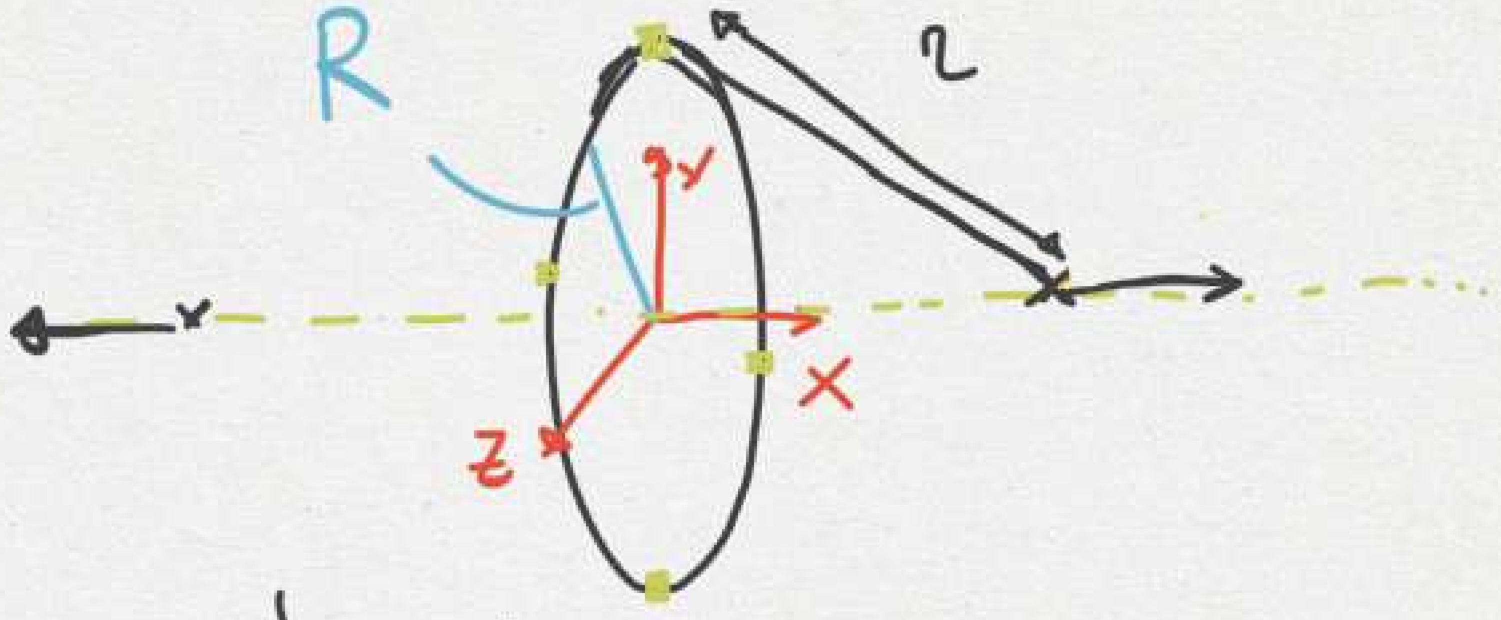
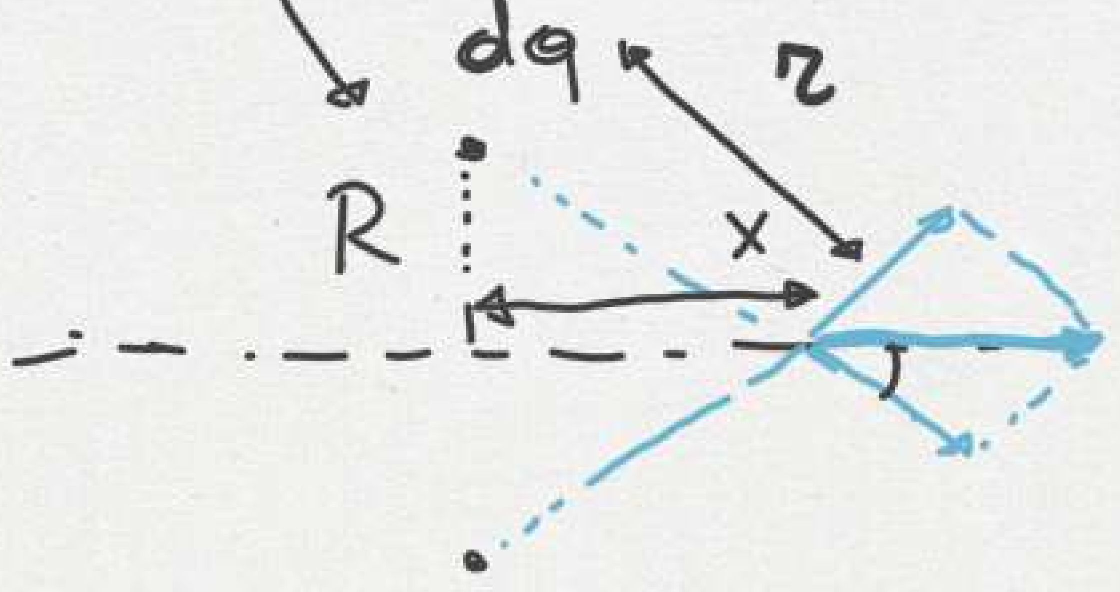
ESERCIZIO 5



$$\vec{E}(x,0,0) = ?$$

$$E_y = E_z = 0$$

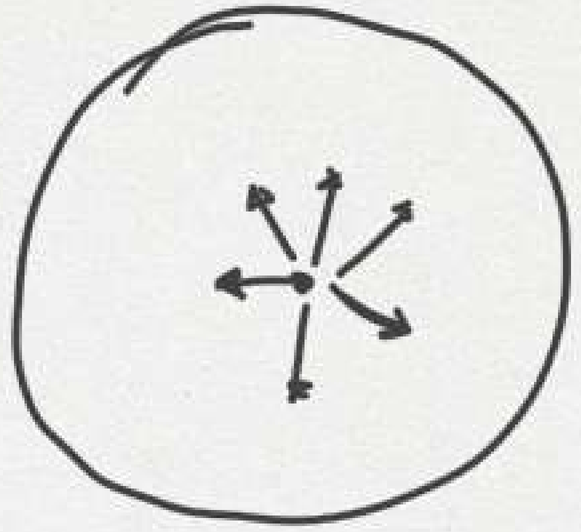
$$E_x = \int dE_x = \int_{\text{ANELLO}} \frac{dq}{4\pi\varepsilon_0}\frac{x}{r^3} = \frac{q}{4\pi\varepsilon_0}\frac{x}{r^3} = \boxed{\frac{q}{4\pi\varepsilon_0}\frac{\boxed{x}}{(x^2+R^2)^{3/2}}}$$

