Project 2: Analysis of a Roll Forge

ME-42: Machine Design (Gary Leisk)

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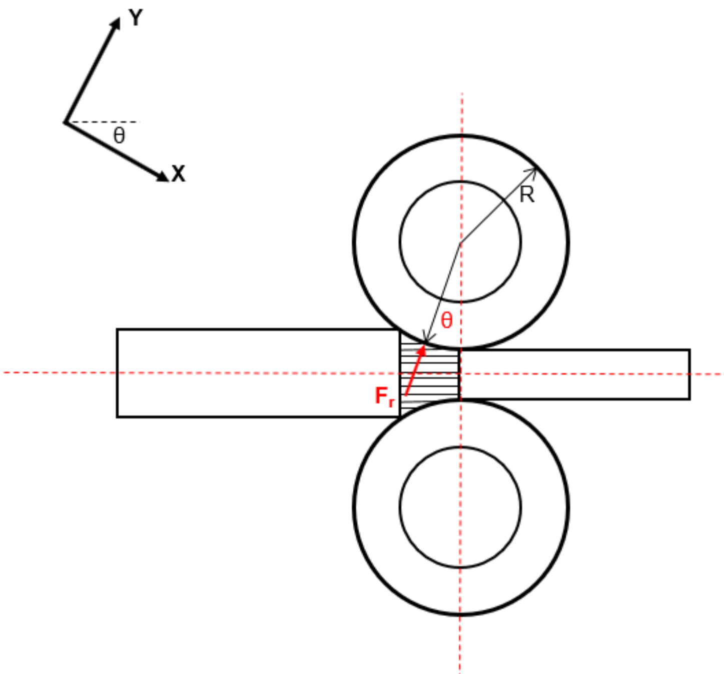
Reaction at the 4 Bearings

In our analysis, the goal of the roll forge was to fully form and mold a series of 4 interconnected rods that would be separated later in the manufacturing process. In order to first give them shape, the roll forge uses molds to shape the softened metal into the desired shape. These softened billets are being compressed, stretched, and molded by the semi-circular die, resulting in a force that acts perpendicular to where the die and billets make contact. This force is then transferred to the 4 bearings that are holding the rollers in place.



This is a cross section, looking down the shaft of the roller, of the forces between the die and the billets. As the billets are given shape, only a certain amount of material is in contact with the die at any given amount during the process.

In order to simplify the calculations in our analysis, we took the resultant force "F_r" to be a single force at a point "θ" degrees from the center of the roller. This force "F_r" acts equally on both the upper and lower rollers, but in opposite directions. Alternatively, it is also possible to describe the resultant force as a distributed load acting on the section of circle that is in contact with the rollers.

There are two major things to specify about this description of the forces, however. As the goal is to determine the reactions at the 4 bearings, it's important to note that our free body diagram shows the forces between the billets and the rollers. However, this model still works because F_r would be transferred from the roller onto its pair of bearings. Essentially, the result force on each of the bearings would be 50% of F_r (0.075 MN per bearing) but in the *opposite* direction that is displayed in the above free body diagram. We elected to leave this diagram as is due to the simple understanding, and because this diagram is effective in explaining deflection later on in analysis.

The second thing to note about this diagram actually relates to deflection. We shifted the axis by θ degrees in order to avoid dealing with an unknown angle in the calculations for deflection. This means that our deflection is not completely vertical, but in the direction of the resultant force.

Static Failure Safety Factor of the Roller

Considering a single cycle of operation for the rollers, a maximum local normal stress and shear was found. These values are then used in determining principal stresses, which are then used in failure theory for ductile materials. In particular, distortion energy failure theory was used to find the ratio between yield strength and factor of safety. Thus, the factor of safety was compared to the loading conditions applied on the rollers. Ultimately, it was found that, with the given dimensions and determined safety factor, failure does not occur.

The moment was determined from the reactions along the end of the bearings, with length 1.086m. The torque experienced was given. In calculating principal stresses,

$$r := 0.13\ m \qquad pi := \pi$$

$$M := 383.097\ kN \cdot m \qquad I := \left(\frac{1}{4}\right) \cdot pi \cdot r^4 = \left(2.243 \cdot 10^{-4}\right)\ m^4$$

$$T := 25.7\ kN \cdot m \qquad J := \left(\frac{1}{2}\right) \cdot pi \cdot r^4 = \left(4.486 \cdot 10^{-4}\right)\ m^4$$

$$\sigma_x := \frac{M}{I} \cdot r = \left(2.22 \cdot 10^{8}\right)\ Pa \qquad \tau := \frac{T}{J} \cdot r = \left(7.447 \cdot 10^{6}\right)\ Pa$$

