

## HW 4: Graphical Models, Author: Lorenzo Men- doza

### Question 1

1)

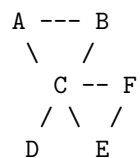
- A: 2 (C, B)
- B: 2 (C, A)
- C: 5 (A, B, D, E, F)
- D: 1 (C)
- E: 2 (C, F)

2)

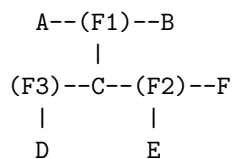
E: (C, F)

3)

$C_1: \{A, B, C\}$  ,  $C_2: \{C, F, E\}$  ,  $C_3: \{C, D\}$



4)



5)

- $F1(A, B, C) = P(A) * P(B) * P(C|A, B) = P(A, B, C)$
- $F2(C, F, E) = P(E|C) * P(F|C) * P(C) = P(E, F|C)$
- $F3(C, D) = P(D|C)$

### Question 2

1)

$P(X)P(Y)$ :

$$P(X = 0) = 0.6$$

$$P(X = 1) = 0.4$$

$$P(Y = 0) = 0.592$$

$$P(Y = 1) = 0.408$$

$$P(X = 0)P(Y = 0) = 0.3552$$

$$P(X = 1)P(Y = 0) = 0.2368$$

$$P(X = 0)P(Y = 1) = 0.2448$$

$$P(X = 1)P(Y = 1) = 0.1632$$

$$P(X, Y):$$

$$P(X = 0, Y = 0) = 0.336$$

$$P(X = 1, Y = 0) = 0.256$$

$$P(X = 0, Y = 1) = 0.264$$

$$P(X = 1, Y = 1) = 0.144$$

$$\therefore P(X)P(Y) \neq P(X, Y)$$

**2)**

$$P(X, Y|Z):$$

$$P(X = 0, Y = 0|Z = 0) = 0.192$$

$$P(X = 0, Y = 0|Z = 1) = 0.144$$

$$P(X = 0, Y = 1|Z = 0) = 0.048$$

$$P(X = 0, Y = 1|Z = 1) = 0.216$$

$$P(X = 1, Y = 0|Z = 0) = 0.192$$

$$P(X = 1, Y = 0|Z = 1) = 0.064$$

$$P(X = 1, Y = 1|Z = 0) = 0.048$$

$$P(X = 1, Y = 1|Z = 1) = 0.096$$

$$P(X|Z)P(Y|Z):$$

$$P(X|Z)$$

$$P(X = 0|Z = 0) = 0.24$$

$$P(X = 0|Z = 1) = 0.36$$

$$P(X = 1|Z = 0) = 0.24$$

$$P(X = 1|Z = 1) = 0.16$$

$$P(Y|Z)$$

$$P(Y = 0|Z = 0) = 0.384$$

$$P(Y = 0|Z = 1) = 0.096$$

$$P(Y = 1|Z = 0) = 0.208$$

$$P(Y = 1|Z = 1) = 0.312$$

$$P(X|Z)P(Y|Z)$$

$$P(X = 0|Z = 0)P(Y = 0|Z = 0) = 0.192$$

$$P(X = 0|Z = 1)P(Y = 0|Z = 1) = 0.144$$

$$P(X = 0|Z = 0)P(Y = 1|Z = 0) = 0.048$$

$$P(X = 0|Z = 1)P(Y = 1|Z = 1) = 0.216$$

$$P(X = 1|Z = 0)P(Y = 0|Z = 0) = 0.192$$

$$P(X = 1|Z = 1)P(Y = 0|Z = 1) = 0.064$$

$$P(X = 1|Z = 0)P(Y = 1|Z = 0) = 0.048$$

$$P(X = 1|Z = 1)P(Y = 1|Z = 1) = 0.096$$

$$\therefore P(X, Y|Z) = P(X|Z)P(Y|Z)$$

**3)**

$$P(X) :$$

$$P(X = 0) = P(X = 0, Y = 0, Z = 0) + P(X = 0, Y = 0, Z = 1) + P(X = 0, Y = 1, Z = 0) + P(X = 0, Y = 1, Z = 1) = 0.192 + 0.144 + 0.048 + 0.216 = 0.6$$

$$P(X = 1) = P(X = 1, Y = 0, Z = 0) + P(X = 1, Y = 0, Z = 1) + P(X = 1, Y = 1, Z = 0) + P(X = 1, Y = 1, Z = 1) = 0.192 + 0.064 + 0.048 + 0.096 = 0.4$$

$$P(Y|Z) :$$

$$P(Z = 0) = P(X = 0, Y = 0, Z = 0) + P(X = 0, Y = 1, Z = 0) + P(X = 1, Y = 0, Z = 0) + P(X = 1, Y = 1, Z = 0) = 0.192 + 0.048 + 0.192 + 0.048 = 0.48$$

