Final Project
Quantitative Macroeconomics

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Introduction

One of the greatest answers that economists have always been trying to answer is the one regarding the explanation of the Great Divergence characterising the development of countries.
It has been argued by historians that before the great boom that Europe experienced during the Industrial Revolution, there were countries that reached a technological superiority way higher than the European one. An example is the situation in Asia in the 11^{th} and 12^{th} century. In that period China had been employing water-driven machinery in textiles and coal for iron smelting, resulting in iron outputs that were only matched in Europe seven hundred years later.
So, how is it possible that there had been such a strong turnover?
This paper is precisely trying to give a theoretical explanation to such question by proposing a model based on Ashraf and Galor (2012). The main objective is to construct a theoretical model that would help economists understand the forces that drive the economy from a pre-industrialization era, characterized by Malthusian stagnation, to the industrialized economy characterised by sustained economic growth.

This research argues that a basic factor determining the dynamics of development is the balance between cultural assimilation and cultural diffusion embodied in a society. This view may seem extravagant by a conservative reader, but this paper shows it importance in the path of development.
The underlying idea is that the main driver of economical development is human capital. Human capital itself directly impacts technological progress, but what are the characteristics of a society that make it enhancing its human capital more than others? The answer given by this research is that human capital can be transmitted by the old generation to the actual, or it can be created by thee current generation. In particular, the model divides the total population into two categories: conformists and non-conformists. Conformists have the ability to transmit human capital intergenerationally, while non-conformists help the economy develop new technologies. The problem is that an economy only composed by conformists

enhances the efficiency with which society operates with respect to the available production technologies, but it would never develop new technologies. While, a society composed by many non-conformists will always create new technologies, but would rarely transmit these advancements to future generations. It is thus clear the importance that a balanced population has in terms of development.

In a Malthusian economy, the technological process is so slow that cultural assimilation is fundamental in order to make the economy work near its efficiency frontier. But then in order to make the passage to industrialization, cultural diffusion is fundamental, because it gives the possibility to embody new technologies from foreigners in order to include them into the society.

The following section tries to model this process.

Theoretical Model

This model theorizes a perfectly competitive overlapping-generations economy in the process of development, where economic activity extends over infinite discrete time.

1 Firm's Problem

The production side of the economy is divided in to two sectors: Rural (R) and Industrial (M). The rural sector is the only one operating at early ages of development, because the industrial one is not yet profitable for entrepreneurs.
The output in R at period t is governed by a Cobb-Douglas production function:

$$Y_t^R = (A_t R)^{1-\alpha}(L_t^R)^\alpha \tag{1}$$

where $\alpha \in (0,1)$, L_t^R is the amount of labor employed in the rural sector in period t, and A_t^R is the level of rural productivity in period t.

The production function in M at t is the following:

$$Y_t^M = A_t^M L_t^M \tag{2}$$

Total labor in period t is

$$L_t = L_t^R + L_t^M \tag{3}$$

1.1 Output of Firm's Problem

The profit of M is therefore equal to $A_t^M L_t^M - L_t^M w_t^M = 0$ (because of perfect competition), where w_t^M is the level of wages in M at time t. It follows that at equilibrium:

$$w_t^M = A_t^M \tag{4}$$

In R, profit is equal to $(A_t^R)^{1-\alpha}(L_t^R)^\alpha - L_t^R w_t^R = 0$ (perfect competition). It follows that in equilibrium:

$$w_t^R = \left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha} \tag{5}$$

From Equation 5 it follows that the inverse demand for labor in the rural sector increases without bound as employment decreases in that sector, following that the rural sector will be operative in every period. On the other hand, the M sector will only exist if entrepreneurs in M are able to provide to workers a wage that is at least as high as the one in R:

$$w_t^M \geq w_t^R$$

$$A_t^M \geq \left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha}$$

It follows that the **Manufacturing Productivity threshold** for the economic viability of the manufacturing sector is the following:

$$A_t^M \geq \left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha} = \hat{A}_t^M \tag{6}$$

From which follows the **Population threshold** for the economic viability of the manufacturing sector:

$$L_t^R \geq \left(\frac{1}{A_t^M}\right)^{\frac{1}{1-\alpha}} A_t^R$$

Thus, given Equation 3:

$$L_t \geq \left(\frac{1}{A_t^M}\right)^{\frac{1}{1-\alpha}} A_t^R = \hat{L}_t \tag{7}$$

