Stochastic Methods for Finance

Report 4: Greeks

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Abstract:

In this report, our objective is to calculate, represent and analyse the behaviour of the Greeks associated with a suitably constructed asset in a BS model.

Introduction:

The term "Greeks" in finance refers to a set of criteria used to measure various aspects of risk in options trading. These metrics, sometimes known as "option Greeks," give traders and investors critical information about how changes in market conditions and other factors might affect the value of an option.

The Greeks are generated from mathematical models that are used to calculate an option's theoretical value depending on numerous inputs such as the underlying asset price, time to expiration, interest rates, and volatility. Each Greek indicates a different component of an option's risk, and traders utilize these measurements to make educated judgments about whether to purchase or sell options, as well as how to manage their risk exposure.
Five main Greeks are used in options trading: Delta, Gamma, Theta, Vega, and Rho. Each Greek measures a different type of risk and provides valuable information about the potential impact of various market conditions on an option's value. Understanding the Greeks is essential for anyone looking to trade options, as it can help them to make more informed and profitable trading decisions.

1. **Delta (** *Derivative of the price function with respect to the asset price* **)**: Delta measures the sensitivity of an option's price to changes in the price of the underlying asset. Delta is expressed as a number between 0 and 1 for call options, and between -1 and 0 for put options. A Delta of 0.5, for example, means that for every $1 change in the underlying asset price, the option price will change by $0.50 in the same direction.

2. **Gamma (** *Derivative of the delta with respect to the asset price* **)**: Gamma measures the rate of change of an option's Delta in response to changes in the price of the underlying asset. Gamma is highest for at-the-money options and decreases as the option moves further into the money or out of the money. A high Gamma indicates that Delta is likely to change quickly in response to changes in the underlying asset price.

3. **Theta (** *Derivative of the price function with respect to time* **)**: Theta measures the sensitivity of an option's price to the passage of time. Theta is expressed as a negative number, as time decay works against the value of an option over time. The closer an option gets to expiration, the faster its value will decay.

4. **Vega (** *Derivative of the price function with respect to the volatility* **)**: Vega measures the sensitivity of an option's price to changes in volatility. Options with higher Vega are more sensitive to changes in volatility, which can be a significant driver of option prices.

5. **Rho (** *Derivative of the price function with respect to the risk-free rate* **)**: Rho measures the sensitivity of an option's price to changes in interest rates. Rho is typically less important than the other Greeks, as interest rate changes tend to have a smaller impact on option prices than other factors.

Results obtained with Excel's VBA:

- **Parameters Settings 1st part:**

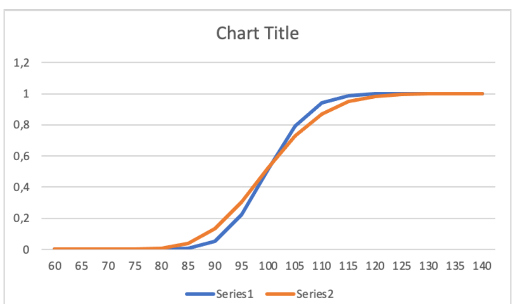
We built the following ad hoc data set for the development of our model:

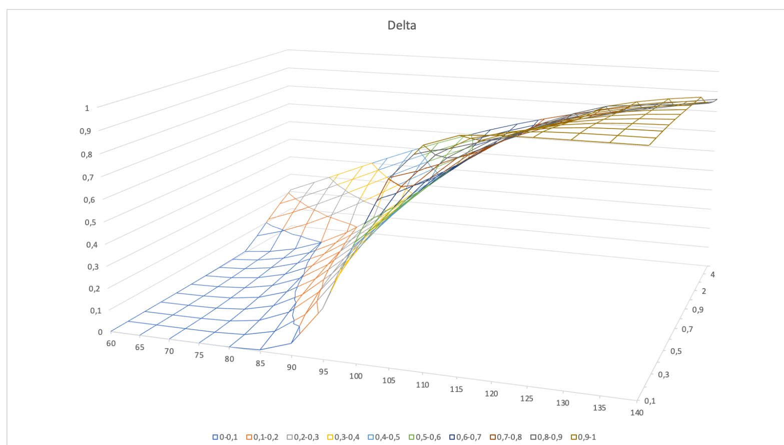
Type	Value
Stock Price (USD)	100
Risk Free Rate	0,01
Volatility	0,2
Dividend Yield	0

- **Implementation of the Greek's formula and 3D surface:**

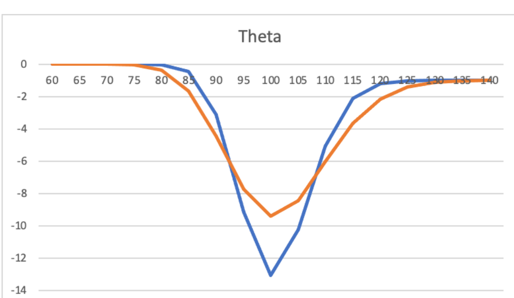
For each greek, we show the 2d chart in which there are two trajectories relating to maturity of 0.1 (blue) and maturity of 0.2 (orange) and the 3d surface where the variables are the strike price and the time to maturity (we placed a strike range of 60 to 140 and maturity of 0.1 up to 1 and then 2,3,4,5).

