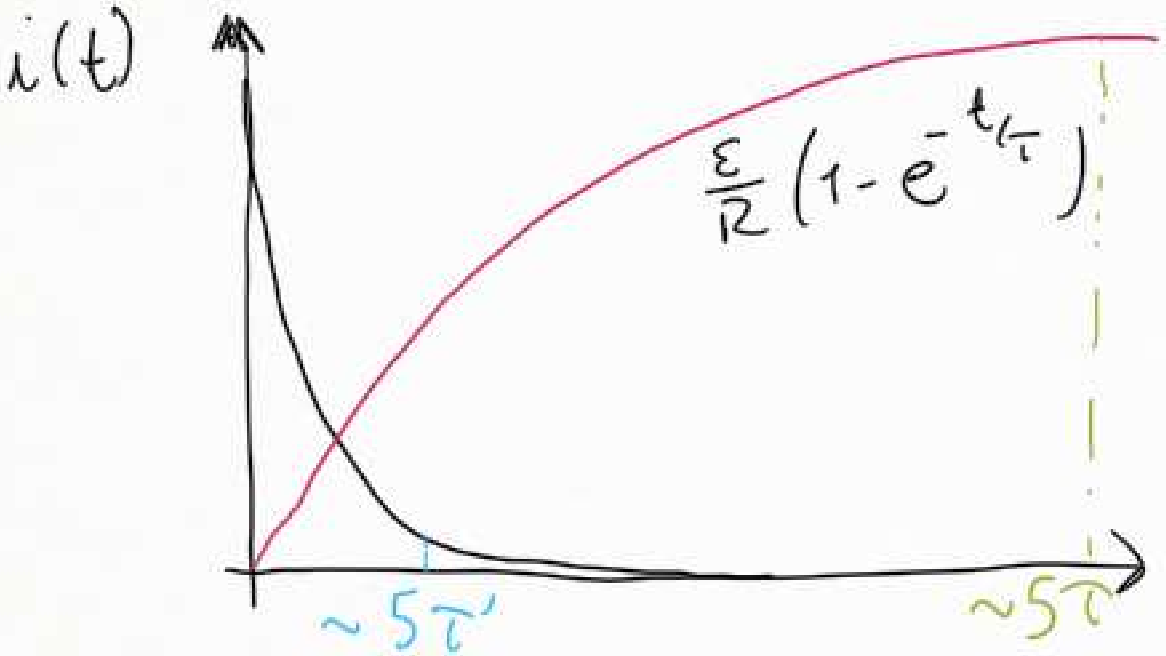
$R' \gg R$

$\mathcal{E}_L \gg \mathcal{E}$ a tempi brevi

$$R' i = \mathcal{E}_L = -L\frac{di}{dt} \Rightarrow -\frac{R'}{L}\,dt = \frac{di}{i} \Rightarrow -\frac{R'}{L}t = \log\frac{i(t)}{i(0)} \Rightarrow$$

$$e^{-\frac{R'}{L}t} = \frac{i(t)}{i(0)} \Rightarrow i(t) = i(0)\,e^{-\frac{R'}{L}t} = \frac{\mathcal{E}}{R}\,e^{-\frac{R'}{L}t} \equiv \frac{\mathcal{E}}{R}\,e^{-t/\tau'}, \quad \tau' \equiv \frac{L}{R'}$$

$\tau' \ll \tau$



$$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau'}$$

$$\mathcal{E}_L(t) = -L\frac{di}{dt} = \frac{R'}{R}\mathcal{E} e^{-t/\tau'} \Rightarrow$$

$$\mathcal{E}_L(0) = \frac{R'}{R}\mathcal{E} \gg \mathcal{E}$$ perché $R' \gg R$

$$\boxed{e^{-t/\tau}}$$ esp. negativo

τ → tempo caratteristico

ENERGIA MAGNETICA

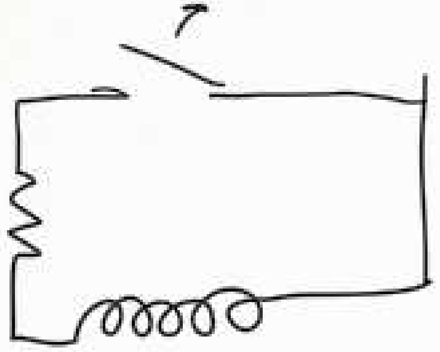
circuito RL

$$Ri = \mathcal{E} + \mathcal{E}_L \Rightarrow \mathcal{E} = Ri - \mathcal{E}_L$$

$$P = \mathcal{E} i = (Ri - \mathcal{E}_L) i = Ri^2 - \mathcal{E}_L i = \underbrace{Ri^2}_{\text{dissipata nella resistenza}} + \overbrace{Li \frac{di}{dt}}^{\text{immagazzinata nell'induttanza}}$$

$$dW = P\,dt \Rightarrow W(t) = \int_{i_1}^{i_2} L i\, di = \frac{1}{2} L i_2^2 - \frac{1}{2} L i_1^2$$

quantità di energia che il generatore fornisce all'induttanza nell'intervallo $\Delta t_{12} = t_2 - t_1$



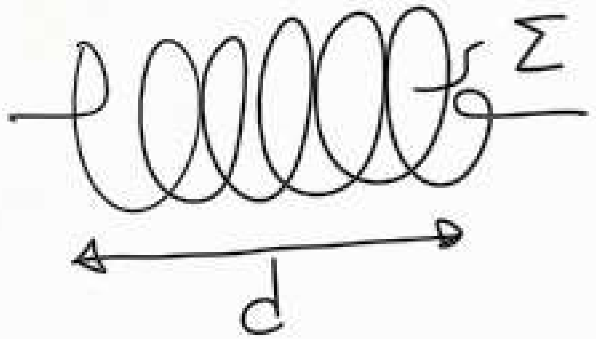
$$i(t) = \frac{\mathcal{E}}{R} e^{-t/\tau'} \implies$$

$$i_\infty^2 = i_{(0)}^2$$

$$W = \int_0^\infty R' i^2 dt = R' \frac{\mathcal{E}^2}{R^2} \int_0^\infty e^{-2\frac{R'}{L}t} dt = \frac{1}{2} L \frac{\mathcal{E}^2}{R^2} = \boxed{\frac{1}{2} L i_\infty^2}$$

questa quantità è uguale all'energia che era stata immagazzinata durante la chiusura del circuito

$$U_L = \frac{1}{2} L i^2$$



$B = \mu_0 n i$, $N = n d$, $\tau = \Sigma d$ volume del solenoide

$\Phi_S(\vec{B}) = \mu_0 n i \Sigma \implies$

$\Phi(\vec{B}) = N\,\Phi_S(\vec{B}) = n d\,\Phi_S(\vec{B}) = \mu_0 n^2 i \Sigma d = \mu_0 n^2 i \tau \implies$

$L = \mu_0 n^2 \tau \implies$

$$U_L = \frac{1}{2} L i^2 = \frac{1}{2} \overset{\frac{B^2}{\mu_0}}{\mu_0 n^2 i^2}\,\tau = \overset{u_m}{\overset{\equiv}{\boxed{\frac{1}{2}\frac{B^2}{\mu_0}}}}\,\tau \doteq u_m \tau \implies$$

$$\boxed{u_m = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\mu_0 H^2 = \frac{1}{2} B H} \implies \boxed{U_m = \int_\tau \frac{1}{2}\frac{B^2}{\mu_0}\, d\tau}$$