1.2 Labor market equilibria

The labor market defines the existence of the industrial sector M. In particular, we saw from Equations 6 and 7 that certain thresholds have to be met in order to have M

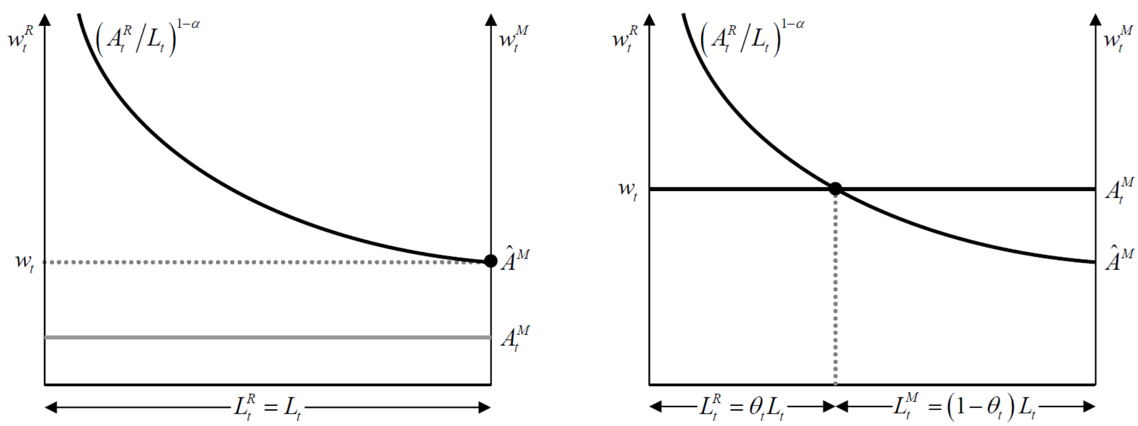
economically viable.
It follows that the equilibrium share of population working in R at time t is the following:

$$\theta_t = \frac{L_R}{L_t} = \begin{cases} 1, & \text{if } A_t^M < \hat{A}_t^M \\ \left(\frac{1}{A_t^M}\right)^{\frac{1}{1-\alpha}} \frac{A_t^R}{L_t}, & \text{if } A_t^M \geq \hat{A}_t^M \end{cases} \tag{8}$$

Finally, the equlibrium wage at t is:

$$w_t = \begin{cases} w_t^R = \left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha}, & \text{if } A_t^M < \hat{A}_t^M \\ w_t = A_t^M, & \text{if } A_t^M \geq \hat{A}_t^M \end{cases} \tag{9}$$

Equation 8 and 9 are showed in Figure 1:



(a) The $A_t^M < \hat{A}_t^M$ Case (b) The $A_t^M > \hat{A}_t^M$ Case

Figure 1: The Labor Market Equilibrium Conditional on the Level of Productivity of Labor in the Manufacturing Sector (source: Ashraf and Galor (2012)).

2 Individual's Problem

Being this an overlapping-generations model, in every time t the size of the working population changes according to its growth rate n_t. Each individual has one parent, and people live for two periods, where at period t they are in the labor force L_t, while at t-1 they are in childhood. So, each working individual has to maximize his utility at each time based on his consumption and on the "cost" of raising one child (τ):

$$\max_{n_t} \left[u_t = (c_t)^\gamma (n_t)^{1-\gamma}\right] \tag{10}$$

subject to the budget constraint:

$$c_t + \tau n_t \leq w_t \tag{11}$$

By substituting c_t inside Equation 10, the maximization gives the following result:

$$n_t = \frac{1-\gamma}{\tau} w_t \tag{12}$$

Finally, the equilibrium growth rate of the working population is the following:

$$n_t = \begin{cases} \frac{1-\gamma}{\tau}\left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha}, & \text{if } A_t^M < \hat{A}_t^M \\ \frac{1-\gamma}{\tau} A_t^M, & \text{if } A_t^M \geq \hat{A}_t^M \end{cases} \tag{13}$$

3 Dynamics of Macroeconomic Variables

As stressed out in the Introduction, the process of development is determined by several cultural factors that impact the growth of human capital (knowledge). Then, the dynamics of knowledge themselves impact the growth of productivity in each of the two sectors (A_t^M and A_t^R) and of the population size (L_t).
Finally, these dynamics all together determine the equilibria in both the Malthusian and Industrial Economies, as much as in the transition process between them.