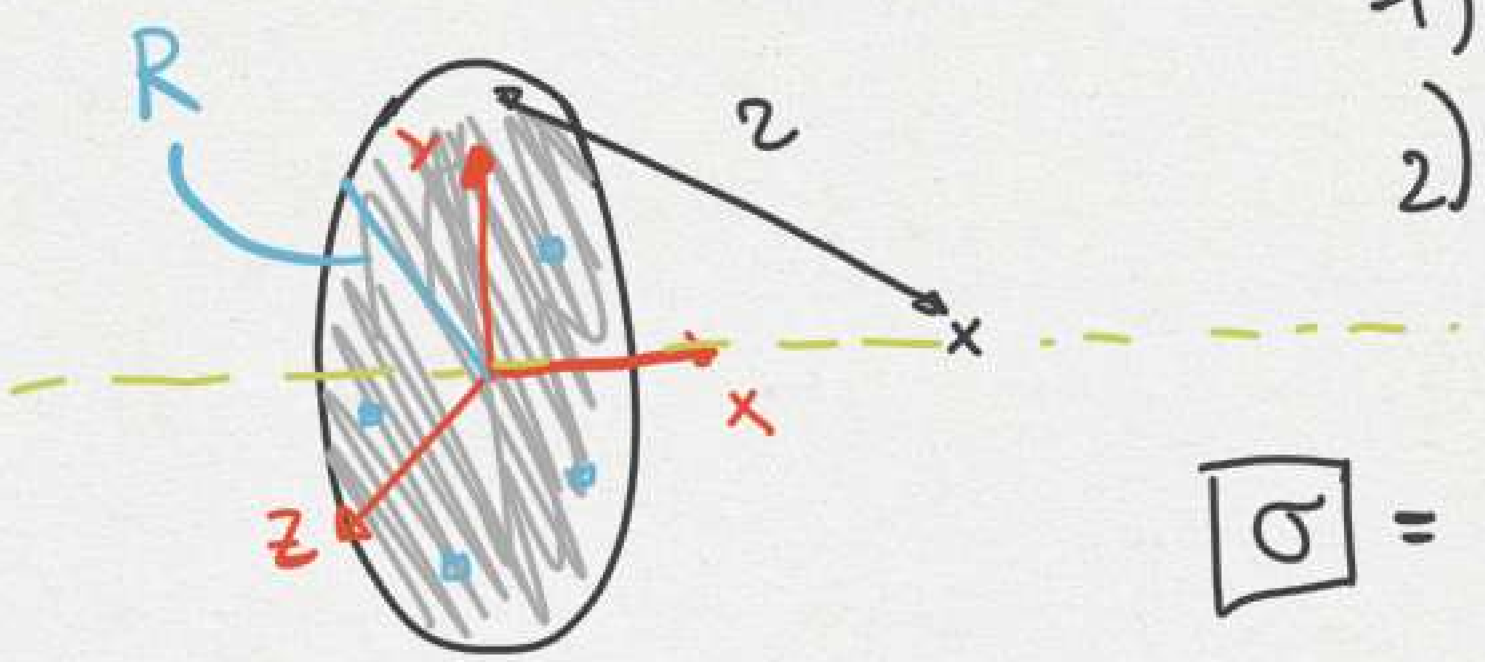


$$\boxed{dE_x = \frac{dq}{4\pi\varepsilon_0}\frac{x}{r^3}}$$

$$r = \sqrt{x^2+R^2}$$

$q>0$		$q<0$	
$x>0$	$x<0$	$x>0$	$x<0$
$E_x>0$	$E_x<0$	$E_x<0$	$E_x>0$



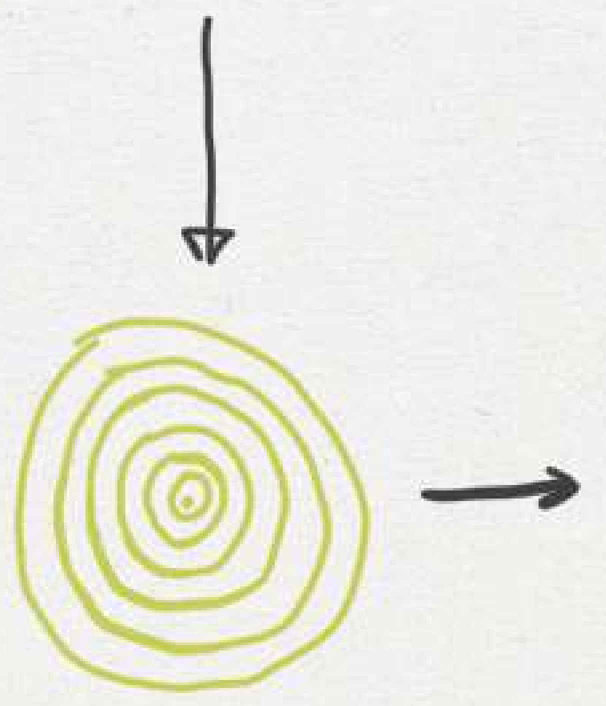


1) $\vec{E}(x,0,0) = ?$

2) $\vec{E}(x,0,0) \xrightarrow[R\to\infty]{} ?$

$$\underbrace{\left(-\frac{1}{z^3}\right)_0^R = \left(\frac{1}{z^3}\right)_R^0}$$

$$\boxed{\sigma} = \frac{q}{\Sigma} = \frac{q}{\pi R^2}$$



$$dq = \sigma\, d\Sigma = \sigma\, \overline{2\pi R' dR'}$$

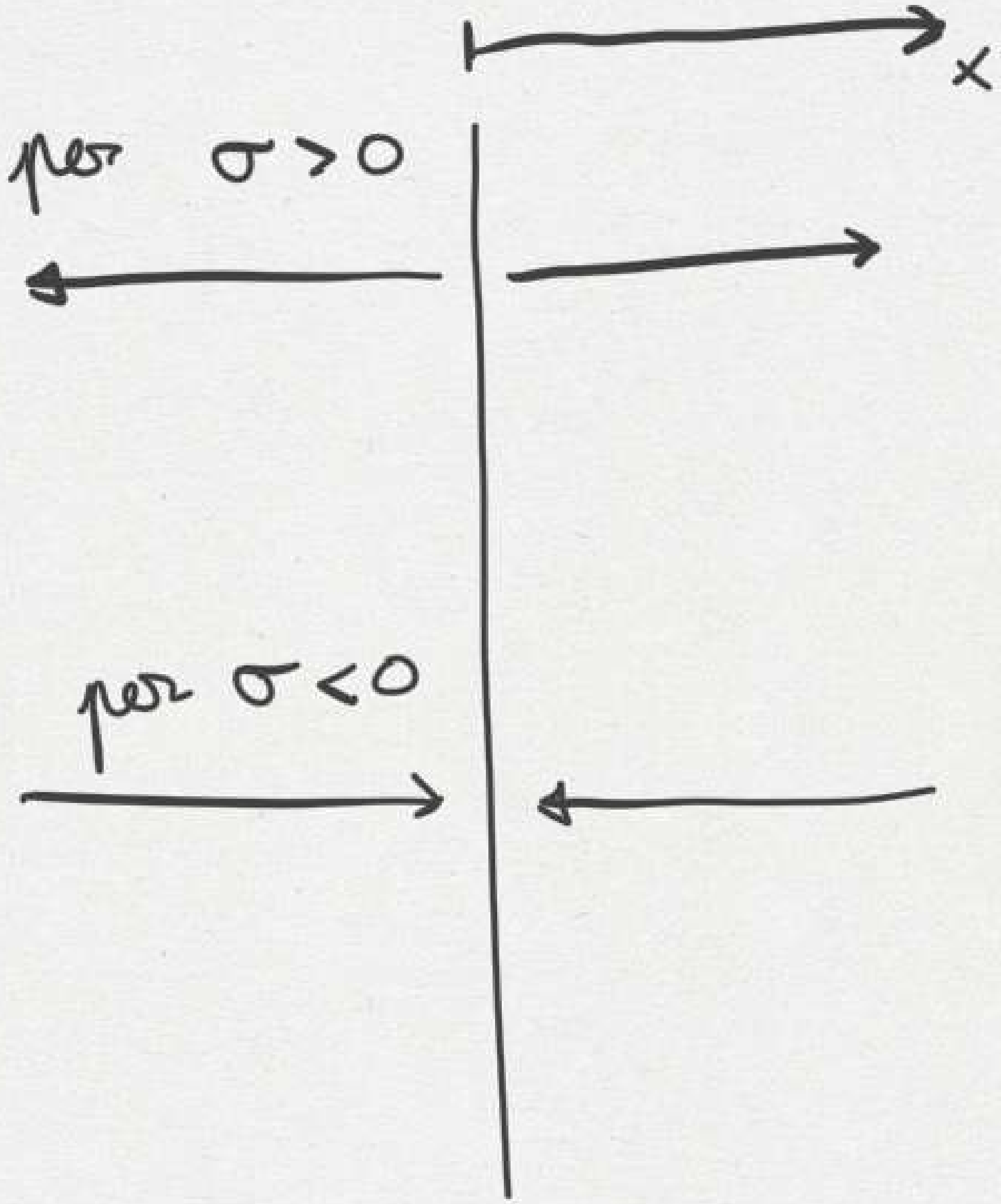
$$dE_x = \frac{dq}{4\pi\varepsilon_0}\frac{x}{z^3} = \frac{\cancel{2\pi} R'\sigma\, dR' x}{2\cancel{4\pi}\varepsilon_0 z^3} = \frac{\sigma x}{2\varepsilon_0}\frac{R' dR'}{z^3} = \frac{\sigma x}{2\varepsilon_0}\frac{R' dR'}{(R'^2+x^2)^{3/2}} \Rightarrow$$

