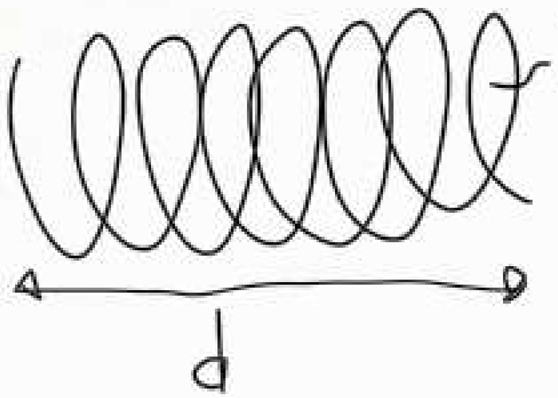
- CODICE OPIS

  C9YWJ7ZT

- ESONERO: 23/12 ORE 11 (DURATA: 2 ORE)
  - ISCRIVETEVI SUL SITO!
  - 2 ESERCIZI
  - ARGOMENTO: MAGNETOSTATICA + ELETTROMAGNETISMO



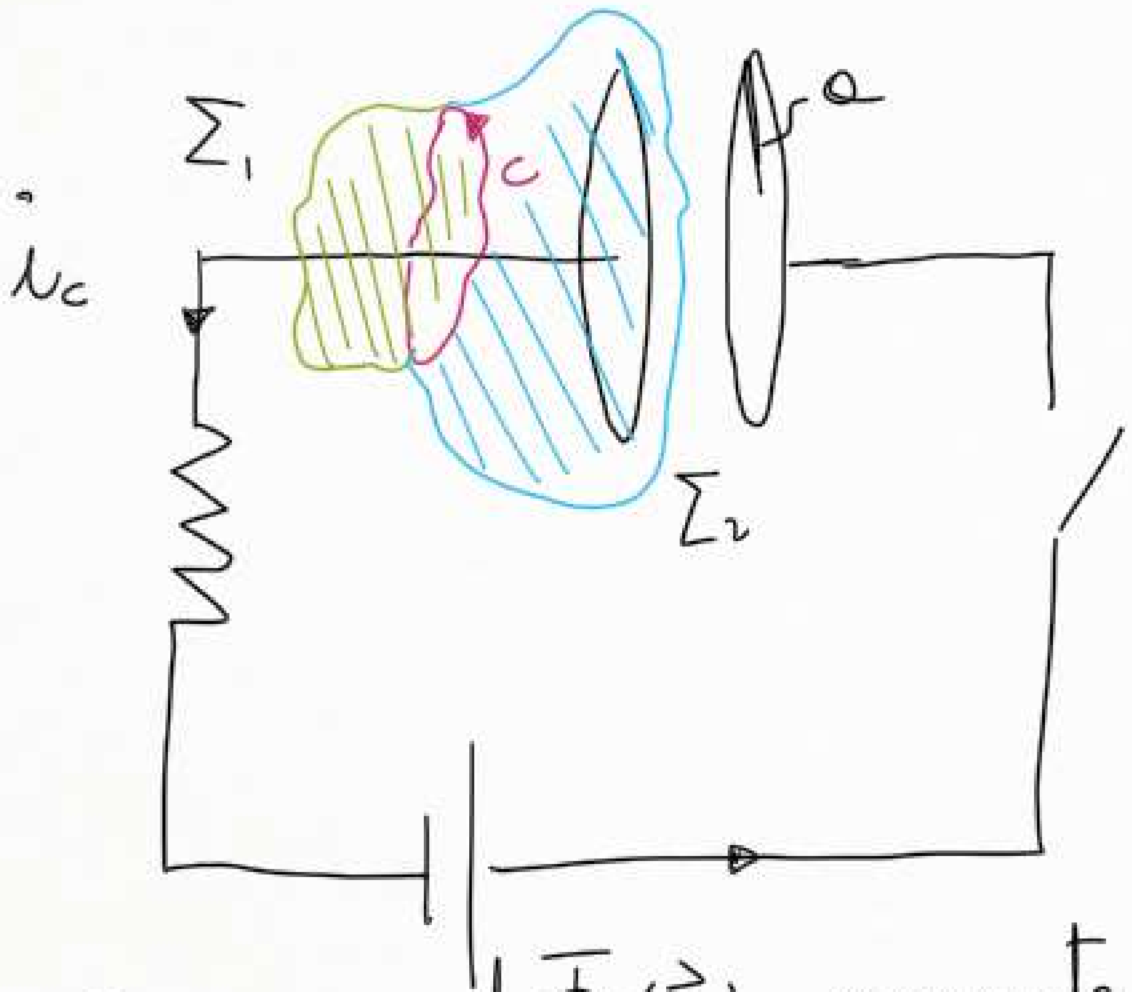
$\Sigma, n, N = nd, B = \mu_0 n i$

$$\Phi_s = \mu_0 n i \Sigma \Rightarrow \Phi = N \Phi_s = n d \Phi_s = \mu_0 n^2 i \underbrace{\Sigma d}_{=\tau} \Rightarrow = \mu_0 n^2 i \tau$$

$$L = \mu_0 n^2 \tau, \quad U_L = \frac{1}{2} L i^2 = \frac{1}{2} \underbrace{\mu_0 n^2 i^2}_{=\frac{B^2}{\mu_0}} \tau = \frac{1}{2} \frac{B^2}{\mu_0} \tau$$

$$U_L = \frac{1}{2} \frac{B^2}{\mu_0} \tau = u_m \tau, \quad \boxed{u_m = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \mu_0 H^2 = \frac{1}{2} BH}$$

