COMM 475 – Investment Policy
Introduction, Review, and Preview

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*I am grateful to Rob Heinkel and Bill Tilford for sharing their COMM475 lecture notes with me. These notes are largely based on the material they developed.

Instructor

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Course Description

This course provides tools for the portfolio construction process, beginning with establishing objectives, whether for an individual investor or an institutional investor.

- Demonstrate techniques to set **strategic** allocation of funds across broad asset classes
- Explore **dynamic, tactical** strategies that may be added on to the strategic allocation
- Determine how to **assess** the success of strategies in meeting objectives

Common theme – Factors

- The two most important words for an investor are ***bad times***
- "Factor risks" is the set of bad times that span asset classes
- Focusing on asset classes (bonds/stocks/private equity etc.) is too crude and misses why assets earn returns
- Assets are a bundle of factors
- On average you earn a risk premium for holding factor risk to compensate for the fact that factors do bad in bad times!
- In sum
  1. Asset owners need to know their bad times
  2. Factors carry a premium to reward losses in bad times.
     Factors, not asset classes is what matter!

Evaluation

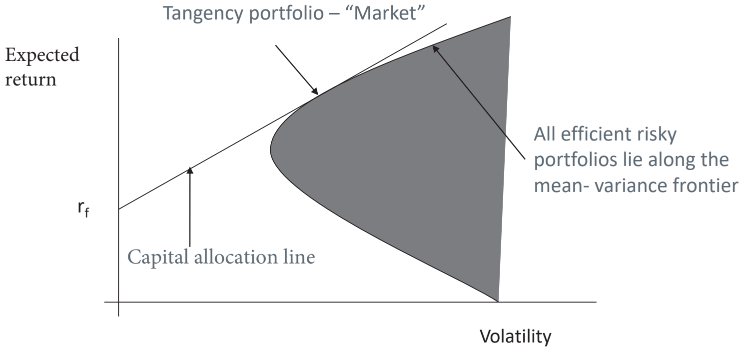
Component	Weight
Assignment #1	10%
Assignment #2	10%
Midterm Exam #1	20%
Midterm Exam #2	20%
Participation	15%
Final Exam	25%
Total	**100%**

- Teams will present course material through the class. Same teams will work together on Assignments as the presentations

Materials

- **Canvas**
  - Syllabus
  - Course Notes (Jupyter notebooks)
  - Lecture notes, Python code, readings (limited use of Excel)
  - Industry materials
- **Jupyter notebooks**
  - Google Colab (cloud)
    https://colab.research.google.com/?utm_source=scs-index
  - Anaconda (local)
    https://www.anaconda.com/download

Review – The Mean-Variance Efficient Frontier



- Harry Markowitz (diversification). We should invest in the efficient frontier, but does not specify where
- Bill Sharpe (CAPM) tells us where in the frontier we should invest

Review – Capital Asset Pricing Model (CAPM)

$$\mathbb{E}[r_s] = r_f + \beta_s(\mathbb{E}[r_m] - r_f) \quad \beta_s = \frac{\mathrm{Cov}\,(r_s, r_m)}{\mathrm{Var}\,(r_m)}$$

- r_s - Return of stock, asset or portfolio s
- r_f - Return of risk-free asset
- r_m - Return of the Market

1. CAPM states that in the mean-variance asset allocation problem with a risk-free asset, all investors hold some combination of the risk-free asset and the tangency portfolio, which must be the market portfolio
2. So the "risk" of any individual asset, and its expected return "reward" depends on its covariance with the market portfolio

Review – Capital Asset Pricing Model (CAPM)

- Tool for consensus expected returns of stocks, assets, investments within companies
- Allows for the separation of returns
  - **Unsystematic**, stock-specific returns
  - **Systematic** returns (market)
- Despite its simplicity and flaws, it is the workhorse in finance for CFOs, investment managers, investment bankers, academics

Review – Revisit the Capital Asset Pricing Model (CAPM)

- The CAPM is a theory about "expected returns", $\mathbb{E}[r_s]$
- Empirically, we state the CAPM model as a "regression" equation

$$r_s = r_f + \beta_s(r_m - r_f) + e_s \tag{1}$$

- Taking the variance of the return equation:

$$\mathrm{Var}\left(r_s\right) = \beta_s^2 \sigma_m^2 + \sigma_{e_s}^2$$

- A measure of the systematic risk of an investment through its **beta** (β_s)
- Unsystematic risk component $\sigma_{e_s}^2$
- Total risk is the sum of the two components
- Unsystematic risk can be diversified away in a well-diversified portfolio of individual stocks

Review – Revisit the Capital Asset Pricing Model (CAPM)

Investors demand a high **expected return** ($\mathbb{E}[r_S]$) for high **systematic risk** (β_S):

$$\mathbb{E}[r_s] = r_f + \beta_s \underbrace{(\mathbb{E}[r_m] - r_f)}_{\equiv \lambda}$$

Where $\lambda = \mathbb{E}[r_m] - r_f$ is the "**market price of risk**"

- Above equation describes the **Security Market Line** in the $(\beta_s, \mathbb{E}[r_s])$-space

Review – From CAPM to Multi-Risk Factor Model

- Start from
$$r_s = r_f + \beta_s(r_m - r_f) + e_s$$
Add and subtract $\beta_s\mathbb{E}[r_m]$ on the right-hand-side:
$$r_s = \underbrace{r_f + \beta_s\left(\mathbb{E}[r_m] - r_f\right)}_{=\mathbb{E}[r_s] \text{ from CAPM}} + \beta_s \underbrace{\left(r_m - \mathbb{E}[r_m]\right)}_{\textbf{Market Risk factor shock}} + e_s$$
- So
$$r_s = \mathbb{E}[r_s] + \beta_s I_m + e_s$$
- For the CAPM, a stock's actual (ex-post) realized return is decomposed into:
  - The **expected** return, $\mathbb{E}[r_s]$
  - Stock's **actual** return due to an unexpected return on the market portfolio
  - Stock's **idiosyncratic** return

Review – From CAPM to Multi-Risk Factor Model

$$r_s = \mathbb{E}[r_s] + \beta_s I_m + e_s$$

- Unexpected return (surprise) comes from
  - unexpected return on the whole market $I_m \neq 0$
  - unexpected firm-specific news, $e_s \neq 0$.
- Do we believe that the **only** systematic risk factor is the unexpected return (shock) to the whole market portfolio (β_s)?
- Macroeconomic activity? GDP? Interest rates?
- A **Multi-Risk Factor (MRF) model** allows many risk factors not just the market, as in the CAPM.
- The origin of MRF are in Ross (1976)'s Arbitrage Pricing Theory (APT).

Preview – Investment Policy

- Pensions, DB (defined benefit)
- Sovereign wealth funds (e.g. Norway)
- Success of investment policy determines peoples' retirement standard of living
- Size of portfolios, billions
- Do professional, "active" managers beat a well-diversified index?

Preview – Portfolio Management Process

- Client Objectives (preferences)
- Asset Class Choices (factors exposure)
- Strategic Asset Allocation (SAA)
- Portfolio Construction
- Risk Management
- Performance Objectives and Measurement

Institutional Investment

- Institutional Investment is important. Pension earnings determine retirement income!
- Example: Assume a 35-year old will save $5,000 per year for 30 years, then buy a 15-year annuity (they die at 80)

Age	35	36	37	...	64	65	66	67	...	79	80
Cash Flow	$+c$	$+c$	$+c$	...	$+c$	$+c$	$-w$	$-w$		$-w$	$-w$

- Where $c = \$5,000$ and w depends upon our pension earnings from age 36 to 65

Institutional Investment

- **Case A**: suppose the pension earns 3% per year
  - $FV = 5,000 \times \frac{1.03^{30}-1}{0.03} = \$237,877$ is the accumulated account at age 65
- Invest the lump sum and draw down an annuity w for 15 years (from age 66 to 80)
- Assuming a 5% return in retirement, this yields
  - $\$237,877 = w \times \frac{1-1.05^{-15}}{0.05} \implies$ **\$22,918** per year in retirement

Institutional Investment

- **Case B:** suppose the pension earns 10% per year
  - $F = 5,000 \times \frac{1.10^{30} - 1}{0.10} = \$822,470$ is the accumulated account at age 65
- Invest the lump sum and draw down an annuity *w* for 15 years (from age 66 to 80)
- Assuming a 5% return in retirement, this yields
  - $\$822,470 = w \times \frac{1 - 1.05^{-15}}{0.05} \implies$ **\$79,239** per year in retirement
- Earning 10% vs 3% allows us to triple the retirement annuity
- Institutional Investment matters!

COMM 475 – Investment Policy
Review and Preview

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Institutional Investment

Table: Largest Pension **Plans** in Canada

Pension	**Assets**	**As of**
Canada Pension Plan Investment Board	$576 B	Sept 30, 2023
Caisse de Dépôt et Placement du Québec (CDPQ)	$424 B	June 30, 2023
Ontario Teachers' Pension Plan	$247 B	Dec 31, 2022
British Columbia Investments (BCI)	$215 B	March 31, 2023
Public Service Pension Plan	$170 B	March 31, 2022

Source: Various pension plan annual reports

Institutional Investment

Table: Largest Pension Investment **Managers** in Canada (as of December 31, 2020)

Firm	**Assets**
TD Global Investment Solutions	$137.6 B
BlackRock Asset Management Canada	$133.5 B
Brookfield Asset Management	$78.5 B
PH&N Institutional	$66.3 B
Manulife Investment Management	$46.1 B

Source: Benefits Canada Sruvey, 2023

Active versus Passive Canadian Equity Investing

Table: **Average** Canadian Equity Returns (1990 Q1 to 2022 Q2)

Percentile	**Return per Quarter**	**Return per Year**
25th Percentile	3.93%	15.72%
Median manager	**2.43%**	**9.72%**
75th Percentile	0.96%	3.84%
TSX average return	**2.17%**	**8.68%**

Source: RBC Investor & Treasury Services

- TSX index (2.17%) on average 3rd quartile performer
- Median Manager added 26 basis points quarterly ($2.43\% - 2.17\% = 0.26\%$), or about 1.00% per year
- Who is the median manager?

Active versus Passive Canadian Equity Investing

Table: **Average** Canadian Equity Returns vs. TSX index (1990 Q1 to 2022 Q2)

TSX Index performance	**Percent of time**
In the 1st Quartile	8%
In the 2nd Quartile	42%
In the 3rd Quartile	29%
In the 4th Quartile	21%
	100%

Source: RBC Investor & Treasury Services

- TSX Index above median about half the time and below median about half
- It is hard to beat the market!

Active versus Passive Canadian Equity Investing

Table: Canadian Equity Returns when **Negative** TSX Index Return (1990 Q1 to 2022 Q2)

Quartile	**Return per Quarter**
1^{st} Quartile	-3.73%
Median manager	-5.50%
3^{rd} Quartile	-7.03%
TSX Index	-6.27%

- Median active managers outperformed by 77 basis points per quarter ($6.27\% - 5.50\% = 0.77\%$), or about 3.1% per year
- Downside protection can be powerful given the compounding of returns
- Who is the median manager?

Active versus Passive Canadian Equity Investing

Table: Canadian Equity Returns vs. **Negative** TSX Index Return (1990 Q1 to 2022 Q2)

TSX Index performance	Percent of time
In the 1st Quartile	7%
In the 2nd Quartile	31%
In the 3rd Quartile	31%
In the 4th Quartile	31%
	100%

Source: RBC Investor & Treasury Services

- TSX Index was below median 62% of the time
- Some evidence of active management providing "downside protection"
- Who is providing downside protection?

Strategic vs. Tactical Asset Allocation

- **Strategic Asset Allocation (SAA). Long-run** weights in asset classes for an investor (also known as “policy weights”)

- **Tactical Asset Allocation (TAA). Short-run** deviation from policy weights to take advantage of market inefficiencies (also known as “market timing”). Very hard to do!
  - Market inefficiency
  - Time varying risk premium

Does Tactical Asset Allocation (TAA) Add Value?

- Stock market performance will at times grow faster or grow slower than its long-run average growth rate. Is this behaviour a market inefficiency or a time-vary equity risk premium (ERP)? Both!
  - **Market inefficiency:** stock market "over-reactions", too high, too low
    - Too high / too low, reverting back to its long-run mean after overshoot
    - "smart" investor wants to buy low and sell high against the long run average, other investors are too busy over / under reacting (behavioural finance)
    - If **all** investors buy low and sell high, we get market efficiency

Does Tactical Asset Allocation (TAA) Add Value?

- Stock market performance will at times grow faster or grow slower than its long-run average growth rate. Is this behaviour a market inefficiency or a time-vary equity risk premium (ERP)? Both!
- **Time-varying equity risk premium in an efficient market:**

$$\mathbb{E}[r_s] = r_f + \beta_s \lambda$$

- Lambda (λ) vary over time? Yes! (Recall: $\lambda = \mathbb{E}[r_m] - r_f$)
  - Low market levels, investors are poor and are more risk averse
    - They demand higher than normal expected return (“buy the dip”)
  - High market levels, investors are wealthy and less risk averse
    - They demand lower than normal expected return (“sell at the top”)
- TAA can is a contrarian strategy (hard to do!)

Does Tactical Asset Allocation (TAA) Add Value?

- Return serial correlations across size deciles (Robert Haugen's study)

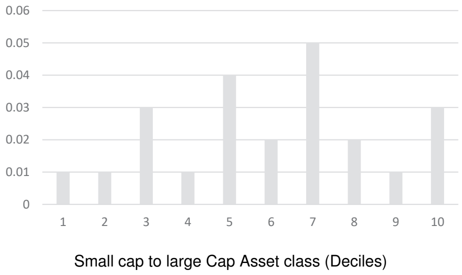

Figure: Serial correlation of **1-year** returns

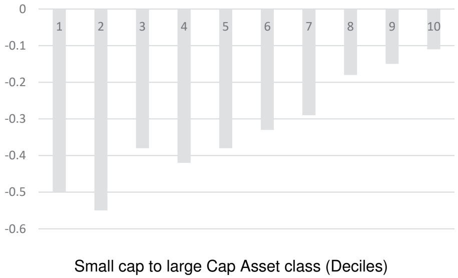

Figure: Serial correlation of **4-year** returns

- Large **negative** correlations ("mean reversion"), especially in small cap, e.g., Russell 2000 (market inefficiency?)

Client Objectives

- **Individual Investor Goal:** Maximize the amount of retirement savings available at retirement (a specific date), subject to some limit on the range of possible outcomes (Risk Management)
  - Example: A dynamic SAA: Life-cycle investing
  - **Institutional Investor Goal:** Meet the obligations of the pension plan (a very long-maturity stream of future payments) with the minimum plan sponsor contributions and a limit on the riskiness of those contributions - there is no specific date

Asset Class choices

- Define a broad range of investment vehicles
  - Criteria:
    - Internal homogeneity
    - Low correlation with other classes
    - Different economic drives
- Examples:
  - Canadian equities, International equities
  - Canadian bonds, Global bonds
  - Real return assets like Real Estate, Real return bonds
  - Private Equity, Private Debt
  - Infrastructure

Strategic Asset Allocation (SAA)

- The choice of a long-term asset allocation over allowed asset classes
  - Can be constructed from:
    - Mean-Variance optimization (and other forms of optimization)
    - Asset-Liability model
- Classic example "60/40", 60% stocks / 40% bonds

Portfolio Construction

- Passive versus Active Investing
  - Passive:
    - Strategic Asset Allocation (SAA)
  - Active:
    - Tactical Asset Allocation (TAA)
    - Insured Asset Allocation (IAA)
    - Security Selection
- Balanced versus specialist structure (if using an active style)
- Style diversification or not
- Examples: passive, active (long only), active (130/30) and passive plus market-neutral

Risk Management

- Limits on Divergence from "policy", examples:
  - Limits on asset allocation deviation from SAA mix
  - Limits on sectoral allocations in bond and equity portfolios
  - Limits on amount of diversification in any asset class
  - Direct limits on "tracking error" relative to a policy benchmark

Performance Measurement

- Is the client's objective met satisfactorily?
- Did the manger add value in ways that she was supposed to?
- Performance attribution tools

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Portfolio Construction

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Outline

- Portfolio Construction - Mean Variance
- Portfolio Construction - Risk Parity
- Portfolio Construction - Multi-Risk Factor Model (MRF)

Portfolio Construction Techniques

- Mean-Variance Analysis (MVA)
- Risk Parity Portfolio Construction (RP)
- Multi-Risk Factor Model (MRF)
  - “Factor investing”

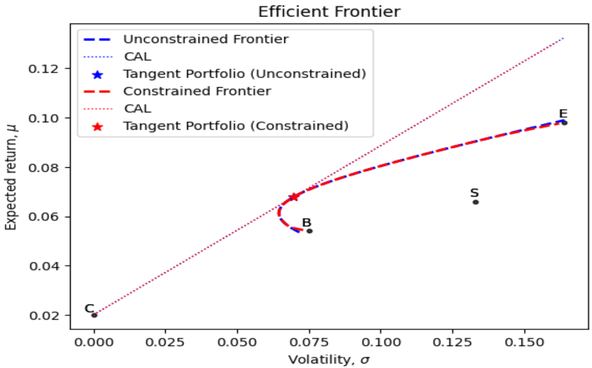
Portfolio Construction Mean-Variance

i	**Expected return**, $\mathbb{E}[r_i]$	**Volatility**, $\sigma[r_i]$
C	2.0%	0.0%
B	5.4%	7.5%
S	6.6%	13.3%
E	9.8%	16.4%

Correlations, $\rho_{i,j}$				
	C	B	S	E
C	1.00	0.00	0.00	0.00
B		1.00	0.08	0.06
S			1.00	0.14
E				1.00

- C = cash, B = bonds, S = public equity, E = private equity
- 75%-85% of total performance comes from Asset Mix
- Please see Jupyter notebook: “`LN_02_Portfolio_Construction.ipynb`” for details on Mean-Variance analysis

Mean-Variance frontier



- Unconstrained Tangency portfolio: B: 55.01%, S: 19.00%, E: 25.99%
- Constrained Tangency portfolio: B: 55.01%, S: 19.00%, E: 25.99%

Portfolio with a given target return

We can achieve a 8% return in three ways:

Portfolio	C	B	S	E
Unconstrained risky-only	0.00%	25.31%	21.04%	53.65%
Constrained risky-only	0.00%	25.31%	21.04%	53.65%
Cash + risky assets	−25.74%	69.16%	23.89%	32.69%

- Risky-only portfolios contain a lot of “macro” economic risk (S and E) relative to Cash+risky portfolio

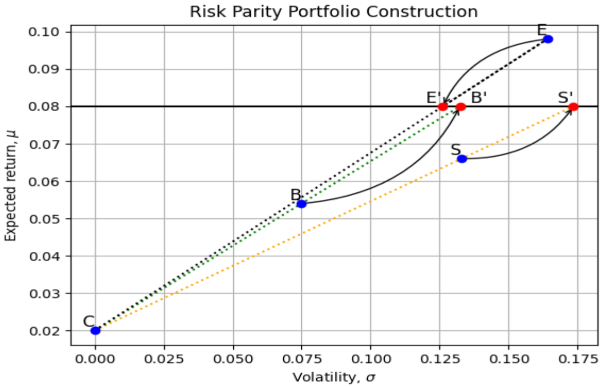
Risk Parity portfolio

- Special case of mean factor investing
- The Risk Parity (RP) approach to portfolio construction seeks to diversify the **sources of risk.**
- The key idea is that different asset classes might be capturing different sources of "factor risks" such as **economic risk** or **inflation risk.**
- **Implementation**: choose asset weights proportional to the **inverse** of variance of volatility, i.e., overweight less volatile assets.
- Originally advocated by Qian (*Journal of Investment Management*, 2006), where each asset class is held in weights to produce equal volatilities.
- Popularized by Ray Dalio's' hedge fund *Bridgwater Associates* in their “All Weather Fund”.

Portfolio Construction Risk Parity (RP)

- **Implementation**: use cash to lever up or down risk in each asset class so that all have the same expected (target) return or the same target volatility.
- Choose how much to allocate to each adjusted asset class
  - Doing so means choosing how to allocate your "**risk budget**" across assets
- **Example.** Suppose Bonds have expected return of 5.4% and cash has return of 2%. We want to target a return of 8%.
  - Find x: $x \times 5.4\% + (1 - x) \times 2\% = 8\% \implies x = 1.7647 \approx 1.76$
  - That is, we need to "lever up" bonds to achieve a 8% expected return
  - Each dollar we invest in the "adjusted bond" class (B') is equivalent to $\$1 \times 1.7647 = \$1.7647$ in the original bond B and a short position -0.7647 in cash (T-bill)

Portfolio Construction Risk Parity (RP)



Please see Jupyter notebook: "`LN_02_Portfolio_Construction.ipynb`" for details on Risk-Parity portfolios

Risk Parity Portfolio Formation

Desired Expected Return (Target) is 8%

Original Asset Class	$\mathbb{E}[r]$	$\sigma[r]$	r_f	Risky weight	C weight	Total
B	5.40%	7.50%	2%	1.76	-0.76	1
S	6.60%	13.30%	2%	1.30	-0.30	1
E	9.80%	16.40%	2%	0.77	0.23	1

Risk-adjusted Asset Class	$\mathbb{E}[r]$	$\sigma[r]$
B'	8.00%	13.23%
S'	8.00%	17.34%
E'	8.00%	12.62%

Deciding a “risk budget”

- S' has more volatility than B'
- S' and E' have different sources or risk (Economic risk) than B' (Inflation risk)
- Risk budget: Downplay S' (and E') and overstate B'
- E.g., given a $6b fund, chose $ in **Risk Adjusted** Asset Classes as follows

Asset Class	**$ Invested (billions)**
B’	**$4.00**
S’	**$1.00**
E’	**$1.00**
Total	**$6.00**

- \$4 in $B' \implies \$4 \times 1.7647 \times \approx \$7.06$ in B and $\$4 \times (-0.7647) \approx -\$3.06$ in C
- \$1 in $S' \implies \ldots$ See calculations in `LN_02_Portfolio_Construction.ipynb`

Risk Parity Portfolio

This implies the following investment in the **Original** Asset Classes

	\$ risky	\$ t-bill	\$ risk-adjusted	% risky	% t-bill	Total %
B	7.058824	-3.058824	4.000000	**1.176471**	-0.509804	0.666667
S	1.304348	-0.304348	1.000000	**0.217391**	-0.050725	0.166667
E	0.769231	0.230769	1.000000	**0.128205**	0.038462	0.166667
Total	9.132402	-3.132402	6.000000	1.522067	**-0.522067**	1.000000

- The **risk-parity portfolio weights** in the actual asset classes B, S, E, C associated with a 4/1/1 allocation to the risk-adjusted asset classes B', S', E' are:

B:	117.65%
S:	21.73%
E:	12.82%
C:	-52.20%

Portfolio Construction Risk Parity

- Risk Parity allows us to manage exposures to broader "Risk Factors"
- Example Risk Factors
  - Economic Risk (GDP - % change over time)
    - S and E more sensitive to Economic shocks
  - Inflation Risk (CPI - % change over time)
    - B is more sensitive to Inflation shocks
  - Warning!
    - Risk parity overweights assets that have low volatilities
    - Past volatility tend to be low when today's prices are high
    - Hence with risk parity your portfolio loads on expensive assets!