3D Mohr's Circle for Principal Stresses

$$\sigma_1 := \frac{\sigma_x}{2} + \left(\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2\right)^{.5} = \left(2.223 \cdot 10^{8}\right)\ Pa$$

$$\sigma_3 := \frac{\sigma_x}{2} - \left(\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2\right)^{.5} = -2.495 \cdot 10^{5}\ Pa$$

$$\sigma_2 := 0\ Pa$$

Distortion Energy - 3D Stress

$$\sigma' := \left(\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right)^{.5} = (2.224 \cdot 10^8)\ Pa$$

For a given 1040 Q&T Steel $S_y := 593\ MPa$

Factor of safety -

$$n := \frac{S_y}{\sigma'} = 2.666$$

For a given 1060 Q&T Steel

$$S_{y'} := 669\ MPa$$

Factor of safety -

$$n' := \frac{S_{y'}}{\sigma'} = 3.008$$

If a higher safety factor is desired, changing material to one which possesses a higher yield strength is recommended.

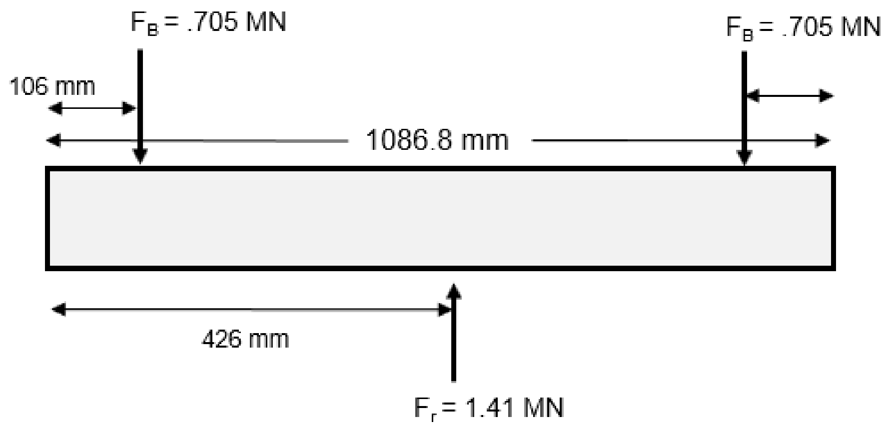
Peak Deflection of Rollers

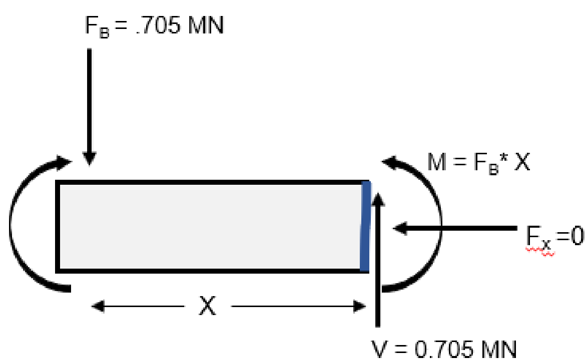
As explained above, the resultant force from the billets to the rollers is at an off angle from the standard axis of the rollers. However, shifting our coordinate axis by θ degrees allows us to use Castigliano's to determine the magnitude of the total deflection.

In terms of loading conditions, we determined that bending shear and bending moment were present in both of the rollers, while torsion is not. Even though the rollers are literally rolling, there is no fastened end to the rollers. The end attached to the gear is spinning due to the gear, so we are treating the roller as being simply supported and having no impact on the deflection of the rollers during the cycle.

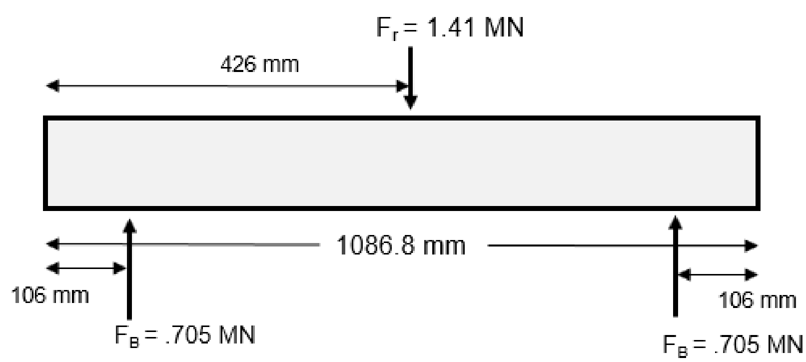
These following free body diagrams are drawn using the provided dimensions and calculated forces.

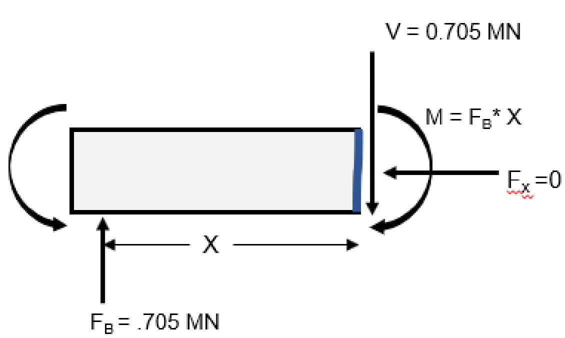
Upper Roller:





Lower Roller:





“ F_B” is the force of the bearings onto the rollers, and since the magnitude of the force on the rollers is simply in opposite directions, the free body diagrams are mirror images of each other.