$$\boxed{U_m = \int_\tau u_m \, d\tau = \int_\tau \frac{1}{2} \frac{B^2}{\mu_0} \, d\tau}$$



$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i_c = \mu_0 \underbrace{\int_{\Sigma_1(c)} \vec{J}_c\cdot\hat{n}\, d\Sigma}_{i_c}$$

$$\int_{\Sigma_2} \vec{J}_c\cdot\hat{n}\, d\Sigma = 0$$

$$i_s = \varepsilon_0 \frac{d\Phi(\vec{E})}{dt}$$ corrente di spostamento $\Rightarrow$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0\left(i_c + \overbrace{\varepsilon_0 \frac{d\Phi(\vec{E})}{dt}}^{\text{corr di spostamento}}\right)$$ Legge di Ampère-Maxwell

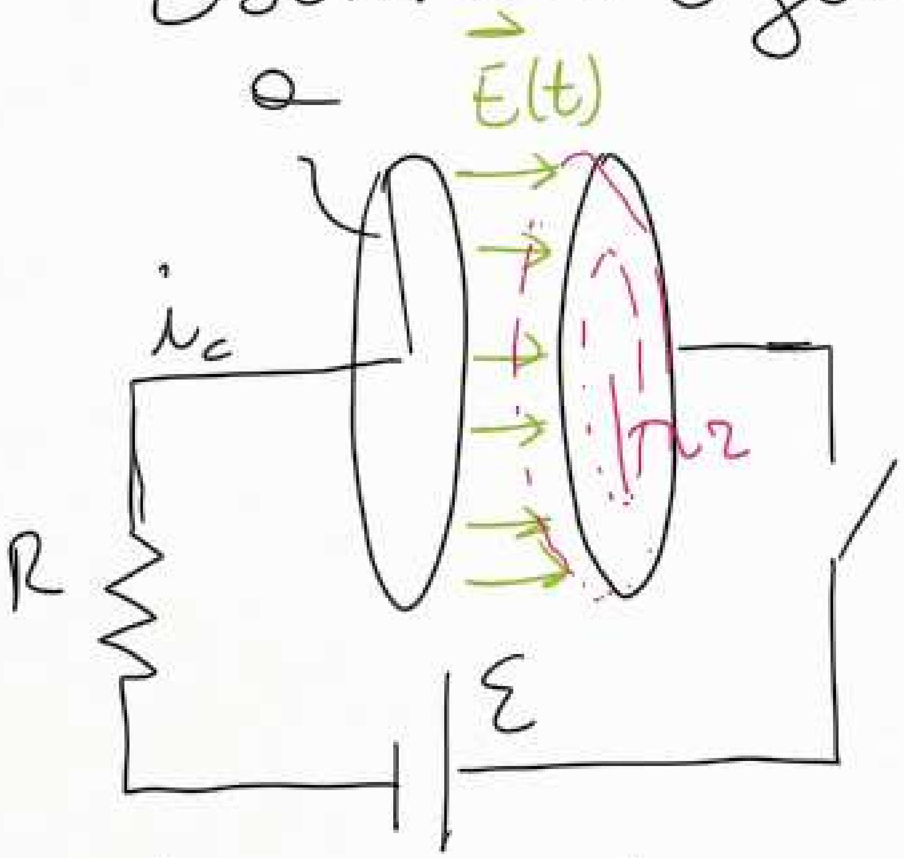
$$\oint \vec{B} \cdot d\vec{s} = \underbrace{\mu_0 \varepsilon_0}\frac{d\Phi(\vec{E})}{dt} = \frac{1}{c^2}\frac{d\Phi(\vec{E})}{dt}\ , \quad \mu_0 \varepsilon_0 = \frac{1}{c^2}$$