From CAPM to Multi-Risk Factor (MRF) Model

- Recall that in a CAPM world a stock's actual (ex-post) realized return is:

$$r_s = \mathbb{E}[r_s] + \beta_s I_m + e_s$$

where

  - $\mathbb{E}[r_s] = r_f + \beta_s(\mathbb{E}[r_m] - r_f)$, **expected** return from CAPM model
  - $I_m \equiv r_m - \mathbb{E}[r_m]$ is market risk factor **shock** (What is the mean of I_m?)
  - e_s is the stock's **idiosyncratic** return

- The expected return $\mathbb{E}[r_s]$ from the CAPM is concisely written as

$$E[r_s] = r_f + \beta_s \underbrace{(\mathbb{E}[r_m] - r_f)}_{\equiv\lambda} = r_f + \beta_s\lambda$$

where $\lambda \equiv E[r_m] - r_f$ is referred to as the **Market Risk Premium**

Multi-Risk Factor (MRF) Model

- Expand our risk factors to include risks beyond the market. Example:
  - MACR - "economic risk", percent changes in GDP
    - $I_1 =$ **surprise** change in GDP
  - INFL - percent changes in CPI
    - $I_2 =$ **surprise** change in inflation (change in CPI)
  - ILLQ - "illiquidity risk", example is changes in bid-ask spreads
    - Harder to trade . . . Risk of not being able to sell, or forced to sell LOW
    - $I_3 =$ **surprise** change in bid-ask spreads

Multi-Risk Factor (MRF) Model

- MRF model "return generating function" is then

$$r_s = \mathbb{E}(r_s) + \beta_{1s} I_1 + \beta_{2s} I_2 + \beta_{3s} I_3 + e_s$$

where, similar to the CAPM

$$\mathbb{E}[r_s] = r_f + \underbrace{\beta_{1s}\lambda_1 + \beta_{2s}\lambda_2 + \beta_{3s}\lambda_3}_{\text{risk premium}}$$

- Note:
  - β_{1s}, β_{2s}, β_{3s} are the **exposures** to the risk factors ("risk quantities")
  - λ_1, λ_2, λ_3 are the **compensation** per unit of risk esposure ("risk prices")
  - $\beta_{1s}\lambda_1 + \beta_{2s}\lambda_2 + \beta_{3s}\lambda_3$ is the **risk premium** above the riskless return

Multi-Risk Factor (MRF) Model

- What **data** do we need to implement the models?
  - **Ex-Ante**: $\mathbb{E}[r_s] = r_f + \beta_{1s}\lambda_1 + \beta_{2s}\lambda_2 + \beta_{3s}\lambda_3$
    - Market data: $r_f, \lambda_1, \lambda_2, \lambda_3, \sigma_1, \sigma_2, \sigma_3$
    - Asset data: $\beta_{1s}, \beta_{2s}, \beta_{3s}$
    - Asset j volatility (assuming factors are uncorrelated):

$$\sigma[r_j] = \sqrt{\beta_{1,j}^2\sigma_1^2 + \beta_{2,j}^2\sigma_2^2 + \beta_{3,j}^2\sigma_3^2}$$

    - Asset j Sharpe Ratio

$$\text{Sharpe ratio} = \frac{E[r_j] - r_f}{\sigma[r_j]}$$

  - **Ex-Post**: $r_s = \mathbb{E}(r_s) + \beta_{1s}I_1 + \beta_{2s}I_2 + \beta_{3s}I_3 + e_s$
    - Shocks: I_1, I_2, I_3

Multi-Risk Factor Model

	Betas of asset class j, β_{ij}						
Risk Factor, i	**Cash, C**	**Bonds, B**	**Stocks, S**	**PE, E**	λ_i	σ_i	r_f
$i = 1$ MACR	0	0.3	0.9	1.1	0.10	0.14	0.02
$i = 2$, INFL	0	-0.3	0.4	0.6	-0.08	0.05	0.02
$i = 3$, ILLQ	0	0.5	0.3	-0.4	-0.04	0.12	0.02

Please see Jupyter notebook: "`LN_02_Portfolio_Construction.ipynb`" for details on the Multi Risk Factor model.

Multi-Risk Factor Model

- Ex-ante:

$$\mathbb{E}[r_B] = r_f + \beta_{1B}\lambda_1 + \beta_{2B}\lambda_2 + \beta_{3B}\lambda_3$$
$$= 0.02 + 0.3 \times 0.1 + (-0.3) \times (-0.08) + 0.5 \times (-0.04) = 5.40\%$$

- Ex-post:

$$r_B = \mathbb{E}[r_B] + \beta_{1B} I_1 + \beta_{2B} I_2 + \beta_{3B} I_3 + e_B$$

- Suppose Inflation experience a positive shock: $I_2 = 5\%$. Then, because our exposure to inflation is $\beta_{2B} = -0.3$ and the price of inflation risk is negative $\lambda_2 = -0.08$
  - Our ex-post return from holding bonds is **lower** by $-0.3 \times (0.05) = -1.5\%$

Factor exposure

- Holding a portfolio of assets is like holding a portfolio of factor exposures!
- Given our portfolio 25% B, 21% S, 54% E, and 0% C. What is the portfolio risk exposure (portfolio beta) to the MACR, INFL, and ILLIQ factors?
- **Portfolio beta** is a "**portfolio OF betas**"!
- E.g., for the MACR factor, $i = 1$, the exposure is

$$\begin{aligned}\text{Exposure to MACR} &= x_B \times \beta_{1B} + x_S \times \beta_{1S} + x_E \times \beta_{1E} + x_C \times \beta_{1C} \\ &= 0.25 \times 0.3 + 0.21 \times 0.9 + 0.54 \times 1.1 + 0 \times 0 = 0.8580\end{aligned}$$

- Hence holding a (25% B, 21% S, 54% E, and 0% C) portfolio is equivalent of **holding 0.8580 units of MACR risk.**
- From this we expect to harvest a risk premium of $0.8580 \times \lambda_1 = 0.08580$ or 8.50%.
- Please see Jupyter notebook: "`LN_02_Portfolio_Construction.ipynb`" for further details.

Asset Liability Model (Preview)

- We get to chose the characteristics of the Assets
- We **DON'T** get to chose the characteristics of the Liabilities
- Think about Liability Betas
  - CPPIB
  - Can we get Asset Betas and Liabilities the same? Similar?
- Asset−Liabilities = our funding surplus or deficit

COMM 475 – Investment Policy
Multi-Risk Factor Model

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Outline

- Multi-Risk Factor Model (MRF)
- Interpreting the signs of the risk prices. $\lambda_i < 0$?!?
- Total Wealth, Change and Volatility of Wealth

Multi-Risk Factor Model

Ex-Post equation

$$r_s = \mathbb{E}[r_s] + \beta_{1s} I_1 + \beta_{2s} I_2 + e_s, \quad \text{where} \quad I_i = (r_i - \mathbb{E}[r_i]), \quad i = 1, 2$$

- I_1 and I_2 are **systematic shocks**, "Actual$-$Expected"
  - E.g., Industrial Product growth: Expected 3% but got 1% so our shock is 2%
  - These are consider systematic risks (like the market) because they will effect **all** kinds of assets
- e_s is an asset specific, or idiosyncratic shock

Multi-Risk Factor Model

Ex-ante: $\mathbb{E}[r_s] = r_f + \beta_{1s}\lambda_1 + \beta_{2s}\lambda_2$

Ex-post: $r_s = \mathbb{E}[r_s] + \beta_{1s}I_1 + \beta_{2s}I_2 + e_s$

$Var[r_s]$: $\sigma_s^2 = \beta_{1s}^2\sigma_1^2 + \beta_{2s}^2\sigma_2^2 + \sigma_{e_s}^2$ [if I_1 and I_2 are ucorrelated shocks]

	Model Decomposition		
	Stock specific	**Capital markets**	**Portfolio choice (n stocks)**
Ex-Ante	β_{is}	$r_f, \lambda_i, \sigma_i^2$	$\{x_s, \beta_{is}\} \Rightarrow \beta_{ip} = \sum_{s=1}^n x_{is}$
Ex-Post	e_s	I_i	$\{x_s, r_s\} \Rightarrow r_p = \sum_{s=1}^n x_s r_s$

- β_{ip} is the portfolio **exposure (beta)** to factor i

Multi-Risk Factor Model

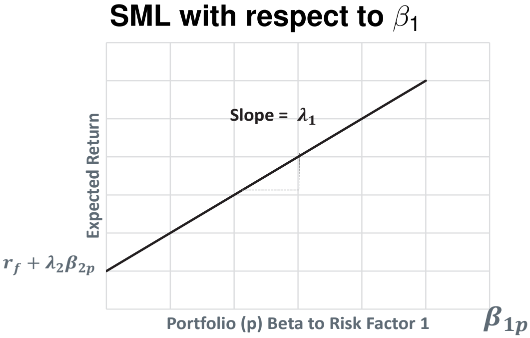
- What do Investment Managers do?
  - Target Prices, Value, Growth, etc
  - They form their "**own**" $\mathbb{E}[r_s]$ and can compare to our "**fair**" $\mathbb{E}[r_s]$ defined above
  - "**Alpha**" is created by looking at the differences for a given level of risk
    - Long/overweight if Manager $\mathbb{E}[r_s] > $ fair $\mathbb{E}[r_s]$
    - Short/underweight if Manager $\mathbb{E}[r_s] < $ fair $\mathbb{E}[r_s]$
- Fair $\mathbb{E}[r_s]$ acts as an anchor. See CPPIB "Total Portfolio Approach".

Multi-Risk Factor Model

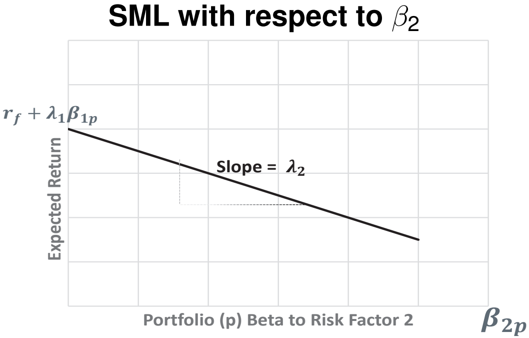
- Ex-ante model
$$\mathbb{E}[r_s] = r_f + \beta_{1s}\lambda_1 + \beta_{2s}\lambda_2$$
with $r_f = 2\%$, $\lambda_1 = 10\%$, $\lambda_2 = -6\%$
- *Example 1.* Exposure **only to factor 2**: $\beta_{1s} = 0$, $\beta_{2s} = 0.1$
  - $\mathbb{E}[r_s] = 2\% + 0 \times 10\% + 0.1 \times (-6\%) = \mathbf{1.4\%}$
- *Example 2.* **Increase exposure to factor 1**: $\beta_{1s} = 1$, $\beta_{2s} = 0.1$
  - $\mathbb{E}[r_s] = 2\% + 1 \times 10\% + 0.1 \times (-6\%) = \mathbf{11.4\%}$
- *Example 3.* **Reduce exposure to factor 1 and 2**: $\beta_{1s} = 0.8$, $\beta_{2s} = 0$
  - $\mathbb{E}[r_s] = 2\% + 0.8 \times 10\% + 0 \times (-6\%) = \mathbf{10.0\%}$
- *Example 4.* **Increase exposure to factor 2**: $\beta_{1s} = 0.8$, $\beta_{2s} = 1$
  - $\mathbb{E}[r_s] = 2\% + 0.8 \times 10\% + 1 \times (-6\%) = \mathbf{4.0\%}$

Multi-Risk Factor Model

- Portfolio ex-ante return: $\mathbb{E}[r_p] = r_f + \beta_{1p}\lambda_1 + \beta_{2p}\lambda_2$



Risk-enanching factor



Insurance factor

Multi-Risk Factor Model

- $\mathbb{E}[r_p] = r_f + \underbrace{\beta_{1p}\lambda_1 + \beta_{2p}\lambda_2}_{\text{risk premium}}$
- What does $\lambda_1 > 0$ mean? Factor 1 is a "**risk-enhancing factor**"
  - You want to be compensated (through **high** expected return) for exposing yourself to risk factor shocks I_1.
  - This is because the risk factor *i* **loses money** when times are **bad** for you, e.g., when you lose our job.
- What does $\lambda_2 < 0$ mean? Factor 2 is an "**insurance factor**"
  - You are willing to pay (through **low** expected return) for exposing yourself to risk factor shocks I_2.
  - This is because the risk factor *i* **makes money** when times are **bad** for you, e.g., when you lose your job.

Total Wealth

- Total wealth of an investor is the sum of **financial** (your portfolio) and **non-financial** wealth (your human capital, i.e., PV of future wage income).

	Wealth	
	$t = 0$	$t = 1$
Financial Portfolio	P_0	$P_1 = P_0(1 + r_p)$
Human Capital	H_0	H_1
Total Wealth	$W_0 = P_0 + H_0$	$W_1 = P_0(1 + r_p) + H_1$

- Worry about the risk or volatility of W_1, **change** in wealth ΔW
- Not much control on the volatility of H_1 but we can control volatility of P_1

$$\Delta W = W_1 - W_0 = \Delta H + P_0 r_p$$

Total Wealth

- Change in total wealth:

$$\begin{aligned} \Delta W &= \Delta H + P_0 r_p \\ r_p &= \mathbb{E}[r_p] + \beta_{1p} I_1 + \beta_{2p} I_2, \quad [e_p \approx 0 \text{ in a well diversified portfolio}] \\ \implies \Delta W &= \Delta H + P_0 [E(r_p) + \beta_{1p} I_1 + \beta_{2p} I_2] \end{aligned}$$

- **"Bad times"** is when my human capital suffers a negative shock, $\Delta H < 0$
- If $cov(I_1, \Delta H) > 0$, factor 1 is **risky**: it loses money in bad times.
  - $\lambda_1 > 0$: I want to **be paid** (high expected return) for holding Factor 1 risk
  - Example: Factor 1 can be industrial production or GDP (can you think of why?)
- If $cov(I_2, \Delta H) < 0$, factor 2 is a **hedge**: it makes money in bad times.
  - $\lambda_2 < 0$: I am willing to **pay** (accept low expected return) to holding Factor 2 risk
  - Example: Factor 2 can be interest rates (can you think of why?)

Total Wealth – Summary

- Change in total wealth

$$\Delta W = \Delta H + P_0[\mathbb{E}(r_p) + \beta_{1p} I_1 + \beta_{2p} I_2], \quad \text{where} \quad \mathbb{E}[r_p] = r_f + \lambda_1 \beta_{1p} + \lambda_2 \beta_{2p}$$

- $COV(I_1, \Delta H) > 0$: **Risk-Enhancing Factor** $\implies \lambda_1 > 0$
  - $\beta_{1p} > 0 \implies$ **More volatile** ΔW
  - $\beta_{1p} < 0 \implies$ **Less volatile** ΔW
- $COV(I_2, \Delta H) < 0$: **Insurance Factor** $\implies \lambda_2 < 0$
  - $\beta_{2p} > 0 \implies$ **Less volatile** ΔW
  - $\beta_{2p} < 0 \implies$ **More volatile** ΔW

Total Wealth - Example

- Change in total wealth

$$\Delta W = \Delta H + P_0[\mathbb{E}(r_p) + \beta_{1p} I_1 + \beta_{2p} I_2], \text{ where } \mathbb{E}[r_p] = r_f + \lambda_1 \beta_{1p} + \lambda_2 \beta_{2p}$$

- Factors: $I_1 = \Delta GDP$, $I_2 = \Delta$Expected Inflation
- Risk prices $\lambda_1 = 0.10$, $\lambda_2 = -0.04$. Risk-free rate: $r_f = 0.05$.
- Two stocks:
  - Stock A: High risk. $\beta_{1A} = 0.8$, $\beta_{2A} = -0.5$
  - Stock B: Low risk. $\beta_{1B} = -0.2$, $\beta_{2B} = 0.2$
- Ex-ante expected return
  - $\mathbb{E}[r_A] = 0.05 + 0.8 \times 0.10 + (-0.5) \times (-0.04) = 15\%$
  - $\mathbb{E}[r_B] = 0.05 + (-0.2) \times 0.10 + 0.2 \times (-0.04) = 2.2\%$

Total Wealth - Example

- Factors: $I_1 = \Delta GDP$, $I_2 = \Delta$Interest Rates
- Risk prices $\lambda_1 = 0.10$, $\lambda_2 = -0.04$. Risk-free rate: $r_f = 0.01$.
- Two stocks:
  - Stock X: Utility, $\beta_{1X} = 0.1$, $\beta_{2X} = -0.2$
  - Stock B: Consumer Discretionary. $\beta_{1Y} = 0.9$, $\beta_{2Y} = 0.2$
- Ex-ante expected return
  - $\mathbb{E}[r_X] = 0.01 + 0.1 \times 0.10 + (-0.2) \times (-0.04) = 2.8\%$
  - $\mathbb{E}[r_Y] = 0.01 + 0.9 \times 0.10 + 0.2 \times (-0.04) = 9.2\%$
- Ex-post: $I_1 = -0.1$, $I_2 = -0.02$
  - $r_X = \mathbb{E}[r_X] + 0.1 \times (-0.1) + (-0.2) \times (-0.02) = 2.2\%$
  - $r_Y = \mathbb{E}[r_Y] + 0.9 \times (-0.1) + 0.2 \times (-0.02) = -0.2\%$
- Ex-post the utility X had a more attractive return

Total Wealth - Portfolio variance and volatility

- Recall

$$r_p = \mathbb{E}[r_p] + \beta_{1p} I_1 + \beta_{2p} I_2$$

where β_{ip}, $i = 1, 2$ are the portfolio betas with respect to factor $i = 1, 2$.

- Hence, because shocks I_1 and I_2 are independent, portfolio variance is

$$\sigma_p^2 = \beta_{1p}^2 \sigma_1^2 + \beta_{2p}^2 \sigma_2^2, \quad \text{where} \qquad \sigma_i^2 = Var(I_i), \quad i = 1, 2$$

- Portfolio volatility is

$$\sigma_p = \sqrt{\sigma_p^2}$$

- Portfolio betas β_{ip} depends on the weights in the different asset classes (e.g., CPPIB holds little Canadian Equity, why?)

COMM 475 – Investment Policy
The Chen, Roll, and Ross Model

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Outline

- Multi-Risk Factor Model (MRF)
- Chen, Roll, and Ross (1986) Model

Multi-Risk Factor Model & Total Wealth

- Change in total wealth

$$\Delta W = \Delta H + P_0[\mathbb{E}(r_p) + \beta_{1p} I_1 + \beta_{2p} I_2], \quad \text{where} \quad \mathbb{E}[r_p] = r_f + \lambda_1 \beta_{1p} + \lambda_2 \beta_{2p}$$

- $COV(I_1, \Delta H) > 0$: **Risk-Enhancing Factor** $\Longrightarrow \lambda_1 > 0$
  - $\beta_{1p} > 0 \Longrightarrow$ **More volatile** ΔW
  - $\beta_{1p} < 0 \Longrightarrow$ **Less volatile** ΔW
- $COV(I_2, \Delta H) < 0$: **Insurance Factor** $\Longrightarrow \lambda_2 < 0$
  - $\beta_{2p} > 0 \Longrightarrow$ **Less volatile** ΔW
  - $\beta_{2p} < 0 \Longrightarrow$ **More volatile** ΔW
- **How do we estimate β_{1p}, β_{2p}, λ_1, and λ_2?**

The Chen, Roll, and Ross (1986) Model

- 20 portfolios made up of all NYSE stocks grouped by market cap size
  - i.e. first portfolio was made up of the smallest 5% of the NYSE, 20^{th} portfolio was the largest 5% of the NYSE
- 5 Economic Risk Factors
  - Industrial Production
  - Expected Inflation
  - Unexpected Inflation
  - Interest Risk Premium
  - Term Structure Risk Premium
- CRR use 371 months of data from January 1953 to November 1983
- In what follows we look at a stylized version of CRR with three portfolios and two factors.

Chen, Roll, and Ross

	Portfolio Returns data			Economic shocks data	
	A	**B**	**C**	**Shock 1**	**Shock 2**
$t = 1$	$r_{A,1}$	$r_{B,1}$	r_{C1}	$I_{1,1}$	$I_{2,1}$
$t = 2$	$r_{A,2}$	$r_{B,2}$	r_{C2}	$I_{1,2}$	$I_{2,2}$
$t = 3$	$r_{A,3}$	$r_{B,3}$	r_{C3}	$I_{1,3}$	$I_{2,3}$
...	...	...	...	...	...
$t = T$	$r_{A,T}$	$r_{B,T}$	$r_{C,T}$	$I_{1,T}$	$I_{2,T}$
Average	$\bar{r}_A$	$\bar{r}_B$	$\bar{r}_C$	$\bar{I}_1 = 0$	$\bar{I}_1 = 0$

- Step 1 (betas). Estimate **many time-series regression,** one for **every** $j = A, B, C$

$$r_{j,t} = \alpha_j + \beta_{1,j} I_{1,t} + \beta_{2,j} I_{2,t}, \ t = 1, \dots, T \quad \Longrightarrow \hat{\beta}_{1,j}, \ \hat{\beta}_{2,j} \ \text{(and } \hat{\alpha}_j\text{)}$$

Chen, Roll and Ross: Economic Forces and the Stock Market

	Portfolio Returns data			Economic shocks data	
	A	B	C	Shock 1	Shock 2
$t=1$	$r_{A,1}$	$r_{B,1}$	r_{C1}	$I_{1,1}$	$I_{2,1}$
$t=2$	$r_{A,2}$	$r_{B,2}$	r_{C2}	$I_{1,2}$	$I_{2,2}$
$t=3$	$r_{A,3}$	$r_{B,3}$	r_{C3}	$I_{1,3}$	$I_{2,3}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$t=T$	$r_{A,T}$	$r_{B,T}$	$r_{C,T}$	$I_{1,T}$	$I_{2,T}$
Average	$\bar{r}_A$	$\bar{r}_B$	$\bar{r}_C$	$\bar{I}_1=0$	$\bar{I}_1=0$

- Step 2 (lambdas). Estimate **one cross-sectional regression:**

$$\bar{r}_j - r_f = \alpha + \hat{\beta}_{1,j}\lambda_1 + \hat{\beta}_{2,j}\lambda_2, \quad j = A, B, C \implies \hat{\lambda}_1,\ \hat{\lambda}_2 \ \text{(and } \hat{\alpha}\text{)}$$

  – Note: in the real world you want way more than 3 portfolios to estimate two prices of risk! (CRR use 20 portfolios to estimate 5 lambdas)

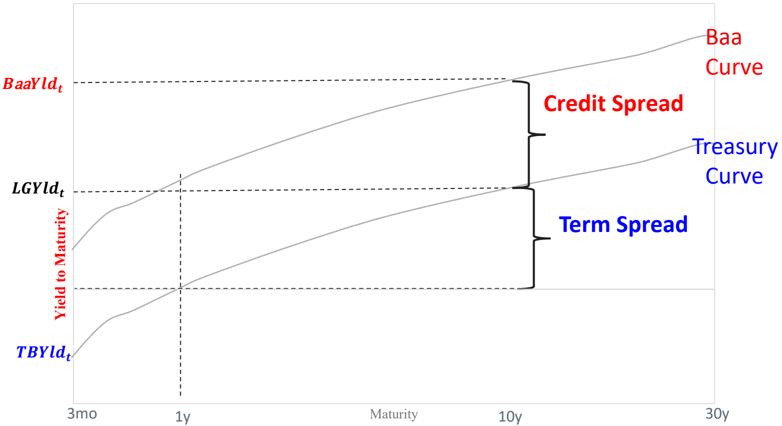
Chen, Roll and Ross: 5 Monthly Economic Risk Factors