$$P(Z = 1) = P(X = 0, Y = 0, Z = 1) + P(X = 0, Y = 1, Z = 1) + P(X = 1, Y = 0, Z = 1) + P(X = 1, Y = 1, Z = 1) = 0.144 + 0.216 + 0.064 + 0.096 = 0.52$$

$$P(Y = 0|Z = 0) = P(Y = 0, Z = 0)/P(Z = 0) = 0.384/0.48 = 0.8$$

$$P(Y = 1|Z = 0) = P(Y = 1, Z = 0)/P(Z = 0) = 0.096/0.48 = 0.2$$

$$P(Y = 0|Z = 1) = P(Y = 0, Z = 1)/P(Z = 1) = 0.208/0.52 = 0.4$$

$$P(Y = 1|Z = 1) = P(Y = 1, Z = 1)/P(Z = 1) = 0.312/0.52 = 0.6$$

$$P(Z|X) :$$

$$p(x = 0, z = 0) = p(0, 0, 0) + p(0, 1, 0) = 0.192 + 0.048 = 0.24$$

$$p(x = 1, z = 0) = p(1, 0, 0) + p(1, 1, 0) = 0.192 + 0.048 = 0.24$$

$$p(x = 0, z = 1) = p(0, 0, 1) + p(0, 1, 1) = 0.144 + 0.216 = 0.36$$

$$p(x = 1, z = 1) = p(1, 0, 1) + p(1, 1, 1) = 0.064 + 0.096 = 0.16$$

$$p(z = 0|x = 0) = p(x = 0, z = 0)/p(x = 0) = 0.24/0.6 = 0.4$$

$$p(z = 1|x = 0) = p(x = 0, z = 1)/p(x = 0) = 0.36/0.6 = 0.6$$

$$p(z = 0|x = 1) = p(x = 1, z = 0)/p(x = 1) = 0.24/0.4 = 0.6$$

$$p(z = 1|x = 1) = p(x = 1, z = 1)/p(x = 1) = 0.16/0.4 = 0.4$$

$$P(X)P(Z|X)P(Y|Z) = P(X, Y, Z)$$

$$p(x = 0)p(z = 0|x = 0)p(y = 0|z = 0) = 0.6 * 0.4 * 0.8 = 0.192 = p(0, 0, 0)$$

$$p(x = 0)p(z = 1|x = 0)p(y = 0|z = 1) = 0.6 * 0.6 * 0.4 = 0.144 = p(0, 0, 1)$$

$$p(x = 0)p(z = 0|x = 0)p(y = 1|z = 0) = 0.6 * 0.4 * 0.2 = 0.048 = p(0, 1, 0)$$

$$p(x = 0)p(z = 1|x = 0)p(y = 1|z = 1) = 0.6 * 0.6 * 0.6 = 0.216 = p(0, 1, 1)$$

$$p(x = 1)p(z = 0|x = 1)p(y = 0|z = 0) = 0.4 * 0.6 * 0.8 = 0.192 = p(1, 0, 0)$$

$$p(x = 1)p(z = 1|x = 1)p(y = 0|z = 1) = 0.4 * 0.4 * 0.4 = 0.064 = p(1, 0, 1)$$

$$p(x = 1)p(z = 0|x = 1)p(y = 1|z = 0) = 0.4 * 0.6 * 0.2 = 0.048 = p(1, 1, 0)$$

$$p(x = 1)p(z = 1|x = 1)p(y = 1|z = 1) = 0.4 * 0.4 * 0.6 = 0.096 = p(1, 1, 1)$$

4)

$$\bar{|\mathbf{X}|}$$

$$\frac{1}{V}$$

$$\bar{|\mathbf{Z}|}$$

$$\frac{1}{V}$$

$$\bar{|\mathbf{Y}|}$$

### Question 3

Approaches:

1.  $(A', B') = \operatorname{argmax}(\psi(A, B))$
2.  $A'' = \operatorname{argmax}(P(A)), B'' = \operatorname{argmax}(P(B))$

Potential Function 1 (Same Results):

A	B	$\psi_1(A, B)$
0	0	0
0	1	1/2
1	0	1/2
1	1	0

Results:

$$(A', B') = (0, 1) \text{ or } (1, 0)$$

$$(A'', B'') = (0, 1) \text{ or } (1, 0)$$

$$\therefore (A', B') = (A'', B'')$$

Potential Function 2 (Distinct Results):