$$E_x = \int_0^R \underbrace{\frac{\sigma x}{2\varepsilon_0}}\ \frac{R' dR'}{\underbrace{(R'^2+x^2)^{3/2}}_{z^3}} = \frac{\sigma x}{2\varepsilon_0}\left(-\frac{1}{\sqrt{R'^2+x^2}}\right)_0^R = \underbrace{\frac{\boxed{\sigma x}}{\boxed{2\varepsilon_0}}}_{>0}\underbrace{\left(\frac{1}{|x|} - \frac{1}{\sqrt{R^2+x^2}}\right)}_{>0}$$

$$E_x = \frac{\sigma}{2\varepsilon_0}\left(\underbrace{\frac{x}{|x|}}_{\pm 1} - \underbrace{\frac{x}{\sqrt{x^2+R^2}}}_{\sim \frac{1}{R} \xrightarrow[R\to\infty]{} 0}\right) \xrightarrow[R\to\infty]{} \frac{\sigma}{2\varepsilon_0}\frac{x}{|x|} = \frac{\sigma}{2\varepsilon_0}\,\mathrm{sgn}(x)$$

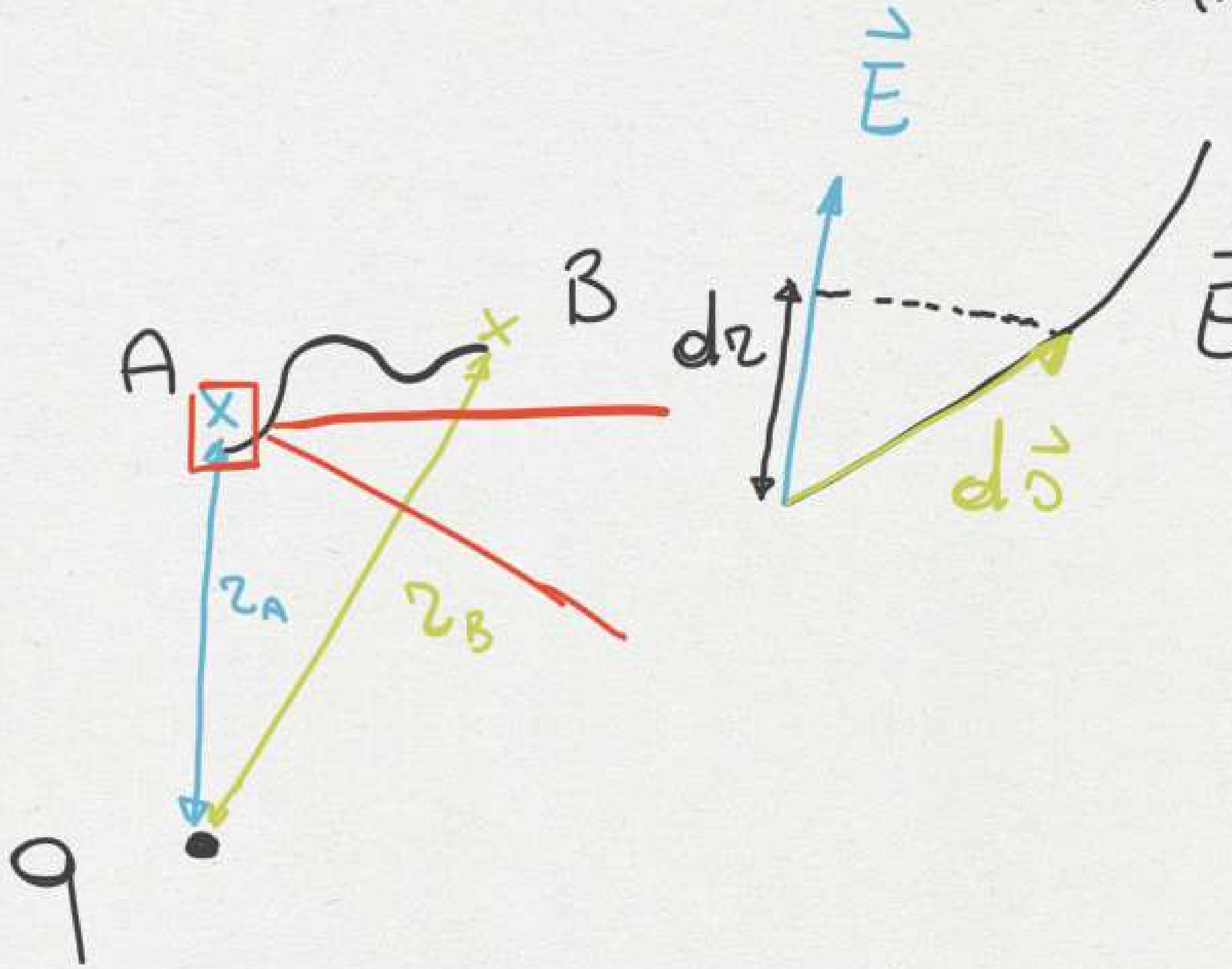
$$\Downarrow$$

$$\boxed{|E_x| = \frac{\sigma}{2\varepsilon_0}}$$



POTENZIALE

$$\Delta V_{AB} \equiv \underline{V(B) - V(A)} = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_{r_A}^{r_B} E\,dr = -\int_{r_A}^{r_B} \frac{q}{4\pi\varepsilon_0}\frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_{r_A}^{r_B} =$$



$$\vec{E}\cdot d\vec{s} = E\,dr$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) =$$

$$= \underbrace{\frac{q}{4\pi\varepsilon_0}\frac{1}{r_B}}_{V(B)} - \underbrace{\frac{q}{4\pi\varepsilon_0}\frac{1}{r_A}}_{V(A)}$$

$$\boxed{V(R) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R} + \text{cost}}$$

