Stochastic Methods for Finance

Report 3: Pricing with different Models

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Abstract:

In this report, our goal is to analyse and compare different methods for pricing a European call option.
In particular, we have used 2 multi-step methods, the Binomial method and the Leiser-Reimer method and we are going to study their convergence to the famous Black-Scholes model.
To do that we used Excel's VBA software and developed our model using a purpose-built dataset.

Introduction:

Pricing a derivative is a fundamental process in finance and mathematical finance. It involves assigning a monetary value to a derivative, a financial instrument whose price depends on the value of an underlying asset. The importance of this process lies primarily in the ability to value financial instruments accurately and thus to make effective and well-informed investment decisions.
Mathematical models are used to price a derivative, including the Binomial model, the Leisen-Reimer model and the Black-Scholes model, which allow the derivative's price to be calculated on the basis of several parameters such as the time remaining to maturity, the volatility of the underlying asset, the interest rate, etc. However, each model has differences in the assumptions and methodologies adopted, which can lead to different price values for the same derivative.
The Binomial model, for example, is based on time-step management and a schematisation of the market into two possible choices (up or down), while the Leisen-Reimer model is based on a correction of the binomial distribution, eliminating the possibility of negative values of the underlying asset.
The Black-Scholes model, on the other hand, uses an analytical formula based on the theory of partial derivative equations and the concept of the geometric Brownian process to calculate the price of the derivative quickly and efficiently, although it may be less accurate in some circumstances (given the simplification of the assumptions for the construction of such a theory).

Results obtained with Excel's VBA:

- **Parameters Settings:**

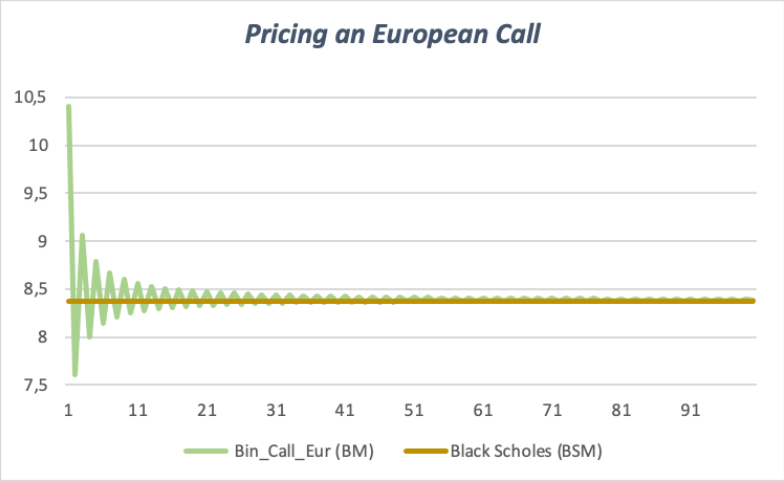
We built the following ad hoc data set for the development of our model:

Type	Value
Stock Price (USD)	123
Exercise price (USD)	123
Rate	0,02
Sigma	0,33
Time	0,25

- **Implementation of our models:**

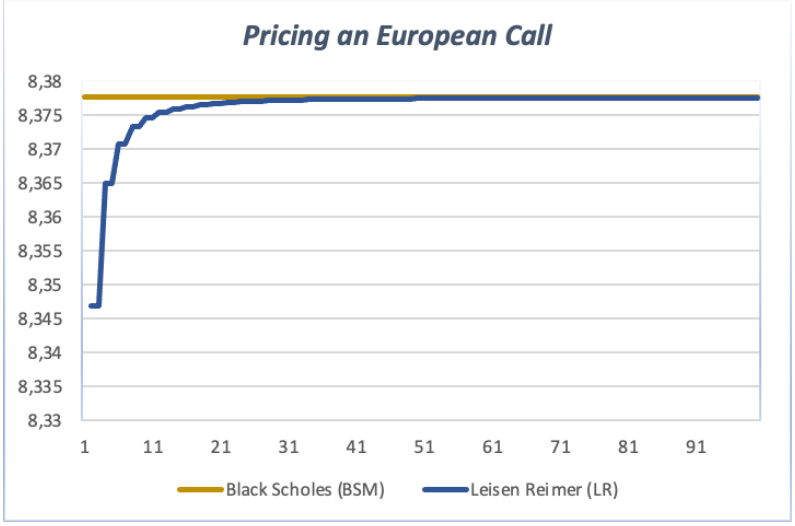
Binomial Model

Using Virtual basics, implement the binomial and Black-Scholes model for a range of steps from 1 to 100. Recall that the Black-Scholes model is not based on a discretisation in steps of the evolution of time and will therefore remain constant as they vary.
Putting the results obtained in a single graph (figure below) we can see how the binomial model assumes a very wide oscillatory trend for the first steps but converges to the Black-Scholes model as they increase.
It is interesting to note that the estimated price of the Black-Scholes model is approximately USD 8.38, while the binomial model with a single step estimates a price of USD 10.41 and with two steps the estimated price is USD 7.61, a long way from the value of the price obtained with the Black-Scholes model.



