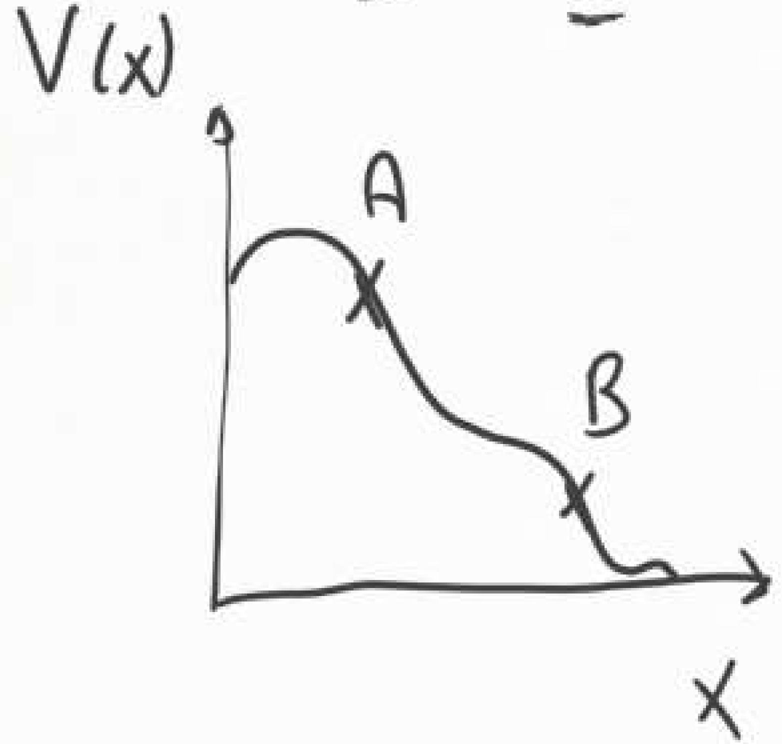
$$U(t) = U_e(t) + U_k(t) = \text{cost}$$

