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Generalized method and its application in the higher-order nonlinear Schrodinger equation in nonlinear optical fibres

Zhenya Yan

Department of Mathematics, University of British Columbia, Vancouver, BC, Canada V6T 1Z2

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Abstract

A generalized method, which is called the generally projective Riccati equation method, is presented to find more exact solutions of nonlinear differential equations based upon a coupled Riccati equation. As an application of the method, we choose the higher-order nonlinear Schrodinger equation to illustrate the method. As a result more new exact travelling wave solutions are found which include bright soliton solutions, dark soliton solution, new solitary waves, periodic solutions and rational solutions. The new method can be extended to other nonlinear differential equations in mathematical physics.

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1. Introduction

It is important to seek more exact solutions of nonlinear partial differential equations (NLPDEs) in mathematical physics. Many powerful methods have been presented such as Backlund transformation [1], Darboux transformation [2], the extended Jacobian elliptic function expansion method [3], the tanh method [4], the sine–cosine method [5,6], the homogeneous balance method [7], the direct reduction method [8] and so on.

Generally speaking, these methods can be classified into two classes: One is called the direct method that is applied to seek exact solutions by using some ansatz, for example

$$u(x,t) = \sum_{i=0}^{n} a_i f^i(\xi), \quad \xi = x - vt, \tag{1}$$

with $f(\xi) = \tanh\xi, \operatorname{sech}\xi, \tan\xi, \sec\xi, \operatorname{sn}\xi, \operatorname{cn}\xi, \operatorname{dn}\xi$, etc. The extended Jacobian elliptic function expansion method [3] and the tanh method [4] belong to the class.

The other one is called the indirect method that is applied to seek exact solutions by using the following transformation:

$$u(x,t) = F(\phi_1, \phi_2, \ldots), \quad G(\phi_1, \phi_2, \ldots) = 0. \tag{2}$$

If one can determine the functions ϕ_i from $G(\phi_1, \phi_2, \ldots) = 0$, then he can obtain the solutions from the transformation $u = F(\phi_1, \phi_2, \ldots)$. Backlund transformation, Darboux transformation and sine–cosine method belong to the class.

E-mail address: zhenya_yan@yahoo.com (Z. Yan).

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In 1992, Conte et al. [9] presented an indirect method to seek more new solitary wave solutions of NLPDEs that can be expressed as a polynomial in two elementary functions which satisfy a projective Riccati equation [10]. We simply introduce the method as follows:

For the given NLPDEs, say, two variables x, t,

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0, \tag{3}$$

under the transformation $u(x,t) = u(\xi)$, $\xi = x + \lambda t$, (3) reduces to be

$$Q(u, u', u'', u''', \ldots) = 0. \tag{4}$$

They assume that (4) has the solution

$$u(\xi) = \sum_{l=0}^{1} \sum_{j=0}^{P-1} c_{j,l}\sigma^j(\xi)\tau^l(\xi), \quad (c_{P,0}, c_{P-1,1}) \neq (0,0), \tag{5}$$

with

$$\sigma(\xi) = \frac{K}{\cosh(\xi) + \mu}, \quad \tau(\xi) = \frac{\sinh(\xi)}{\cosh(\xi) + \mu} \tag{6}$$

which satisfy

$$\sigma'(\xi) = -\sigma(\xi)\tau(\xi), \quad \tau'(\xi) = -\tau^2(\xi) - \frac{\mu}{K}\sigma(\xi) + 1, \tag{7}$$

$$\left(\frac{1}{\tau(\xi)} - \frac{\mu}{K}\right)^2 - \frac{\tau^2(\xi)}{\sigma^2(\xi)} = K^{-2}. \tag{8}$$

The method had been applied to find many solitary wave solutions of many equations [9,11]. But they only obtained solitary wave solutions.

In this paper we improve the method to be a more powerful such that it can be used to obtain more types of solutions which contain not only the results obtained by using the above-mentioned method but also other types of solutions such as singular solitary wave solutions, periodic wave solution and rational solutions.