3.1 Knowledge

Knowledge is only created by the interaction of conformists and non-conformists. This assumption is viable since it is logical to assume that only stimuli coming from the interaction between two different ideologies can create new knowledge.
In particular, it is assumed that in this economy people work in pairs. Thus, the amount of knowledge created in period t is equal to the probability that two heterogeneous people work together $2\omega(1-\omega)$, where ω is the share of non conformists in the working population L_t), multiplied by the amount of working couples $\frac{L_t}{2}$. It follows that the Δk_t is:

$$\Delta k_t = \frac{L_t}{2} 2\omega(1-\omega) = L_t^\lambda \omega(1-\omega) \tag{14}$$

where $\lambda \in (0, 1)$ defines that only part of the heterogeneous couples is finally capable of producing valuable new knowledge.

3.2 Productivities of R and M

The growth of A_t^M and A_t^R is directly caused by the dynamics of knowledge.
In particular, in both sectors productivities are enhanced by the creation of new knowledge Δk_t, but at the same time the non-conformist fraction of the working society (ω) diminishes technological advancement because is not able to transmit it to the new generation (due to

the effect of their lack of cultural assimilation). The productivities in the two sectors follow the following dynamics:

$$A_{t+1}^{M} = (1-\omega)A_t^M + \omega(1-\omega)L_t^{\lambda}A_t^M = \tilde{A}_t^M \tag{15}$$

$$A_{t+1}^{R} = (1-\omega)A_t^R + \omega(1-\omega)L_t^{\lambda}(A_t^R)^{\beta} = \tilde{A}_t^R \tag{16}$$

where $\beta > 0$ and $\lambda + \beta < 1$, making sure that there is more complementarity to productivity in the industrial sector compared to the rural.

3.3 Working Population

From equation 13 comes the dynamics of the population size of the economy:

$$L_{t+1} = n_t L_t = \begin{cases} \frac{1-\gamma}{\tau}\left(\frac{A_t^R}{L_t^R}\right)^{1-\alpha} L_t^{\alpha} = \tilde{L}_t^R, & \text{if } L_t < \hat{L}_t \\ \frac{1-\gamma}{\tau} A_t^M L_t = \tilde{L}_t^M, & \text{if } L_t \geq L_t \end{cases} \tag{17}$$

4 Process of Development

In this section all the findings of the previous sections are unified, and the evolution from an initial Malthusian economy to an Industrial Economy is described.
Following the interpretation of Equation 6, it is clear that the existence of the industrial sector depends on the productivity of M being relatively higher than the productivity of R and of the amount of working people at time t. It follows that it is convenient to show the Process of Development in a graph having A_t^R on the x-axis and L_t on the y-axis, so that by conditioning on the different possible levels of A_t^M it is possible to visualize the state of the economy. In order to do so, the following three geometric elements have to be defined:

- **MM (Conditional Malthusian Frontier)**:
  The MM delimits the pairs (A_t^R, L_t) given A_t^M between an economy where M exists ($A_t^M \geq \hat{A}_t^M$), and an economy where it does not ($A_t^M < A_t^M$). It follows that it is the geometric locus in (A_t^R, L_t) where, given A_t^M:

$$L_t = \left(\frac{1}{A_t^M}\right)^{\frac{1}{1-\alpha}} A_t^R$$

$$L_t = \hat{L}_t \tag{18}$$

- **AA (Constant Productivity of R)**:
  The AA delimits the pairs (A_t^R, L_t) where $A_{t+1}^R > A_t^R$ to the pairs where $A_{t+1}^R < A_t^R$. It is there fore the locus (A_t^R, L_t) where:

$$A_{t+1}^R = A_t^R$$

$$L_t = \Big(\frac{1}{1-\omega}\Big)^{\frac{1}{\lambda}} (A_t^R)^{\frac{1-\beta}{\lambda}}$$

$$L_t = L_t^{AA} \tag{19}$$

- **LL (Constant Working Population)**:
The LL delimits the pairs (A_t^R, L_t) where $L_{t+1} > L_t$ to the ones where $L_{t+1} < L_t$. It is therefore the locus (A_t^R, L_t) where:

$$L_{t+1} = L_t$$

$$L_t = \Big(\frac{1-\gamma}{\tau}\Big)^{\frac{1}{1-\alpha}} A_t^R$$

$$L_t = L_t^{LL} \tag{20}$$

4.1 Malthusian Economy vs Industrial Economy

Now the question is: When is the economy in a Malthusian state, and when in an industrial state?
The answer lies in the relationship between the MM and the LL. The reason is that, in a situation of pairs (A_t^R, L_t) lying above the MM, the population grows only if $L_{t+1} - L_t > 0$, which according to Equation 17 is only possible if:

$$\frac{1-\gamma}{\tau} A_t^M L_t - L_t > 0$$

$$A_t^M > \frac{\tau}{1-\gamma} \tag{21}$$

From Equation 20 we know that $L_{t+1} - L_t > 0$ if:

$$L_t < L_t^{LL} \tag{22}$$

So, the economy is in a state of permanent growth if both Equation 21 and Equation 22 are satisfied (i.e the MM is below the LL). On the other hand, if MM is above LL, the economy does not allow L_t to grow, so even if the first pair (A_t^R, L_t) is above MM, the economy would bring it back to the Malthusian Steady State, where LL=AA.

These findings are showed by Figures 2 and 3:

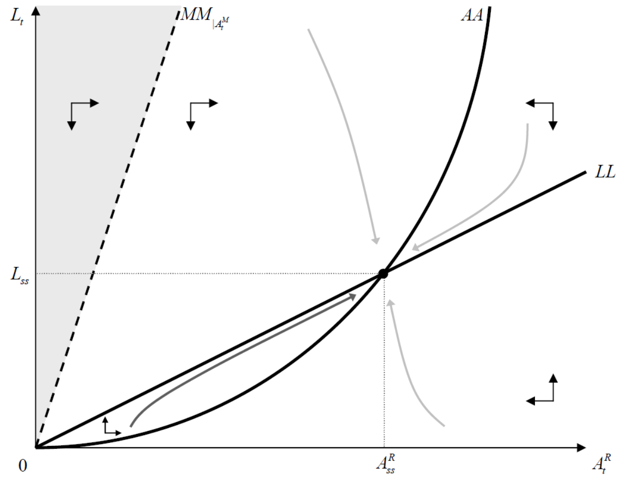


Figure 2: Malthusian Economy (source: Ashraf and Galor (2012)).

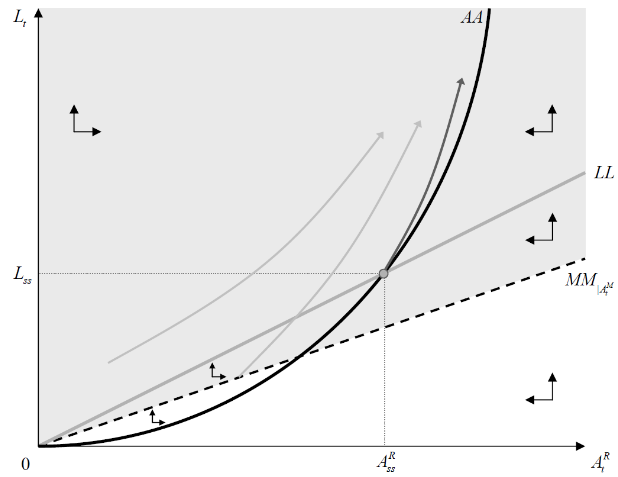


Figure 3: Industrial Economy (source: Ashraf and Galor (2012)).

As inferred before, a Malthusian Economy (Figure 2) gravitates around the SS (A_{SS}^R, L_{SS}). In particular, in the region above LL the population is too large and the wage too low to sustain a growth of population, driving the economy down to SS. The same happens below the LL, where the population is too low compared to wages, so it grows towards SS. This pattern is perfectly in line with the literature commenting the dynamics of Malthusian Economies. The value of L_{SS} is computed as the intersection between the AA and the LL:

$$L_{SS} = \left[\left(\frac{1-\gamma}{\tau} \right)^{\frac{1-\beta}{1-\alpha}} (1-\gamma) \right]^{\frac{1}{1-\lambda-\beta}} \tag{23}$$

On the other hand, in the Industrial Economy showed in Figure 3 it is clear that as specified by Equations 21 and 22, the economy has developed to Industrial, which in turns makes it grow indefinitely.