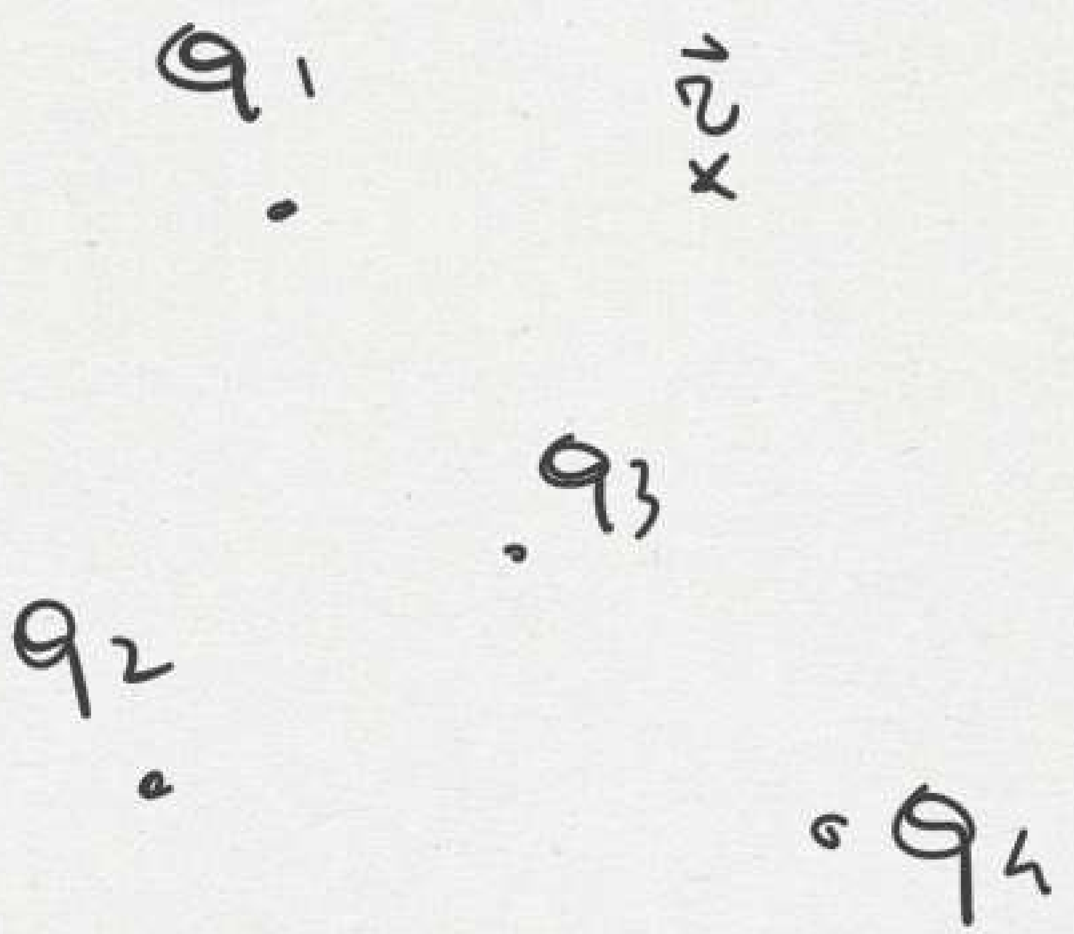
$$V(R) \xrightarrow[R\to\infty]{} 0 \Rightarrow \text{cost} = 0$$

$$W(B) = V(B) + C$$
$$W(A) = V(A) + C$$
$$\Rightarrow W(B) - W(A) = V(B) - V(A)$$

$$\vec{E} = \sum_i \vec{E}_i \quad , \quad \Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{s} = -\int_A^B \sum_i \vec{E}_i \cdot d\vec{s} = -\sum_i \int_A^B \vec{E}_i \cdot d\vec{s} =$$

$$= \underbrace{\sum_i \Delta V_{AB}^{(i)}}_{}$$

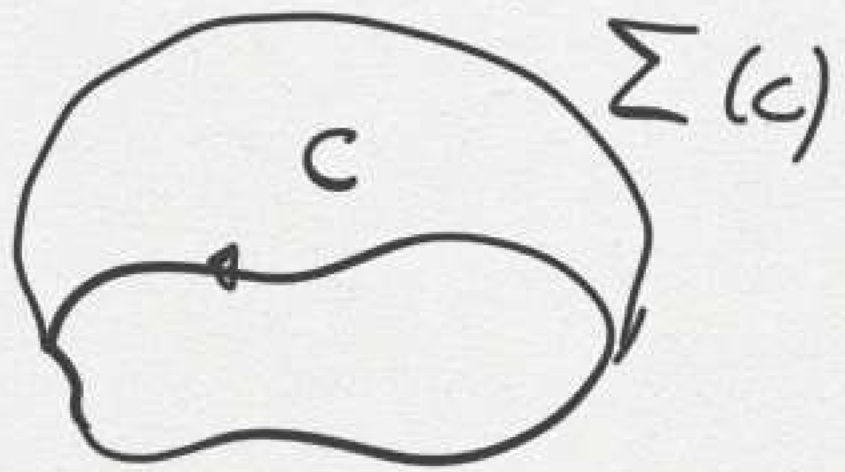
$$\boxed{V(\vec{r})} = \sum_i V_i(\vec{r}) \longrightarrow \int_\tau dV = \underbrace{\int_\tau \frac{dq}{4\pi\varepsilon_0} \frac{1}{r}}_{} + \text{cost}$$



$\times \; B\,(x+dx,\, y+dy,\, z+dz)$, $d\vec{s}\,(dx,\, dy,\, dz)$

$A = (x,y,z)$ $\times$

$$\rightarrow \; dV = V(x+dx,\, y+dy,\, z+dz) - V(x,y,z) = -\vec{E}\cdot d\vec{s} = \underbrace{-E_x}\,dx \;\underbrace{-E_y}\,dy \;\underbrace{-E_z}\,dz$$



$$\rightarrow \; dV(x,y,z) \equiv \underbrace{\frac{\partial V}{\partial x}}\,dx + \underbrace{\frac{\partial V}{\partial y}}\,dy + \underbrace{\frac{\partial V}{\partial z}}\,dz$$

$$\Downarrow$$

$$\boxed{\vec{E} = \left(-\frac{\partial V}{\partial x},\, -\frac{\partial V}{\partial y},\, -\frac{\partial V}{\partial z}\right) = -\vec{\nabla} V}\,, \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x},\, \frac{\partial}{\partial y},\, \frac{\partial}{\partial z}\right)$$

$$\oint_c \vec{E}\cdot d\vec{s} = \int_{\Sigma(c)} \vec{\nabla}\times\vec{E}\cdot\hat{n}\, d\Sigma = 0 \;\Longrightarrow\; \vec{\nabla}\times\vec{E} = 0$$