Given and Calculated Values

$C := 1.11$
$E := 200\ GPa$
$I := 0.000224\ m^4$
$M := F \cdot x$

$F := \frac{1.41}{2}\ MN$
$T := 25.7\ kN \cdot m$
$a := 0.5434\ m$
$V := F$

$A := 0.053\ m^2$
$G := 140.85\ GPa$
$J := 0.000449\ m^4$

Castigliano's Theorem (Bending and Shear)

$$\delta_t := \int_0^a \frac{1}{E \cdot I} (F \cdot x)(x)\, dx + \int_0^a \frac{C}{A \cdot G} V\, dx = \left(8.986 \cdot 10^{-4}\right)\ m$$

Angle of Deflection

$$\theta := \int_0^a \frac{M}{E \cdot I}\, dx \qquad \theta := \int_0^a \frac{(F \cdot x)}{E \cdot I}\, dx = 0.019$$

We used Castigliano’s Theorem to find the total deflection of the rollers. We only took into consideration normal bending and shear bending when doing the calculations. The only other bending that we could have potentially used was torsion. However, we decided not to include it in the calculation of the deflection because the angle that is calculated from the torsion equation is the angle at which the roller is rotated. That does not affect the vertical deflection of the beam.

The slope of the rollers at the bearings was calculated using the method of integration. So we took: $\frac{M}{EI} = \frac{d^2y}{dx^2}$ and integrated it to give us $\theta = \frac{dy}{dx}$, both of which are part of the method of integration. The angel that was calculated was θ = 0.019°. With such a small angel, we have no reason to believe that the bearings will fail or break.

Fatigue Failure Safety Factor of the Roller

To find the safety factor guarding against fatigue failure, we used the Goodman criterion. We chose this method over the many others because it allows for our results to be conservative with the simplicity that we have assumed up to this point in this analysis of the forge roller. Other methods would not have been as conservative, like the Morrow criterion, or they would have been best for finite-life situations, like the Smith-Watson Topper criterion.

In order to use Goodman's criterion, we had to make some assumptions about the material. We are assuming that the rollers were machined, that the diameter is 254mm instead of 260mm to calculate k_b, and that it has a reliability of 95%.

Calculating Needed Values

$$\sigma_{max} := \left(2.223 \cdot 10^{8}\right)\ \mathbf{Pa} \qquad \sigma_{min} := \left(-2.495 \cdot 10^{5}\right)\ \mathbf{Pa}$$

$$\sigma_a := \left| \frac{\left(\sigma_{max} - \sigma_{min}\right)}{2} \right| = \left(1.113 \cdot 10^{8}\right)\ \mathbf{Pa} \qquad \sigma_m := \frac{\left(\sigma_{max} + \sigma_{min}\right)}{2} = \left(1.11 \cdot 10^{8}\right)\ \mathbf{Pa}$$

$$k_a := 0.6843 \qquad k_b := 0.633 \qquad k_e := 0.868$$

$$S_{ut} := 965\ \mathbf{MPa} \qquad k_c := 1 \qquad k_d := 1 \qquad S'_e := 482.5\ \mathbf{MPa}$$

Finding the Safety Factor (Goodman Criterion)

$$S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S'_e = \left(1.814 \cdot 10^{8}\right)\ \mathbf{Pa} \qquad n_f := \frac{1}{\left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}\right)} = 1.373$$

From our calculations, and by using the assumptions that we set in place, our final safety factor for fatigue is 1.373. This is a low safety factor so the possibility of failure is higher than what is desired. To increase this safety factor, the size of the rollers would have to increase.

Gear Torques and Speeds

The torques, speeds, and overall gear train of the system can be determined simply by using gear ratios. Given the torque and speed of the rollers, which are equivalent on the gear that they're attached to (Gear 4), it was a simple matter of working backwards from the information we were provided. The overall Train Value of the gear system is equivalent to the speed of the driving gear (Gear 1) divided by the speed of the output gear (Gear 4). All calculated values are included in the table below.

$$\text{Gear Ratio} = \frac{W_1}{W_2} = \frac{N_1}{N_2} = \frac{d_2}{d_1}$$

$$\text{Gear Ratio} = \frac{T_{out}}{T_{in}}$$

W = Angular Velocity

N = Gear speed (rpm)

d = Gear diameter

T = Torque

Gear	**Diameter** (mm)	**Gear Speed** (rev/second)	**Torque** (KN*m)
4 (provided)	396	0.5	25.7
3	880	0.225	57.11
2	175	1.13	11.37
1	736	0.267	48.12
Overall Train Value		1.87