2. Summary of the generalized method

The key idea of our method is to extend the projective Riccati equation (7) to be a more general form

$$\sigma'(\xi) = \epsilon\sigma(\xi)\tau(\xi), \quad \tau'(\xi) = R + \epsilon\tau^2(\xi) - \mu\sigma(\xi), \qquad \epsilon = \pm 1, \tag{9}$$

where $' = \mathrm{d}/\mathrm{d}\xi$, R, μ are constants. When $\epsilon = -1$, $R = 1$, $\mu \to \mu/K$, (9) becomes (7). It is easy to see that (9) admits the first integral with $R \neq 0$,

$$\tau^2(\xi) = -\epsilon\left[R - 2\mu\sigma(\xi) + \frac{(\mu^2 - 1)}{R}\sigma^2(\xi)\right]. \tag{10}$$

We know that (9) admits the following solutions:

Case 1. When $\epsilon = -1$, $R \neq 0$,

$$\begin{cases} \sigma_1(\xi) = \dfrac{R\,\mathrm{sech}(\sqrt{R}\xi)}{\mu\,\mathrm{sech}(\sqrt{R}\xi) + 1}, \quad \tau_1(\xi) = \dfrac{\sqrt{R}\tanh(\sqrt{R}\xi)}{\mu\tanh(\sqrt{R}\xi) + 1}, \\ \sigma_2(\xi) = \dfrac{R\,\mathrm{csch}(\sqrt{R}\xi)}{\mu\,\mathrm{csch}(\sqrt{R}\xi) + 1}, \quad \tau_2(\xi) = \dfrac{\sqrt{R}\coth(\sqrt{R}\xi)}{\mu\coth(\sqrt{R}\xi) + 1}. \end{cases} \tag{11}$$

Case 2. When $\epsilon = 1$, $R \neq 0$,

$$\begin{cases} \sigma_3(\xi) = \dfrac{R\sec(\sqrt{R}\xi)}{\mu\sec(\sqrt{R}\xi) + 1}, \quad \tau_3(\xi) = \dfrac{\sqrt{R}\tan(\sqrt{R}\xi)}{\mu\tan(\sqrt{R}\xi) + 1}, \\ \sigma_4(\xi) = \dfrac{R\csc(\sqrt{R}\xi)}{\mu\csc(\sqrt{R}\xi) + 1}, \quad \tau_4(\xi) = -\dfrac{\sqrt{R}\cot(\sqrt{R}\xi)}{\mu\cot(\sqrt{R}\xi) + 1}. \end{cases} \tag{12}$$

Case 3. When $R = \mu = 0$,

$$\sigma_5(\xi) = \frac{C}{\xi} = C\epsilon\tau_5(\xi), \quad \tau_5(\xi) = \frac{1}{\epsilon\xi}, \tag{13}$$

where C is a constant. It is clear to see that (6) is special case of (11)–(13). Based upon the above analysis, we assume that (4) has the following two solutions:

Type 1. When $R \neq 0$,

$$u(\xi) = \sum_{i=1}^{n} \sigma^{i-1}(\xi)[A_i\sigma(\xi) + B_i\tau(\xi)] + A_0 \tag{14}$$

where σ, τ satisfy (9) and (10).

Type 2. When $R = \mu = 0$,

$$u(\xi) = \sum_{i=0}^{n} A_0\tau^i(\xi) \tag{15}$$

where τ satisfies

$$\tau'(\xi) = \tau^2(\xi) \tag{16}$$

which can be obtained from (9), (10) and (14).

The parameter n can be found by balancing the highest-order linear term with nonlinear term in (3) or (4). (If n is not a positive integer, then we first make the transformation $u = w^n$.) Substituting (9), (10) and (14) (or (15) and (16)) into (4) yields a set of algebraic equations for $\sigma^j(\xi)\tau^i(\xi)$, $j = 0, 1, \ldots;\ i = 0, 1$ ($\tau^l(\xi),\ l = 0, 1, \ldots$). Setting the coefficients of these terms $\sigma^j(\xi)\tau^i(\xi)$ (or $\tau^l(\xi)$) to zero yields a set of over-determined algebraic equations in v, A_i, B_i, R and μ. If we can find solutions A_i, B_i of the set of equations, then we can obtain many solutions of (3) according to (11)–(15).

In what follows we choose the higher-order nonlinear Schrodinger equation describing propagation of ultrashort pulses in nonlinear optical fibres [12]

$$\frac{\partial\Psi}{\partial z} = \mathrm{i}\alpha_1\frac{\partial^2\Psi}{\partial t^2} + \mathrm{i}\alpha_2\Psi|\Psi|^2 + \alpha_3\frac{\partial^3\Psi}{\partial t^3} + \alpha_4\frac{\partial\Psi|\Psi|^2}{\partial t} + \alpha_5\Psi\frac{\partial|\Psi|^2}{\partial t} \tag{17}$$

to illustrate the above-mentioned method. Where Ψ is slowly varying envelope of the electric field, the subscripts z and t are the spatial and temporal partial derivative in retard time coordinates, and α_1, α_2, α_3, α_4 and α_5 are the real parameters related to the group velocity (GVD), self-phase modulation (SPM), third-order dispersion (TOD), self-steepening, and self-frequency shift arising from stimulated Raman scattering, respectively. Some properties of the equation, as well as many versions of it have been studied [12–18]. Up to now, the bright, dark and the combined bright and dark solitary waves and periodic waves were found of (17) and its special case. In this paper we apply the above-mentioned method to (17) such that more new exact solutions are obtained.

