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Relativistic Gross-Pitaevskii equation and the cosmological Bose Einstein Condensation -Quantum Structure in Universe-

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Abstract. We do not know 96% of the total matter in the universe. A model is proposed in which Dark Energy is identified as Bose-Einstein Condensation. Global cosmic acceleration and rapid local collapse into black holes (Dark Matter) are examined. We also propose a novel mechanism of inflation due to the steady flow of condensation, which is free from slow-roll conditions for the potential.

1. Introduction

The standard LCDM model works perfectly. But they are mostly in the linear stage. For example, the temperature fluctuations in the sky are calculable from the primordial density fluctuations, and they actually satisfactorily describe the observations. Large scale power spectrum of density fluctuations can also be calculable from the primordial density fluctuations, and they satisfactorily describe the observations.

However, there are some missing essentials in the standard LCDM model as follows.

1. We don't know 96% of the cosmic contents that are simply called Dark Energy (DE) and Dark Matter (DM).
2. Many peculiarities in the non-linear regime exist such as very early formation of objects (re-ionization at $z \approx 20$), bias mechanism, and the first star formation, etc.

In order to improve the LCDM model in the above points, we should be aware of the following basic facts:

A. DE has negative pressure: $p < 0$ to promote accelerated expansion. This fact comes from the Einstein equations: $\ddot{a}(t) = -(4\pi G/3)(\rho + 3p)a(t)$, $(\dot{a}/a)^2 = 8\pi G\rho/3$.

B. DE and DM are almost the same amount; their energy density ratio is about 3:1.

Up to now, well known matter which shows negative pressure would be the classical scalar field ϕ. This scalar field is often used in various cosmological models without specifying its origin. Here we try to identify this scalar field as the macroscopic mean filed of Bose Einstein condensation (BEC) with attractive interaction. This means, according to the Bogoliubov prescription, the classical field Φ_0 in the decomposition $\hat{\Phi} = \Phi_0 + \hat{\phi}$ corresponds to the scalar field ϕ we consider now in the context of cosmology. Thus we now propose a model in which Bose Einstein Condensation (BEC) of some boson works as DE [1]. Moreover DM will naturally turn out to be the locally collapsed BEC.

2. Cosmic BEC mechanism

BEC is possible if thermal de Broglie length becomes larger than the mean separation of particles [2], or

$$\lambda_{dB} \equiv \left(\frac{2\pi\hbar^2}{mkT}\right)^{1/2} > r \equiv n^{-1/3} \quad \text{i.e. } kT < \frac{2\pi\hbar^2 n^{2/3}}{m}. \tag{1}$$

On the other hand the cosmic particle number evolution: $n = n_0\left(\frac{m}{2\pi\hbar^2}\frac{T}{T_0}\right)^{3/2}$ has the same temperature dependence. Therefore for example, if the boson temperature is equal to the radiation temperature at $z = 3000$, its present critical temperature and the energy density are $T_{cr} = 0.0027K$ and $\rho = \rho_{now} = 9.44\ 10^{-30}\,\mathrm{g/cm}^3$. Then the above BEC condition becomes $m < 1.96\mathrm{eV}$. In this argument, we used, $\rho \propto a^{-3} \propto T^{3/2}$ for matter and $\rho \propto a^{-4} \propto T^4$ for radiation assuming adiabatic cosmic expansion.

3. Relativistic Gross-Pitaevski Equation

BEC is usually described by the Gross-Pitaevskii equation which has a form of non-linear Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi + g|\psi|^2\psi, \tag{2}$$

where $\psi(\vec{x},t)$ is the condensate classical function, $V(\vec{x})$ is the potential, $g = 4\pi\hbar^2 a/m$, and a is the s-wave scattering length. However we need a relativistic version because we are considering the extreme resume of the state equation. Relativistic version of GP equation is easily obtained as

$$\frac{\partial^2\Phi}{\partial t^2} - \Delta\Phi + m^2\Phi - \lambda(\Phi^*\Phi)\Phi = 0, \tag{3}$$

which is just the Klein-Gordon equation with quartic self-interaction.