1. Monthly Industrial Production, $MP_t = \ln\left(\frac{IP_t}{IP_{t-1}}\right)$, with $IP_t =$ ind. prod. month t
2. Changes in Expected Inflation, $DEI_t = \mathbb{E}_t[Infl_{t+1}] - \mathbb{E}_{t-1}[Infl_t]$
3. Unexpected Inflation, $UI_t = Infl_t - \mathbb{E}_{t-1}[Infl_t]$
4. Interest Risk Premium, $URP_t = BaaRet_t - LGBRet_t$,
   - $BaaRet_t$: **return** on "Baa and under" bond portfolio
   - $LGBRet_t$: **return** on long-term goverbnent bond portfolio
   - Note: $URP_t \uparrow \Longrightarrow$ **Credit spreads** $\downarrow$ **("good times").** Recall: yields $\neq$ returns!
5. Term Structure Risk Premium, $UTS_t = LGBRet_t - TBRet_{t-1}$ (slope of TS)
   - Note: $UTS_t \uparrow \Longrightarrow$ **Term spread** $\downarrow \Longrightarrow$ **yield curve flattens ("bad times"?)**

Chen, Roll and Ross: Economic Forces and the Stock Market

Ciurrent yield curve of government bonds (from WSJ)

Idealized Yield Curve Example for CRR



Chen, Roll and Ross: Summary of factor Shocks

- $I_{MP,t} = MP_t - \overline{MP}$, where $\overline{MP}$ = historical average
  - $I_{MP,t} > 0 \implies$ **Industrial Production increasing**
- $I_{DEI,t} = DEI_t - \overline{DEI}$
  - $I_{DEI,t} > 0 \implies$ **Inflation Expection higher**
- $I_{UI,t} = UI_t - \overline{UI}$
  - $I_{UI,t} > 0 \implies$ **Actual Inflation higher than Expected**
- $I_{URP,t} = URP_t - \overline{URP}$
  - $I_{URP,t} > 0 \implies$ **Credit Spreads got smaller**
- $I_{UTS,t} = UTS_t - \overline{UTS}$
  - $I_{UTS,t} > 0 \implies$ **Yield Curve got flatter**

Chen, Roll and Ross: Economic Forces and the Stock Market

- Data
  - $r_{p,t}$ - return on portfolio p in month t for 20 different portfolios by NYSE market cap, smallest to largest market cap in 5% buckets
  - $I_{i,t}$ the shock, or change in risk factor i over month t
  - Monthly data from January 1953 to November 1983
- Need to estimate our λ's (Risk Premiums /Market Price of Risk)
  - λ_{MP}-Industrial Production
  - λ_{DEI}-Inflation Expectations
  - λ_{UI}-Actual Inflation
  - λ_{URP}- Credit Spreads
  - λ_{UTS}-Term Spreads

Chen, Roll and Ross: Economic Forces and the Stock Market

- To obtain our λ_i i= 1 to 5, we run a **two-stage regression** for each month
  - **Step 1.a**: Choose a start date: i.e., t =January 1958 (1953 + 5 years)
  - **Step 1.b**: Use previous 60 months to estimate factor sensitivities $\hat{\beta}_{i,p}$, $i = 1, \ldots, 5$
    - $r_p = \alpha_p + I_1\beta_{1p} + I_2\beta_{2p} + I_3\beta_{3p} + I_4\beta_{4p} + I_5\beta_{5p} + e_p$, $p = 1, \ldots, 20$
    - **20 time series regressions** for each NYSE market cap portfolio
  - **Step 2**: estimate **12 cross-sectional regressions** (one for each of the months in 1958) using 20 returns $r_{p,t}$, $p = 1, \ldots, 20$, and $20 \times 5 = 100$ betas, $\hat{\beta}_{i,p}$
    - $r_{p,t} = \alpha_t + \hat{\beta}_{1,p}\lambda_{1,t} + \hat{\beta}_{2,p}\lambda_{2,t} + \hat{\beta}_{3,p}\lambda_{3,t} + \hat{\beta}_{4,p}\lambda_{4,t} + \hat{\beta}_{5,p}\lambda_{5,t} + e_p$,
    - Get $\hat{\lambda}_{i,t}$ $i = 1 \ldots 5$ for t = Jan 1958, Feb 1958,. . . , Dec 1958
  - **Step 3:** Go back to Step 1.a. Set start date = Jan 1959. Repeat until Nov 1983, (T = 311 months).
- The λ_i's estimates are the sample means of $\hat{\lambda}_{i,t}$: $\hat{\lambda}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{i,t}$, $i = 1 \ldots, 5$.

Chen, Roll and Ross: Economic Forces and the Stock Market

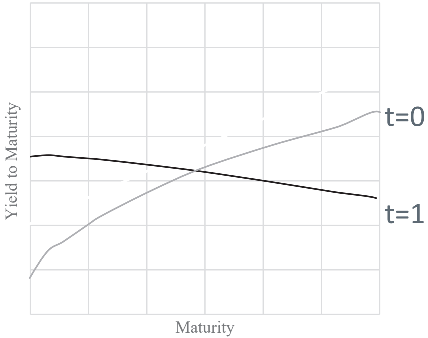
- After the two stage regression we get our estimates of $\lambda's$
- Summary of 5 Economic Risk Factors Risk Premiums (Table 4, Panel B, p. 396)
  - $\hat{\lambda}_{MP} = 13.589\%^{**}$ - Industrial Production
  - $\hat{\lambda}_{DEI} = -0.125\%$ - Inflation Expectations
  - $\hat{\lambda}_{UI} = -0.629\%^{*}$ - Unexpected Inflation
  - $\hat{\lambda}_{URP} = 7.205\%^{**}$ - Credit Spreads
  - $\hat{\lambda}_{UTS} = -5.21\%^{*}$ - Term Spreads
- These represent averages across all the months (**: significant at the 5% level, *: significant at the 10% level)

Chen, Roll and Ross: Economic Forces and the Stock Market

- How is our intuition for the Lambdas?
- $\lambda_{MP} = 13.589\%$ Industrial production is a Risk Enhancing Factor
  - $Cov(I_{MP}, \Delta H) > 0$
- $\lambda_{URP} = 7.205\%$
  - If $I_{URP} > 0$ then Credit Spreads lower
  - Human Capital?
  - Remember H is the Present Value of future income, some people's income is riskier than others
  - $I_{URP} > 0 \implies$ Lower Credit Spreads $\implies$ Lower Discount Rate $\implies \Delta H > 0$
  - $Cov(I_{URP}, \Delta H) > 0$

Chen, Roll and Ross: Interpreting risk premia

- $\lambda_{UTS} = -5.211\%$ Insurance Factor
  - If $I_{UTS} > 0$ then Yield Curve is flatter
  - Flat or inverted curve typically means economic peak / stock market peak
    - higher uncertainty (?)
    - Human capital riskier (?)
    - higher discount rate (?), $\Delta H < 0$
    - $Cov(I_{UTS}, \Delta H) < 0$



COMM 475 – Investment Policy
Client Objectives: Strategic Asset Allocation (SAA)

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Outline

- Individuals' Objectives – SAA
  - Life-cycle Investing
- Institutional Objectives – SAA
  - Defined Benefit (DB) Pensions Structure
  - Pension plans's SAA
  - Funding Surplus
  - Funding Surplus Risk Management
  - Choosing an Asset Mix

Client Objectives - Strategic Asset Allocation (SAA)

- SAA is a long-term decision
- Client Preferences
  - "Mean-Variance" tastes
  - **Individual Client:** Specific investment horizon (retirement day)
  - **Institutional Client:** Very long (infinite??) stream of payments

Client Objectives - Strategic Asset Allocation (SAA)

- **Individual** Risk/Reward Analysis
  - **Objective**: choose preferred distribution for start of **retirement wealth**
  - **Variables**: Asset Class Weights (SAA)
  - **Dynamic** strategy: Life-cycle investing can account for personal human capital
- **Institutional** Risk/Reward Analysis
  - Institution has a **Liability** = PV of all promised future payments
  - **Objective**: trustees choose the risk/reward trade-off of the Funding Surplus (FS), the "wealth" of the institution:

$$FS = Assets - Liability$$

  - **Variables**: Asset Class Weights (SAA)
  - Pension Plan Structures:
    - Defined Benefits (DB) Plans
    - Defined Contribution (DC) Plans

Client Objectives: Individuals

- Two assets with different risk profile
  - Safe asset A: $\mathbb{E}[r_p] = 0.02 = 2\%$; $\sigma[r_p] = 0.01 = 1\%$
  - Risky asset B: $\mathbb{E}[r_p] = 0.06 = 6\%$; $\sigma(r_p)$=$0.167 = 16.7\%$
- **Long-horizon planning:** Range of 20-year Wealth per \$1 invested:
- **Asset A:**
  - $\mathbb{E}[W] = (1.02)^{20} = \$1.486$
  - $\sigma[W] = (0.01)\sqrt{20} = \$0.0447$
  - Upper Bound $= 1.486 + 2 \times 0.0447 = \$1.575$
  - Lower Bound $= 1.486 - 2 \times 0.0447 = \$1.397$
- **Asset B:**
  - $\mathbb{E}[W] = (1.06)^{20} = \$3.207$
  - $\sigma[W] = (0.167)\sqrt{20} = \$0.747$
  - Upper Bound $= 3.207 + 2 \times 0.747 = \$4.701$
  - Lower Bound $= 3.207 - 2 \times 0.747 = \$1.713$

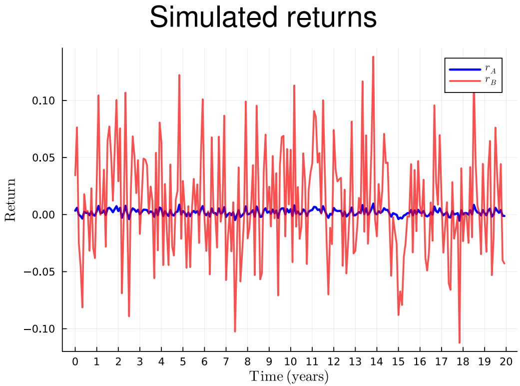
Client Objectives: Individuals

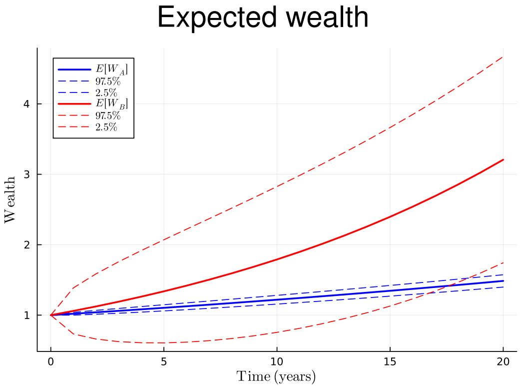
- **Short-horizon planning:** Suppose we are approaching retirement: Range of 5-year Wealth per $1 invested

- **Asset A:**
  - $\mathbb{E}[W] = (1.02)^5 = \$1.104$
  - $\sigma(W) = (0.01) \times \sqrt{5} = \$0.0224$
  - Upper Bound $= 1.104 + 2 \times 0.0224 = \$1.1488$
  - Lower Bound $= 1.104 - 2 \times 0.0224 = \$1.0592$

- **Asset B:**
  - $\mathbb{E}[W] = (1.06)^5 = \$1.338$
  - $\sigma(W) = (0.167) \times \sqrt{5} = \$0.3734$
  - Upper Bound $= 1.338 + 2 \times 0.3734 = \$2.0848$
  - Lower Bound $= 1.338 - 2 \times 0.3734 = \$0.5911$

Client Objectives: Individuals





Individual SAA Decision: Life-Cycle Investing

- Individual Wealth at time (t) is the sum of financial wealth (P_t) and human capital value (H_t):

$$W_t = P_t + H_t$$

- Scenario: Suppose H_t is "**Bond-like**". What if Individual wants her TOTAL Wealth to always be 50% Bonds (B) and 50% Stocks (S)?

$$B_t + H_t = .5W_t \implies B_t = .5W_t - H_t \implies S_t = P_t - B_t$$

- Age 25 Example: $P_{25} = \$50,000$, $H_{25} = \$950,000$, $W_{25} = \$1,000,000$.

$$B_{25} = .5 \times 1,000,000 - 950,000 = -\$450,000 \text{ [Borrow!]}$$
$$S_{25} = \$50,000 - (-\$450,000) = \$500,000$$

- It might be hard to borrow! So the best one can do is $B_{25} = 0$, $S_{25} = \$50,000$.

$$\text{\% stock in portfolio:} \frac{S_{25}}{P_{25}} = \frac{50,000}{50,000} = 100\%$$

Individual SAA Decision: Life-Cycle Investing

- Age 40 Example: $P_{40} = \$700,000$, $H_{40} = \$500,000$, $W_{40} = \$1,200,000$.

$$B_{40} = .5 \times 1,200,000 - 500,000 = \$100,000$$
$$S_{40} = \$700,000 - \$100,000 = \$600,000$$

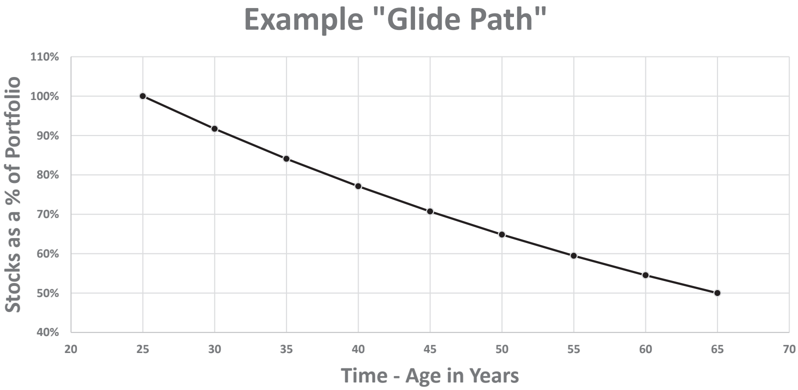
$$\text{\% stock in portfolio:} \frac{S_{40}}{P_{40}} = \frac{600,000}{700,000} = 86\%$$

- Age 65 Example: $P_{65} = \$900,000$, $H_{65} = \$0$, $W_{65} = \$900,000$.

$$B_{65} = .5 \times 900,000 - 0 = \$450,000$$
$$S_{40} = \$900,000 - \$450,000 = \$450,000$$

$$\text{\% stock in portfolio:} \frac{S_{65}}{P_{65}} = \frac{450,000}{900,000} = 50\%$$

Individual SAA Decision: Life-Cycle Investing



Institutional Objectives - (SAA)

- Types of Pension Plans
  - **Defined Contribution (DC)** Pension Plan
    - Sponsor (e.g., employer) promises to make contributions until retirement
    - NO explicit Liability but an **implicit** Liability
    - No pension promise (explicit) but pension trustees still need to work to raise the member's standard of living to an appropriate level (implicit).
  - **Defined Benefit (DB)** Pension Plan
    - Sponsor promises to make contributions AND a level of retirement payments
    - Here we have an explicit Liability - present value of all promised payments

Defined Benefit Pension Structure

A	L FS

Think Balance Sheet!

$$A = L + FS$$
$$FS = A - L$$

- Assets (A): market value of all contributions and retained earnings + present value of future promised contributions
- Liabilities (L): Present value of all promised payments (working and retired)
- Funding Surplus (FS): Difference between Assets and Liabilities
  - FS $>$ 0: plan is “funded”
  - FS $<$ 0: plan is “underfunded”. The sponsor will need to increase contributions

Defined Benefit Pension Structure

A	L FS

Think Balance Sheet!

$$A = L + FS$$
$$FS = A - L$$

- Example of a DB promise:
  - First Retirement Payment = Final Annual Salary $\times$# of Years of Service $\times$2.5%
  - i.e. $\$50,000 \times 20 \times 2.5\% = \$25,000$ = Half of your Final Salary paid every year
  - L= PV of all the string of future payments for all the members

Defined Benefit SAA Example

	Present Value @ 5%	$t = 1$	$t = 2$	$t = 3$
+ Contributions (from Sponsor)	27.23	10	10	10
- Liability	-40.41	-10	-15	-20
+ Investments	13.17			
Total (Funding Surplus)	0.00			

Contrib.	27.23	40.41	L
Investment	13.17	**0.00**	FS
Total (A)	40.41	40.41	

Defined Benefit SAA Example

	Present Value @ 5%	$t = 1$	$t = 2$	$t = 3$
+ Contributions (from Sponsor)	27.23	10	10	10
- Liability	40.41	10	15	20
+ Investments	13.17	0	5	10
Pay shortfall		0	5	10
Remaining Investments		0	0	0
Total (Funding Surplus)	0.00			

- Scenario 1: Buy **Strip** Bonds to pay 5 at $t = 2$ and 10 at $t = 3$
  - $Cost = \frac{5}{(1.05)^2} + \frac{10}{(1.05)^3} = 13.17$
  - Would just meet all its expected payments (liability)
  - Strategy known as cash flow matching, it "immunizes" the Funding Surplus: FS=0 today and for the future

Defined Benefit SAA Example

	Present Value @ 5%	$t = 1$	$t = 2$	$t = 3$
+ Contributions (from Sponsor)	27.23	10	10	10
- Liability	40.41	10	15	20
+ Investments	13.17	14.49	15.94	12.03
Pay shortfall		0	5	10
Remaining Investments		14.49	10.94	2.03
Total (Funding Surplus)	0.00			

- Scenario 2: Instead of Strips, Invest $13.17 in an equity portfolio with an expected return of 10% per period.
- Expect to have $2.03, t=3 means only need to contribute $\$10 - 2.03 = \$7.97$. "Contribution Holiday" for sponsor
- If the equity portfolio actual return is less than 10% (lose everything?!?) then contributions increase to make up shortfall

Defined Benefit Pension Structure

Think Balance Sheet!

$$A = L + FS$$
$$FS = A - L$$

- Assets (A): market value of all contributions and retained earnings + present value of future promised contributions
- Liabilities (L): Present value of all promised payments (working and retired)
- Funding Surplus (FS): Difference between Assets and Liabilities
  - FS $>$ 0 means the plan is "funded"
  - FS $<$ 0 means the plan is "underfunded" , sponsor will need to increase contributions

How a DB Plan Liability Can Change, Example 1

	Present Value @ 5%	$t = 1$	$t = 2$	$t = 3$
+ Contributions (from Sponsor)	27.23	10	10	10
- Liability	40.41	10	15	20
+ Investments	13.17			
Total (Funding Surplus)	0.00			
New Liability (GDP surge)	51.17	12	20	25

- GDP surge; marginal productivity of labour to rise; wages to rise; future pension benefits to rise
- Discount rate remains = 5%
- $L(\%change) = \frac{51.17}{40.41} - 1 = 26.6\%$
- Creates a funding shortfall (FS < 0)

How a DB Plan Liability Can Change, Example 2

	Present Value @ 5%	$t = 1$	$t = 2$	$t = 3$
+ Contributions (from Sponsor)	27.23	10	10	10
- Liability	40.41	10	15	20
+ Investments	13.17			
Total (Funding Surplus)	0.00			
New Liability @ 7%	38.77	10	15	20

- Rise in interest rates causes discount rate for the liability to rise to 7%
- Discount rate on future contributions remains = 5%
- $L(\%change) = \frac{38.77}{40.41} - 1 = -4.1\%$
- Creates a funding surplus (FS $>$ 0)

DB Objective: Funding Surplus Risk Management

t=0	
Assets A_0	Liability L_0
	Funding Surplus
	$FS_0 = A_0 - L_0$

Assets earn $1 + r_A$

Liability changes $1 + r_L$

t=1	
$A_1 = A_0(1 + r_A)$	$L_1 = L_0(1 + r_L)$
	$FS_1 = A_1 - L_1$

$$FS_1 = A_1 - L_1 = A_0(1 + r_A) - L_0(1 + r_L) = A_0 - L_0 + r_A A_0 - r_L L_0$$

And so

$$FS_1 - FS_0 = r_A A_0 - r_L L_0$$

Return to Funding Surplus is r_{FS}

$$r_{FS} = \frac{FS_1 - FS_0}{FS_0} = \left(\frac{A_0}{FS_0}\right) r_A - \left(\frac{L_0}{FS_0}\right) r_L = \alpha r_A + (1 - \alpha) r_L$$

with $\alpha \equiv \left(\frac{A_0}{FS_0}\right) > 1$ and $(1 - \alpha) \equiv -\left(\frac{L_0}{FS_0}\right) < 0$

DB Objective: Example FS Risk Management

t=0	
$A_0 = 100$	$L_0 = 80$
	$FS_0 = 20$

Assets earn $r_A = -10\%$

Liability changes $r_L = 10\%$

t=1	
$A_1 = 90$	$L_1 = 88$
	$FS_1 = 2$

$$r_{FS} = \frac{FS_1 - FS_0}{FS_0} = \frac{2-20}{20} = -90\%$$

$$A_0 r_A = 100 \times (-10\%) = -\$10$$

\$ Changes: $L_0 r_L = 80 \times 10\% = \$8$ $\quad A_0 r_A - L_0 r_L = -10 - 8 = -\$18$

$$FS_0 r_{FS} = 20 \times (-90\%) = -\$18$$

$$r_{FS} = \left(\frac{A_0}{FS_0}\right) r_A - \left(\frac{L_0}{FS_0}\right) r_L = \left(\frac{100}{20}\right)(-10\%) - \left(\frac{80}{20}\right)(10\%) = -50\% - 40\% = -90\%$$

Client Objectives: Review Multi-Risk Factor Model

Review:

- Multi-Risk Factor RGF: $r_s = \mathbb{E}[r_s] + \beta_{1,s} I_1 + \beta_{2,s} I_2 + e_s$
- Multi-Risk Factor SML: $\mathbb{E}[r_s] = r_f + \beta_{1,s}\lambda_1 + \beta_{2,s}\lambda_2$
- Multi-Risk Factor Risk: $\sigma_s^2 = \beta_{1,s}^2\sigma_1^2 + \beta_{2,s}^2\sigma_2^2$

DB Objective: Funding Surplus Risk Management

Leverage our knowledge of Multi-Risk Factor Model

$$
\begin{aligned}
r_A &= \mathbb{E}[r_A] + \beta_{1A} I_1 + \beta_{2A} I_2 + e_A \\
r_L &= \mathbb{E}[r_L] + \beta_{1L} I_1 + \beta_{2L} I_2 + e_L \\
\mathbb{E}[r_A] &= r_f + \beta_{1A}\lambda_1 + \beta_{2A}\lambda_2 \\
\mathbb{E}[r_L] &= r_f + \beta_{1L}\lambda_1 + \beta_{2L}\lambda_2
\end{aligned}
$$

- As before our "shocks" with expected values of 0 are the risk factor shocks, i.e. I_1 might be GDP risk and I_2 might be interest rate risk
- We **cannot** chose our Liability Betas
  - Liability Betas are defined by the features of the pension plan promises
- BUT, **we can chose our Asset Betas**!