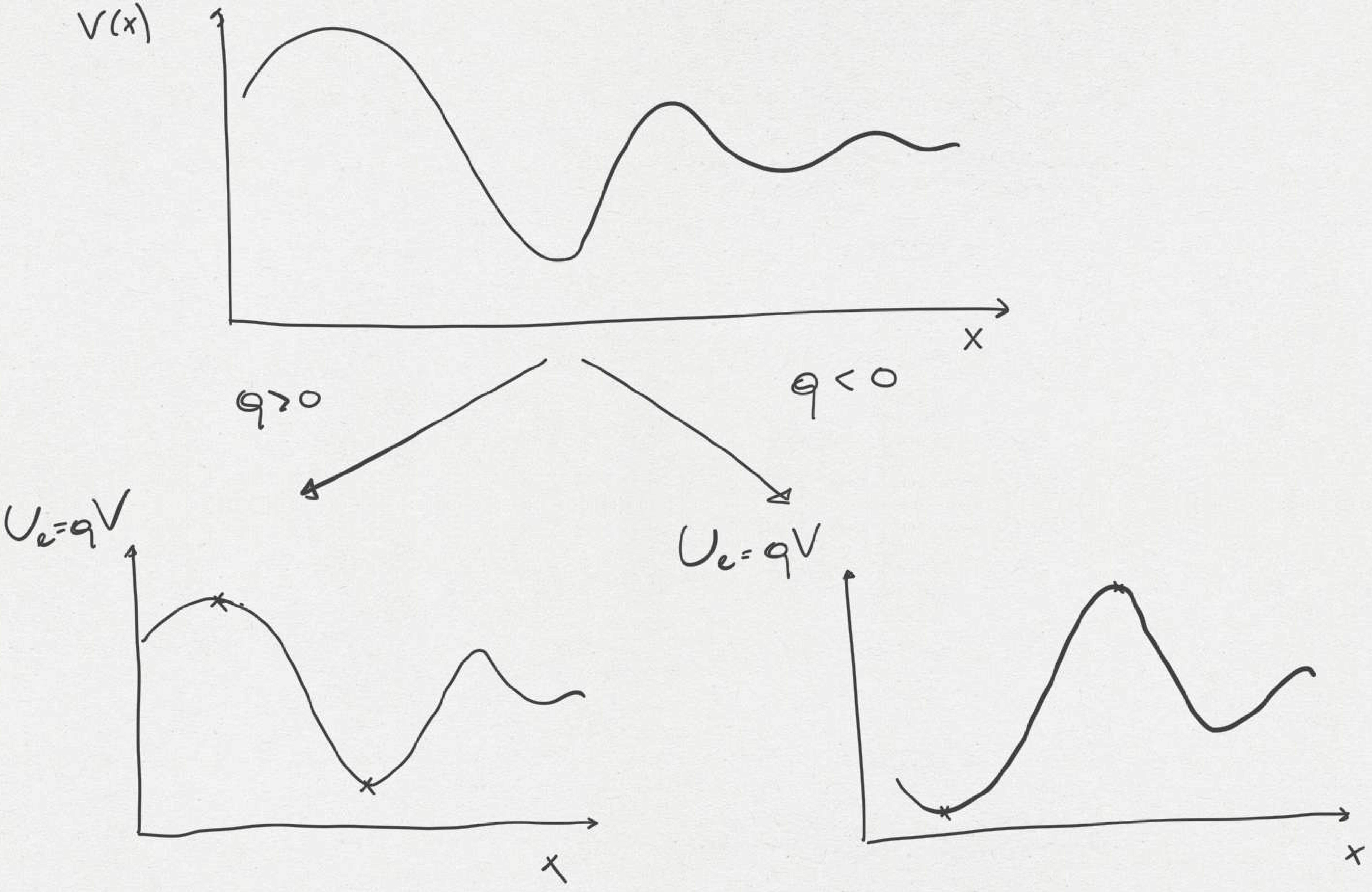
campo conservativo

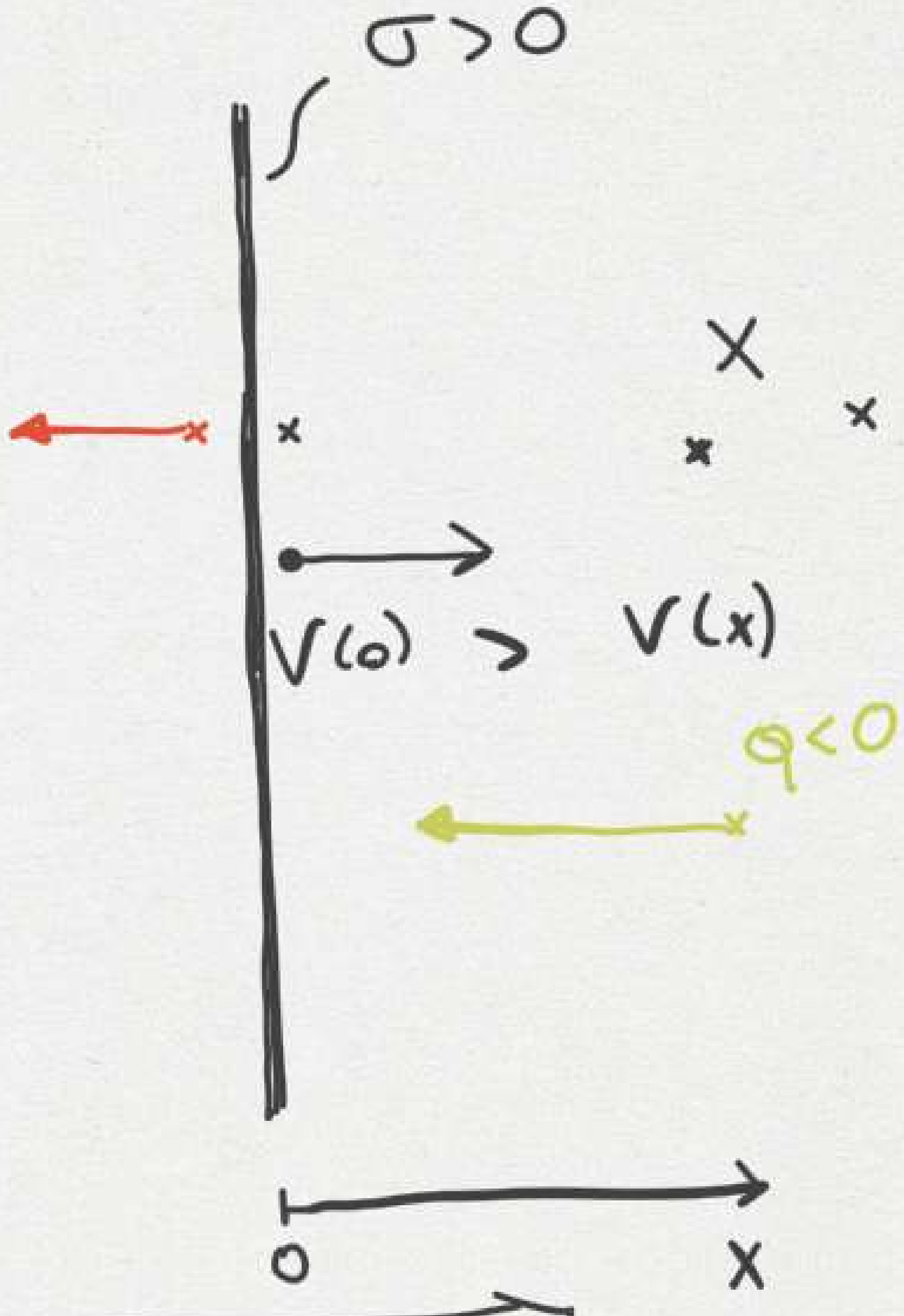
$\Updownarrow$

campo irrotazionale

$$\Delta U_e = q_0 \, \Delta V$$

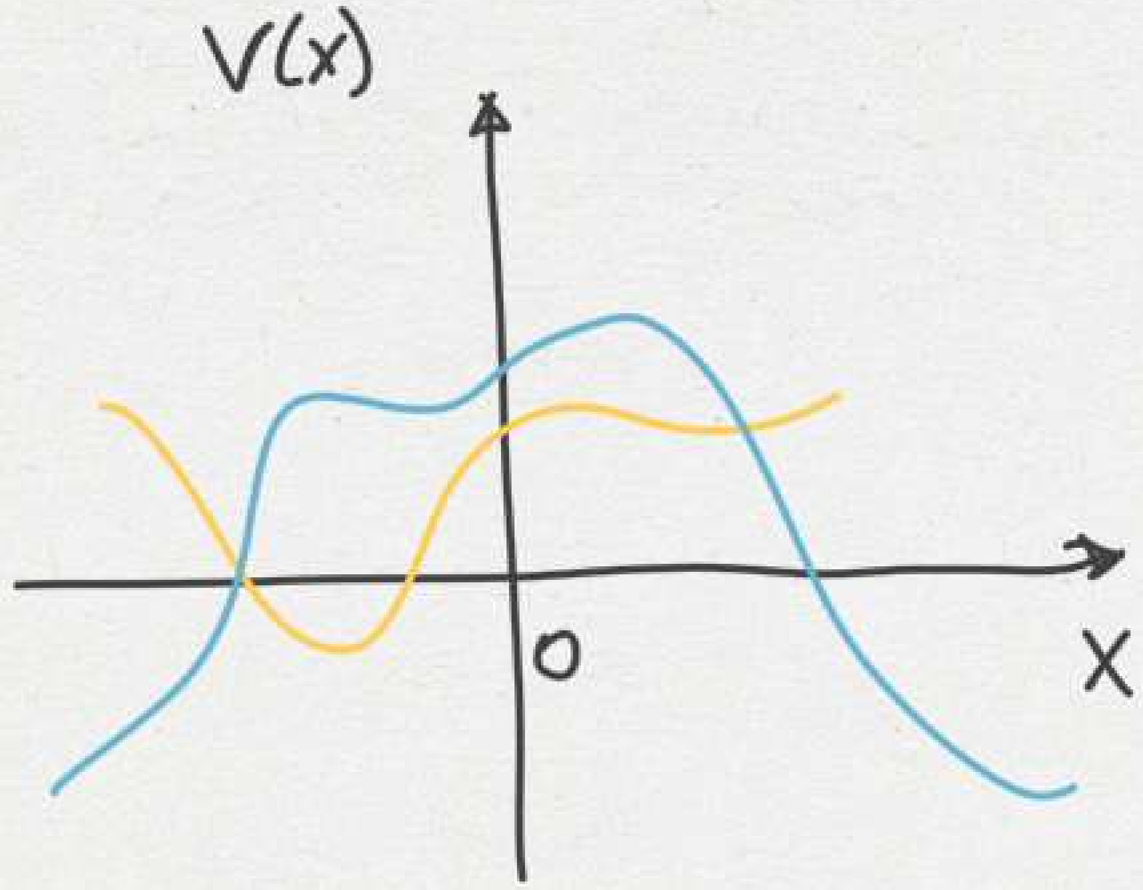
$$\underbrace{m g \, h(x)} = U_e \qquad \underbrace{"V" = \frac{U_e}{m}}$$