4. Gradual process of BEC

We suppose the condensation slowly proceeds with constant speed Γ because the cosmic evolution is almost adiabatic. Further, it turns out that the BEC suddenly becomes unstable after it reaches the critical energy density. This instability is represented by a intermittent decay term Γ'. Then our basic set of equations becomes

$$\begin{aligned} H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}(\rho_g + \rho_\phi + \rho_l) \\ \dot{\rho}_g &= -3H\rho_g - \Gamma\rho_g \\ \dot{\rho}_\phi &= -6H(\rho_\phi - V) + \Gamma\rho_g - \Gamma'\rho_\phi \\ \dot{\rho}_l &= -3H\rho_l + \Gamma'\rho_\phi \end{aligned} \tag{4}$$

where

$$V = \frac{m^2}{2}\Phi^*\Phi + \frac{\lambda}{4}(\Phi^*\Phi)^2 \qquad (\lambda < 0) \tag{5}$$

The essence of the cosmic BEC model is not the relativistic form of GP Eq(3), but the slow and steady condensation flow represented by Γ.

There are two interesting regimes for Eq.(4).

(1) Over-hill regime: This is realized for too faster condensation flow. The corresponding fixed point of Eq.(4) is

$$\phi \to \infty, \rho_\phi \to 0, H \to 0, a \to a_*. \tag{6}$$

Analysis of linear stability [3] reveals the BEC Jeans instability sets in for $k < k_J$ where

$$k_J^2 = -\lambda a_0^2 + \sqrt{(\lambda a_0^2)^2 + 16\pi G m^2 a_0^2 \omega^2} \tag{7}$$

Moreover, since the weak energy condition is eventually violated $-p_\phi > \rho_\phi$, uniform mode of BEC would finally collapse, with speed Γ', to form local structure whose energy density is ρ_l. This effect is represented in the last line in Eq.(4).
(2)Mini-inflationary regime: This regime appears when the condensation flow balances with the potential force. In this mode, the fixed point is

$$\phi \to \phi_*, H \to H_*, \rho \to 0, \dot{\phi} \to 0\,. \tag{8}$$

Linear analysis reveals that this is a stable fixed point.

Simultaneously, this fixed point is a new type of inflation, supported by the steady flow of condensation. The ordinary inflation requires $V'(\phi) \approx 0$ and $V(\phi) \neq 0$ (slow-roll condition). On the other hand, our new type inflation is free from such extra conditions. The essence of our inflation is the relation $\dot{V} = \Gamma \rho_g$, that simply represents the balance between the condensation flow and the potential force, and this balance is automatically established at the fixed point. In this sense, this new inflation is robust independent of the coupling, potential V, and initial condition, etc.

As a whole, the entire cosmic evolution starts from the Over-Hill regime followed by the final Mini-Inflationary regime. A typical numerical calculation is depicted in Figure 1.

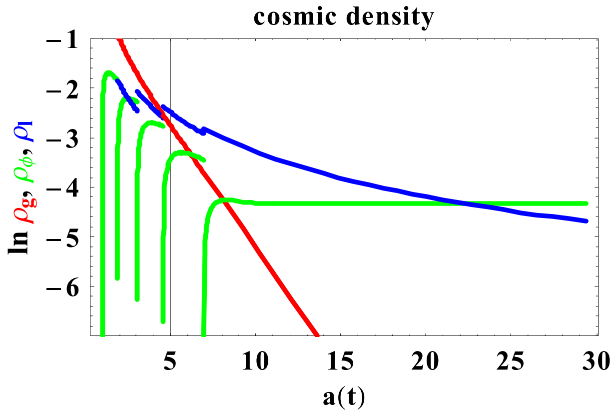

Figure 1. Cosmic evolution of various energy densities in typical numerical calculation of Eq.(4). Boson gas (ρ_g) gradually condenses into BEC (ρ_ϕ) component with speed Γ. BEC (ρ_ϕ) collapses when it reaches the critical value and forms localized objects (ρ_l) with speed Γ'.

