HW 4: Graphical Models, Author: Lorenzo Mendoza

Question 1

1)

- A: 2 (C, B)
- B: 2 (C, A)
- C: 5 (A, B, D, E, F)
- D: 1 (C)
- E: 2 (C, F)

2)

E: (C, F)

3)

C_1:$\{A, B, C\}$, C_2:$\{C, F, E\}$,C_3: $\{C, D\}$

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A --- B
 \   /
   C -- F
  / \  /
 D   E
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4)

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       A--(F1)--B
            |
     (F3)--C--(F2)--F
       |        |
       D        E
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5)

- F1(A,B,C) = P(A)*P(B)*P(C|A,B) = P(A,B,C)
- F2(C,F,E) = P(E|C)*P(F|C)*P(C) = P(E,F|C)
- F3(C,D) = P(D|C)

Question 2

1)

$P(X)P(Y)$:

$P(X=0)=0.6$

$P(X=1)=0.4$

$P(Y=0)=0.592$

$P(Y=1)=0.408$

$P(X=0)P(Y=0)=0.3552$

$P(X=1)P(Y=0)=0.2368$

$P(X=0)P(Y=1)=0.2448$

$P(X=1)P(Y=1)=0.1632$

$P(X,Y)$:

$P(X=0,Y=0)=0.336$

$P(X=1,Y=0)=0.256$

$P(X=0,Y=1)=0.264$

$P(X=1,Y=1)=0.144$

$\therefore P(X)P(Y) \neq P(X,Y)$

2)

$P(X,Y|Z)$:

$P(X=0,Y=0|Z=0)=0.192$

$P(X=0,Y=0|Z=1)=0.144$

$P(X=0,Y=1|Z=0)=0.048$

$P(X=0,Y=1|Z=1)=0.216$

$P(X=1,Y=0|Z=0)=0.192$

$P(X=1,Y=0|Z=1)=0.064$

$P(X=1,Y=1|Z=0)=0.048$

$P(X=1,Y=1|Z=1)=0.096$

$P(X|Z)P(Y|Z)$:

$P(X|Z)$

$P(X=0|Z=0)=0.24$

$P(X=0|Z=1)=0.36$

$P(X=1|Z=0)=0.24$

$P(X=1|Z=1)=0.16$

$P(Y|Z)$

$P(Y=0|Z=0) = 0.384$

$P(Y=0|Z=1) = 0.096$

$P(Y=1|Z=0) = 0.208$

$P(Y=1|Z=1) = 0.312$

$P(X|Z)P(Y|Z)$

$P(X=0|Z=0)P(Y=0|Z=0) = 0.192$

$P(X=0|Z=1)P(Y=0|Z=1) = 0.144$

$P(X=0|Z=0)P(Y=1|Z=0) = 0.048$

$P(X=0|Z=1)P(Y=1|Z=1) = 0.216$

$P(X=1|Z=0)P(Y=0|Z=0) = 0.192$

$P(X=1|Z=1)P(Y=0|Z=1) = 0.064$

$P(X=1|Z=0)P(Y=1|Z=0) = 0.048$

$P(X=1|Z=1)P(Y=1|Z=1) = 0.096$

$\therefore P(X,Y|Z) = P(X|Z)P(Y|Z)$

3)