3. Exact solutions of (17)

According to the above steps, to seek travelling wave solutions of (17), we make the gauge transformation

$$\Psi(z,t) = \psi(\xi)\exp[\mathrm{i}(kz - wt)], \quad \xi = t - \lambda z, \tag{18}$$

where k, w, λ are constants to be determined later. Substituting (18) into (17) yields a complex ODE of $\psi(\xi)$, the real and imaginary parts of which, respectively, read

$$(\alpha_1 - 3\alpha_3 w)\psi'' + (\alpha_3 w^3 - \alpha_1 w^2)\psi + (\alpha_2 - \alpha_4 w)\psi^3 = 0, \tag{19a}$$

$$\alpha_3\psi''' + (2\alpha_1 w - 3\alpha_3 w^2 + \lambda)\psi' + (3\alpha_4 + 2\alpha_5)\psi^2\psi' = 0. \tag{19b}$$

It is easy to see that (19a), (19b) becomes a equation

$$\psi'' + \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}\psi + \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\psi^3 = 0 \tag{20}$$

under the constraint conditions

$$w = \frac{\alpha_1(3\alpha_4 + 2\alpha_5) - 3\alpha_2\alpha_3}{6\alpha_3(\alpha_4 + \alpha_5)}, \tag{21}$$

$$k = -\frac{1}{\alpha_3}[(\alpha_1 - 3\alpha_3 w)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda) - \alpha_1 w^2] + w^3. \tag{22}$$

By balancing the highest-order linear term ψ'' and the nonlinear ψ^3 in (20), we obtain $n = 1$ in (14) and (15). In what follows we will obtain solutions of (17) by considering (20).

3.1. Solitary waves and periodic waves

According to the above-mentioned method, we assume (20) has the solutions in the form with $R \neq 0$,

$$\psi(\xi) = A_0 + A_1\sigma(\xi) + B_1\tau(\xi), \tag{23}$$

where A_0, A_1, B_1 are constants to be determined later. $\sigma(\xi)$ and $\tau(\xi)$ satisfying (9) and (10).

With the aid of Maple, substituting (9), (10) and (23) into (20) and collecting all terms with the same power in $\sigma^j(\xi)\tau^i(\xi)$, $j = 0, 1, 2, 3, 4; i = 0, 1$. Setting the coefficients of these terms $\sigma^j(\xi)\tau^i(\xi)$ to zero yields a set of over-determined algebraic polynomials with respect to the unknowns λ, R, A_0, A_1, B_1, namely

$$\text{const}: \ \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3} A_0 + \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(A_0^3 - 3\epsilon A_0 B_1^2\right) = 0, \tag{24.1}$$

$$\sigma(\xi): \ \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3} A_1 + \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(3A_0^2 A_1 - 3\epsilon A_1 B_1^2 R + 6\epsilon\mu A_0 B_1^2\right) - \epsilon A_1 R = 0, \tag{24.2}$$

$$\tau(\xi): \ \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3} B_1 + \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(3A_0^2 B_1 - \epsilon B_1^3 R\right) = 0, \tag{24.3}$$

$$\sigma(\xi)\tau(\xi): \ \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(6A_0 A_1 B_1 + 2\epsilon\mu B_1^3\right) + \epsilon\mu B_1 = 0, \tag{24.4}$$

$$\sigma^2(\xi): \ \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(3A_0 A_1 A_1^2 + 6\epsilon\mu A_1 B_1^3 - 3A_0 B_1^2 \epsilon \frac{\mu^2 - 1}{R}\right) + 3\epsilon\mu A_1 = 0, \tag{24.5}$$

$$\sigma^2(\xi)\tau(\xi): \ \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(3A_1^2 B_1 - \epsilon B_1^3 \frac{\mu^2 - 1}{R}\right) - 2\epsilon B_1 \frac{\mu^2 - 1}{R} = 0, \tag{24.6}$$

$$\sigma^3(\xi): \ \frac{3\alpha_4 + 2\alpha_5}{3\alpha_3}\left(A_1^3 - 3\epsilon A_1 B_1^2 \frac{\mu^2 - 1}{R}\right) - 2\epsilon A_1 \frac{\mu^2 - 1}{R} = 0. \tag{24.7}$$