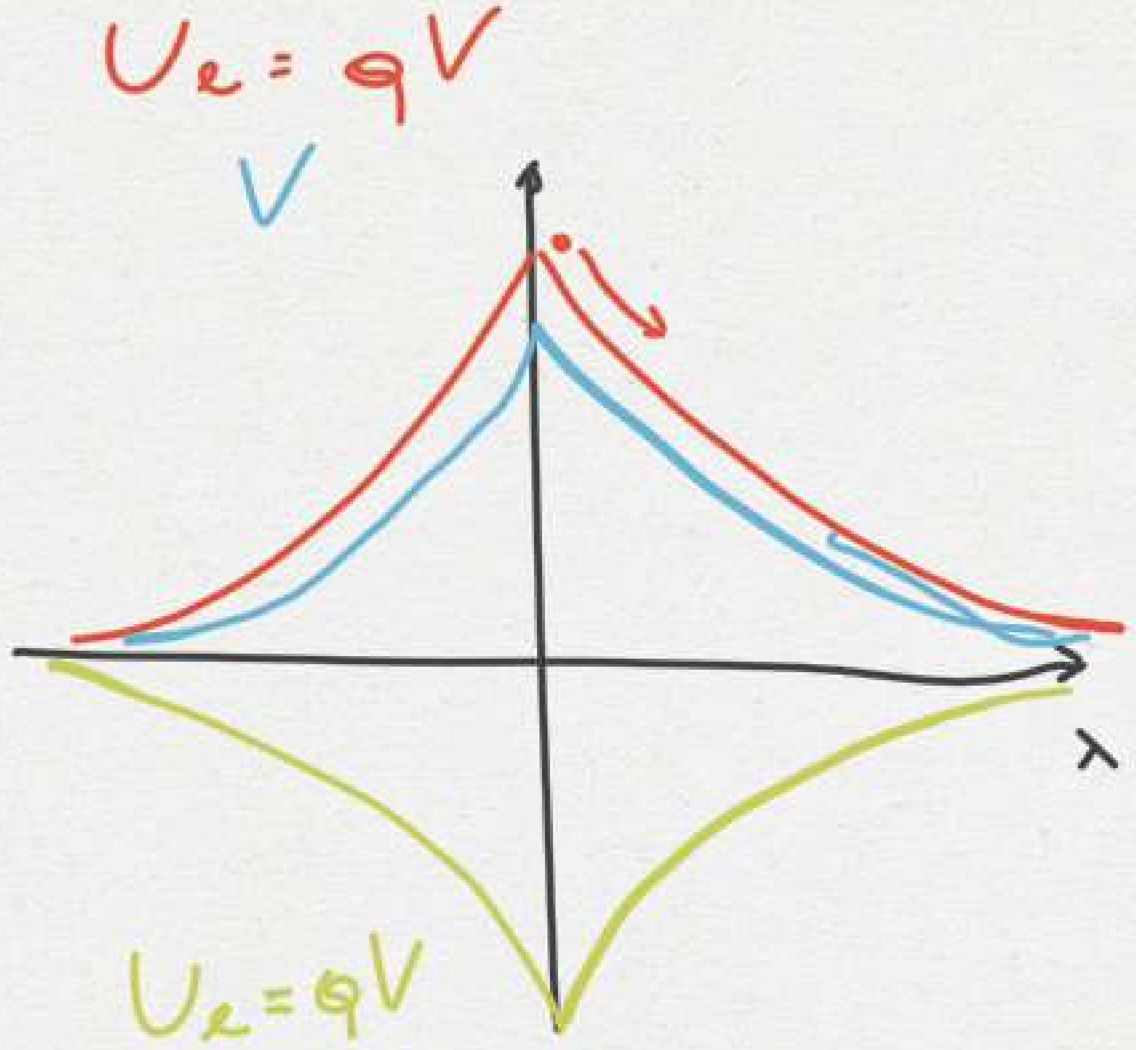

V(x)
x
q>0
q<0
U_e=qV
x
U_e=qV
x



$$E_x = \mathrm{sgn}(x)\,\frac{\sigma}{2\varepsilon_0}$$

$$V(x) = ?$$

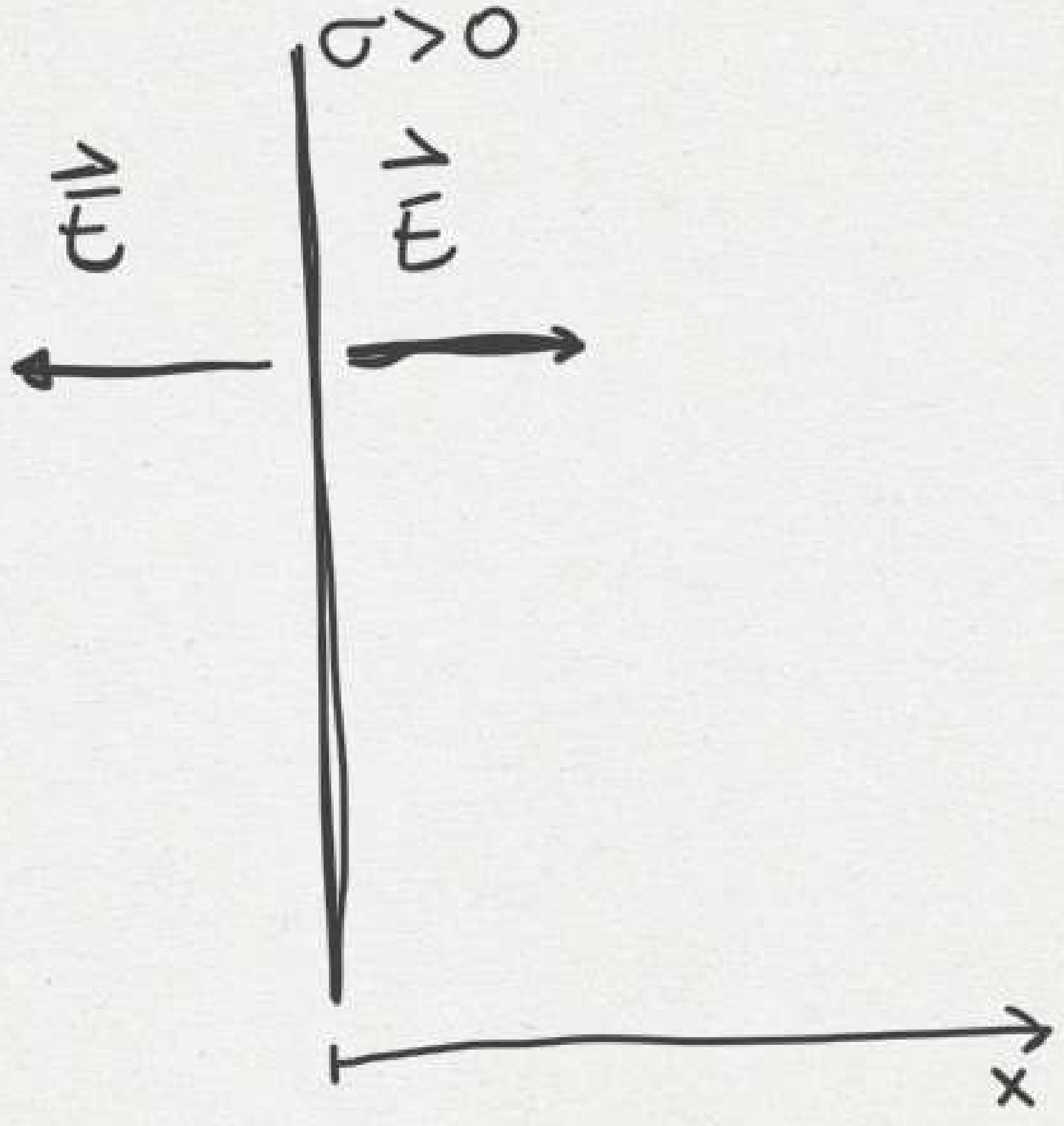




$$\boxed{\Delta V = V(x) - V(0)} = \underbrace{-\int_0^x \vec{E}\cdot d\vec{s}} = -\int_0^x E\,dx' = -\int_0^x \frac{\sigma}{2\varepsilon_0}\,dx' = -\frac{\sigma}{2\varepsilon_0}\,x \equiv V(x)$$