DB Objective: Funding Surplus Risk Management

Plug the 4 Multi-Risk Factor Model equations into r_{FS}:

$$\begin{aligned} r_{FS} &= \alpha r_A + (1-\alpha) r_L \\ &\dots \\ &= r_f + \beta_{1FS}\lambda_1 + \beta_{2FS}\lambda_2 + \beta_{1FS} I_1 + \beta_{2FS} I_2 + \alpha e_A + (1-\alpha) e_L \end{aligned}$$

where

$$\begin{aligned} \beta_{1FS} &= \alpha\beta_{1A} + (1-\alpha)\beta_{1L} = \left(\frac{A_0}{FS_0}\right)\beta_{1A} - \left(\frac{L_0}{FS_0}\right)\beta_{1L} \\ \beta_{2FS} &= \alpha\beta_{2A} + (1-\alpha)\,\beta_{2L} = \left(\frac{A_0}{FS_0}\right)\beta_{2A} - \left(\frac{L_0}{FS_0}\right)\beta_{2L} \end{aligned}$$

are the **funding surplus betas** with respect to the two factors.

DB Objective: Funding Surplus Risk Management

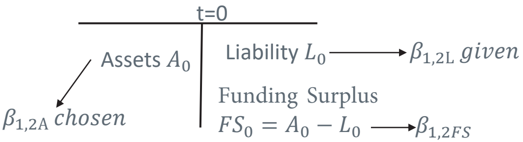
To understand these Beta equations, for β_{1FS}, multiply through by FS_0,

$$FS_0\beta_{1FS} = A_0\beta_{1A} - L_0\beta_{1L}$$

- This equation is about funding surplus **dollar** volatility:
  FS dollar volatility = Asset dollar volatility $-$ Liability dollar volatility
- This is what pension plan trustees care about!
- For example, we want $\beta_{1FS} = 0$, must choose β_{1A} such that $A_0\beta_{1A} - L_0\beta_{1L} = 0$
- These definitions get us our expected funding surplus return and its variance

$$\mathbb{E}[r_{FS}] = r_f + \beta_{1FS}\lambda_1 + \beta_{2FS}\lambda_2$$
$$\sigma^2[r_{FS}] = \beta_{1FS}^2\sigma^2[I_1] + \beta_{2FS}^2\sigma^2[I_2]$$

DB Objective: Funding Surplus Risk Management

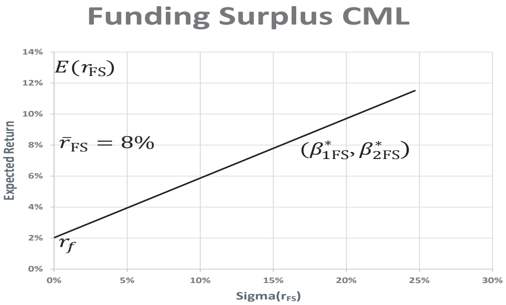


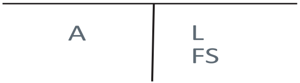
$$\beta_{1FS} = \left(\frac{A_0}{FS_0}\right)\beta_{1A} - \left(\frac{L_0}{FS_0}\right)\beta_{1L} \implies FS_0\beta_{1FS} = A_0\beta_{1A} - L_0\beta_{1L}$$

$$\beta_{2FS} = \left(\frac{A_0}{FS_0}\right)\beta_{2A} - \left(\frac{L_0}{FS_0}\right)\beta_{2L} \implies FS_0\beta_{2FS} = A_0\beta_{2A} - L_0\beta_{2L}$$

Funding Surplus CML (Capital Market Line)

- CML: mean-volatility **efficient frontier** of the funding surplus
  - Find β_{1FS}, β_{2FS} that **minimize** $\sigma^2[r_{FS}]$, **subject to** $\mathbb{E}[r_{FS}]$ = target return
  - Familiar problem but in the **“beta” space** instead of “asset weight” space!
  - For every set of portfolio weights there is a corresponding set of betas!





Determining the Optimal Asset Risk Exposure

- Step 1: The trustees know $A_0, L_0, FS_0 = A_0 - L_0$ and β_{1L}, β_{2L}
  - Estimate β_{1L}, β_{2L} from a regression of historical liability percent changes and our historical shocks (our I's)
- Step 2: Trustees set desired $\mathbb{E}[r_{FS}] = \bar{r}_{FS}$
- Step 3: Trustees calculates the ex-ante funding surplus Capital Market Line
  - Similar to our efficient frontier from CAPM but multiple Betas, that is,

$$\min_{\beta_{1FS}, \beta_{2FS}} \sigma^2[r_{FS}], \quad \text{s.t. } \mathbb{E}[r_{FS}] = \bar{r}_{FS}$$

- Step 4: From the FS "optimal" betas β^*_{1FS}, β^*_{2FS} recover Asset Betas β_{1A}, β_{2A}:

$$\beta^*_{1FS} = \left(\frac{A_0}{FS_0}\right) \beta_{1A} - \left(\frac{L_0}{FS_0}\right) \beta_{1L} \text{ and } \beta^*_{2FS} = \left(\frac{A_0}{FS_0}\right) \beta_{2A} - \left(\frac{L_0}{FS_0}\right) \beta_{2L}$$

DB Objective: Optimal Asset Betas

The problem

$$\min_{\beta_{1FS},\beta_{2FS}} \sigma[r_{FS}], \quad s.t. \;\; \mathbb{E}[r_{FS}] = \bar{r}_{FS}$$

has the following solution: ▸ Proof

$$\beta^*_{1FS} = \frac{\lambda_1/\sigma_1^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f), \quad \text{where } \sigma_1^2 \equiv \sigma^2[I_1] \text{ and } \sigma_2^2 \equiv \sigma^2[I_2]$$

$$\beta^*_{2FS} = \frac{\lambda_2/\sigma_2^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f)$$

so that: $\sigma^*_{FS} \equiv \sigma^*[r_{FS}] = \frac{\mathbb{E}[r_{FS}]-r_f}{\sqrt{\lambda_1^2/\sigma_1^2+\lambda_2^2/\sigma_2^2}}$ and $\mathbb{E}[r_{FS}] = r_f + \lambda_1\beta^*_{1FS} + \lambda_2\beta^*_{2FS} = \bar{r}_{FS}$.

What are: λ_1/σ_1 and λ_2/σ_2? What do they remind you of?

DB Objective: Optimal Asset Betas

- Recall the relation between betas of asset, liability and FS ($A = L + FS$):

$$\beta^*_{1A} = \left(\frac{FS_0}{A_0}\right)\beta^*_{1FS} + \left(\frac{L_0}{A_0}\right)\beta^*_{1L}$$
$$\beta^*_{2A} = \left(\frac{FS_0}{A_0}\right)\beta^*_{2FS} + \left(\frac{L_0}{A_0}\right)\beta^*_{2L}$$

- If I wanted to **immunize** ($\beta^*_{1FS} = 0$, $\beta^*_{2FS} = 0$) the pension fund, what are my Asset Betas?

$$\beta^*_{1A} = \left(\frac{L_0}{A_0}\right)\beta^*_{1L}, \quad \text{and} \quad \beta^*_{2A} = \left(\frac{L_0}{A_0}\right)\beta^*_{2L}$$

DB pension plan Asset Risk Exposure: Two-Factor Example

Relevant Capital Market Data:

$$r_f = 6\%, \lambda_1 = 16\%, \lambda_2 = 6\%, \sigma_1^2 = 0.09, \sigma_2^2 = 0.05$$

Relevant Pension Plan Data:

$$\beta_{1L} = 0.8, \beta_{2L} = 1.3, A_0 = 100, L_0 = 70, \ \text{ so } \ FS_0 = 100 - 70 = 30$$

Calculations:

$$\frac{\lambda_1}{\sigma_1^2} = 1.7778, \frac{\lambda_1^2}{\sigma_1^2} = 0.28444, \ \frac{\lambda_2}{\sigma_2^2} = 1.2, \frac{\lambda_2^2}{\sigma_2^2} = 0.0720 \implies \frac{\lambda_1^2}{\sigma_1^2} + \frac{\lambda_2^2}{\sigma_2^2} = 0.35644$$

$$\beta^*_{1FS} = \left(\frac{1.7778}{0.35644}\right)(\bar{r}_{FS} - 6\%) = 4.9877(\bar{r}_{FS} - 6\%)$$

$$\beta^*_{2FS} = \left(\frac{1.2}{0.35644}\right)(\bar{r}_{FS} - 6\%) = 3.3666(\bar{r}_{FS} - 6\%)$$

DB pension plan Asset Risk Exposure: Two-Factor Example

Calculations:

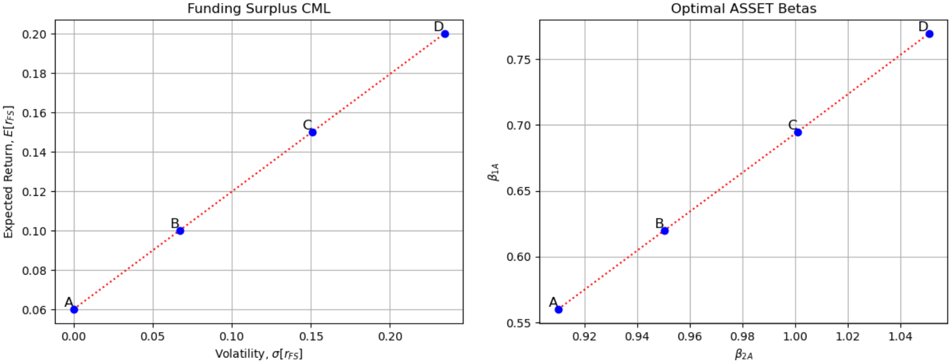
$$\beta^*_{1FS} = 4.9877(\bar{r}_{FS} - 6\%) \quad \sigma^*_{FS} = \frac{\bar{r}_{FS} - 6\%}{\sqrt{0.35644}} = 1.675(\bar{r}_{FS} - 6\%)$$
$$\beta^*_{2FS} = 3.3666(\bar{r}_{FS} - 6\%)$$

$$\beta^*_{1A} = \left(\frac{30}{100}\right)\beta^*_{1FS} + \left(\frac{70}{100}\right)\beta^*_{1L} = 0.3\beta^*_{1FS} + 0.7(.8)$$

$$\beta^*_{2A} = \left(\frac{30}{100}\right)\beta^*_{2FS} + \left(\frac{70}{100}\right)\beta^*_{2L} = 0.3\beta^*_{2FS} + 0.7(1.3)$$

Scenarios	$\bar{r}_{FS}$	β^*_{1FS}	β^*_{2FS}	σ^*_{FS}	β^*_{1A}	β^*_{2A}
A	6%	0	0	0	0.56	0.91
B	10%	0.1995	0.1347	0.067	0.6199	0.9504
C	15%	0.4489	0.303	0.1508	0.6947	1.001
D	20%	0.6983	0.4713	0.2345	0.7695	1.051

DB pension plan Asset Risk Exposure: Two-Factor Example



- Note that both factor 1 and 2 are risk-enhancing factors (both lambdas > 0)
- To achieve a higher $\bar{r}_{FS}$ we need to increase exposure in each factors (asset betas increase)

DB pension plan Asset Risk Exposure: Second Example

Relevant Capital Market Data: - Factor #1 is GDP, Factor #2 is Unexpected Inflation:

$$r_f = 4\%, \lambda_1 = 15\%, \lambda_2 = -5\%, \sigma_1 = 0.25, \sigma_1^2 = 0.0625, \sigma_2 = 0.2, \sigma_2^2 = 0.04$$

Relevant Pension Plan Data:

$$\beta_{1L} = 0.65, \beta_{2L} = 0.44, A_0 = 2300, L_0 = 1750, \text{ so } FS_0 = 550$$

Calculations:

$$\frac{\lambda_1}{\sigma_1^2} = 2.4, \frac{\lambda_1^2}{\sigma_1^2} = 0.36, \quad \frac{\lambda_2}{\sigma_2^2} = -1.25, \frac{\lambda_2^2}{\sigma_2^2} = 0.0625 \Longrightarrow \frac{\lambda_1^2}{\sigma_1^2} + \frac{\lambda_2^2}{\sigma_2^2} = 0.4225$$

$$\beta_{1FS}^* = \left(\frac{2.4}{0.4225}\right)(\bar{r}_{FS} - 4\%) = 5.6805(\bar{r}_{FS} - 4\%)$$

$$\beta_{2FS}^* = \left(\frac{-1.25}{0.4225}\right)(\bar{r}_{FS} - 4\%) = -2.9586(\bar{r}_{FS} - 4\%)$$

DB pension plan Asset Risk Exposure: Two-Factor Example

Calculations:

$$\beta^*_{1FS} = 5.6805(\bar{r}_{FS} - 4\%) \qquad \sigma^*_{FS} = \frac{\bar{r}_{FS} - 4\%}{\sqrt{0.4225}} = 1.538462(\bar{r}_{FS} - 4\%)$$
$$\beta^*_{2FS} = -2.9586(\bar{r}_{FS} - 4\%)$$

$$\beta^*_{1A} = \left(\frac{550}{2300}\right)\beta^*_{1FS} + \left(\frac{1750}{2300}\right)\beta^*_{1L} = 0.2391\beta^*_{1FS} + 0.7609(0.65)$$

$$\beta^*_{2A} = \left(\frac{550}{2300}\right)\beta^*_{2FS} + \left(\frac{1750}{2300}\right)\beta^*_{2L} = 0.2391\beta^*_{2FS} + 0.7609(0.44)$$

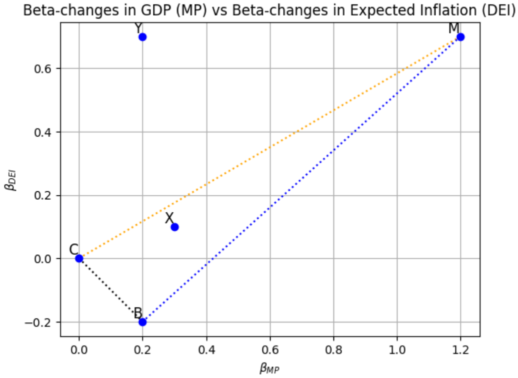
Scenarios	$\bar{r}_{FS}$	β^*_{1FS}	β^*_{2FS}	σ^*_{FS}	β^*_{1A}	β^*_{2A}
A	6%	0.1136	-0.0592	3.10%	0.5217	0.3206
B	15%	0.6249	-0.3254	16.90%	0.644	0.257

- To achieve a higher $\bar{r}_{FS}$ we need to **lower** the asset beta for factor 2!

SAA Decision: Optimal Asset Betas to an Asset Mix

- A pension plan is concerned with GDP and inflation risk.
- Following Chen, Roll, and Ross, the plan used the following two factors
  - MP: Growth in industrial production
  - DEI: changes in expected inflation
- The possible investable asset classes are
  - Cash (C), $\beta_{1C} = \beta_{2C} = 0$
  - A bond portfolio (B), $\beta_{1B} = 0.2$, $\beta_{2B} = -0.2$.
  - An equity portfolio (M), $\beta_{1M} = 1.2$, $\beta_{2M} = 0.7$.
- See graph on next page

SAA Decision: Optimal Asset Betas to an Asset Mix



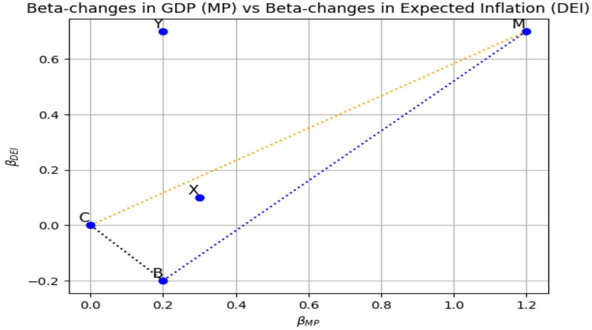
- Combining points B, C, M with non-negative weights we can obtain any point in the interior of the triangle!

SAA Decision: Optimal Asset Betas to an Asset Mix

- Example. Solve for point X: $\beta_X = (\beta_{MP}, \beta_{DEI}) = (0.3, 0.1)$.
- Let $X_B, X_M, X_C = 1 - X_M - X_B$ be the weights. Two equations with two unknowns
  - MP Risk: $0.3 = .2X_B + 1.2X_M + 0(1 - X_B - X_M)$
  - DEI Risk: $0.1 = -.2X_B + 0.7X_M + 0(1 - X_B - X_M)$
- Solution is

$$X_M = 21.05\%$$
$$X_B = 23.7\%$$
$$X_C = 55.25\%$$

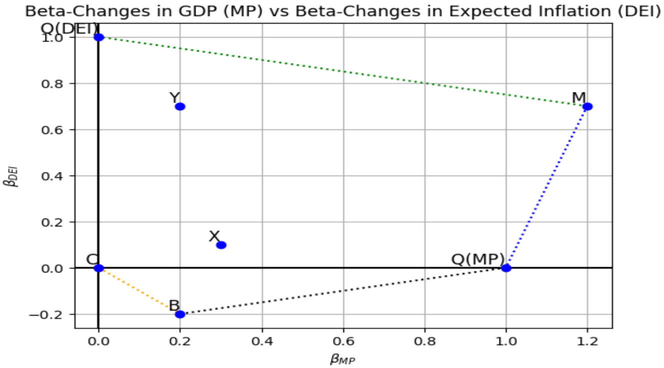
SAA Decision: Optimal Asset Betas to an Asset Mix



- What about point Y, $\beta_Y = (0.2, 0.7)$?
  $X_M = 47.37\%$, $X_B = -184.21\%$ and $X_C = 236.84\%$!
- Many pension plans cannot short! What to do?

SAA Decision: Optimal Asset Betas to an Asset Mix

- Suppose an intermediary forms two factor portfolios (ETFs): QMP and QDEI [This is what BlackRock does!]



SAA Decision: Optimal Asset Betas to an Asset Mix

- How are QMP and QDEI created? Lots of leverage! (see graph below)
- Solving for the asset mix that replicates Q_{MP} and Q_{DEI} we get
  - $Q_{MP} = (X_C = -136.8\%, X_B = 184.2\%, X_M = 52.6\%)$
  - $Q_{DEI} = (X_C = 363.2\%, X_B = -315.8\%, X_M = 52.6\%)$
- BlackRock can do so, pension plans cannot!

SAA Decision: Optimal Asset Betas to an Asset Mix

- Now we can solve for $Y : \beta_Y = (.2, .7)$ with **no shorting** [BlackRock did the shorting for us] using the 5 possible assets. For example, take QDEI, M and C:
  - MP Risk: $.2 = 0X_{QDEI} + 1.2X_M + 0(1 - X_{QDEI} - X_M)$
  - DEI Risk: $.7 = 1X_{QDEI} + 0.7X_M + 0(1 - X_{QDEI} - X_M)$
- Solving yields $X_C = 25\%, X_M = 16.67\%$ and $X_{QDEI} = 58.33\%$.
- Even easier use cash and the two Q factor portfolios:
  - MP Risk: $.2 = 1X_{QMP} + 0X_{QDEI} + 0(1 - X_{QDEI} - X_{QMP})$
  - DEI Risk: $.7 = 0X_{QMP} + 1X_{QDEI} + 0(1 - X_{QDEI} - X_{QMP})$
- Solving yields $X_C = 10\%, X_{QMP} = 20\%$ and $X_{QDEI} = 70\%$

SAA Decision: Asset-Liability Management (ALM) Examples

Relevant Capital Market Data:

$$r_f = 2\%, \lambda_{MP} = 16\%, \lambda_{DEI} = 6\%, \sigma^2_{MP} = 0.09, \sigma^2_{DEI} = 0.05$$

Example 1, Pension Plan Data:

$$\beta_{MP,L} = 0.5, \beta_{DEI,L} = 0.25, A_0 = 900, L_0 = 700, \; so \; FS_0 = 200$$

Calculations:

$$\beta^*_{MP,FS} = \left(\frac{1.778}{0.35644}\right)(\bar{r}_{FS} - 2\%) \quad \beta^*_{DEI,FS} = \left(\frac{1.2000}{0.35644}\right)(\bar{r}_{FS} - 2\%)$$

$$\beta^*_{MP,A} = \left(\frac{200}{900}\right)\beta^*_{MP,FS} + \left(\frac{700}{900}\right)(0.50) = 0.2222\beta^*_{MP,FS} + 0.3889$$

$$\beta^*_{DEI,A} = \left(\frac{200}{900}\right)\beta^*_{DEI,FS} + \left(\frac{700}{900}\right)(0.25) = 0.2222\beta^*_{DEI,FS} + 0.1944$$

SAA Decision: Asset-Liability Management (ALM) Examples

- For $\bar{r}_{FS} = 4\%$, this requires:

$$\beta^*_{MP,FS} = 0.0997, \quad \beta^*_{DEI,FS} = 0.0673, \quad \beta^*_{MP,A} = 0.41, \quad \beta^*_{DEI,A} = 0.21$$

- Using the 5 assets from the graph, we can get this FS growth rate in many ways
  - E.g., using C, B, and M we get

$$X_C = 58.2\%, X_B = 9.2\%, X_M = 32.6\%$$

- More simply, using C, QMP, and QDEI

$$X_C = 38\%, X_{QMP} = 41\%, X_{QDEI} = 21\%$$

  - Note: X_{QMP} and X_{QDEI} are equal to asset betas, $\beta^*_{MP,A} = 0.41$, $\beta^*_{DEI,A} = 0.21$ Why?

Appendix. Optimal FS betas – Proof

We need to solve

$$\min_{\beta_{1FS},\beta_{2FS}} \sigma[r_{FS}], \quad s.t. \;\; \mathbb{E}[r_{FS}] = \bar{r}_{FS}$$

where $\sigma[r_{FS}] = \sqrt{\beta_{1,FS}^2\sigma_1^2 + \beta_{2,FS}^2\sigma_2^2}$ and $\mathbb{E}[r_{FS}] = r_f + \beta_{1,FS}\lambda_1 + \beta_{2,FS}\lambda_2$

Note that the β_{1FS}, β_{2FS} that miminize $\sigma[r_{FS}]$ also minimizes $\sigma^2[r_{FS}]$.

Therefore, for ease of calculation we work with $\sigma^2[r_{FS}]$ instead of $\sigma[r_{FS}]$.

Optimal FS betas – Proof (cont'd)

The Lagrangian for the problem

$$\min_{\beta_{1FS},\beta_{2FS}} \sigma^2[r_{FS}], \quad s.t.\ \mathbb{E}[r_{FS}] = \bar{r}_{FS}$$

is

$$\mathcal{L} = \beta_{1,FS}^2\sigma_1^2 + \beta_{2,FS}^2\sigma_2^2 + \eta\left(\bar{r}_{FS} - r_f - \beta_{1,FS}\lambda_1 - \beta_{2,FS}\lambda_2\right),$$

with η denoting the Lagrange multiplier.