$P(X):$

$P(X=0) = P(X=0,Y=0,Z=0)+P(X=0,Y=0,Z=1)+P(X=0,Y=1,Z=0)+P(X=0,Y=1,Z=1) = 0.192+0.144+0.048+0.216 = 0.6$

$P(X=1) = P(X=1,Y=0,Z=0)+P(X=1,Y=0,Z=1)+P(X=1,Y=1,Z=0)+P(X=1,Y=1,Z=1) = 0.192+0.064+0.048+0.096 = 0.4$

$P(Y|Z):$

$P(Z=0) = P(X=0,Y=0,Z=0)+P(X=0,Y=1,Z=0)+P(X=1,Y=0,Z=0)+P(X=1,Y=1,Z=0) = 0.192+0.048+0.192+0.048 = 0.48$

$P(Z=1) = P(X=0,Y=0,Z=1)+P(X=0,Y=1,Z=1)+P(X=1,Y=0,Z=1)+P(X=1,Y=1,Z=1) = 0.144+0.216+0.064+0.096 = 0.52$

$P(Y=0|Z=0) = P(Y=0,Z=0)/P(Z=0) = 0.384/0.48 = 0.8$

$P(Y=1|Z=0) = P(Y=1,Z=0)/P(Z=0) = 0.096/0.48 = 0.2$

$P(Y=0|Z=1) = P(Y=0,Z=1)/P(Z=1) = 0.208/0.52 = 0.4$

$P(Y=1|Z=1) = P(Y=1,Z=1)/P(Z=1) = 0.312/0.52 = 0.6$

$P(Z|X):$

$p(x=0,z=0) = p(0,0,0) + p(0,1,0) = 0.192 + 0.048 = 0.24$

$p(x=1,z=0) = p(1,0,0) + p(1,1,0) = 0.192 + 0.048 = 0.24$

$p(x=0,z=1) = p(0,0,1) + p(0,1,1) = 0.144 + 0.216 = 0.36$

$p(x=1,z=1) = p(1,0,1) + p(1,1,1) = 0.064 + 0.096 = 0.16$

$p(z=0|x=0) = p(x=0,z=0)/p(x=0) = 0.24/0.6 = 0.4$

$p(z=1|x=0) = p(x=0,z=1)/p(x=0) = 0.36/0.6 = 0.6$

$p(z=0|x=1) = p(x=1,z=0)/p(x=1) = 0.24/0.4 = 0.6$

$p(z=1|x=1) = p(x=1,z=1)/p(x=1) = 0.16/0.4 = 0.4$

$P(X)P(Z|X)P(Y|Z) = P(X,Y,Z)$

$p(x=0)p(z=0|x=0)p(y=0|z=0) = 0.6 * 0.4 * 0.8 = 0.192 = p(0,0,0)$

$p(x=0)p(z=1|x=0)p(y=0|z=1) = 0.6 * 0.6 * 0.4 = 0.144 = p(0,0,1)$

$p(x=0)p(z=0|x=0)p(y=1|z=0) = 0.6 * 0.4 * 0.2 = 0.048 = p(0,1,0)$

$p(x=0)p(z=1|x=0)p(y=1|z=1) = 0.6 * 0.6 * 0.6 = 0.216 = p(0,1,1)$

$p(x=1)p(z=0|x=1)p(y=0|z=0) = 0.4 * 0.6 * 0.8 = 0.192 = p(1,0,0)$

$p(x=1)p(z=1|x=1)p(y=0|z=1) = 0.4 * 0.4 * 0.4 = 0.064 = p(1,0,1)$

$p(x=1)p(z=0|x=1)p(y=1|z=0) = 0.4 * 0.6 * 0.2 = 0.048 = p(1,1,0)$

$p(x=1)p(z=1|x=1)p(y=1|z=1) = 0.4 * 0.4 * 0.6 = 0.096 = p(1,1,1)$

4)

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 _
|X|
 |
 V
 _
|Z|
 |
 V
 _
|Y|
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Question 3

Approaches:

1. $(A', B') = argmax(\psi(A, B))$
2. $A'' = argmax(P(A))$, $B'' = argmax(P(B))$

Potential Function 1 (Same Results):

A	B	ψ_1(A, B)
0	0	0
0	1	1/2
1	0	1/2
1	1	0

Results:

$(A', B') = (0, 1)$ or $(1, 0)$

$(A'', B'') = (0, 1)$ or $(1, 0)$

$\therefore (A', B') = (A'', B'')$

Potential Function 2 (Distinct Results):

A	B	ψ_2(A, B)
0	0	0.4
0	1	0
1	0	0.3
1	1	0.3

Results:

$(A', B') = (0, 0)$

$(A'', B'') = (1, 0)$

$\therefore (A', B') \neq (A'', B'')$

Question 4

1)