$$U_k = \frac{1}{2} m v^2(t), \quad U_e(t) = q_0 V(x(t), y(t), z(t))$$

$$U = \frac{1}{2} m v^2 + q V(x, y, z) = \text{cost}$$

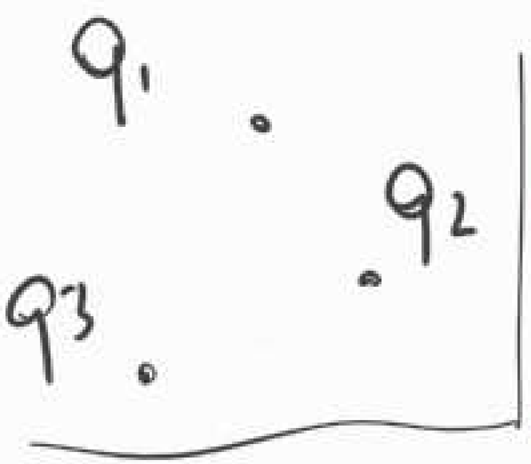


$$\Delta V = V(B) - V(A) < 0$$

1) $q_0 > 0$, $\Delta U < 0 \implies v_B > v_A$

2) $q_0 < 0$, $\Delta U > 0 \implies v_B < v_A$

$$W = -\Delta U_e \, , \quad W_{AB} = \int_A^B \vec{F}_e \cdot d\vec{s} \, ,$$

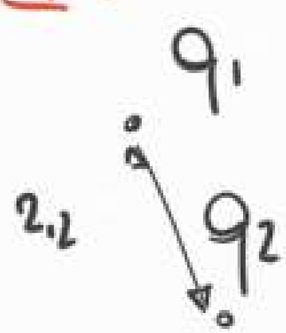


$\boxed{\text{I PASSO}}$

$\cdot \, q_1$

$$W'''_{ext} = 0$$

II PASSO

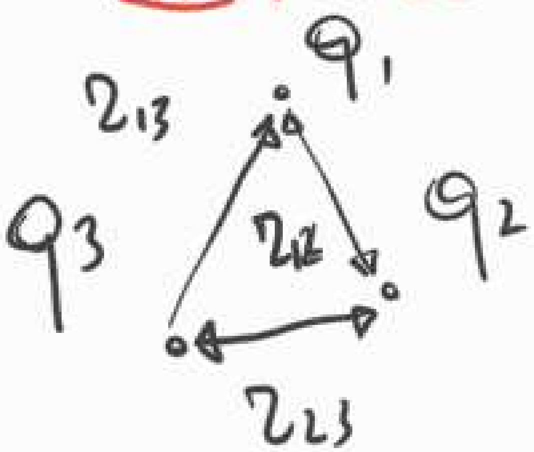


$$W''''_{ext} = \int_{\infty}^{r_2} \vec{F}_{ext} \cdot d\vec{s} = -q_2 \int_{\infty}^{r_2} \vec{E}_1 \cdot d\vec{s} =$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r_{12}} = \Delta U_e = q_2 \Delta V =$$

$$= -W$$

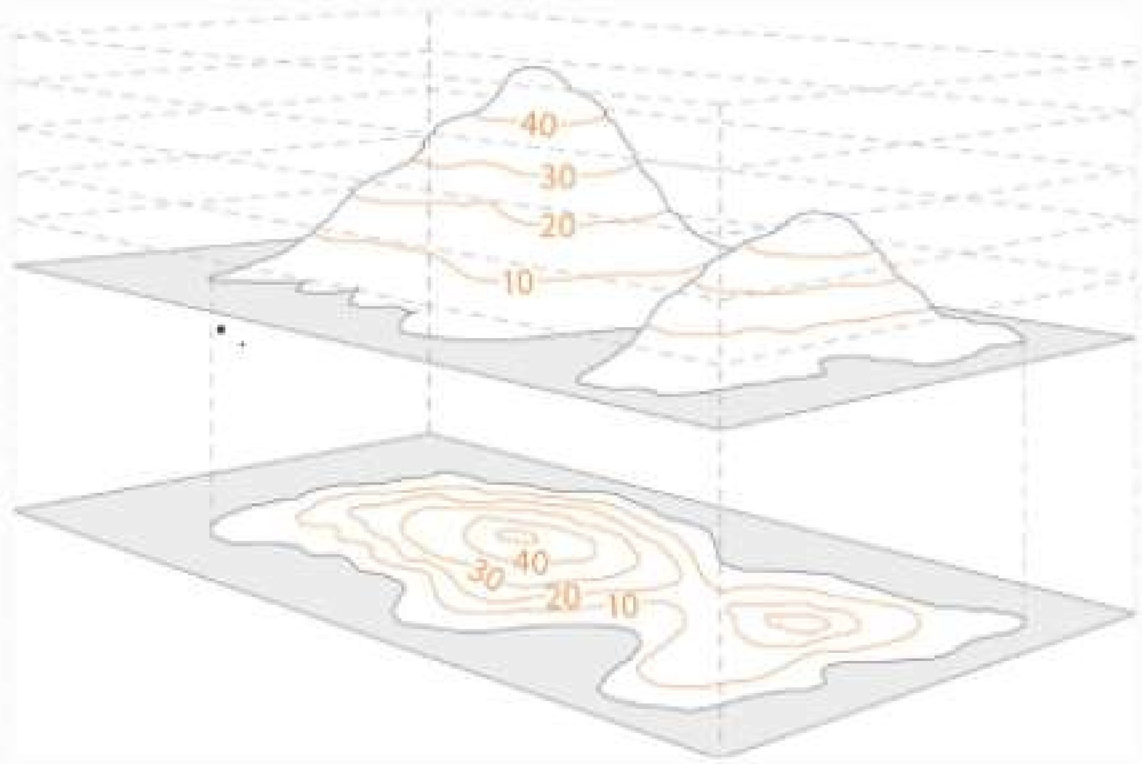
III PASSO



$$W_{ext}^{(III)} = \int_{\infty}^{r_3} \vec{F}_{ext}\cdot d\vec{s} = -\int_{\infty}^{r_3}\vec{F}_e\cdot d\vec{s} = -q_3\int_{\infty}^{r_3}(\vec{E}_1+\vec{E}_2)\cdot d\vec{s}$$