A	B	$\psi_2(A, B)$
0	0	0.4
0	1	0
1	0	0.3
1	1	0.3

Results:

$$(A', B') = (0, 0)$$

$$(A'', B'') = (1, 0)$$

$$\therefore (A', B') \neq (A'', B'')$$

## Question 4

1)

$$\mu_{E \rightarrow f3} = 1$$

2)

$$\mu_{D \rightarrow f2} = 1$$

3)

$$\mu_{f1 \rightarrow D} = \sum_{A,B} (\psi(A, B, D) * \mu_{A \rightarrow f1} * \mu_{B \rightarrow f1}) = \sum_{A,B} (\psi(A, B, D))$$

- $\mu_{f1 \rightarrow D=0} = \psi(0, 0, 0) + \psi(0, 1, 0) + \psi(1, 0, 0) + \psi(1, 1, 0) = 0.62$
- $\mu_{f1 \rightarrow D=1} = \psi(0, 0, 1) + \psi(0, 1, 1) + \psi(1, 0, 1) + \psi(1, 1, 1) = 0.39$

$$\therefore \mu_{f1 \rightarrow D} = [0.62, 0.39]$$

4)

$$P(D=0) = A \ B \ C \ E \ \psi_1(A, B, D=0) * \psi_2(C, D=0) * \psi_3(D=0, E)$$

$$P(D=1) = A \ B \ C \ E \ \psi_1(A, B, D=1) * \psi_2(C, D=1) * \psi_3(D=1, E)$$

Substituting the values from the given tables, we get:

$$P(D=0) = (0.08+0.08+0.08+0.02) * 0.60 * (0.40+0.60) + (0.40+0.05+0.06+0.24) * 0.40 * (0.90+0.10)$$

$$P(D=1) = (0.08+0.08+0.08+0.02) * 0.10 * (0.40+0.60) + (0.40+0.05+0.06+0.24) * 0.90 * (0.90+0.10)$$

After calculating these values, we get:

$$P(D=0) = 1.056 \ P(D=1) = 1.584$$

Since these probabilities must sum to 1, we need to normalize them:

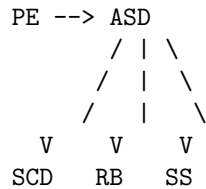
$$P(D=0) = 1.056 / (1.056 + 1.584) \approx 0.400$$

$$P(D=1) = 1 / (1 + (1.056/1.584)) \approx 0.600$$

So,  $P(D=0) \approx 0.400$  and  $P(D=1) \approx 0.600$ .

## Question 5

Directed graphical models, such as Bayesian networks, can be used to study Autism Spectrum Disorder (ASD) by identifying the causal relationships between different symptoms or characteristics of ASD. For example, a directed graphical model could be used to identify the causal relationships between social communication difficulties, repetitive behaviors, sensory sensitivities, and other symptoms of ASD. Consider a directed graphical model with the following variables:



- ASD (Autism Spectrum Disorder): The outcome variable of interest. This variable indicates the presence or absence of ASD.
- SCD (Social Communication Difficulties): This variable measures social communication difficulties, such as difficulties with nonverbal communication or social interactions. SCD is a potential predictor of ASD because individuals with ASD often have difficulties with social communication.
- RB (Repetitive Behaviors): This variable measures repetitive behaviors, such as repetitive movements or rituals. RB is a potential predictor of ASD because individuals with ASD often display repetitive behaviors.
- SS (Sensory Sensitivities): This variable measures sensory sensitivities, such as sensitivity to noise or light. SS is a potential predictor of ASD because individuals with ASD often have sensory sensitivities.
- PE (Pregnancy Environment): This variable measures the pregnancy environment, which could include factors such as exposure to alcohol, drugs, or environmental toxins, as well as genetic predisposition to ASD. PE is a potential predictor of ASD because the pregnancy environment may influence the probability of having ASD. The conditional probability distributions (or potential functions) for this model can be defined as follows:
  - $P(\text{ASD} \mid \text{PE})$ : The probability of having ASD given the values of the predictor variable PE.
  - $P(\text{SCD} \mid \text{ASD})$ : The probability of having social communication difficulties given the presence of ASD.
  - $P(\text{RB} \mid \text{ASD})$ : The probability of having repetitive behaviors given the presence of ASD.
  - $P(\text{SS} \mid \text{ASD})$ : The probability of having sensory sensitivities given the presence of ASD.

This model can be used to answer a variety of questions related to ASD and its potential predictors, such as: \* How does the pregnancy environment influence the probability of having ASD? \* Can social communication difficulties, repetitive behaviors, or sensory sensitivities be used as early indicators of ASD? \* How do the different predictors interact to influence the probability of having ASD? \* How can interventions be tailored based on the specific combination of symptoms and predictors in individuals with ASD?