The first-order conditions with respect to $\beta_{1,FS}$ and $\beta_{2,FS}$ are

$$2\beta_{1,FS}\sigma_1^2 = \eta\lambda_1 \implies \beta_{1,FS} = \eta\frac{\lambda_1}{2\sigma_1^2} \tag{1}$$

$$2\beta_{2,FS}\sigma_1^2 = \eta\lambda_2 \implies \beta_{2,FS} = \eta\frac{\lambda_2}{2\sigma_2^2} \tag{2}$$

Optimal FS betas – Proof (cont'd)

The first-order condition with respect to η is

$$\bar{r}_{FS} - r_f = \beta_{1,FS}\lambda_1 + \beta_{2,FS}\lambda_2 \tag{3}$$

Substituting $\beta_{1,FS}$ and $\beta_{2,FS}$ from equations (1) and (2) in equation (3) we have

$$\bar{r}_{FS} - r_f = \eta\left(\frac{\lambda_1^2}{2\sigma_1^2} + \frac{\lambda_2^2}{2\sigma_2^2}\right) \implies \eta = \frac{\bar{r}_{FS} - r_f}{\frac{\lambda_1^2}{2\sigma_1^2} + \frac{\lambda_2^2}{2\sigma_2^2}} \tag{4}$$

Substituting the expression of η from equation (4) in equations (1) and (2) and simplifying, we obtain that the opimal betas are:

$$\beta^*_{1,FS} = \frac{\lambda_1/\sigma_1^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f), \quad \text{and} \quad \beta^*_{2,FS} = \frac{\lambda_2/\sigma_2^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f).$$

□

Capital Market Line

Subtstituting the expressions of $\beta^*_{1,FS}$ and $\beta^*_{2,FS}$ in the definition of FS volatility, $\sigma[r_{FS}]$ we have

$$
\begin{aligned}
\sigma[r_{FS}] &= \sqrt{(\beta^*_{1,FS})^2\sigma_1^2 + (\beta^*_{2,FS})^2\sigma_2^2} \\
&= \sqrt{\frac{\lambda_1^2/\sigma_1^4}{\left(\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2\right)^2} \times (\bar{r}_{FS} - r_f)^2 \times \sigma_1^2 + \frac{\lambda_2^2/\sigma_2^4}{\left(\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2\right)^2} \times (\bar{r}_{FS} - r_f)^2 \times \sigma_2^2} \\
&= (\bar{r}_{FS} - r_f)\sqrt{\frac{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2}{\left(\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2\right)^2}} \\
&= \frac{\bar{r}_{FS} - r_f}{\sqrt{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2}} \qquad (5)
\end{aligned}
$$

Capital Market Line (cont'd)

Subtstituting the expressions of $\beta^*_{1,FS}$ and $\beta^*_{2,FS}$ in the definition of FS expected return, $EE[r_{FS}]$ we have

$$\begin{aligned}
\mathbb{E}[r_{FS}] &= r_f + \lambda_1 \beta^*_{1FS} + \lambda_2 \beta^*_{2FS} \\
&= r_f + \frac{\lambda_1^2/\sigma_1^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f) + \frac{\lambda_2^2/\sigma_2^2}{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2} \times (\bar{r}_{FS} - r_f) \\
&= r_f + (\bar{r}_{FS} - r_f) = \bar{r}_{FS} \ \text{ as it should be if } \beta_{1FS} \text{ and } \beta_{2FS} \text{ are optimal!} \qquad (6)
\end{aligned}$$

Combining equations (5) and (6) we obtain

$$\mathbb{E}[r_{FS}] = r_f + \left(\sqrt{\lambda_1^2/\sigma_1^2 + \lambda_2^2/\sigma_2^2}\right) \times \sigma[r_{FS}] \qquad (7)$$

Equation (7) is the CML in a two-factor model. Note that in a 1-factor model equation (7) collapses to λ_1/σ_1. What does this remind you of? Back

COMM 475 – Investment Policy
Tactical Asset Allocation (TAA)

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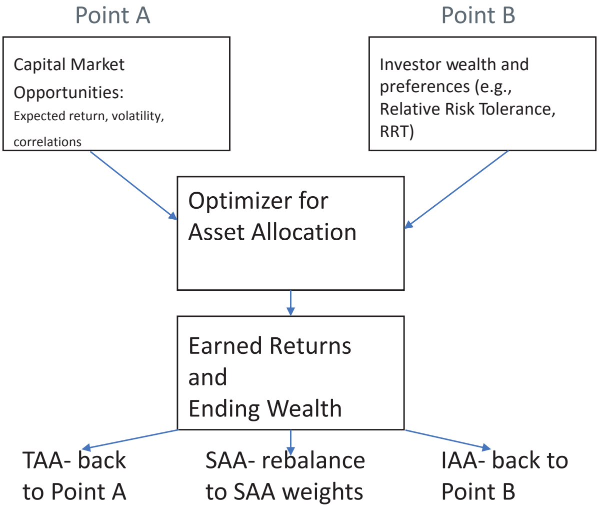
Outline

- Overview:
  - Strategic Asset Allocation (SAA)
  - **Tactical Asset Allocation (TAA)**
  - Insured Asset allocation (IAA)
- Tactical Asset Allocation (TAA)
  - Fama-French (1989) predictive model
  - Chen (1991) predictive model
- Estimating the Equity Risk Premium: breaking it down
- An example of TAA implementation

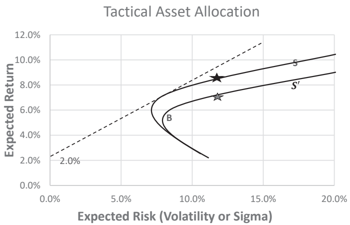
Overview

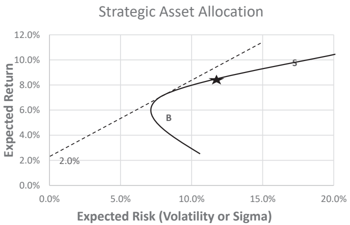
- Subcategories of **Dynamic asset allocation (DAA)**
  - Strategic asset allocation (SAA) – long-term weights
  - Tactical asset allocation (TAA) – short-term weights
  - Insured asset st allocation (IAA) – investors' preferences
- Basic principle for TAA: **price** "mean reversion" $\iff$ negative **return** autocorrelation.
  - TAA is a "contrarian" strategy.
- TAA requires estimating (predicting) the expected risk premium
  - Fama French (1989) – Focus on equity risk premium
  - Chen (1991) – Links predictive variables to business cycle (macro variables)
- Bottom line
  - The equity risk premium (ERP) is related to the business cycle
  - Predicting the equity risk premium is hard!

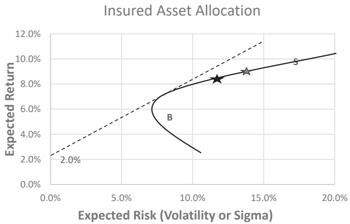
TAA Introduction: William Sharpe's Characterization



SAA, TAA, & IAA







TAA: Frontier moves as new data are used to predict returns
SAA: Frontier does not move - Rebalance to target weights
IAA: Frontier does not move - Agents' preference might change

TAA: Definitions of SAA, TAA, and IAA

- Sharpe's solution for portfolio choice:

$$\max_{X_{St}, X_{Bt}} \mathbb{E}_t[r_p] - \frac{1}{2RRT(W_t)} Var[r_p],$$

where $RRT(W_t)$: relative risk tolerance (function of wealth) and

$$\mathbb{E}_t[r_p] = X_{St}\mathbb{E}_t[r_S] + X_{Bt}\mathbb{E}_t[r_B], \quad X_{St} + X_{Bt} = 1$$

$$Var[r_p] = X_{St}^2\sigma_S^2 + X_{Bt}^2\sigma_B^2 + 2X_{St}X_{Bt}\sigma_{BS}$$

- Solution:

$X_{St} = K_0 + K_{1t} \times RRT(W_t)$, It **must** be on the MV frontier! Why? Suppose not...

$$K_0 = \frac{\sigma_B^2 - \sigma_{BS}}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}}, \quad K_{1t} = \frac{\mathbb{E}_t[r_S] - \mathbb{E}_t[r_B]}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}} > 0 \text{ if } \mathbb{E}_t[r_S] > \mathbb{E}_t[r_B]$$

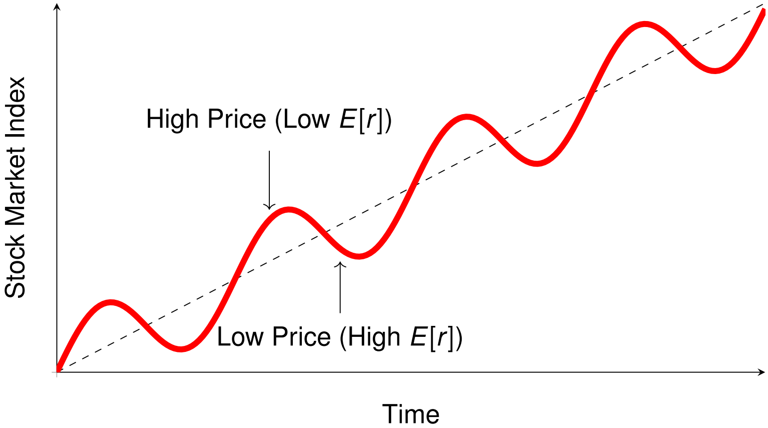
- RRT changes with wealth. Wealth goes up, are you more or less risk tolerant?

Definitions of SAA, TAA, & IAA

$$\boxed{X_{St} = K_0 + K_{1t} \times RRT(W_t)}$$

- **SAA Investing:** $\mathbb{E}[r]$'s, σ's, and RRT are unchanging over time
  - Thus, K_0, K_{1t} and $RRT(W_t)$ constant $\implies$ X_{St} X_{Bt} are the same at each date t.
- **TAA Investing:** Investors alters their $\mathbb{E}[r]$'s and σ's overtime. RRT stays constant
  - Thus, K_{0t} and K_{1t} changes at each date $t \implies X_{St}$ and X_{Bt} change
- **IAA Investing:** $RRT(W_t)$ changes at each date t.
  - Increases in W_t means risk tolerance increases $\implies RRT(W_t)$ increases
  - Investors are more aggressive when they are wealthier
  - Thus, higher W_t means bigger RRT means bigger X_{St}

TAA Example: Negative Serial Correlation (Mean Reversion)



- When do we know that we are at a peak/trough?!?!

Generalize TAA Methodology: Forecast $\mathbb{E}_t[r_S]$

- How do we "predict" returns?
- Run **predictive** regressions:

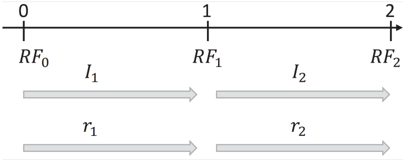
$$r_{s,t+1} = a + b_1 PV_{1,t} + b_2 PV_{2,t} + e_{t+1}, \quad \text{where } PV\text{s are "predictor variables"}$$

  - estimate $\widehat{a}$, $\widehat{b}_1$, and $\widehat{b}_2$ using a desired frequency (monthly, quarterly, annual, 4-years)
  - $\mathbb{E}[r_s] = \widehat{a} + \widehat{b}_1 PV_1 + \widehat{b}_2 PV_2$

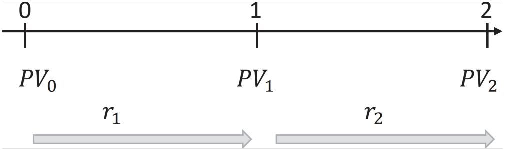
Time	**Predictor variables**		**Stock market return**
	$PV_{1,t}$	$PV_{2,t}$	$r_{S,t+1}$
$t = 1$	$PV_{1,1}$	$PV_{2,1}$	$r_{S,2}$
$t = 2$	$PV_{1,2}$	$PV_{2,2}$	$r_{S,3}$
$t = 3$	$PV_{1,3}$	$PV_{2,3}$	$r_{S,4}$
...	...	...	...
$t = 25$	$PV_{1,25}$	$PV_{2,25}$	$r_{S,26}$

SAA vs TAA modeling

- SAA $\Longrightarrow$ MRF model: Risk Factor Shocks and Returns are **coincident**!

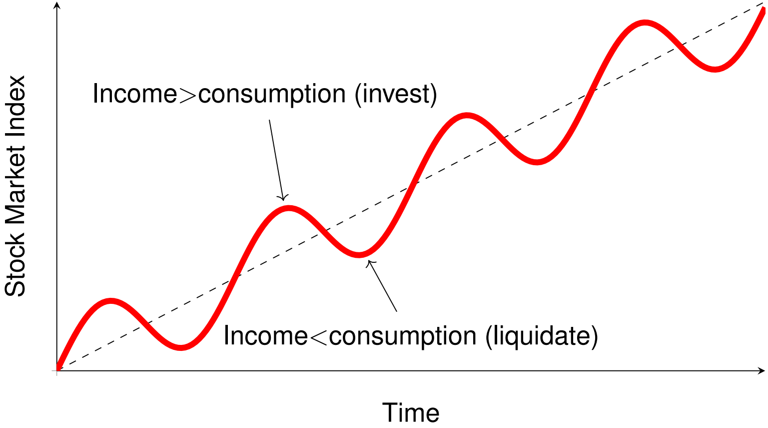


- TAA $\Longrightarrow$ Forecasting model: PV_t **precedes** returns, r_{t+1}!



Fama-French (1989) Predictive Model (FF)

- "Consumption Smoothing" $\Longrightarrow$ time-varying risk premium



Fama-French (FF) Predictive Model

FF Predictor Variables:

1. **DP** = dividend yield on all NYSE stocks (dividend/price)
   - Since dividends are fairly constant, DP is inversely related to price levels
   - Low P $\Longrightarrow$ **High DP** $\Longrightarrow$ **high** $\mathbb{E}[r_S]$
2. **DEF** = "default" spread/risk = credit spread (similar to CRR)
   - Corporate yield less Government yield (Treasuries)
   - **High DEF** $\Longrightarrow$ Corporate Bond prices low relative to Treasury prices $\Longrightarrow$ high corporate yields $\Longrightarrow$ high default rates $\Longrightarrow$ **high** $\mathbb{E}[r_S]$
3. **TERM** = Term spread (slope), similar to CRR
   - Long term Treasury yield$-$Short term Treasury yield
   - Interest rate risk / slope of the yield curve
   - Trough = high TERM, Peak = low or negative TERM
   - **High TERM** $\Longrightarrow$ Economic trough $\Longrightarrow$ **high** $\mathbb{E}[r_S]$

Fama-French (FF) Predictive Model

Looked at predicting returns for 4 asset classes:

1. Aaa = long-term riskless bonds
2. LG = long-term, low-grade, i.e., risky bonds
3. VW = a market cap value weighted stock portfolio (large cap stocks)
4. EW = an equally weighted stock portfolio (small cap stocks)

- Risk spectrum from low to high: Aaa $<$ LG $<$ VW $<$ EW
- Aaa and LG will have interest rate risk. VW and EW as equities may not
- Predictive horizon:
  - Monthly
  - Quarterly
  - Annual
  - 4-year

Fama-French (FF) Predictive Model

- Correlations:
  - DP and DEF are very high 75%
    - Why correlated? What do they have in common?
    - Both are proxies for risky asset prices and they move together
    - Not good to include both at the same time in a predictive regression!
  - TERM behaves differently than DP and DEF
    - time variation in TERM closely related to business cycle.
    - time variation in DP and DEF **persist** beyond business cycles.

Fama-French (FF) Predictive Model Regression Results (Table 3)

$$r_j - r_f = a + b\,PV_1 + c\,PV_2, \quad j = AAA : \text{long-term riskless bonds}$$

	AAA - Asset Class Regression (Monthly data 1941-1987)					
	PVs = DP and TERM			**PVs = DEF and TERM**		
	b	**c**	**R-squared**	**b**	**c**	**R-squared**
Monthly	0.13	0.25	4%	0.23	0.22	4%
t-stat	2.75	2.77		1.87	2.81	
Quarterly	0.36	0.62	6%	0.57	0.55	5%
t-stat	1.91	1.51		1.22	1.51	
One Year	0.4	3.64	39%	1.42	3.25	39%
t-stat	0.75	4.74		1.08	4.41	
Four Years	2.41	3.73	9%	13.4	3.09	34%
t-stat	3.78	1.11		1.85	0.98	

Fama-French (FF) Predictive Model Regression Results (Table 3)

$$r_j - r_f = a + b\,PV_1 + c\,PV_2, \quad j = LG: \text{long-term risky bonds}$$

	LG - Asset Class Regression (Monthly data 1941-1987)					
	PVs = DP and TERM			**PVs = DEF and TERM**		
	b	**c**	**R-squared**	**b**	**c**	**R-squared**
Monthly	0.3	0.31	5%	0.84	0.29	6%
t-stat	3.82	3.32		3.89	2.99	
Quarterly	0.94	0.77	10%	2.72	0.68	11%
t-stat	3.28	1.89		3.25	1.64	
One Year	3.33	3.27	35%	11.2	3.03	45%
t-stat	3.91	4.1		4.37	4.41	
Four Years	12.7	3.27	51%	40.2	2.92	71%
t-stat	4.21	1.45		11.5	1.05	

- Horizon increases $\Longrightarrow$ DP & DEF have improving t-stats
- Horizon increases $\Longrightarrow$ TERM has decreasing t-stats

Fama-French (FF) Predictive Model Regression Results (Table 3)

$$r_j - r_f = a + b\,PV_1 + c\,PV_2, \quad j = VW : \text{large cap stocks}$$

	VW - Asset Class Regression (Monthly data 1941-1987)					
	PVs = DP and TERM			PVs = DEF and TERM		
	b	c	R-squared	b	c	R-squared
Monthly	0.4	0.48	3%	0.52	0.46	2%
t-stat	2.88	3.29		1.43	3.21	
Quarterly	1.31	1.13	6%	2.18	1.09	4%
t-stat	2.93	2.17		1.61	2.03	
One Year	5.49	1.64	16%	11	1.75	9%
t-stat	3.45	0.94		2.12	0.99	
Four Years	18.5	-2.4	60%	42	-1.6	43%
t-stat	5.26	-0.75		3.94	-0.4	

- Horizon increases $\Longrightarrow$ DP & DEF have improving t-stats
- Horizon increases $\Longrightarrow$ TERM has decreasing t-stats

Fama-French (FF) Predictive Model Regression Results (Table 3)

$$r_j - r_f = a + b\,PV_1 + c\,PV_2, \quad j = EW : \text{small cap stocks}$$

	EW - Asset Class Regression (Monthly data 1941-1987)					
	PVs = DP and TERM			PVs = DEF and TERM		
	b	**c**	**R-squared**	**b**	**c**	**R-squared**
Monthly	0.53	0.51	3%	0.91	0.48	2%
t-stat	2.99	2.97		1.77	2.83	
Quarterly	1.78	1.17	6%	3.7	1.08	4%
t-stat	3.03	1.82		1.84	1.62	
One Year	7.96	1.33	18%	18.6	1.31	13%
t-stat	3.67	0.66		2.79	0.61	
Four Years	23.4	-2.67	50%	57.4	-1.97	42%
t-stat	3.18	-0.64		3.78	-0.49	

- Horizon increases$\Longrightarrow$DP & DEF have improving t-stats
- Horizon increases$\Longrightarrow$TERM has decreasing t-stats

Fama-French (FF) Predictive Model Summary Results

- Almost all the "b" and "c" regression coefficients are positive (as predicted)
- Risky assets (LG, VW and EW)
  - Horizon increases $\Longrightarrow$ DP & DEF have improving t-stats
  - Horizon increases $\Longrightarrow$ TERM has decreasing t-stats
  - Conclusion: DP and DEF are better predictors of risk asset returns at longer horizons. TERM does not get better as horizon lengthens.
  - Consistent with the pattern of persistence for DP and DEF

Fama-French (FF) Predictive Model Summary Results

- At 1-year and 4-year horizons
  - DP and DEF coefficients increase along the risk spectrum, LG$\Longrightarrow$VW$\Longrightarrow$EW
  - TERM coefficients not sensitive to risk of asset class
  - Conclusion: DP and DEF are better predictors for riskier assets, TERM's power of prediction does not depend on risk

Coefficients (b, c) from 4-year regression

	DP	DEF	TERM
LG	12.7	40.2	3.10
VW	18.5	42.0	-2.00
EW	23.4	57.4	-2.32

Note: in the table TERM is the average of the two regressions

Fama-French (FF) Predictive Model Summary Results

- Expected return on both stocks and bonds are lower when economic conditions are strong and higher when economic conditions are weak.
- DP, DEF, and TERM track component of expected returns that are **common** across assets.
- Findings consistent with “consumption smoothing” hypothesis
- But also consistent with variation in capital investment opportunity (see Chen’s model)

The Chen (1991) Predictive Model

- **Main idea**: relate the predictive variable in FF to the **business cycle**
  - The business cycle is related to the predictor variables
  - Predictor variables are related to the Equity Risk Premium (both Chen and FF)
  - **Therefore:** business cycle and Equity Risk Premium are related
- **Hypotheses**:
  - At business cycle **trough** (vice-versa at peak)
    - **Future** GNP growth **is expected** to be high (that's why we are in a trough!)
    - Short-term rates are low / yield curve steep
    - Equity expected return is **high**
  - At Business cycle **peak** (vice-versa at a trough)
    - **Past** GNP growth **has been** high (that's how we got to a peak!)
    - Recent Industrial production has been high
    - Equity expected return is **low**

Chen Predictive Model-Quarterly data (1954-1986)

- Predictor Variables (PVs):
  - DP_{t-1} Dividend yield on NYSE stocks (FF's DP)
  - URP_{t-1} 10-year all-Corp bond yield$-$10-year Aaa bond yield (FF's DEF)
  - UTS_{t-1} 10 year Gov't bond yield$-$T-bill yield (FF's TERM)
  - YPL_{t-1} **Annual growth in industrial production**
  - TB_{t-1} **30-day T-bill yield**
- Macro Variable:
  - $GNPG_t$ Quarterly GNP growth
- Variable to predict: Market Return:
  - EVW_t Value-weighted NYSE stock portfolio - T-bill yield (FF's VW)

    $EVW_t = a + b \times PVs_{t-1} + e_t$

Chen Predictive Model–Results I (Table III)

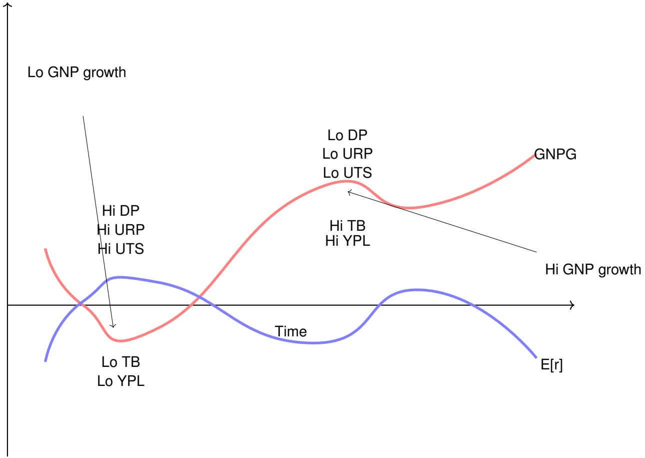
- **Predictabilty regression:** Excess Market return vs. one-quarter lagged PVs

Predictive regressions			
	b	**t-stat**	**R-squared**
DP	2.13	2.86	4%
URP	5.58	2.22	3%
UTS	1.86	2.85	8%
YPL	−0.32	−2.75	6%
TB	−0.47	−2.14	3%

- Positive DP, URP, and UTS. Same as FF
- High YPL$\Longrightarrow$low Equity Risk Premium
  - Industrial production is highest at business cycle peaks, when stock prices are usually high$\Longrightarrow$Expect slower stock market growth in the future
- High TB$\Longrightarrow$low Equity Risk Premium
  - Short rates are highest at business cycle peaks, when stock prices are usually high$\Longrightarrow$Expect slower stock market growth in the future

Chen Predictive Model – Equity risk premium and Business cycle

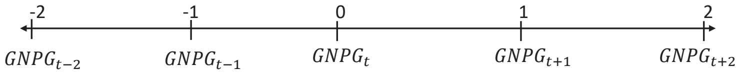
- Time-varying Equity Risk Premium around the business cycle



Chen Predictive Model – Lead/Lag analysis

- How does macro variables (GNP growth) relate to predictor variables?

$$GNPG_{t+lead} = a + b \times PVs_{t-1} + e_{t+lead}, \quad lead = -4, -3, \ldots, 0, 1, \ldots 4$$



- When *lead* $= -1$, contemporaneous relation, as in CRR
- When *lead* < -1, we are using the “future” to predict the “past”!
  - This tells us how the past GNP growth relates to current PV

Chen Predictive Model–Results II (Table IV)

$$GNPG_{t+lead} = a + b \times PVs_{t-1} + e_{t+lead}, \quad lead = -4, -3, \ldots, 0, 1, \ldots 4$$

Table IV			Coefficients and t-stats							
Lead	**YPL**	**t**	**URP**	**t**	**DP**	**t**	**UTS**	**t**	**TB**	**t**
-4	0.07	7.2	-1.05	**-3.5**	-0.19	**-1.6**	-0.08	-0.8	-0.01	-0.2
-3	0.11	14.3	-1.24	**-3.7**	-0.32	**-2.6**	-0.15	-2	0.0	0.1
-2	0.11	14.3	-1.51	**-4.7**	-0.43	**-3.8**	0.02	0.2	-0.03	-0.6
-1	0.09	7.4	-1.41	**-4.3**	-0.46	**-4.0**	0.03	0.4	-0.03	-0.8
0	0.04	2.5	-1.25	**-3.4**	-0.44	**-3.7**	0.25	**3.3**	-0.08	**-2.4**
1	-0.01	-0.4	-1.04	**-2.4**	-0.27	**-2.4**	0.32	**4.9**	-0.1	**-3.0**
2	-0.02	-1.7	-0.09	-0.2	-0.06	-0.5	0.33	**4.6**	-0.1	**-3.0**
3.0	-0.03	-2.1	0.09	0.2	0.01	0.1	0.27	**3.0**	-0.08	**-2.1**

- DP and UPR do not have forecasting power for future GNP growth
- UTS uncorrelated to past GNP but has forecasting power

Chen Predictive Model - Conclusions

- The business cycle (GNPG) is related to the predictor variables PV
  - High DP and high URP (“price” variables): **past** GNPG has been low
  - High TB and low UTS (“interest rate” variables): **predict** low GNPG in the future
- DP and URP are price variable that look **back** at GNP growth
  - Lower current levels of GNP are associated with lower stock and bond prices, thus leading to high DP and URP
- TB and UTS are interest rates variables that look **forward** at GNP growth.
- As in FF, high DP, URP, UTS signals higher future ERP
- High TB predicts low ERP

A word of caution on TAA

- TAA can be
  - A “contrarian” strategy: look for reversals (e.g., Value)
  - A “momentum” strategy: look for continuation (e.g., Momentum)
- Timing the market is hard! TAA is hard to do!