$$= \frac{q_1 q_3}{4\pi\varepsilon_0}\frac{1}{r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0}\frac{1}{r_{23}} \Rightarrow$$

$$W_{ext} = W_{ext}^{(I)} + W_{ext}^{(II)} + W_{ext}^{(III)} = \boxed{\sum_{i>j}\frac{q_i q_j}{4\pi\varepsilon_0}\frac{1}{r_{ij}} = U_e}$$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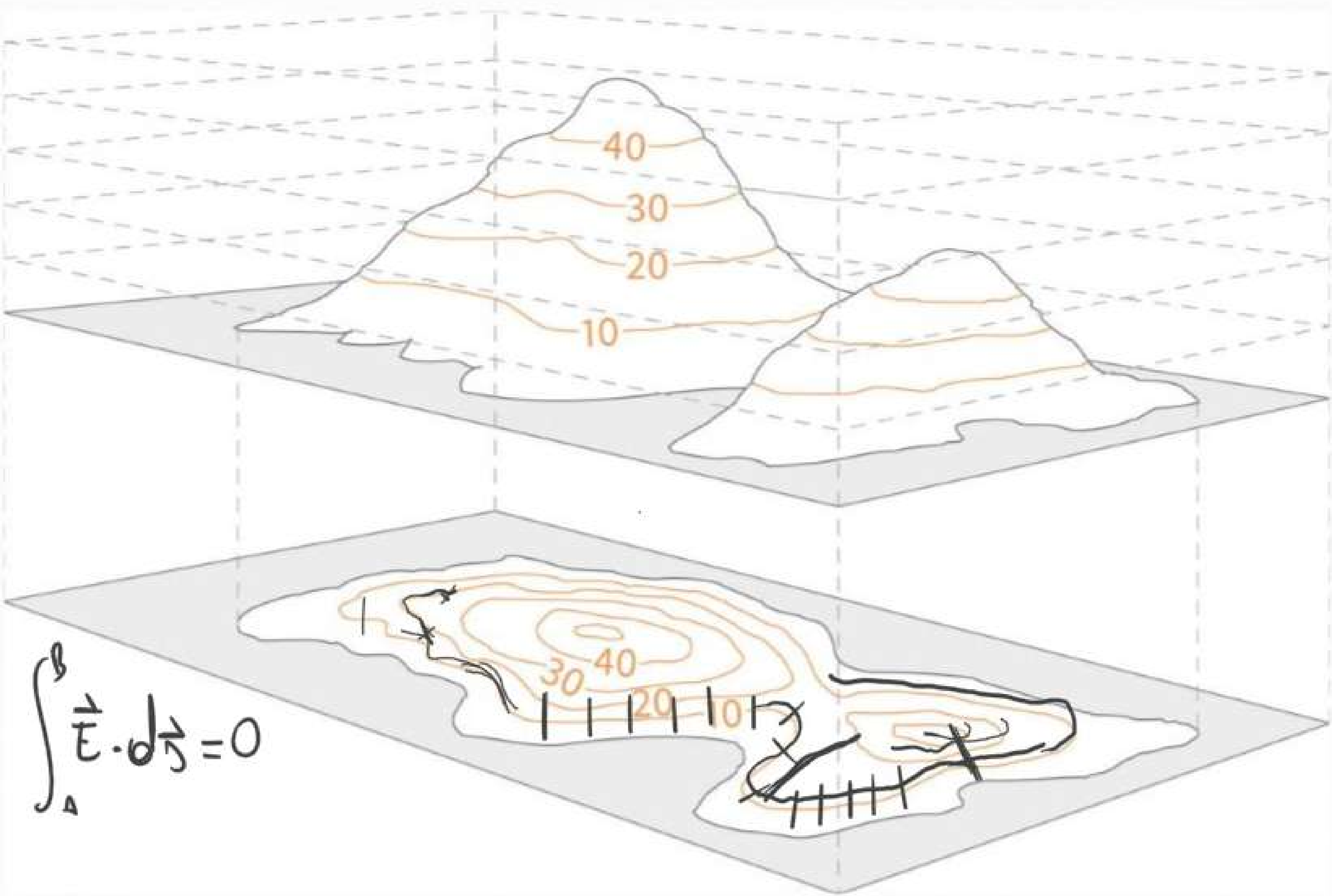
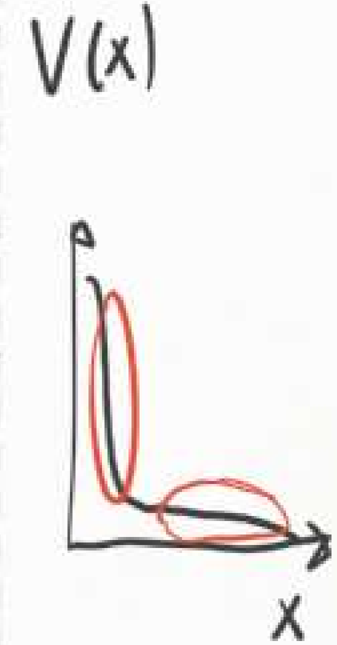
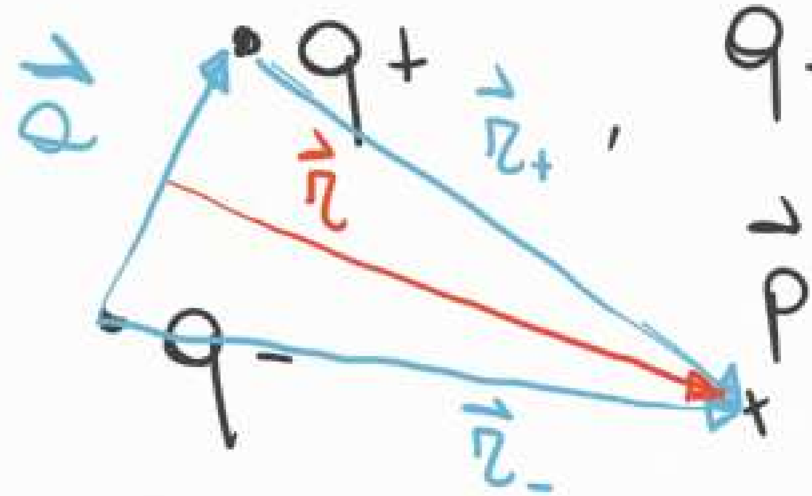