5. Collapse of BEC

In the linear stage, the relativistic GP equation reduces to the Euler equation for fluid:

$$\varepsilon \frac{\partial \vec{p}}{\partial t} + \vec{\nabla}\left(\frac{\vec{p}^2}{2m} + \frac{1}{2}\mu^2 + \frac{\lambda}{12}A^2 + \frac{\hbar^2}{2A}\Box A\right) = 0\,, \tag{9}$$

if we decompose $\phi = Ae^{iS}$ and defining $p_\mu = -\partial_\mu S = (\varepsilon, -\vec{p})$, $\vec{p} = m\gamma\vec{v}$, and $\gamma = \left(1-\left(\vec{v}^2/c^2\right)\right)^{-1/2}$. Therefore if λ is negative, the interaction yields $p \propto -b\rho^2 \quad (b>0)$ and this promotes the collapse of BEC. After the dominance of ρ_ϕ, the linear perturbation equation for the gauge invariant quantity yields $\ddot{\delta}_k + 2H\dot{\delta}_k = -\left(3H^2 + 2(k/a)^2\right)\delta_k$, if we suppose $b\rho \approx 1$ as the extreme case. Then, small scale mode becomes extremely unstable and grows very rapidly as $\delta \propto a(t)^{k/aH}$. Further collapse of BEC would yield boson stars or black holes. Thus many compact clumps are rapidly formed after each collapse of BEC at the smallest scale. In this sense black holes are found everywhere in the universe and they are made from Dark Energy.

6. Observational tests for the model

Large scale structure predicted by LCDM model will not be affected in our model. This is because exceedingly violent instability at small scales, which is the most characteristic property of our model, does not leave trace in the large scales within its too short time interval. This is demonstrated by the following argument. Suppose the condensation collapses at the scale $l = a/k \equiv \tilde{k}^{-1} \ll H^{-1}$. Then, for $a \approx t^{2/3}$, $\delta_k \propto a^{2\tilde{k}/H} = t^{4\tilde{k}/(3H)}$. The time duration of the collapse, i.e. the time necessary for the condensation to disappear, is $\Delta t \approx l/c$ [1]. The fluctuations are very unstable only within this time. During this time duration, the fluctuation of the scale l can grow

$$\frac{\delta(t+\Delta t)}{\delta(t)} = (1+\frac{H}{\tilde{k}})^{4\tilde{k}/(3H)} \approx e^{4/3} \approx 3.8\,. \tag{10}$$

For much larger scale $10 \times l$, the growing rate is only $\delta(t+\Delta t)/\delta(t) \approx 1.14$.

The condensation is supposed to collapse well after the decoupling time: i.e. $t_{dec} < \Gamma^{-1}$. Therefore the temperature inhomogeneity generated within the decoupling era would not be changed from the standard LCDM model. However the integrated Sacks-Wolfe effect may slightly modify the spectrum.

In the present BEC cosmology, many black holes are formed in the universe. We first have to calculate the mass function of these black holes, though we have not yet succeeded to do so. This is mainly because the shock wave, which is naturally formed in the BEC collapse, would increase the BEC temperature to about the mass of the particle $T \approx GM^2/(NR) \approx m$. In this relativistic regime, the particle number is not conserved and the BEC would melt. Therefore we have to generalize the relativistic GP equation to include dynamics of boson gas as well.

7. Summary of BEC Cosmological model

BEC Cosmology can be summarized as follows. Initially dominated boson gas, which behaves as global DM slowly condensates to form BEC, which behaves as DE. When it exceeds the critical value, it collapses into localized objects such as black holes, which behave as localized DM. After repeating BEC collapse several times, the universe eventually enters into the Mini-inflationary regime and the cosmic expansion is accelerated. Since these rapid collapses take place very fast and well after the photon-decoupling, there is no strong violation of linear results, such as the power spectrums of the density fluctuations and CMB, in LCDM model.

On the other hand in the condensed matter physics, experimental and theoretical efforts are actively revealing unique properties of BEC. Among them, the boson-nova phenomenon is remarkable [4, 5]. We would like to expect similar interesting structures in the universe, especially in relation with the universal cosmic jets in various scales and the quasar formation in the very early (for example, $z \approx 17$) stage of the universe.

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