$\mu_{E \to f3} = 1$

2)

$\mu_{D \to f2} = 1$

3)

$\mu_{f1\to D} = \sum_{A,B}(\psi(A,B,D) * \mu_{A\to f1} * \mu_{B\to f1}) = \sum_{A,B}(\psi(A,B,D)$

- $\mu_{f1\to D=0} = \psi(0,0,0) + \psi(0,1,0) + \psi(1,0,0) + \psi(1,1,0) = 0.62$
- $\mu_{f1\to D=1} = \psi(0,0,1) + \psi(0,1,1) + \psi(1,0,1) + \psi(1,1,1) = 0.39$

$\therefore\ \mu_{f1\to D} = [0.62, 0.39]$

4)

P(D=0) = A B C E ψ_1(A,B,D=0) * ψ_2(C,D=0) * ψ_3(D=0,E)

P(D=1) = A B C E ψ_1(A,B,D=1) * ψ_2(C,D=1) * ψ_3(D=1,E)

Substituting the values from the given tables, we get:

P(D=0) = (0.08+0.08+0.08+0.02) * 0.60 * (0.40+0.60) + (0.40+0.05+0.06+0.24) * 0.40 * (0.90+0.10)

P(D=1) = (0.08+0.08+0.08+0.02) * 0.10 * (0.40+0.60) + (0.40+0.05+0.06+0.24) * 0.90 * (0.90+0.10)

After calculating these values, we get:

P(D=0) = 1.056 P(D=1) = 1.584

Since these probabilities must sum to 1, we need to normalize them:

P(D=0) = 1.056 / (1.056 + 1.584) ≈ 0.400

P(D=1) = 1 / (1 + (1.056/1.584)) ≈ 0.600

So, P(D=0) ≈ 0.400 and P(D=1) ≈ 0.600.

Question 5

Directed graphical models, such as Bayesian networks, can be used to study Autism Spectrum Disorder (ASD) by identifying the causal relationships between different symptoms or characteristics of ASD. For example, a directed graphical model could be used to identify the causal relationships between social communication difficulties, repetitive behaviors, sensory sensitivities, and other symptoms of ASD. Consider a directed graphical model with the following variables:

```
PE --> ASD
      / | \
     /  |  \
    /   |   \
  V     V    V
SCD    RB   SS
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- ASD (Autism Spectrum Disorder): The outcome variable of interest. This variable indicates the presence or absence of ASD.
- SCD (Social Communication Difficulties): This variable measures social communication difficulties, such as difficulties with nonverbal communication or social interactions. SCD is a potential predictor of ASD because individuals with ASD often have difficulties with social communication.
- RB (Repetitive Behaviors): This variable measures repetitive behaviors, such as repetitive movements or rituals. RB is a potential predictor of ASD because individuals with ASD often display repetitive behaviors.
- SS (Sensory Sensitivities): This variable measures sensory sensitivities, such as sensitivity to noise or light. SS is a potential predictor of ASD because individuals with ASD often have sensory sensitivities.
- PE (Pregnancy Environment): This variable measures the pregnancy environment, which could include factors such as exposure to alcohol, drugs, or environmental toxins, as well as genetic predisposition to ASD. PE is a potential predictor of ASD because the pregnancy environment may influence the probability of having ASD. The conditional probability distributions (or potential functions) for this model can be defined as follows:
  - P(ASD | PE): The probability of having ASD given the values of the predictor variable PE.
  - P(SCD | ASD): The probability of having social communication difficulties given the presence of ASD.
  - P(RB | ASD): The probability of having repetitive behaviors given the presence of ASD.
  - P(SS | ASD): The probability of having sensory sensitivities given the presence of ASD.

This model can be used to answer a variety of questions related to ASD and its potential predictors, such as: * How does the pregnancy environment influence the probability of having ASD? * Can social communication difficulties, repetitive behaviors, or sensory sensitivities be used as early indicators of ASD? * How do the different predictors interact to influence the probability of having ASD? * How can interventions be tailored based on the specific combination of symptoms and predictors in individuals with ASD?