Parameterized Algorithms for Matrix Completion with Radius Constraints

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Problem Statement

For $i = 1 \dots n$:

$S[i]$ string on alphabet Σ admitting wildcards $*$

$$S = \begin{pmatrix} \mathtt{b} & \mathtt{a} & \mathtt{n} & \mathtt{a} & \mathtt{n} & \mathtt{a} \\ \mathtt{a} & * & \mathtt{n} & * & \mathtt{n} & * \\ \mathtt{b} & * & * & \mathtt{c} & \mathtt{z} & \mathtt{a} \\ \mathtt{z} & \mathtt{a} & \mathtt{z} & * & \mathtt{n} & * \\ \mathtt{b} & \mathtt{a} & * & \mathtt{a} & \mathtt{c} & \mathtt{a} \\ \mathtt{b} & * & \mathtt{n} & \mathtt{c} & * & \mathtt{z} \end{pmatrix} \in \left(\Sigma \cup \{*\}\right)^{n \times \ell}$$

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Minimum Radius Matrix Completion (MinRMC)

Input: Matrix $S \in (\Sigma \cup \{*\})^{n \times \ell}$ and $d \in \mathbb{N}$.
Output: $\exists T$ completion of S and $\exists v \in \Sigma^{\ell}$ s.t. $\delta(v, T) \leq d$?

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Minimum Local Radius Matrix Completion (MinLRMC)

Input: Matrix $S \in (\Sigma \cup \{*\})^{n \times \ell}$ and $d \in \mathbb{N}$.

Output: $\exists T$ completion of S and $\exists i \leq n$ s.t. $\delta\,(T[i], T) \leq d$?

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Constraint Radius Matrix Completion (ConRMC)

Input: Matrix $S \in (\Sigma \cup \{*\})^{n \times \ell}$ and $d_1 \dots d_n \in \mathbb{N}$.
Output: $\exists T$ completion of S and $\exists v \in \Sigma^{\ell}$ s.t. $\forall i \; \delta(v, T[i]) \leq d_i$?

Related Work

Stringology and ML meet in the middle!

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ICML '18: Parameterized Algorithms for Matrix Completion

Dataset contains missing entries

Fill the gaps optimizing some metrics (e.g. rank of the matrix)

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CPM '14: Parameterized Closest String with Wildcards

Closest string $\equiv$ MinRMC for a wildcard-free S

Applications in Bioinformatics

- Motif finding
- PCR primer design
- Genetic probe design

Hardness Results

All Problems are Intractable

- Closest String is $\mathcal{NP}$-hard, even for $|\Sigma| = 2$
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Fixed Parameter Tractability

$f(n,k) = \mathcal{O}\left(g(k)n^{\mathcal{O}(1)}\right)$

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Fixed Parameter Tractability

$$f(n,k) = \mathcal{O}\left(g(k)n^{\mathcal{O}(1)}\right) \iff f(n,k) = \mathcal{O}^*\left(g(k)\right)$$

State of Art