Estimating the Equity Risk Premium

- Estimating the equity risk premium is a hard problem!
- Like all hard problems it helps to break it down into parts hoping that some are easier to tackle than others
- Similar idea as in a Fermi's problem:
  - How many piano tuners are there in Chicago?

TAA: Estimating the Equity Risk Premium Directly

$$\text{Equity Risk Premium } (ERP) = \mathbb{E}[r_s] - r_f = \mathbb{E}[r_s] - \mathbb{E}[r_b]$$

- Where we can think of risk free rate as a riskless bond portfolio b with $\mathbb{E}[r_b]$
- Bond portfolio will have a maturity equal to the forecast horizon of ERP. Often we take $\mathbb{E}[r_b] =$ yield to maturity

$$\mathbb{E}[r_s] = \frac{D_1 + R_1 + MVE_1 - MVE_0}{MVE_0}$$

- D_1 = total dividends paid on the index
- R_1 = total amount spent by companies on stock **repurchases** in the index
- MVE_0 = total market value of index at start of period, $t = 0$
- MVE_1 = total market value of index at end of period, $t = 1$

TAA: Estimating the Equity Risk Premium Directly

- Share Repurchases: Define $R_1 = -\Delta(S) \times k$ where
  - $\Delta(S) = S_1 - S_0$ with S_t number of shares at time t.
  - k: price at which shares are repurchased
- Let $MVE_t = P_t \times S_t$ where P_t is the price per share at time t.
- Then we can rewrite

$$\mathbb{E}[r_s] = \frac{D_1 + R_1 + MVE_1 - MVE_0}{MVE_0} = \underbrace{\frac{d_1}{P_0}}_{\text{Div. yield}} + \underbrace{\frac{\Delta(S)}{S_0} \times \frac{P_1 - k}{P_0}}_{\text{Repurchase adjustment, } ra} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital gains}}$$

with d_1 = dividends per share, D_1 / S_0

TAA: Estimating the Equity Risk Premium Directly

$$\mathbb{E}[r_s] = \underbrace{\frac{d_1}{P_0}}_{\text{Div. yield}} + \underbrace{\frac{\Delta(S)}{S_0} \times \frac{P_1 - k}{P_0}}_{\text{Repurchase adjustment, } ra} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital gains}}$$

- The key component in estimating $\mathbb{E}[r_s]$ is the capital gain return
- Rewrite $P_t = \frac{P_t}{E_t} \times E_t = p_t \times E_t$, where $E_t =$ earnings per share, $p_t \equiv \frac{P_t}{E_t}$.
- Then capital gains can be re-express as follows

$$\text{Capital Gains} = \frac{P_1 - P_0}{P_0} \approx g + i + rp$$

  - $g =$ expected real growth in earnings
  - $i =$ expected inflation ($\Longrightarrow E_1 = E_0(1 + g + i)$)
  - $rp =$ repricing effect $\frac{p_1}{p_0} - 1 =$ change in the P/E ratio over the period ($\Longrightarrow p_1 = p_0(1 + rp)$)

TAA: Estimating the Equity Risk Premium Directly

- So we have

$$ERP = \mathbb{E}[r_s] - \mathbb{E}[r_b] = \underbrace{\frac{d_1}{P_0} + ra}_{\text{Income return}} + \underbrace{g + i + rp}_{\text{capital gains}} - \mathbb{E}[r_b]$$

- Can we forecast the individual components?
- Long-Run example (from BlackRock)
  - Income return = dividend yield + repurchase ($\frac{d_1}{P_0} + ra$) $= 4.4\%$
  - Real Earnings Growth (g) $= 1.7\%$
  - Inflation (i) $= 3.1\%$
  - P/E repricing (rp) $= 1.5\%$
  - ERP $= 4.4\% + 1.7\% + 3.1\% + 1.5\% - bondyield = 10.7\% - 4.0\% = 6.7\%$

Estimating the repricing effect

- Perhaps the most difficult estimate is the P/E repricing
- How can we forecast the P/E miultiple?
- From the Gordon's growth formula we know

$$P_0 = \frac{D_1}{r-g} = \frac{xE_1}{r-g} \implies \frac{P_0}{E_1} = \frac{x}{r-g}$$

  where
  - x = payout ratio
  - r = risk-adjusted discount rate, and g = perpetual growth rate of earnings
- So you expect a positive repricing effect if you expect
  - Payout ratio x to increase
  - Risk to decrease (reflected in r)
  - Growth to increase (reflected in g)

TAA: S&P 500 P/E ratio and Shiller's P/E ratio

- S&P P/E ratio. Based on trailing twelve month earnings.
- Shiller's P/E ratio. Based on average inflation-adjusted earnings from the previous 10 years. Cyclically Adjusted PE Ratio (CAPE Ratio).

An example of TAA implementation

Recall: Sharpe's solution for portfolio choice

$$X_{St} = K_0 + K_{1t} \times RRT(W_t)$$

Where,

$$X_{St} = K_0 + K_{1t} \times RRT(W_t), \quad \text{where } RRT(W_t) \text{ is Relative Risk Tolerance}$$
$$K_0 = \frac{\sigma_B^2 - \sigma_{BS}}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}}, \quad K_{1t} = \frac{\mathbb{E}_t[r_S] - \mathbb{E}_t[r_B]}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}} > 0 \text{ if } \mathbb{E}_t[r_S] > \mathbb{E}_t[r_B]$$

Assuming:

- Bonds with $\mathbb{E}[r_B]$ and σ_B, Stocks with $\mathbb{E}[r_S]$, σ_S, and covariance σ_{BS}
- RRT is increasing in W_t: wealth goes up, investor more aggressive

TAA: Simple TAA implementation

- Data:

$$\mathbb{E}[r_B] = 5\%, \sigma_B = 10\%$$
$$\mathbb{E}[r_S] = 10\%, \sigma_B = 20\% \text{ and correlation } \rho_{BS} = 0.6$$

- **Backing out RRT.** Assume portfolio weights: $X_B = 50\%$, $X_S = 50\%$. Using Sharpe's model we get

$$K_0 = \frac{\sigma_B^2 - \sigma_{BS}}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}} = -\frac{0.002}{0.026} = -0.077$$
$$K_{1t} = \frac{\mathbb{E}[r_S] - \mathbb{E}[r_B]}{\sigma_B^2 + \sigma_S^2 - 2\sigma_{BS}} = \frac{0.05}{0.26} = 1.923$$

  At the SAA mix: $X_S = 50\% = -0.077 + 1.923 \times RRT \implies RRT = 0.3$

- We want to place TAA on top of our long-run SAA.

TAA: Simple TAA implementation

- The client's TAA mix can be calculated for any ERP forecast
- As $\mathbb{E}_t(r_S) - \mathbb{E}[r_B]$ changes, K_{1t} changes and so X_S changes
  - If $\mathbb{E}[r_S] = 12\% \implies K_{1t} = \frac{0.070}{0.026} = 2.692$ and $X_S = 73.1\%$
  - If $\mathbb{E}[r_S] = 8\% \implies K_{1t} = \frac{0.030}{0.026} = 1.154$ and $X_S = 26.9\%$
- TAA will move equity weights above and below our SAA weight of 50% for ERP forecasts above and below 10%.

COMM 475 – Investment Policy
Style Investing

Lorenzo Garlappi

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Outline

- Simple investment factors
- Dynamic investment factors
- Style Investing
- Recent empirical evidence

Simple investment factors

- Equity
- Bonds

- Cheap index management delivers these **long-only** factors at essentially zero cost in very large size
- Based on market capitalization weights – represent the aggregate **average** investor

Dynamic factors

- Theory and long investing experience have identified classes of assets that have consistently higher (or lower) average returns than the market portfolio
- Dynamic factors take **long** position in securities with similar characteristics, which tend to comove with each other, and offsetting **short** positions in securities with the opposite characteristics
- The market portfolio, by definition, has no dynamic factor exposure
- The average investor, who holds the market, does not practice dynamic factor investing!

Dynamic factors

- Value-Growth Premium = Value stocks minus growth stocks
- Size Premium = Small stocks minus large stocks
- Momentum Premium = Winning stocks minus losing stocks
- Illiquidity Premium = Illiquid securities minus liquid securities
- Credit Risk Premium = Securities with high default risk minus securities with low default risk
- Low Volatility Risk Premium = Stocks with low volatility minus stocks with high volatility

Style Investing

Starting in the 1980's increasing findings of "anomalies" (or nonzero Lambdas as per our MRF models) relative to CAPM

- Size, Book to Market (B/M), Earnings to Price (E/P), etc.
- Fama and French (1992, 1993) are two papers that consolidated knowledge in this area (Fama-French Three-Factor MRF)
- Today a vast array of "anomalies" have been presented in the finance literature
- Look at Ken French's website to get an idea of the portfolios used to "beat" the CAPM.

Style Investing

- Practitioners developed many “Style products” to address CAPM failures.
- Empirical challenges to the CAPM and the market being represented by indices like the Dow Jones Industrials, or the S&P 500 led to the creation of “Style Indices”
- Some of the original style indices
  - Ibbotson Small stock index (Roger Ibbotson and Rex Sinquefield, 1976)
    - Stocks, Bonds, Bills, and Inflation (SBBI)
  - Russell 2000 index (Frank Russell Company , 1984)
  - Russell Growth and Value indexes (1987)

Style Investing

- The first empirical attempts at factor models focused on economically motivated factors (Recall our CRR MRF)
  - Chen, Roll, and Ross (1986): industrial production, expected inflation, unexpected inflation, credit spread, term spread
- Using corporate data, Fama and French (1992, 1993) provide robust empirical results for two **styles** (or factors): **Size and Book to Market.** Confirming Ibbotson and Russell. Their methods using quintile analysis that became industry and academic standard for data analysis
- Today, factor investing has exploded across the industry with many active strategies, ETFs, "**Smart Beta**" strategies, Thematic or Direct indexing, etc. (see BlackRock presentation)

Style Investing

- Three-Factor MRF (Fama-French) return generating function (RGF):

$$r_p - r_f = \alpha_p + \beta_{1p}\left(r_m - r_f\right) + \beta_{2p}(r_v) + \beta_{3p}(r_s) + \varepsilon_p$$

  Where
  - α_p is the "alpha" of the portfolio (the constant in linear regressions, also unique excess return for the portfolio)
  - r_m is the return of the equity market
  - r_v **long value stocks and short growth** stocks (Value Bias)
  - r_s **long small-cap stocks and short large-cap** stocks (Size Bias)
  - r_f risk-free rate
- Style investing usually refers to **within** asset classes, we will focus our MRF on the equity portfolio

Style Investing

- The Expected Return for this Three-Factor MRF would be:

$$\mathbb{E}[r_p] - r_f = \alpha_p + \beta_{1p}\lambda_1 + \beta_{2p}\lambda_2 + \beta_{3p}\lambda_3$$

Where

  - λ_j? CAPM assumes $\lambda_2 = \lambda_3 = 0$
  - Blackrock, Fama-French and others show there **IS** additional return, $\lambda_j > 0$ over long periods of time

Style Investing

- Recall our original work for Multi-Risk Factor (MRF)
  - GDP? Interest Rates?
    - Important because it relates to our human capital
    - “risk-enhancing” (positive lambda)
    - “insurance” (negative lambda)
  - If $\lambda_2 > 0$ and $\lambda_3 > 0$ then our human capital models:
    - Correlation with **Value** stock return with human capital gains is positive
    - Correlation with **Small-cap** stock return with human capital gains is positive
- Social & Technological changes mean Tech companies now dominate overall employment?? i.e. Human Capital now dominated by **Growth** / **Large Cap** stocks
  - $\lambda_2 < 0$ and $\lambda_3 < 0$??

Style Investing

- **Value** (see Blackrock's presentation), $\lambda_2 > 0$
  - Risk-based explanation
    - Value stocks have more fixed costs - "rewarded risk"
    - Higher operating leverage
    - More volatility $\Longrightarrow$ investors expect to be compensated higher expected return
  - Behavioural Bias explanation
    - Stocks with recent poor returns are overlooked (value)
    - Market focuses on popular (growth) stocks
    - Value stocks undervalued $\Longrightarrow$ higher expected return
    - Growth stocks overvalued $\Longrightarrow$ lower expected return

Style Investing

- **Size** (see Blackrock's presentation), $\lambda_3 > 0$
  - Risk-based explanation
    - Small Cap stocks are more "fragile" to economic swings- "rewarded risk"
    - More volatility $\implies$ investors expect to be compensated higher expected return
  - Behavioural Bias explanation
    - Small cap stocks are less "known"
    - Market focuses on large-cap stocks
    - Small Cap stocks undervalued $\implies$ higher expected return
    - Large cap stocks overvalued $\implies$ lower expected return
  - Empirical evidence (like Blackrock, Fama-French 3 factor model) shows
    - $\lambda_2, \lambda_3 > 0$

Style Investing

- Using our MRF return generating function (RGF), equity fund manager's actual return are coming from:

$$r_p - r_f = \alpha_p + \beta_{1p}\big(r_m - r_f\big) + \beta_{2p}(r_v) + \beta_{3p}(r_s) + \varepsilon_p$$

$r_p - r_f$:

- Market Beta: $\beta_{1p}(r_m - r_f)$
- "Smart" Beta $\beta_{2p}(r_v) + \beta_{3p}(r_s)$
- Stock Selection (alpha): $\alpha_p + \varepsilon_p$

– What we want is to find a manager with $\alpha_p > 0$ (manager skill) but stock selection also includes our ε_p which is noise, so hard to measure skill

Style Investing: Recent Empirical Evidence

MSCI Canada Value Index
MSCI Canada Growth Index
MSCI Canada Small Cap Index
MSCI Canada Standard Core Index

- Value and Growth defined by 8 financial characteristics from balance sheet and income statement
- Large Cap is largest 70% of total market, small cap the rest
- Standard Core (like SP/TSX index), 2,050 of the largest Canadian stocks

Style Investing: Recent Empirical Evidence

- Because HML (value) and SMB (size) are based on traded asset their λ are easy to estimate
  - $\hat{\lambda}_{HML}$: Average return of value minus growth indices
  - $\hat{\lambda}_{SMP}$: Average return of small minus large cap indices
- Historically these $\hat{\lambda}$ have been positive

COMM 475 – Investment Policy
Insured Asset Allocation (IAA)

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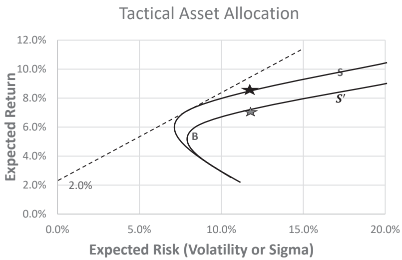
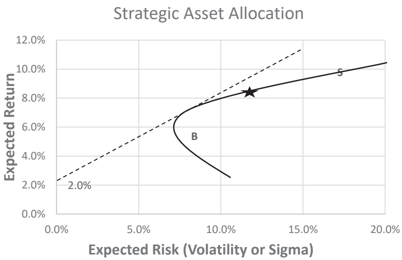
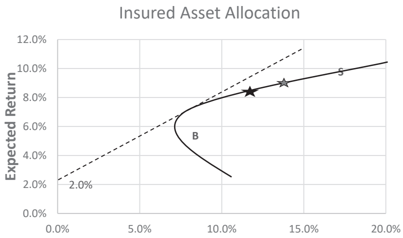
Outline

- Review of definitions: SAA, TAA, IAA
- Why do asset allocation weights change under IAA?
- Criteria for IAA success
- Characterizing risky asset dynamics – The Binomial tree model
- Examples of IAA strategies
  - Buy & Hold
  - Stop-loss
  - Market-traded Puts and Calls
  - Synthetic options created by dynamic trading strategies
  - Constant Proportion Portfolio Insurance

Review: SAA, TAA, and IAA







$$X_{St} = K_0 + K_{1t} * RRT(W_t)$$

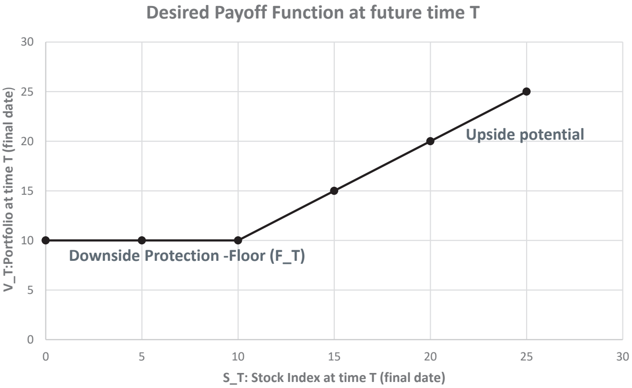
Review of definitions: SAA, TAA, and IAA

- **SAA Investing:** $\mathbb{E}[r]$'s, σ's, and RRT are unchanging over time
  - Thus, X_{St} X_{Bt} are the same at each date t.
- **TAA Investing:** Investors alters their $\mathbb{E}[r]$'s, σ's overtime. RRT stays constant
  - Thus, K_{1t} changes at each date $t \Longrightarrow X_{St}$ and X_{Bt} change
- **IAA Investing:** $RRT(W_t)$ changes at each date t.
  - Increases in W_t means risk tolerance increases $\Longrightarrow \mathrm{RRT}(W_t)$ increases
  - Investors are more aggressive when they are wealthier
  - Thus, higher W_t means bigger RRT means bigger X_{St}

Insured Asset Allocation Overview

- Also a Dynamic Asset Allocation
- Need to define Criteria for IAA Success
- Rely on a dynamic model for risky assets, e.g., Binomial Tree
- Portfolio weights change because agents' risk tolerance (RRT) change overtime

IAA Example: Downside protection of a portfolio



F: Floor, S: Stock Index, V: Value of Portfolio, Desired Payoff: $V_T = \max\{F_T, S_T\}$

Criteria for IAA Success

Let S_T : Stock index at time T, V_T : Portfolio Value, and F_T : Portfolio floor

1. **Downside Protection**: Probability $(V_T < F_T) = 0$
2. **Full Upside Potential**: When $V_T \geq F_T$ then $\frac{\Delta V_T}{\Delta S_T} = 1$
3. **Path Independence**: V_T depends only on final stock index level S_T (not before or after)

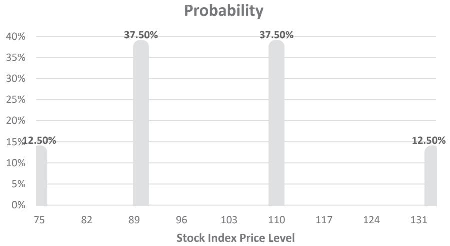
IAA: Binomial Tree for Stock Index

$S_0 = 100$
$u = 1.1$
$d = \frac{1}{u} = 0.09091$
$R_f = 1 + r_f = 1.05$

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
								214.36
							194.87	
						177.16		177.16
					161.05		161.05	
				146.41		146.41		146.41
			133.1		133.1		133.1	
		121		121		121		121
	110		110		110		110	
100		100		100		100		100
	90.91		90.91		90.91		90.91	
		82.64		82.64		82.64		82.64
			75.13		75.13		75.13	
				68.3		68.3		68.3
					62.09		62.09	
						56.45		56.45
							51.32	
								46.65

Binomial Tree - Symmetric Distribution: $q \equiv Pr(up) = Pr(dn)$

$t=0$	$t=1$	$t=2$	$t=3$
			133.10
		121	
	110		110.00
100		100	
	90.91		90.91
		82.64	
			75.13

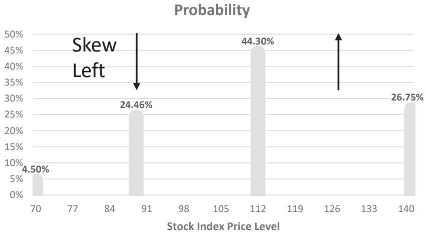


$S_0 = 100$
$u = 1.10$
$d = 1/u = 0.9091$
$R_f = 1 + r_f = 1.05$
$q \equiv Pr(up) = 0.5$

$$Pr(S_3 = 133.10) = q^3 = (0.5)^3 = 12.5\%$$
$$Pr(S_3 = 110.00) = 3 \times q^2 \times (1-q) = 3(0.5)^2 \times (0.5) = 37.5\%$$
$$Pr(S_3 = 90.91) = 3 \times q \times (1-q)^2 = 3(0.5) \times (0.5)^2 = 37.5\%$$
$$Pr(S_3 = 75.13) = (1-q)^3 = (0.5)^3 = 12.5\%$$

Binomial Tree - Left-skewed distribution: $Pr(up) > Pr(dn)$

$t=0$	$t=1$	$t=2$	$t=3$
			141.4
		126	
	112.2		112.2
100		100	
	89.09		89.09
		79.38	
			70.72



$$S_0 = 100$$
$$u = 1.1224$$
$$d = 0.8909$$
$$q \equiv Pr(up) = 0.6443$$

$$Pr(S_3 = 141.4) = q^3 = (0.6443)^3 = 26.75\%$$
$$Pr(S_3 = 112.2) = 3 \times q^2 \times (1-q) = 3(0.6443)^2 \times (0.5) = 44.3\%$$
$$Pr(S_3 = 89.09) = 3 \times q \times (1-q)^2 = 3(0.6443) \times (0.5)^2 = 24.46\%$$
$$Pr(S_3 = 70.72) = (1-q)^3 = (0.6443)^3 = 4.5\%$$

If $q > 0.5 \implies$ left skewness; $q < 0.5 \implies$ right skewness.