With the aid of Maple, we have the following many families of solutions:

Case 1. $\mu = A_0 = A_1 = 0$, $\epsilon = -1$,

$$B_1^2 = \frac{6\alpha_3}{3\alpha_4 + 2\alpha_5}, \quad R = \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{2\alpha_3}.$$

Case 2. $\mu = A_0 = B_1 = 0$, $\epsilon = -1$,

$$A_1^2 = -\frac{6\alpha_3^2}{(3\alpha_4 + 2\alpha_5)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}, \quad R = -\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}.$$

Case 3. $\mu = A_0 = A_1 = 0$, $\epsilon = 1$,

$$B_1^2 = -\frac{6\alpha_3}{3\alpha_4 + 2\alpha_5}, \quad R = -\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{2\alpha_3}.$$

Case 4. $\mu = A_0 = B_1 = 0$, $\epsilon = 1$,

$$A_1^2 = -\frac{6\alpha_3^2}{(3\alpha_4 + 2\alpha_5)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}, \quad R = \frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}.$$

Case 5. $A_0 = 0$, $\epsilon = -1$,

$$A_1^2 = -\frac{3\alpha_3^2(1-\mu^2)}{(3\alpha_4+2\alpha_5)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}, \quad B_1^2 = -\frac{6\alpha_3}{3\alpha_4+2\alpha_5}, \quad R = -\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}.$$

Case 6. $A_0 = 0$, $\epsilon = 1$,

$$A_1^2 = -\frac{3\alpha_3^2(1-\mu^2)}{(3\alpha_4+2\alpha_5)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}, \quad B_1^2 = -\frac{6\alpha_3}{3\alpha_4+2\alpha_5}, \quad R = -\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}.$$

Therefore from (11), (12), (18), (23) and Cases 1–6, we obtained many families of exact travelling wave solution of (17):

Family 1. *Dark soliton solutions*

$$\Psi_1 = \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4+2\alpha_5}}\tanh\left[\sqrt{\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{2\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{25}$$

Family 2. *Singular dark soliton solutions*

$$\Psi_2 = \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4+2\alpha_5}}\coth\left[\sqrt{\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{2\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{26}$$

Family 3. *Bright soliton solutions*

$$\Psi_4 = \sqrt{\frac{6\alpha_3}{3\alpha_4+2\alpha_5}}\mathrm{sech}\left[\sqrt{-\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{27}$$

Family 4. *Singular soliton solutions*

$$\Psi_4 = \sqrt{\frac{6\alpha_3}{3\alpha_4+2\alpha_5}}\mathrm{csch}\left[\sqrt{-\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{28}$$

Family 5. *Periodic wave solutions*

$$\Psi_5 = \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4+2\alpha_5}}\tan\left[\sqrt{-\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{2\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{29}$$

Family 6. *Periodic wave solutions*

$$\Psi_6 = -\sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4+2\alpha_5}}\cot\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{2\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{30}$$

Family 7. *Periodic solutions*

$$\Psi_7 = \sqrt{-\frac{6\alpha_3}{3\alpha_4+2\alpha_5}}\sec\left[\sqrt{\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{31}$$

Family 8. *Periodic solutions*

$$\Psi_8 = \sqrt{\frac{6\alpha_3}{3\alpha_4+2\alpha_5}}\csc\left[\sqrt{\frac{2\alpha_1 w - 3\alpha_3 w^2 + \lambda}{\alpha_3}}(t-\lambda z)\right]\exp[\mathrm{i}(kz-wt)]. \tag{32}$$

Family 9. *New bright and dark soliton solutions*

$$\Psi_9 = \left\{ \sqrt{\frac{(1-\mu^2)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\operatorname{sech}\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \operatorname{sech}\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} + \sqrt{-\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\tanh\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \tanh\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \right\} \exp[\mathrm{i}(kz - wt)]. \tag{33}$$

In particular, when $\mu = \pm 1$, the new dark soliton solution

$$\Psi = \sqrt{-\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\tanh\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\pm \tanh\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \exp[\mathrm{i}(kz - wt)]. \tag{34}$$

Family 10. *New soliton solutions*

$$\Psi_{10} = \left\{ \sqrt{\frac{(1-\mu^2)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\operatorname{csch}\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \operatorname{csch}\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} + \sqrt{-\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\coth\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \coth\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \right\} \exp[\mathrm{i}(kz - wt)]. \tag{35}$$