4.2 Transition from Malthusian to Industrial

As anticipated in the introduction of Section 4, the transition towards an Industrial Economy depends on the level of A_t^M. In particular, as seen in Section 4.1, $A_t^M > \frac{\tau}{1-\gamma}$ determines the Industrial Economy, while $A_t^M < \frac{\tau}{1-\gamma}$ determines a Malthusian Economy. SO, how does a country develop from Malthusian to Industrial?
The answer lies in the growth of A_t^M:

$$g_{t+1} = \frac{A_{t+1}^M - A_t^M}{A_t^M}$$

which from Equation 15:

$$g_{t+1} = \omega[(1-\omega)L_t^\lambda - 1] \tag{24}$$

The growth rate of A_t^M in a Malthusian Economy is then equal to:

$$g_{SS} = \omega[(1-\omega)L_{SS}^{\lambda} - 1]$$

where, plugging L_{SS} from Equation 23, we get:

$$g_{SS} = \omega\left[\left[\left(\frac{1-\gamma}{\tau}\right)^{\frac{\lambda}{1-\alpha}}(1-\omega)\right]^{\frac{1-\beta}{1-\lambda-\beta}} - 1\right] \tag{25}$$

The impact of ω over g_{SS} is showed by Figure 4:

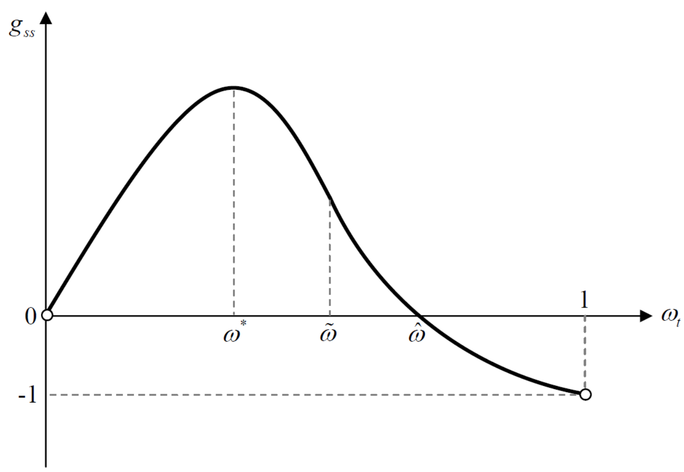


Figure 4: The impact of Cultural Diversity on the Productivity of the Industrial Sector at the Malthusian Steady State (source: Ashraf and Galor (2012)).

It follows that in order to make the economy transit as soon as possible from a Malthusian to an Industrial state, the highest g_{SS} is needed. To do so it is necessary to maximize g_{SS} for ω, and find the optimal ω^*.

5 Conclusion

The primary objective of this paper is to explain, through an overlapping-generation theoretical model, why some countries have been able to develop towards an Industrial Economy before than others. The main driver of this passage has been identified as the heterogeneity of the working population, as it seems logical to assume that only the interaction between people with different points of view can create new ideas, and thus new technologies.
This argument is particularly interesting, since it could be the explanation of the anticipated passage to an Industrial Economy that Europe made with respect to other economies, as the Asiatic ones. Moreover, it can give an explanation to the initial highest technological frontier reached by China during the European Middle Ages. Indeed, since we are assuming that Europe has been able to turn sooner towards Industrialization, we are inferring at the same

time that it has a level of cultural heterogeneity higher than the Chinese one. This in turns implies that being China more culturally homogeneous, accordingly to the theoretical model, it has been able to operate at its technological frontier during the Malthusian epoque. While Europe, due to its cultural fragmentation was not able to do so, but this fragmentation itself helped Europe transiting sooner towards a stable Economic growth.

Lastly, this model is highly interesting for what concerns the theoretical explanation of the passage from an Agricultural to an Industrial Economy, but it is still not able to explain why once Industrialization has been reached some countries are still more advanced than others, and why this pattern does not seem to decelerate. It would be interesting to develop it even more in order to tackle this issue.