$$\Downarrow$$

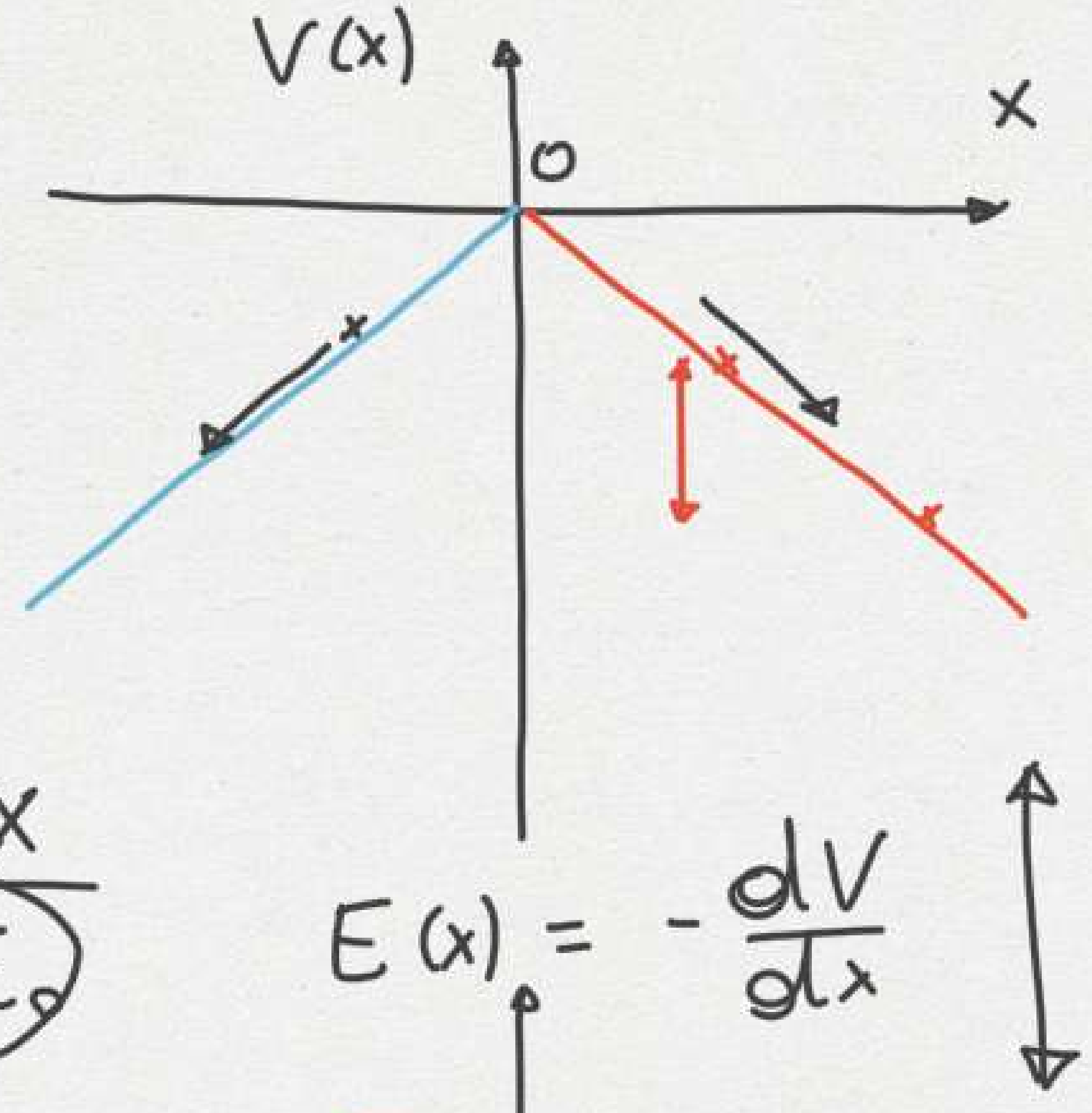
$$V(x) = \Delta V + V(0)$$



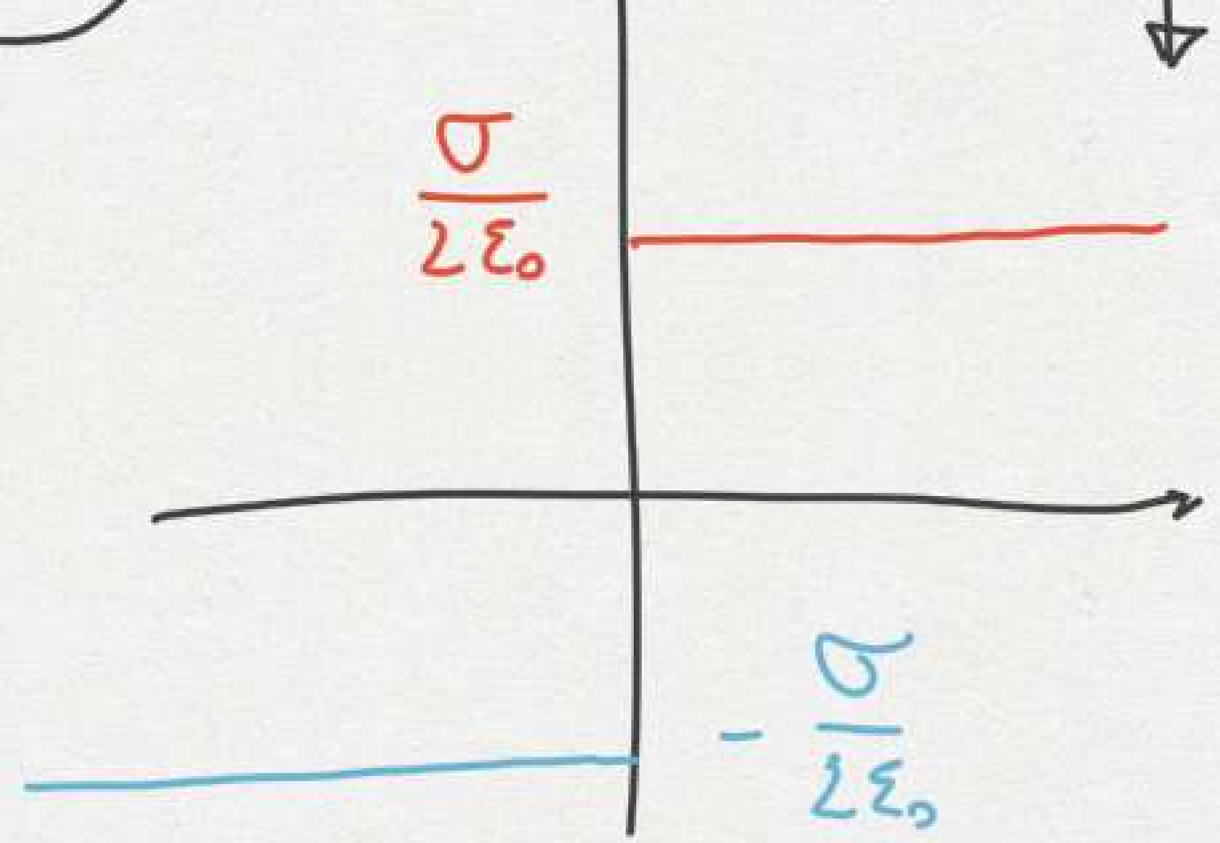
$$V(x>0) = -\frac{\sigma}{2\varepsilon_0}x$$

$$\Delta V(x<0) = -\int_0^x \vec{E}\cdot d\vec{s} =$$

$$= +\int_0^x E\,dx' = \frac{\sigma x}{2\varepsilon_0}$$