$$\int_A^B \vec{E} \cdot d\vec{s} = 0$$





$q_- = -q_+ = -q$

$$V(\vec{P}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_+}{r_+} + \frac{q_-}{r_-}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q}{4\pi\varepsilon_0}\,\frac{r_- - r_+}{r_+ r_-}$$

$\boxed{a \ll r \,,\ \frac{a}{r} \ll 1}$ APPROSSIMAZIONE DI DIPOLO

$\vec{r}_- = \vec{r}_+ + \vec{a} \Rightarrow \vec{r}_+ = \vec{r}_- - \vec{a} \,,\quad r_+ r_- \approx r^2$

$$r_+ = \sqrt{(\vec{r}_- - \vec{a})\cdot(\vec{r}_- - \vec{a})} = \sqrt{r_-^2 + a^2 - 2\vec{a}\cdot\vec{r}_-} = \sqrt{r_-^2 + a^2 - 2 a r_- \cos\theta}$$

$$\simeq r_- \sqrt{1 + \cancel{\frac{a^2}{r_-^2}} - 2\frac{a}{r_-}\cos\theta} \simeq r_- \left(1 - \frac{a}{r_-}\cos\theta\right) = r_- - a\cos\theta$$

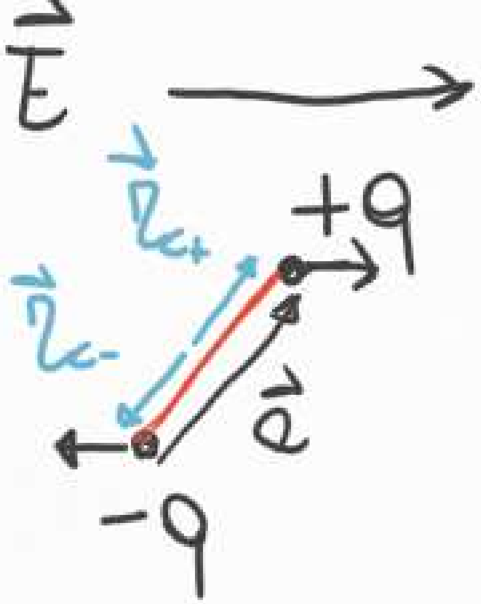
$$\sqrt{1+\varepsilon} \simeq 1 + \frac{1}{2}\varepsilon$$

$$r_+ = r_- - a\cos\theta \Rightarrow r_- - r_+ \simeq a\cos\theta \Rightarrow$$

$$V(\vec{P}) \simeq \frac{q}{4\pi\varepsilon_0}\frac{a\cos\theta}{r^2} = \frac{\vec{p}\cdot\hat{r}}{\underbrace{4\pi\varepsilon_0 r^2}}, \quad \boxed{\vec{p} \equiv \vec{a}\, q}$$

MOMENTO DI DIPOLO

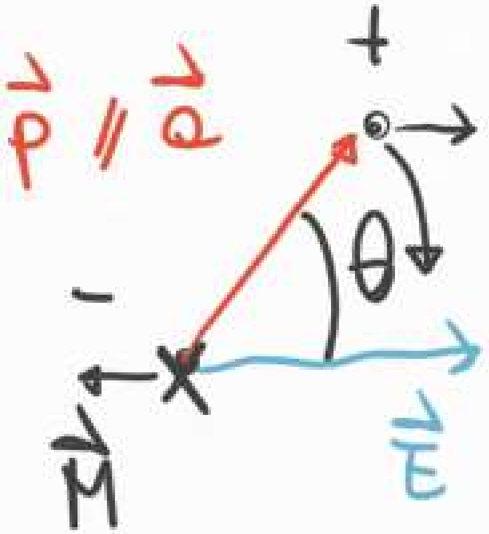
$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3} \quad , \quad E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3} = -\frac{1}{r}\frac{\partial V}{\partial \theta} \quad , \quad E_\phi = 0$$



$$\vec F_{TOT} = q\vec E - q\vec E = 0$$

$$\vec M = \vec r_{c+} \times \vec F_+ + \vec r_{c-} \times \vec F_- = (\vec r_{c+} - \vec r_{c-}) \times \vec F_+ =$$

$$= \vec d \times \vec F_+ = q\vec d \times \vec E = \vec p \times \vec E$$

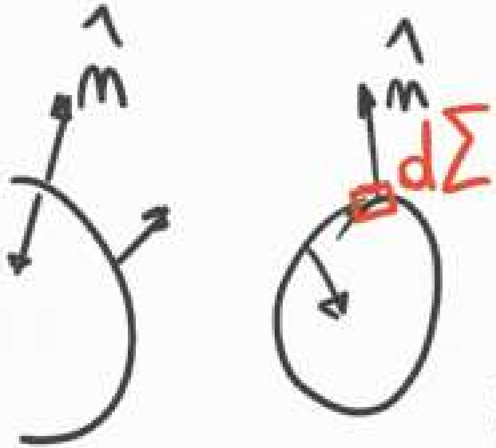


$$\vec M = -pE\sin\theta\,\hat z \,,$$

$$W = \int_{\theta_1}^{\theta_2} M\, d\theta = -pE\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = pE\left(\cos\theta_2 - \cos\theta_1\right) =$$