Suppose we want to insure portfolio at time $t = 8$

$S_0 = 100$
$u = 1.1$
$d = 0.909$
$R_f = 1 + r_f = 1.05$

$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	**t=8**
								214.36
							194.87	
						177.16		**177.16**
					161.05		161.05	
				146.41		146.41		**146.41**
			133.1		133.1		133.1	
		121		121		121		**121**
	110		110		110		110	
100		100		100		100		**100**
	90.91		90.91		90.91		90.91	
		82.64		82.64		82.64		**82.64**
			75.13		75.13		75.13	
				68.3		68.3		**68.3**
					62.09		62.09	
						56.45		**56.45**
							51.32	
								46.65

Possible IAA strategies

1. Buy & Hold
2. Stop-loss
3. Market-traded Puts and Calls
4. Synthetic options created by dynamic trading strategies
5. Constant Proportion Portfolio Insurance

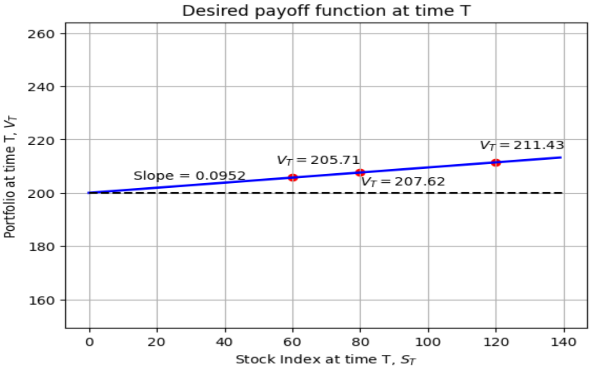
- Data:

$S_0 = \$100$: Initial value of stock index
$V_0 = \$200$: Initial portfolio value
$F_T = \$200$: **Desired value of insured portfolio**
$T = 1$ period
$r_f = 5\%$

IAA: Strategy #1: "Buy and Hold"–Static and Linear

At $t = 0$: Buy $\frac{F_T}{(1+r_f)^T} = \frac{200}{105} = \$190.48$ in T-bills and $V_0 - 190.48 = \$9.52$ in index
$\implies \frac{9.52}{100} = 0.0952$ **units** of the stock index

$S_0 = \$100$
$V_0 = \$200$
$F_T = \$200$
$T = 1$ period
$r_f = 5\%$



IAA: Strategy #1: "Buy and Hold"–Static and Linear

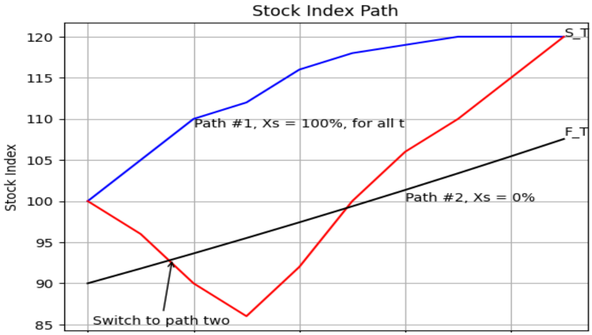
Does Buy & Hold Strategy meet the 3 IAA Criteria?

1. Downside protection: $P(V_T < F_T) = 0$? **Yes**
2. Full upside Potential, $\frac{\Delta V_T}{\Delta S_T} = Slope = 0.0952 < 1$ **NO**
3. Path Independence? **Yes**: V_T depends only on S_T

- Note: Amount of money to satisfy the Floor leaves little for upside potential
- What is the **profit** (not payoff) function from the buy-and-hold strategy?

IAA: Strategy #2: Stop-Loss

- Strategy invests all in stock at the start of period, $X_{s,0} = 100\%$
- Stay in stock unless $V_t < \frac{F_T}{(1+r_f)^{T-t}}$ (Dynamic strategy)
- If so, sell all the stock and buy T-bills, $X_{s,t} = 0$ stay in T-bills to the end T



IAA: Strategy #2: Stop-Loss

Does Stop-Loss Strategy meet the 3 IAA Criteria?

- Downside protection:
  - $P(V_T < F_T) = 0$? **Yes**, assuming no jumps / gaps in S_t
- Full upside Potential:
  - **Yes**, if no switch to T-bill, in which case $V_T > F_T \implies \frac{\Delta V_T}{\Delta S_T} = 1$
  - If switch to T-bill at $t < T$, then $V_T = F_T$ and $\frac{\Delta V_T}{\Delta S_T} = 0$
- Path Independence?
  - **NO**. The value V_T depends on whether the path hit F_t for $t < T$!

IAA: Strategy #2: Stop-Loss

$S_0 = 100,\ V_0 = 200,\ F_3 = 170.10,\ u = 1.2,\ d = 0.9\ 1 + r_f = 1.05$

- Payoff depends upon **HOW** the stock index gets to its S_T

Stock Index, S_t

t=0	*t=1*	*t=2*	*t=3*
			172.8
		144	
	120		129.6
100		108	
	90		**97.2**
		81	
			72.9

Unconstrained portfolio, V_t

	t=0	*t=1*	*t=2*	*t=3*
				345.6
			288	
		240		259.2
	200		216	
		180		194.4
			162	
				145.8
$PV_t(F_3) =$	**146.9**	**154.3**	**162**	**170.1**

Stop-Loss portfolio, V_t

t=0	t=1	t=2	t=3
			345.6
		288	
	240		259.2
200		216	
	180		**194.4**
		162	**170.1**
			170.1

- Suppose S_3=97.20, What is our portfolio V_3? Depends on path:
  - Index does $\{u, d, d\}$ or $\{d, u, d\}$ then no floor is hit and V_3=194.40
  - Index does $\{d, d, u\}$ then we hit the floor $F_2 = 162$ and then V_3=170.10

IAA:Strategy #2: Stop-Loss when Stock Index Price Jumps

$S_0 = 100,\ V_0 = 200,\ F_3 = 198.45$ UP $r = 20\%$ Down $r = -10\%$ $r_f = 5\%$

- Market prices can jump through the floor…think of our trees as "jumps"

Stock Index, S_t

t=0	*t=1*	*t=2*	*t=3*
			172.8
		144	
	120		129.6
100		**108**	
	90		97.2
		81	
			72.9

Unconstrained portfolio, V_t

	t=0	*t=1*	*t=2*	*t=3*
				345.6
			288	
		240		259.2
	200		**216**	
		180		194.4
			162	
				145.8
$PV_t(F_3) =$	**171.43**	**180**	**189**	**198.45**

Stop-Loss portfolio, V_t

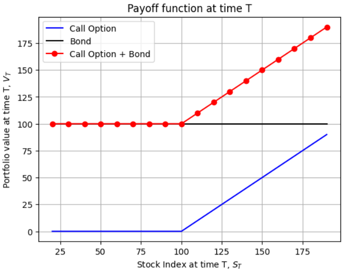
t=0	t=1	t=2	t=3
			345.6
		288	
	240		259.2
200		216	
	180		**194.4**
		189	**198.45**
			198.45

- Suppose $S_2 =$108 and we move to $S_3 = 97.2$ What is our portfolio V_3?
  - We end up **below** the floor because V_t jumped from 216 to 194.4 and were fully invested!

IAA:Strategy #3:Portfolio Insurance with Call options

$S_0 = 100$, $F_T = 100$, $T = 91$-day period $r_f = 10\%$ annual, $\sigma = 20\%$, annual

Black-Scholes **call** option price with strike $K = 100$ is \$5.22



$S(T)$	Tbills	K =100 Call	Total Value
90	100	0	100
95	100	0	100
100	100	0	100
105	100	5	105
110	100	10	110
115	100	15	115

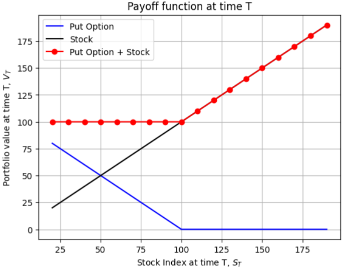
Strategy:

- Buy $K/(1 + r_f)^{1/4} = 97.65$ in T-bills & one Call = \$5.22
- Total Cost is 102.87. Why more than S_0? (See next slide)

IAA:Strategy #3:Portfolio Insurance with Put options

$S_0 = 100$, $F_T = 100$, $T = 91$-day period $r_f = 10\%$ annual, $\sigma = 20\%$ annual

Black-Scholes **put** option price with strike $K = 100$ is 2.87

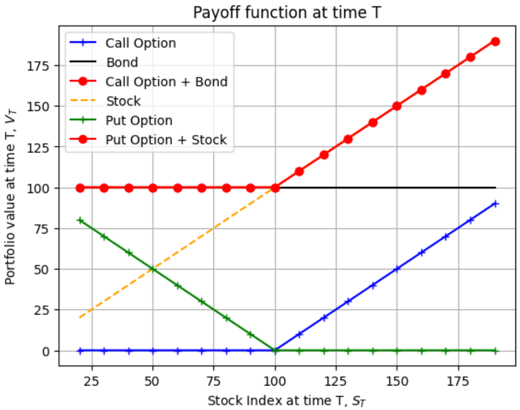


$S(T)$	K =100 Put	Total Value
90	10	100
95	5	100
100	0	100
105	0	105
110	0	110
115	0	115

Strategy:

- Buy Stock Index=100 & Put=2.87
- Total Cost is 102.87
- Put-Call Parity!

IAA: Strategy #3: Portfolio Insurance with Call options



- Put-Call Parity: Stock index + Put Costs the same as Cash+ call

IAA: Strategy #4: Synthetic Options

- IAA strategies involving options might not always be available
- E.g., we cannot always find options with the desired strike and maturity that suits our poftfolio needs
- If options are not available, we can always create **syntentic options** by exploiting the principle of **payoff replication**
- We will do so with an example using the binomial tree model

IAA: Strategy #4: Synthetic Options

$S_0 = 100 \ F_T = 100 \ u = 1.1,\ d = 1/u, 1 + r_f = 1.04,\ p = \frac{1+r_f-d}{u-d} = 0.685714$

Index price

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
								214.36
							194.87	
						177.16		177.16
					161.05		161.05	
				146.41		146.41		146.41
			133.1		133.1		133.1	
		121		121		121		121
	110		110		110		110	
100		100		100		100		100
	90.91		90.91		90.91		90.91	
		82.64		82.64		82.64		82.64
			75.13		75.13		75.13	
				68.3		68.3		68.3
					62.09		62.09	
						56.45		56.45
							51.32	
								46.65

- p is the **risk-neutral probability**:

$$S_t = \frac{pS_{t+1,u} + (1-p)S_{t+1,d}}{1+r_f}, \quad \text{for all } t$$

IAA: Strategy #4: Synthetic Options

Insured portfolio: $C_T = \max\{S_T, F_T\} = F_T + \max\{S_T - F_T, 0\}$, $F_T = 100$

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
								214.36
							194.87	
						177.16		177.16
					161.05		161.05	
				146.41		146.41		146.41
			133.14		133.1		133.1	
		121.2		121.14		121		121
	110.52		110.56		110.48		110	
101.05		101.29		101.53		101.58		**100**
	93.26		93.97		94.92		96.15	
		87.59		89.45		92.46		**100**
			84.81		88.9		96.15	
				85.48		92.46		**100**
					88.9		96.15	
						92.46		**100**
							96.15	
								100

- Payoff of insured portfolio = payoff of call option + 100!
- We can **replicate** this payoff using the index and the bond!

Replicating portfolio

- Because of the binomial structure of the model we know that at each time t the value of a call option can be obtained by holding a **portfolio** of
  - Δ_t units of the index ("hedge ratio")
  - B_t dollars in bonds
- Hence the value of our insured portfolio C_t is

$$C_t = \Delta_t S_t + B_t, \text{for all } t$$

- (Δ_t, B_t) is the **Replicating portfolio**
- If S_t increases: hold more stocks and less bonds
- If S_t decreases: hold less stocks and more bonds

IAA: Strategy #4: Synthetic Options

Hedge ratio: Call option "Delta", Δ_t (# of index shares)

$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$
							1
						1	
					1		1
				1		1	
			0.9943		1		1
		0.9778		0.9793		1	
	0.9478		0.9342		0.9245		1
0.9043		0.8687		0.815		0.7253	
	0.7896		0.6959		0.526		0
		0.5808		0.3815		0	
			0.2767		0		0
				0		0	
					0		0
						0	
							0

- If S_t increases (decreases): hold more (less) stocks

IAA: Strategy #4: Synthetic Options

Dollar bond position B_t: \$ of T-Bills held

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$
							0
						0	
					0		0
				0		0	
			0.8		0		0
		2.89		2.65		0	
	6.26		7.8		8.78		0
10.62		14.42		20.02		29.06	
	21.47		30.7		47.1		96.15
		39.59		57.92		92.46	
			64.02		88.9		96.15
				85.48		92.46	
					88.9		96.15
						92.46	
							96.15

IAA: Strategy #4: Synthetic Options

Option replicating strategy:

- $t = 0$, buy 0.9043 shares at \$100 buy \$10.62 T-Bills. Total=\$101.05
- $t = 1$, $S_1 = 110$:
  - Funds available: $0.9043 \times \$110 + \$10.62 \times 1.04 = \$110.52$
  - Rebalance to $\Delta_1 \times S_1 + 6.26 = \$110.52$. Self-financing
- $t = 1$, $S_1 = 90.91$:
  - Funds available: $0.9043 \times \$90.91 + \$10.62 \times 1.04 = 93.26$
  - Rebalance to $\Delta_1 \times S_1 + 21.47 = \$93.26$. Self-financing

- Note: Buy more of the index when index goes up!
- "Buy high, Sell low"! Convexity!

IAA: Strategy #4: How do we build the replicating portfolio?

Example: $S_0 = 100$, $F_T = 100$, $u = 1.1$, $d = 1/u$, $R_f = 1 + r_f = 1.05$

Index: S_t		
t=0	t=1	t=2
		121
	110	
100		100
	90.91	
		82.64

Insured portfolio: $C_2 = \max\{S_2, F_2\}$		
t=0	t=1	t=2
		121
	110	
101.08		100
	95.24	
		100

Work backward from time $t = 2$.

- At $t = 1, u$ solve

$$\Delta_{1,u} \times 121 + B_{1,u} \times (1.05) = 121$$
$$\Delta_{1,u} \times 100 + B_{1,u} \times (1.05) = 100$$

$$\Delta_{1,u} = 1,\ B_{1,u} = 0 \implies C_{1,u} = \Delta_{1,u} \times 110 + B_{1,u} = 110$$

IAA: Strategy #4: How do we build the replicating portfolio?

Example: $S_0 = 100$, $F_T = 100$, $u = 1.1$, $d = 1/u$, $R_f = 1 + r_f = 1.05$

Index: S_t		
t=0	t=1	t=2
		121
	110	
100		100
	90.91	
		82.64

Insured portfolio: $C_2 = \max\{S_2, F_2\}$		
t=0	t=1	t=2
		121
	110	
101.08		100
	95.24	
		100

Work backward from time $t = 2$.

- At $t = 1, d$ solve

$$\Delta_{1,d} \times 100 + B_{1,d} \times (1.05) = 100$$
$$\Delta_{1,d} \times 82.64 + B_{1,d} \times (1.05) = 100$$

$$\Delta_{1,d} = 0,\ B_{1,u} = 95.24 \implies C_{1,d} = \Delta_{1,d} \times 90.91 + B_{1,d} = 95.24$$

IAA: Strategy #4: How do we build the replicating portfolio?

Example: $S_0 = 100$, $F_T = 100$, $u = 1.1$, $d = 1/u$, $R_f = 1 + r_f = 1.05$

Index: S_t		
t=0	t=1	t=2
		121
	110	
100		100
	90.91	
		82.64

Insured portfolio: $C_2 = \max\{S_2, F_2\}$		
t=0	t=1	t=2
		121
	110	
101.08		100
	95.24	
		100

Work backward from time $t = 2$.

- At $t = 0$ solve

$$\Delta_0 \times 110 + B_0 \times (1.05) = C_{1,u} = 110$$
$$\Delta_0 \times 90.91 + B_0 \times (1.05) = C_{1,d} = 95.24$$

$$\Delta_0 = 0.7732,\ B_0 = 23.76 \implies C_0 = \Delta_0 \times 100 + B_0 = 101.08$$

Replicating portfolio – General formulas

$$
\begin{aligned}
\Delta_t &= \frac{C_{u,t+1} - C_{d,t+1}}{(u-d)S_t} \\
B_t &= \frac{uC_{d,t+1} - dC_{u,t+1}}{(u-d)R_f} \\
C_t &= \Delta_t \times S_t + B_t \\
&= \frac{p \times C_{u,t+1} + (1-p) \times C_{d,t+1}}{R_f}
\end{aligned}
$$

where

$$p \equiv \frac{R_f - d}{u - d}$$

is called the **risk-neutral probability**.

IAA: Strategy #4: Another Synthetic Portfolio Insurance Example

- $S_0 = 100$, $V_0 = 260$, $F_T = 200$, $u = 1.2$, $d = 0.8$ $R_f = 1.04$, $p = \frac{1.04-0.8}{1.2-0.8} = 0.6$.
- **Stop-loss**: $N_0 = \frac{V_0}{S_0} = 2.6$ units of index purchased

S_t

t=0	t=1	t=2	t=3
			172.8
		144	
	120		115.2
100		96	
	80		76.8
		64	
			51.2

unconstrained Wealth, V_t

t=0	t=1	t=2	t=3
			449.28
		374.4	
	312		299.52
260		249.6	
	208		199.68
		166.4	
			133.12
$PV_t(F_T)$	**184.91**	**192.31**	**200**

Stop-Loss, V_t

t=0	t=1	t=2	t=3
			449.28
		374.4	
	312		299.52
260		249.6	
	208		199.68
		166.4	**173.06**
			173.06

Stop loss, N_t

t=0	t=1	t=2
		2.6
	2.6	
2.6		2.6
	2.6	
		0

IAA: Strategy #4: Another Synthetic Portfolio Insurance Example

- $S_0 = 100$, $V_0 = 260$, $F_T = 200$, $u = 1.2$, $d = 0.8$ $R_f = 1.04$, $p = \frac{1.04-0.8}{1.2-0.8} = 0.6$.
- **Portfolio insurance**

S_t

$t=0$	$t=1$	$t=2$	$t=3$
			172.8
		144	
	120		115.2
100		96	
	80		76.8
		64	
			51.2

unconstrained Wealth , V_t

$t=0$	$t=1$	$t=2$	$t=3$
			449.28
		374.4	
	312		299.52
260		249.6	
	208		199.68
		166.4	
			133.12
$= PV_t(F_T)$	**184.91**	**192.31**	**200**

Portfolio Insurance, V_t

$t=0$	$t=1$	$t=2$	$t=3$
			449.28
		374.4	
	312.05		299.52
263.89		249.72	
	218.04		200
		192.31	
			200

$$374.40 = \frac{0.6 \times 449.28 + 0.4 \times 299.52}{1.04}$$

$$249.72 = \frac{0.6 \times 299.52 + 0.4 \times 200}{1.04}$$

$$etc. = etc.$$

IAA: Strategy #4: Another Synthetic Portfolio Insurance Example

Insured portfolio, V_t

$t=0$	$t=1$	$t=2$	$t=3$
			449.28
		374.4	
	312.05		**299.52**
263.89		249.72	
	218.04		**200**
		192.31	
			200

$N_t = \Delta_t \times 2.6$

$t=0$	$t=1$	$t=2$
		2.6
	2.5974	
2.3503		2.5917
	1.7942	
		0

Bond, B_t

$t=0$	$t=1$	$t=2$
		0
	0.36	
28.86		0.92
	74.5	
		192.31

Hedge ratio, Δ_t

$t=0$	$t=1$	$t=2$
		1
	0.999	
0.904		0.9968
	0.6901	
		0

$$N_{2,uu} = \frac{449.28 - 299.52}{(1.2 - 0.8) \times 144} = 2.6 = \Delta_{2uu} \times 2.6$$

$$N_{2,ud} = \frac{299.52 - 200}{(1.2 - 0.8) \times 96} = 2.5917 = \Delta_{2ud} \times 2.6$$

$$etc. = etc.$$

- Hedge ratio Δ_t refer to <u>one unit</u> of the index. $N_t = \Delta_t \times V_0/S_0 = \Delta_t \times 2.6$.
- Note that $V_0 = 260$, so insurance "premium" is $263.89 - 260 = 3.89$.

IAA: Strategy #5: Constant Proportion Portfolio Insurance (CPPI)

- Let D_{st} is \$ investment in stock (s) at date t
- Let F be a constant floor for all t
- Let m is the multiplier (to allow more or less upside potential)

$$D_{st} = m \times (V_t - F)$$

- Must start $V_0 > F$ and CPPI keeps $V_t > F$ for all t

$$X_{st} = \frac{D_{st}}{V_t} = m \times \left(1 - \frac{F}{V_t}\right)$$

- Note $X_{ct} = 1 - X_{st}$ is our cash weight
- This strategy has a "**momentum**" flavor and does not insures against jumps!

IAA: Strategy #5: Constant Proportion Portfolio Insurance (CPPI)

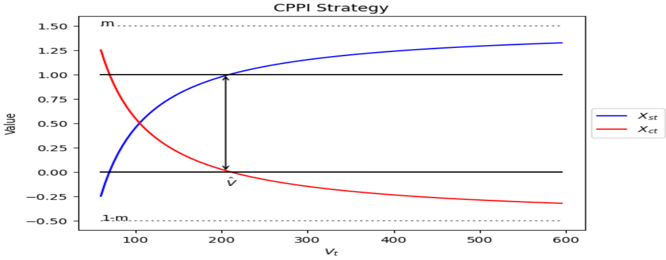


Figure: X_{st} and X_{ct} weights as a function of V_t

$$\lim_{V_t \to \infty} m \times \left(1 - \frac{F}{V_t}\right) = m$$

If $m > 1$ we are levering up our position in the index

Strategy comparison – CPPI with $V_0 = 100$, $m = 2$ and $F = 70$

t	S_t	D_{st}	D_{ct}	N_{st}	$V_t = D_{st} + D_{ct}$	**Desired** $D_{st} = 2 \times (V_t - 70)$
0	100	60	40	0.60	100	60
1	90	54 = 0.6 $\times$ 90	40	0.6	94	48 =2 $\times$ (94-70) < 54 $\Longrightarrow$ Sell
Rebalance (sell)		48	46 =94-48	0.533 = 48/90	94	48
2	80	42.67=0.533 $\times$ 80	46	0.533	88.67	37.34 = 2 $\times$ (88.67-70)< 42.67 $\Longrightarrow$ Sell
Rebalance (sell)		37.34	51.33 = 88.67-37.34	0.4668=37.34/80	88.67	37.34
3	90	42.01 = 0.4668 $\times$ 90	51.33	0.4668	93.34	46.68= 2 $\times$ (93.34-70) > 42.01 $\Longrightarrow$ Buy
Rebalance (buy)		46.68	46.66=93.34-46.68	0.5187=46.68/90	93.34	46.68
4	100	51.87 = 0.5187 $\times$ 100	46.66	0.5187	98.53	57.06 = 2 $\times$ (98.53-70) > 521.87 $\Longrightarrow$ Buy

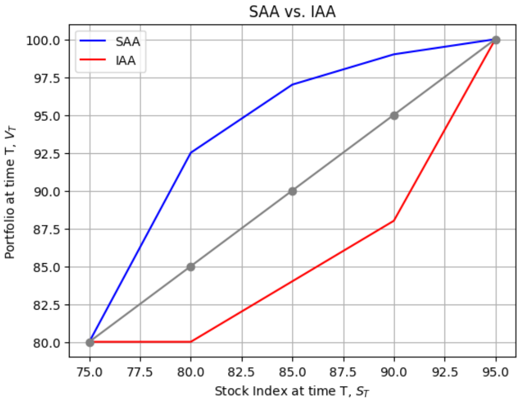
- This strategy is **convex.** It buys after the stock index rises and sells after the stock index falls.
- A **momentum** TAA investor would have the same convex behaviour.