In particular, when $\mu = \pm 1$, the new dark solution solution

$$\Psi = \sqrt{-\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\coth\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\pm \coth\left[\sqrt{\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \exp[\mathrm{i}(kz - wt)]. \tag{36}$$

Family 11. *New periodic solutions*

$$\Psi_{11} = \left\{ \sqrt{\frac{(\mu^2-1)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\sec\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \sec\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} + \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\tan\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \tan\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \right\} \exp[\mathrm{i}(kz - wt)]. \tag{37}$$

In particular, when $\mu = \pm 1$, the new periodic solution

$$\Psi = \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\tan\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\pm \tan\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \exp[\mathrm{i}(kz - wt)]. \tag{38}$$

Family 12. *New periodic solutions*

$$\Psi_{12} = \left\{ \sqrt{\frac{(\mu^2-1)(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\csc\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \csc\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} - \sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\cot\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\mu \cot\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \right\} \exp[\mathrm{i}(kz - wt)]. \tag{39}$$

In particular, when $\mu = \pm 1$, the new periodic solution

$$\Psi = -\sqrt{\frac{3(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{3\alpha_4 + 2\alpha_5}} \frac{\cot\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right]}{\pm\cot\left[\sqrt{-\frac{2(2\alpha_1 w - 3\alpha_3 w^2 + \lambda)}{\alpha_3}}(t - \lambda z)\right] + 1} \exp[\mathrm{i}(kz - wt)]. \tag{40}$$

3.2. Rational solutions

According to the above-mentioned method in Section 2 and (15) and (16), we assume (20) has the solutions in the form with $R \neq 0$,

$$\psi(\xi) = A_0 + A_1\tau(\xi), \tag{41}$$

where A_0, A_1 are constants to be determined later. $\tau(\xi)$ satisfying (16). Substituting (41) and (16) into (20) yields an equation of $\tau^i(\xi)$ $(i = 0, 1, 2, 3)$. Setting the coefficients of these terms to zero, we have a set of equations of A_0, A_1, λ and solve it such that we get

$$\lambda = 3\alpha_3 w^2 - 2\alpha_1 w, \quad A_0 = 0, \quad A_1 = \sqrt{-\frac{6\alpha_3}{3\alpha_4 + 2\alpha_5}}. \tag{42}$$

Therefore we have the solutions from (11), (12), (16), (41) and (42)

$$\Psi_{13}(z,t) = \sqrt{-\frac{6\alpha_3}{3\alpha_4 + 2\alpha_5}} \frac{1}{t - (3\alpha_3 w^2 - 2\alpha_1 w)z} \exp[\mathrm{i}(kz - wt)]. \tag{43}$$

Remark 1. Only the solutions (25)–(27), (29) and (30) of (17) and its versions had been found [12–18].

Remark 2. In [15], Li et al. obtained a new solitary wave solution of (17) which is a solution of combining bright and dark solitary waves. But we obtain new types of solutions (33) and (35) which are different from the solutions by Li et al.

Remark 3. To our knowledge, the rational solution (43) has been not found before.

4. Summary and conclusions

In summary, we have derived many families of exact travelling wave solutions of the higher-order nonlinear Schrodinger equation in optical fibres based upon two transformations, that is, the first one is the transformation (23) with the coupled Riccati equation; the other one is the transformation (41) with the special Riccati equation. The method can be also easy to be extended to other NLPDEs. The paper is shown that the improved method is sufficient to seek more new exact wave solutions of NLPDEs.

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Erratum

Erratum of "Generalized method and its application in the higher-order nonlinear Schrodinger equation in nonlinear optical fibres" (Chaos, Solitons and Fractals 16 (2003) 759–766) ☆

Zhenya Yan

Department of Mathematics, University of British Columbia, Vancouver, B.C., Canada V6T 1Z2

Some misprints are corrected below:

(a) Change "tanh" into "sech" in the denominator of the expression $\tau_1(\xi)$ in (11), the denominator of the second term in (33) and the denominator of expression (34), respectively.
(b) Change "coth" into "csch" in the denominator of the expression $\tau_2(\xi)$ of (11), the denominator of the second term in (35) and the denominator of expression (36), respectively.
(c) Change "tan" into "sec" in the denominator of the expression $\tau_3(\xi)$ of (12), the denominator of the second term in (37), the denominator of expression (38), respectively.
(d) Change "cot" into "csc" in the denominator of $\tau_4(\xi)$ of (12), the denominator of the second term in (39), the denominator of expression (40), respectively.

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E-mail address: zhenya_yan@yahoo.com (Z. Yan).

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