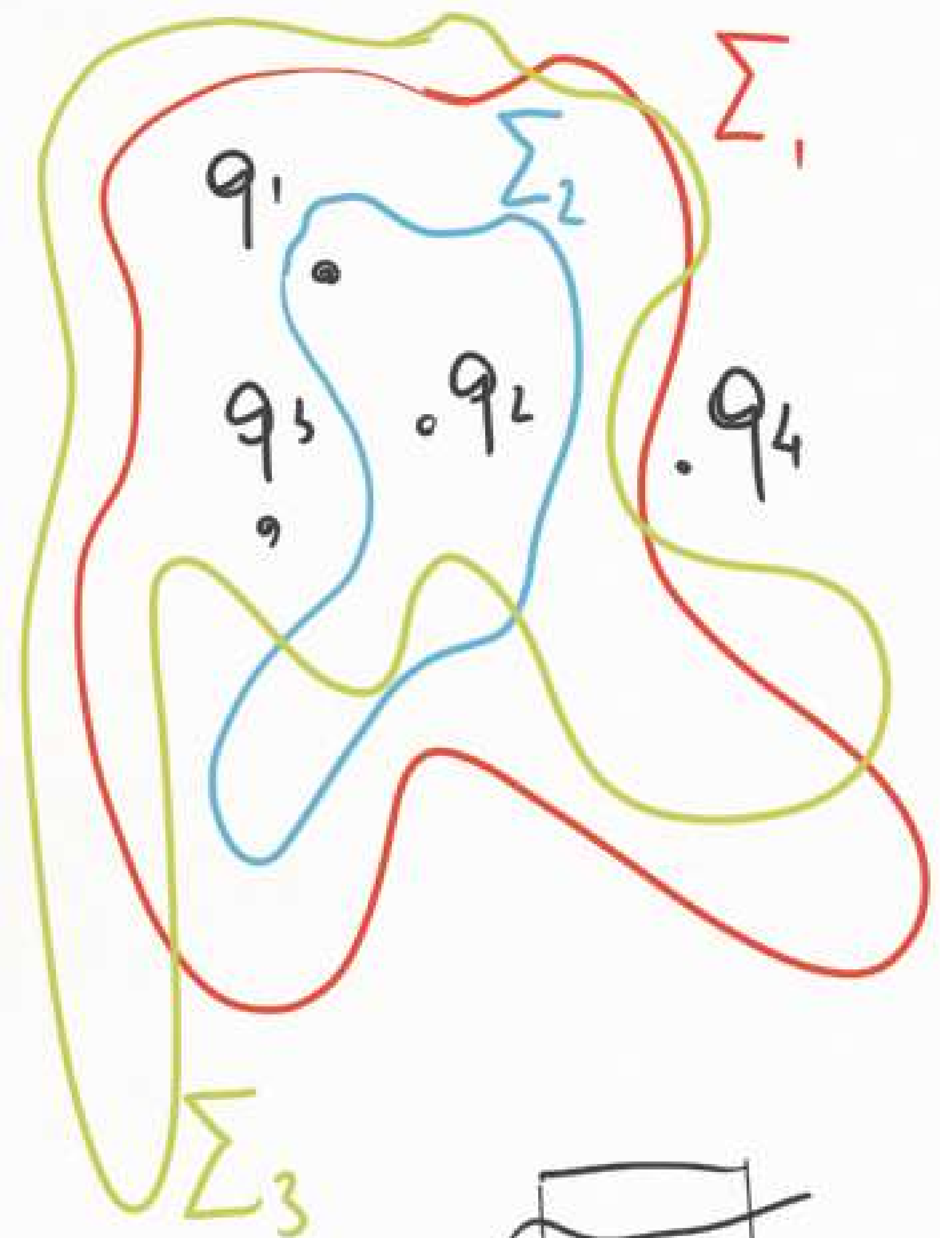
$$= -\Delta U_e \implies U_e = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

TEOREMA DI GAUSS



$$d\Phi(\vec{E}) \equiv \vec{E}\cdot\hat{n}\, d\Sigma$$

$$\boxed{\Phi_\Sigma(\vec{E}) = \int_\Sigma \vec{E}\cdot\hat{n}\, d\Sigma = \boxed{\frac{Q_\Sigma}{\varepsilon_0}}}$$



$$\int_{\Sigma_1} \vec{E}\cdot\hat{n}\,d\Sigma_1 = \frac{q_1+q_2+q_3}{\varepsilon_0} = \int_{\Sigma_3} \vec{E}\cdot\hat{n}\,d\Sigma_3$$

$$\int_{\Sigma_2} \vec{E}\cdot\hat{n}\,d\Sigma_2 = \frac{q_1+q_2}{\varepsilon_0}$$