Strategy comparison – Constant-Mix SAA Strategy, 60/40 Rebalancing

t	S_t	D_{st}	D_{ct}	V_t	$N_{st} = D_{st}/S_t$	Actual $w_s = D_{st}/V_t$
0	100	60	40	100	0.6	0.6
1	90	54 = 0.6 × 90	40	94	0.6	0.5745 $\Longrightarrow$ Buy
Rebalance (buy)		56.4 = 0.6 × 94	37.6 = 0.4 × 94	94	0.6267	0.6
2	80	50.13 = 0.6267 × 80	37.6	87.73	0.6267	0.5714 $\Longrightarrow$ Buy
Rebalance (buy)		52.64 = 0.6 × 87.73	35.09 = 0.4 × 87.73	87.73	0.6580	0.6
3	90	59.22 = 0.6580 × 90	35.09	94.31	0.6580	0.6279 $\Longrightarrow$ Sell
Rebalance (sell)		56.59 = 0.6 × 94.31	37.72 = 0.4 × 94.31	94.31	0.6288	0.6
4	100	62.88 = 0.6288 × 100	37.72	100.60	0.6288	62.61 $\Longrightarrow$ Sell

- This strategy is **concave.** It buys after the stock index falls and sells after the stock index rises.
- A **contrarian** TAA investor would have the same convave behaviour.

SAA vs IAA


SAA vs. IAA
SAA
IAA
100.0
97.5
95.0
92.5
90.0
87.5
85.0
82.5
80.0
75.0
77.5
80.0
82.5
85.0
87.5
90.0
92.5
95.0
Portfolio at time T, V_T
Stock Index at time T, S_T

Summary of SAA, TAA, & IAA Strategies

Payoff Shape	Passive	Forecast Driven	Wealth Drive
Linear	SAA “Buy & Hold” no Rebalance		
Concave	SAA “Constant Mix” (Rebalance: buy low, sell high)	Contrarian/Value TAA (buy low, sell high)	
Convex		Momentum TAA (buy high, sell low)	All IAA Strategies

COMM 475 – Investment Policy
Performance Measurement

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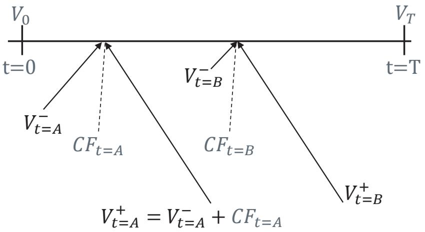
Outline

- Has the Client's objective been achieved?
- Calculating the rate of return on an investment portfolio
  - With intermediate valuations (Geometric return)
  - Without intermediate valuations (IRR, Modified Dietz's formula)
- Practical performance measurement
- Performance measurement from equilibrium models
  - Sharpe Measure / Ratio
  - Jensen Measure
  - Information Ratio
  - Portable Alpha
  - MRF Performance Measure
- Performance attribution analysis
- Assessing TAA skill: the Merton model

Performance Measurement

- Rate of Return
  - With intermediate valuations
    - Geometric Average - “Time-weighted Return”
  - Without intermediate valuations
    - Internal Rate of Return - “Dollar-weighted Return”
    - Modified Dietz Formula
- Notation
  - V_t^- value of a portfolio **before** a cash (in/out)flow CF_t at time t
  - V_t^+ value of a portfolio **after** a cash (in/out)flow at time t: $V_t^+ = V_t^- + CF_t$

Performance Measurement with intermediate valuations



- Returns with Intermediate Valuations: V_t observable.
  - Measure return for all sub-periods

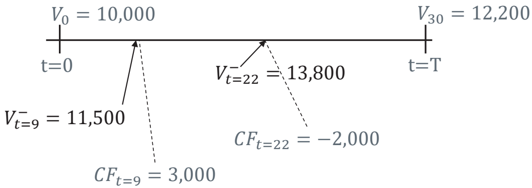
$$r_{t,t+1} = \frac{V^-_{t+1}}{V^+_t} - 1 \implies G = (1 + r_{0,1}) \times (1 + r_{1,2}) \cdots (1 + r_{T-1,T})$$

G is a geometric (time-weighted) return

  - E.g., open-ended mutual fund with daily NAVs (Net Asset Value)

Performance Measurement Example

Timeline of 1 month = 30 days



- $V^{+}_{t=9} = 11,500 + 3,000 = 14,500$; $V^{+}_{t=22} = 13,800 - 2,000 = 11,800$
- $r_{0,9} = \frac{11{,}500}{10{,}000} - 1 = 0.15$; $r_{9,22} = \frac{13{,}800}{14{,}500} - 1 = -0.483$; $r_{22,30} = \frac{12{,}200}{11{,}800} - 1 = 0.0339$
- $G = (1 \underbrace{+0.15}_{=r_{0,9}}) \times (1 \underbrace{-0.483}_{=r_{9,22}}) \times (1 \underbrace{+0.339}_{=r_{22,30}}) - 1 = 0.1316 = 13.16\%$ per month

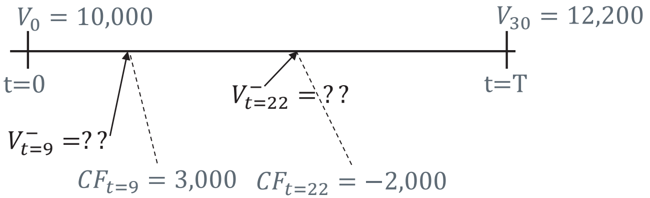
Performance Measurement Example, Mutual Fund NAV

- Assets $A_0 = 10,000$; Units $N_0 = 1,000 \implies NAV_0 = \frac{10,000}{1000} = \$10/\text{unit}$
- $t = 9$: $N^-_{t=9} = 1,000$ & $NAV^-_{t=9} = \frac{V^-_{t=9}}{N^-_{t=9}} = \frac{11,500}{1,000} = \$11.50/\text{unit}$
  $\implies \Delta N_9 = \frac{CF_{t=9}}{NAV_{t=9}} = \frac{3,000}{11.5} = 260.87$ (new units bought)
  $\implies NAV^+_{t=9} = \frac{V^+_{t=9}}{N^+_{t=9}} = \frac{14,500}{1000+\Delta N_9} = \$11.50$ & $N^+_{t=9} = N^-_{t=9} + \Delta N_9 = 1,260.87 = N^-_{t=22}$
- $t = 22$: $NAV^-_{t=22} = \frac{13,800}{1,260.87} = \$10.945 \implies \Delta N_{22} = \frac{-2,000}{10.945} = -182.75$ (units sold)
  $NAV^+_{t=22} = \frac{11,800}{1260.87+\Delta N_{22}} = \$10.945$ & $N^+_{t=22} = N^-_{t=22} + \Delta N_{22} = 1,078.12$
- $t = 30$: $NAV^-_{t=30} = \frac{12,200}{1,078.12} = \$11.316/\text{unit}$
- Hence, return on each share of mutual fund is

$$\text{return} = \frac{11.316}{10} - 1 = 13.16\% \quad \text{per month, the same of } G \text{ computed above}$$

Performance Measurement without Intermediate Valuations

Timeline of 1 month=30 days



- **IRR**–Dollar-weighted Return: The return *IRR* needed to achieve V_T

$$V_0(1+IRR)^T + CF_1(1+IRR)^{T-1} + \ldots + CF_t(1+IRR)^{T-t} \overset{\text{set}}{=} V_T$$

e.g. $10,000(1+IRR)^{\frac{30}{30}} + 3,000(1+IRR)^{\frac{21}{30}} - 2,000(1+IRR)^{\frac{8}{30}} = 12,200$

$\Longrightarrow$ IRR = 10.3856%

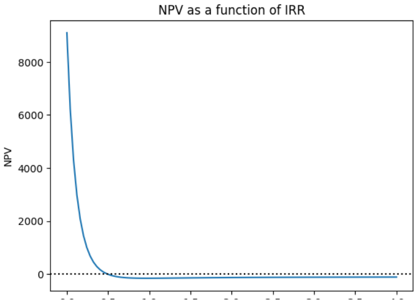
Performance Measurement without intermediate valuations

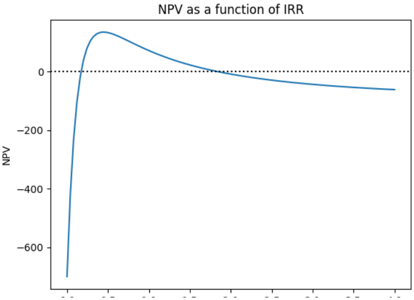
- Warning! The equation

$$V_0(1+IRR)^T + CF_1(1+IRR)^{T-1} + \ldots + CF_t(1+IRR)^{T-t} \overset{\text{set}}{=} V_T$$

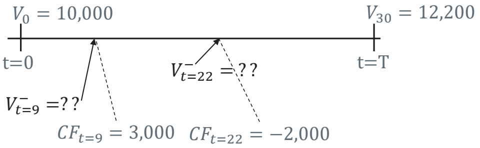
can have **multiple** solution for IRR!

- **IRR-Dollar-weighted Return** may not be well defined!





Performance Measurement without intermediate valuations



Modified Dietz (MD) return: $\frac{\text{Profit/loss}}{\text{Average capital (weighted by time)}}$

- MD return (over a month)=

$$\frac{(V_T - V_0 - \sum CF_t)}{V_0 + \sum \frac{T-t}{T} \times CF_t} = \frac{12,200 - 10,000 - (3,000 - 2,000)}{10,000\,(\times\frac{30}{30}) + 3000 \times (\frac{21}{30}) - 2000 \times (\frac{8}{30})} = \frac{1,200}{11,566.67} = 10.4\%$$

- Geometric is the only correct way to compute returns
- But without intermediate valuations we must use IRR or MD. IRR can have multiple solutions so MD is more commonly used
  However, it is conceptually wrong. Can you see why?

Performance Measurement from equilibrium models

- Has the client's objective been achieved?
- Let's go back to the CAPM return generating function (RGF)

$$r_p - r_f = \alpha_p + \beta_p(r_m - r_f) + \varepsilon_p$$

- $\beta_p(r_m - r_f)$: Market Beta/systematic return
- $\alpha_p + \varepsilon_p$: Stock Selection (alpha)
- $\sigma[r_p - r_f]$: Standard Deviation of portfolio p
- $\frac{r_p - r_f}{\sigma[r_p - r_f]}$: **Sharpe Ratio** of portfolio p
- What we want is to find a manager with $\alpha_p > 0$: manager's **skill.**
- But stock selection also includes our ε_p which is **noise**!
- Hard to separate skill from luck!

Performance Measurement

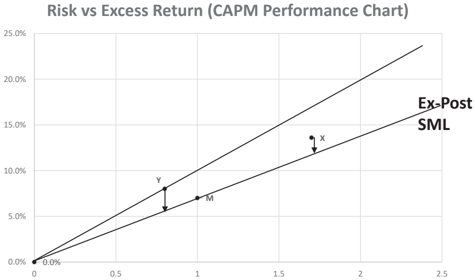
- CAPM RGF:

$$r_p - r_f = \alpha_p + \beta_p(r_m - r_f) + \varepsilon_p$$
$$\mathbb{E}[r_p] - r_f = \alpha_p + \beta_p(\mathbb{E}[r_m] - r_f)$$

- Average return (e.g., over 4 years): $\bar{r}_p - r_f = \alpha_p + \beta_p\left(\bar{r}_m - r_f\right)$ [no adjustment for risk]
- Average **excess** Return: $\alpha_p = \bar{r}_p - r_f - \beta_p(\bar{r}_m - r_f)$
- **Tracking Error**: $w_p = \sigma[\varepsilon_p]$, or residual risk
- **Information Ratio** $IR_p = \alpha_p / w_p$: focus only on unsystematic risk (skill/luck)

Performance Measurement - Risk adjusted Example

	Beta	Excess return
Market	1	7.00%
Manager X	1.7	13.60%
Manager Y	0.8	8.00%



$$\alpha_p = \bar{r}_p - r_f - \beta_p(\bar{r}_m - r_f)$$

$$\alpha_X = 0.136 - 1.7 \times 0.07 = 1.7\%$$

$$\alpha_Y = 0.08 - 0.8 \times 0.07 = 2.4\%$$

- Slope of the SML $= 0.07$ (7%), Distance to SML is portfolio alpha

Performance Measurement - Example

Canadian Equity Mangers (A&B) and TSX (M) Performance

Manager	$\bar{r}_p - r_f$	β_p	σ_p	SR	α_p	w_p	IR
A	4.00%	0.5	11%	36.4%	0.5%	9%	0.056
B	8.00%	1.3	18%	44.4%	−1.1%	3%	−0.367
M	7.00%	1	14%	50%	0%	0%	0

- Raw Returns: $B > M > A$
- Sharpe Ratio: $M > B > A$
- Alpha: $A > M > B$
- Information Ratio: $A > M > B$
- What is the main difference between manager A and B?
- Which manager would you chose? What else we should know before deciding who to hire?

Performance Measurement: Portable Alpha

- "Portable alpha": market-neutral strategy that captures a portfolio value added

- Consider 3 investments:
  - X_A with Manager A: $\mathbb{E}[r_A] = \alpha_p + r_f + \beta_A(\mathbb{E}[r_m] - r_f)$
  - X_M in the Market Portfolio: $\mathbb{E}[r_m] = r_f + 1(\mathbb{E}[r_m] - r_f)$
  - $X_c = 1 - X_A - X_M$ in T-Bills: earn r_f
- Let's make our portfolio **market-neutral**, i.e., set portfolio beta to zero:

$$0 \overset{\text{set}}{=} X_A\beta_A + X_M \times 1 + X_C \times 0 \implies X_M = -X_A\beta_A$$

- Expected portfolio return consists of:

$$\mathbb{E}[r_p] = X_A \mathrm{E}(r_A) + X_M \mathrm{E}(r_M) + (1 - X_A - X_M) r_f = r_f + X_A\alpha_P \implies \text{Portable alpha}$$

- By changing X_A we can keep as much of manager' α_P as we want
- Note α_A is **expected**, not guaranteed!

Performance Measurement with multi-risk factor model

- Same idea as CAPM, just with multiple λ's. See CRR.
- K-factor model RGF:

$$r_p = \mathbb{E}[r_p] + \sum_{k=1}^{K} \beta_{pk} I_k + \varepsilon_p$$

$$\mathbb{E}[r_p] - r_f = \alpha_p + \sum_{k=1}^{K} \beta_{pk} \lambda_k$$

- Portfolio's alpha or value added:

$$\alpha_p = \bar{r}_p - \underbrace{\left(r_f + \sum_{k=1}^{K} \beta_{pk} \lambda_k \right)}_{\text{benchmark}}$$

Performance Measurement: Attribution Analysis

- Attribution analysis is a technique that decomposes a fund's return into components attributable to
  - The **client**'s decision (SAA or policy decision)
  - The **manager's** decision (TAA, security selection)
- Suppose we have information on
  - Manager's portfolio weights and return in each asset class
  - Fund's SAA **policy** weights in each asset class
  - The **benchmark** return in each asst class (e.g., ETF indices)
  - The **typical** SAA weights for **peer funds** in each asset class

Performance Measurement: Attribution

	Weights			Returns	
	(A)	(B)	(C)	(D)	(E)
Asset Class	**Median Fund**	**Fund Policy**	**Mgr's**	**Mgr's**	**Benchmark**
	$\overline{w}_{B,n}$	$w_{B,n}$	w_n	r_n	$r_{B,n}$
Money Markets	5%	0%	**3%**	**4%**	4%
Bonds	45%	30%	**35%**	**3%**	6%
Stocks	50%	70%	**62%**	**11%**	9%

- Fund's return (C$\times$ D):
  $r_p = \sum_{n=1}^{n} w_n r_n = 0.03 \times 0.04 + 0.35 \times 0.03 + 0.62 \times 0.11 = 7.99\%$

Performance Measurement: Attribution

	Weights			Returns	
	(A)	(B)	(C)	(D)	(E)
Asset Class	**Median Fund**	**Fund Policy**	**Mgr's**	**Mgr's**	**Benchmark**
	$\overline{w}_{B,n}$	$w_{B,n}$	w_n	r_n	$r_{B,n}$

- Decomposition of total portfolio return:

$$r_p = C \times D = \underbrace{(\underbrace{A + (B - A)}_{=B} + (C - B))}_{=C} \times \underbrace{(E + (D - E))}_{=D}$$

$$= \underbrace{A \times E}_{\text{Passive}} + \underbrace{(B - A) \times E}_{\text{Policy}} + \underbrace{(C - B) \times E}_{\text{TAA}} +$$

$$+ \underbrace{B \times (D - E)}_{\text{Security Selection}} + \underbrace{(C - B) \times (D - E)}_{\text{Cross product}}$$

Performance Measurement: Attribution

	Weights			Returns	
	(A)	(B)	(C)	(D)	(E)
Asset Class	**Median Fund**	**Fund Policy**	**Mgr's**	**Mgr's**	**Benchmark**
	$\overline{w}_{B,n}$	$w_{B,n}$	w_n	r_n	$r_{B,n}$

- Decomposition of total portfolio return:

	$E : r_{B,n}$	$(D-E) : r_n - r_{B,n}$
$A : \quad \overline{w}_{B,n}$	$A \times E$: Passive	$A \times (D-E)$: Sec. Selection
$(B-A) : w_{B,n} - \overline{w}_{B,n}$	$(B-A) \times E$: Policy	$(B-A) \times (D-E)$: Sec. Selection
$(C-B) : w_n - w_{B,n}$	$(C-B) \times E$: TAA	$(C-B) \times (D-E)$: Cross-product

Performance Measurement: Attribution

Asset Class	Passive $A \times E$	Policy $(B\text{-}A) \times E$	TAA $(C\text{-}B) \times E$	Security Selection $B \times (D\text{-}E)$	Cross Product $(C\text{-}B) \times (D\text{-}E)$
Money Markets	0.20%	−0.20%	0.12%	0%	0%
Bonds	2.70%	−0.90%	0.30%	−0.90%	−0.15%
Stocks	4.50%	1.80%	−0.72%	1.40%	−0.16%
Total	**7.40%**	**0.70%**	−0.30%	0.50%	−0.31%

- Total Value Added $=$ Total Fund Return $-$ Passive $= 7.99\% - 7.40\% = 0.59\%$
- **Client Policy** (SAA) added 0.70% or 70bps to median fund passive return
- **Manager** added $-0.30\% + 0.50\% - 0.31\% = -0.11\%$ or -11bps
- **Total value added** $= 70bps - 11bps = 59bps$.

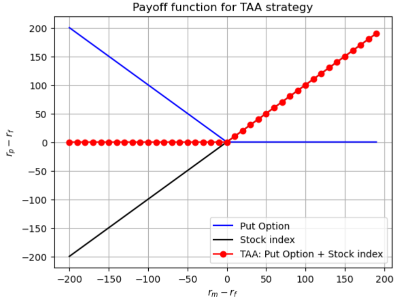
Performance Measurement - Merton “perfect” TAA

- Perfect TAA skill: Getting out of the market when market drops; getting in when it raises
- If manager has a perfect TAA skill, then portfolio is equivalent to

{cash + call} or, by P-C parity, {asset + put}

Performance Measurement - Merton “perfect” TAA

- TAA talent creates a **non-linear** payoff: long asset + long put



- Use this idea to “control” for perfect TAA in performance evaluation

Performance Measurement- Assessing Market-Timing Skill

- Merton regression model to assess TAA

$$r_{pt} - r_{ft} = \alpha_{pt} + \beta_{p1}(r_{mt} - r_{ft}) + \beta_{p2}\max\{0, r_{ft} - r_{mt}\} + \varepsilon_{pt}$$

  - r_{pt}: Portfolio employing TAA; r_{mt}: Stock Market Index return; r_{f_t}: T-Bill return
  - $\max\{0, r_{ft} - r_{mt}\}$: Payoff of a put option on Stock Market Index, with strike r_{ft}
  - β_{p1}: market exposure, like CAPM
  - β_{p2}: fraction of "put options" provided by manager's TAA skill (downside protection)
    - $\beta_{p2} = 1$: perfect TAA skill; $\beta_{p2} = 0$: no TAA skill
- Risk-adjusted value added: $r_{pt} - r_{ft} - \beta_{p1}(r_{mt} - r_{ft})$
- TAA skill return: $\beta_{p2}\max\{0, r_{ft} - r_{mt}\}$
- Security Selection$= \alpha_{pt} + \varepsilon_{pt}$

Performance Measurement: Merton example

Quarters	$r_{pt} - r_{ft}$	$r_{mt} - r_{ft}$	$\max\{0, r_{ft} - r_{mt}\}$
$t = 1$	9%	11%	0
$t = 2$	6%	5%	0
$t = 3$	1%	-4%	4%
$t = 4$	1%	2%	0
$t = 5$	3%	1%	0
$t = 6$	3%	4%	0
$t = 7$	-2%	-3%	3%
$t = 8$	0%	1%	0
$t = 9$	8%	7%	0
$t = 10$	0%	-5%	5%
Average	2.9%	1.9%	1.2%

- Regress $r_{pt} - r_{ft}$ on $r_{mt} - r_{ft}$ and $\max\{0, r_{ft} - r_{mt}\}$:
  $\implies \beta_{p1} = 0.897, \beta_{p2} = 0.802, \alpha_p = 0.0023$
- Manager has systematic risk below market (β_{p1}) and a high fraction of index puts (β_{p2}).

Performance Measurement: Merton Example

Category	Merton Model Return Review		Formula
	Return	Total	
Total Return	2.90%		$r_{pt} - r_{ft}$
Return to Systematic Risk	1.71%		$\beta_{p1} \times \text{avg}(r_{mt} - r_{ft})$
Risk-Adjusted Value added		1.19%	$r_{pt} - r_{ft} - \beta_{p1}(r_{mt} - r_{ft})$
TAA Return	0.96%		$\boldsymbol{\beta}_{p2} \times \text{avg}(Max\{0,(r_{ft} - r_{mt})\})$
Security Selection Return	0.23%		α_p (intercept from regression)
Total		1.19%	$\alpha_p + \boldsymbol{\beta}_{p2} \times \text{avg}(Max\{0,(r_{ft} - r_{mt})\})$

- This manager adds value mostly through TAA but we need to assess statistical significance of coefficient before drawing any conclusion!