con c velocità della luce

$$\updownarrow$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi(\vec{B})}{dt}$$

ESEMPIO: $\vec{B}$ generato dalla carica di un condensatore



$$q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \quad , \quad \tau \equiv RC$$

$$E(t) = \frac{\sigma(t)}{\varepsilon_0} = \frac{q(t)}{\Sigma\, \varepsilon_0} = \frac{q(t)}{\pi a^2 \varepsilon_0} \quad \text{uniforme}$$

$$\oint_{\Sigma(r)} \vec{B} \cdot d\vec{s} = B\, 2\pi r = \mu_0 \varepsilon_0 \frac{d\Phi(\vec{E})}{dt}$$

$$\frac{d}{dt}\Phi(\vec{E}) = \frac{d}{dt}\Sigma E(t) = \pi r^2 \frac{d}{dt} E(t) = \pi r^2 \frac{d}{dt}\frac{q(t)}{\pi a^2 \varepsilon_0} = \frac{r^2}{a^2 \varepsilon_0}\frac{dq(t)}{dt} =$$

$$= \frac{r^2}{a^2 \varepsilon_0}\frac{\mathcal{E}}{R} e^{-t/\tau} \implies B\, 2\pi r = \frac{\mu_0 r^2}{a^2}\frac{\mathcal{E}}{R} e^{-t/\tau} \implies \boxed{B = \frac{\mu_0 r}{2\pi a^2}\frac{\mathcal{E}}{R} e^{-t/\tau}}$$

$$\oint \vec{E} \cdot \hat{n}\, d\Sigma = \frac{Q_{\tau(\Sigma)}}{\varepsilon_0}$$

$$\oint_c \vec{E} \cdot d\vec{s} = -\frac{d\,\Phi_{\Sigma(c)}(\vec{B})}{dt}$$

$$\boxed{\oint_\Sigma \vec{B} \cdot \hat{n}\, d\Sigma = 0}$$

$$\oint_c \vec{B} \cdot d\vec{s} = \mu_0 \left(i + \varepsilon_0 \frac{d_{\Sigma(c)}\Phi(\vec{E})}{dt} \right)$$

Equazioni di Maxwell in forma integrale

$$\oint_\Sigma \vec{E}\cdot\hat{n}\, d\Sigma = 0 \qquad\qquad \oint_c \vec{E}\cdot d\vec{s} = -\frac{d\Phi(\vec{B})}{dt}$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

$$\oint_\Sigma \vec{B}\cdot\hat{n}\, d\Sigma = 0 \qquad\qquad \oint_c \vec{B}\cdot d\vec{s} = \frac{1}{c^2}\frac{d\Phi(\vec{E})}{dt}$$

Queste equazioni hanno due classi di soluzioni:

① $\vec{B} = 0, \quad \vec{E} = 0$

② $\vec{B} = \vec{B}(x,y,z,t)\,, \quad \vec{E} = \vec{E}(x,y,z,t) \rightarrow$ onde elettromagnetiche

$$\oint_c \vec{E}\cdot d\vec{s} = \int_{\Sigma(c)} \vec{\nabla}\times\vec{E}\cdot\hat{n}\,d\Sigma = -\frac{\partial}{\partial t}\int_{\Sigma(c)} \vec{B}\cdot\hat{n}\,d\Sigma \quad \Longrightarrow$$