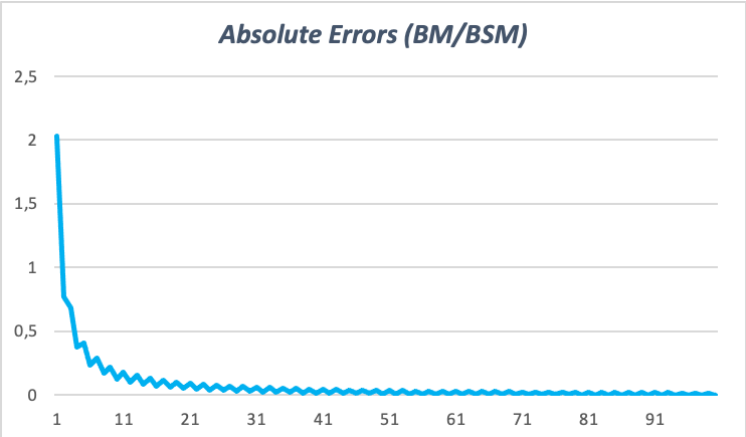
Leisen-Reimer

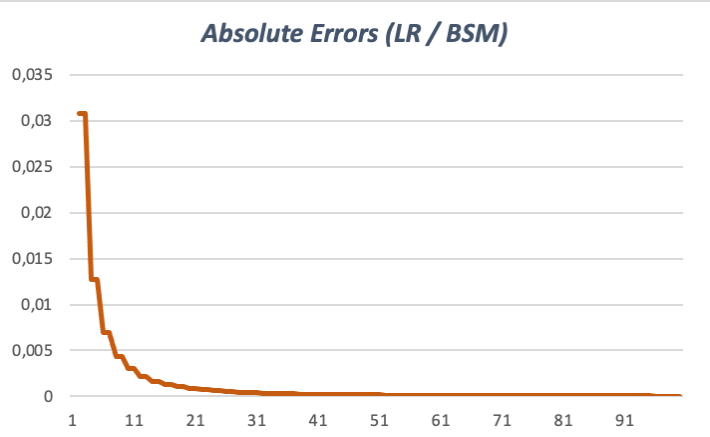
Similarly, we repeat the previous steps by implementing the Leisen-Reimer model.
It can be seen (from the graph below) that the trend is different from the previous one. There is an increasing and converging trend towards the Black-Scholes model, and the price estimated by the single-step model is far more accurate than the previous one.



- **Convergence and absolute errors:**

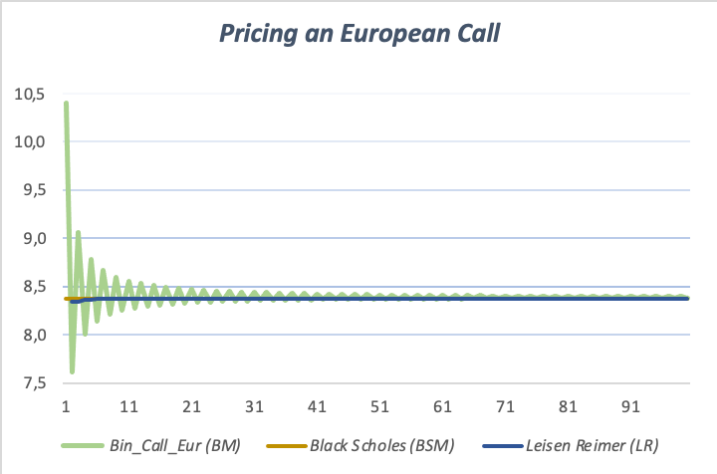
We noticed how the two multi-step models (the binomial and the Leisen-Reimer) compared and differed greatly both in their general trend and in their convergence to the Black-Scholes model.
In this section, we want to understand more precisely how far the two models differ from the BS model and how 'fast' they converge.
To do this, we have calculated the absolute errors at each step of the difference between the multistep model under consideration and the Black-Scholes.
In the graphs below, we can see that both models get considerably closer to the Black-Scholes model in about 20 steps, but if we look closely at the values on the y-axis, we notice that the absolute error that the binomial model manages to obtain in 100 steps (of about 0.0003) the Leisen-Reimer model obtains after only 15 iterations.

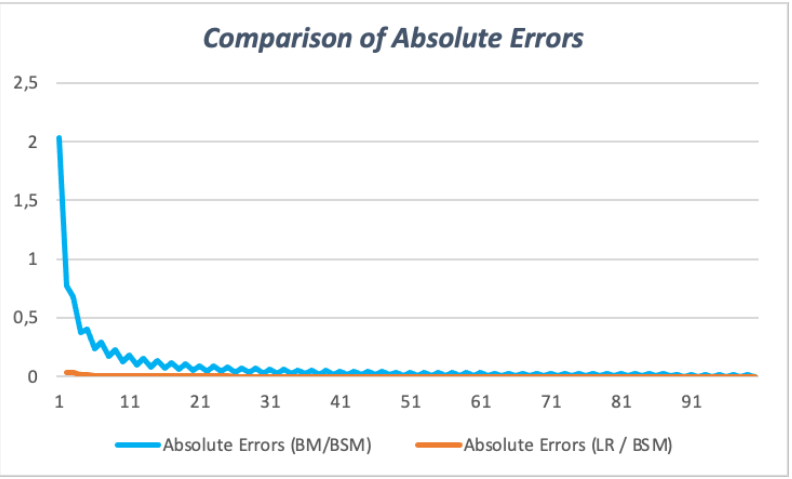




Conclusions:

From the above arguments, it has been inferred that the two multistep models are very different from each other in both trend and speed of convergence (we present the two summary graphs in which all three models are compared).





The Binomial model is popular because it is very intuitive and simple to understand. It, however, due to too large a discretization of the time interval is likely to be inaccurate and differ by no small amount from the Black-Scholes model. Recall that in the world of finance, a small price difference can result in large losses if care is not taken. On the other hand, the newer Leisen-Reimer model is much more accurate and for a not very large number of steps (>20) obtains the price estimate given by the Black Scholes model with an error of about 10^(-5).