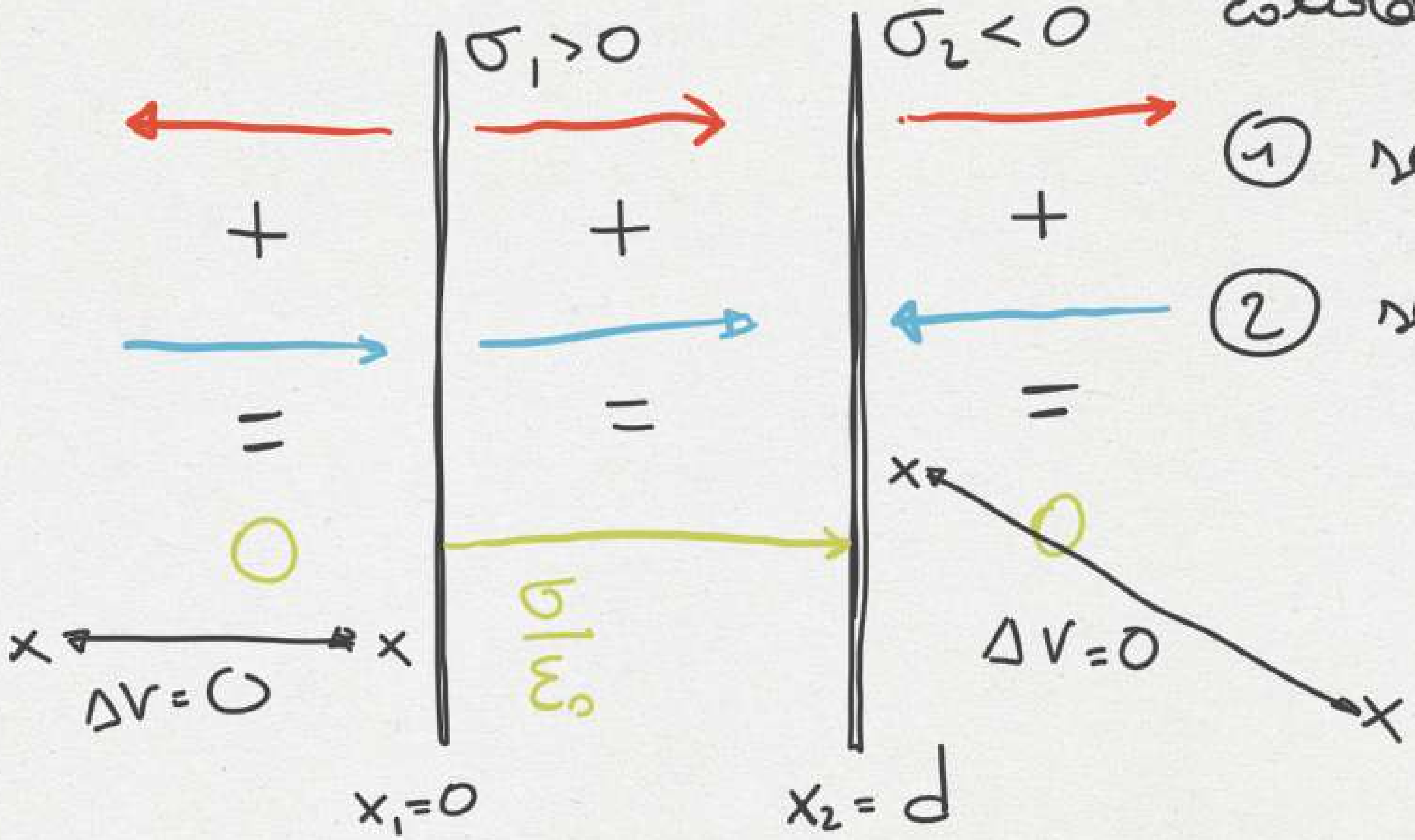
$$E(x) = -\frac{dV}{dx}$$



calcolare e <u>disegnare</u> $V(x)$ e $E(x)$

① se $\sigma_1 = \sigma = -\sigma_2 > 0$

② se $\sigma_1 = \sigma_2 = \sigma > 0$



$$\left|\frac{\sigma}{2\varepsilon_0}\right|$$