STOKES

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

$$\oint_c \vec{B}\cdot d\vec{s} = \int_{\Sigma(c)} \boxed{\vec{\nabla}\times\vec{B}}\cdot\hat{n}\,d\Sigma = \mu_0\left(i_c + \varepsilon_0\frac{d\Phi(\vec{E})}{dt}\right) = \mu_0\overbrace{\int_{\Sigma(c)}\vec{J}\cdot\hat{n}\,d\Sigma}^{i_c} + \varepsilon_0\mu_0\frac{d\Phi(\vec{E})}{dt} =$$

$$= \mu_0\int_{\Sigma(c)}\vec{J}\cdot\hat{n}\,d\Sigma + \varepsilon_0\mu_0\frac{\partial}{\partial t}\int_{\Sigma(c)}\vec{E}\cdot\hat{n}\,d\Sigma = \mu_0\int_{\Sigma(c)}\boxed{\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)}\cdot\hat{n}\,d\Sigma \quad \Longrightarrow$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)}$$

EQUAZIONI DI MAXWELL

① $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}$

② $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

③ $\vec{\nabla} \cdot \vec{B} = 0$

④ $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \right)$

And God said

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

and there was light.

CONSERVAZIONE DELLA CARICA

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} \perp \vec{a},\ \vec{c} \perp \vec{b}$$

$$\vec{c} \cdot \vec{a} = 0,\ \vec{c} \cdot \vec{b} = 0$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$$

$$0 = \vec{\nabla} \cdot \left(\mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \right) = \mu_0 \vec{\nabla} \cdot \vec{J} + \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{E} = \mu_0 \vec{\nabla} \cdot \vec{J} + \mu_0 \frac{\varepsilon_0}{\varepsilon_0} \frac{\partial \rho}{\partial t} \Rightarrow$$

$$\cancel{\mu_0} \vec{\nabla} \cdot \vec{J} = - \cancel{\mu_0} \frac{\varepsilon_0}{\varepsilon_0} \frac{\partial \rho}{\partial t} \Rightarrow \boxed{\vec{\nabla} \cdot \vec{J} = - \frac{\varepsilon_0}{\varepsilon_0} \frac{\partial \rho}{\partial t} = - \frac{\partial \rho}{\partial t}}$$

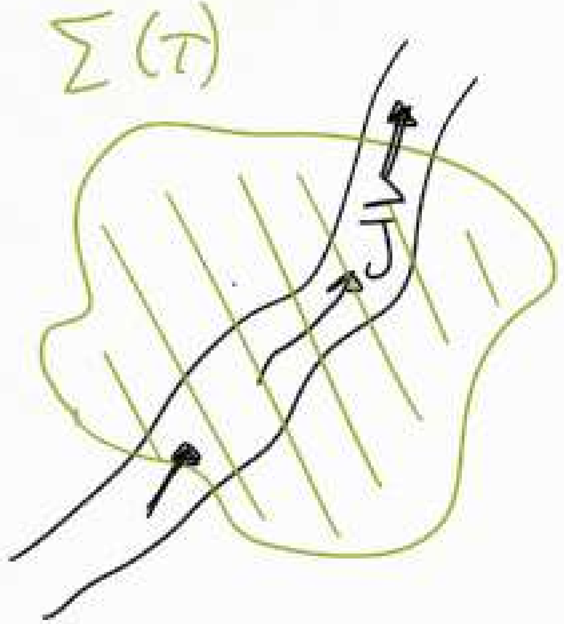
equazione di continuità

$$\boxed{\vec{\nabla} \cdot \vec{J} = - \frac{\partial \rho}{\partial t}} \Rightarrow$$

$$\int_\tau \vec{\nabla}\cdot\vec{J}\, d\tau = -\int_\tau \frac{\partial \rho}{\partial t}\, d\tau = -\frac{\partial}{\partial t}\int_\tau \rho\, d\tau = -\frac{\partial}{\partial t}\int_\tau dq = -\frac{\partial q_\tau}{\partial t}$$

$$\int_\tau \vec{\nabla}\cdot\vec{J}\, d\tau \underset{\uparrow}{=} \oint_{\Sigma(\tau)} \vec{J}\cdot\hat{n}\, d\Sigma = i_\tau \;\mapsto\; i_\tau = -\frac{\partial q_\tau}{\partial t}$$

teorema della divergenza



$$0 = -\frac{\partial q_\tau}{\partial t} \;\mapsto\; q_\tau = \text{costante} \;\mapsto\; \text{conservazione della carica}$$

$\vec{A}$

$$\int_\tau \vec{\nabla} \cdot \vec{A}\, d\tau = \oint_{\Sigma(\tau)} \vec{A} \cdot \hat{n}\, d\Sigma$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\boxed{\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$