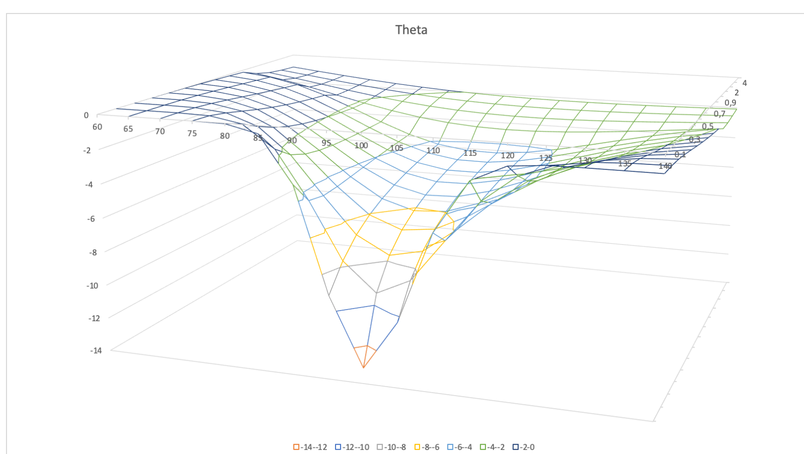
Delta:



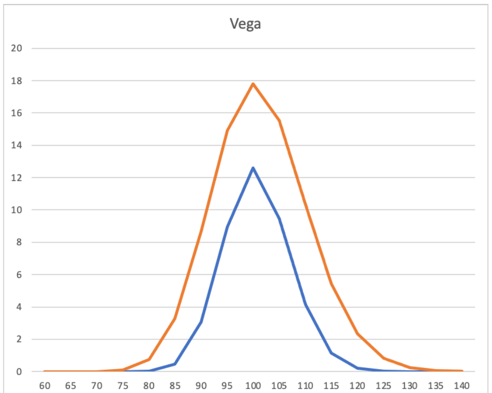


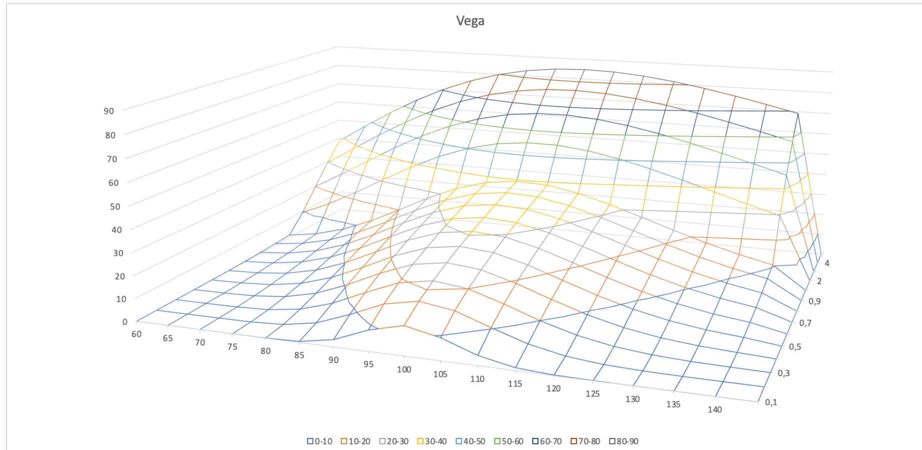
Theta:



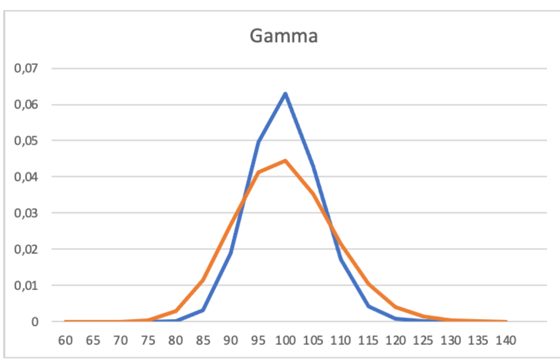


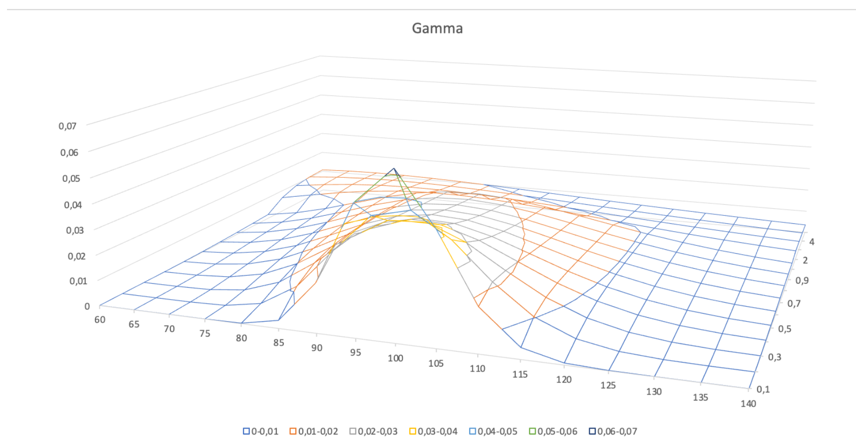
Vega:



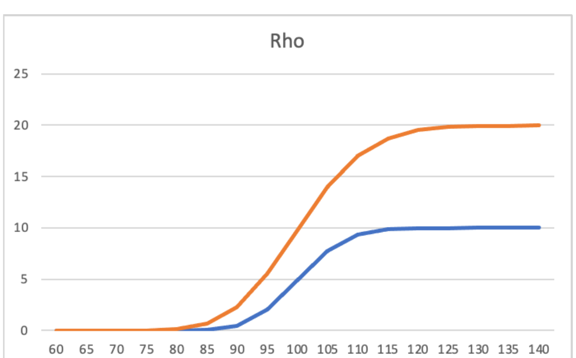


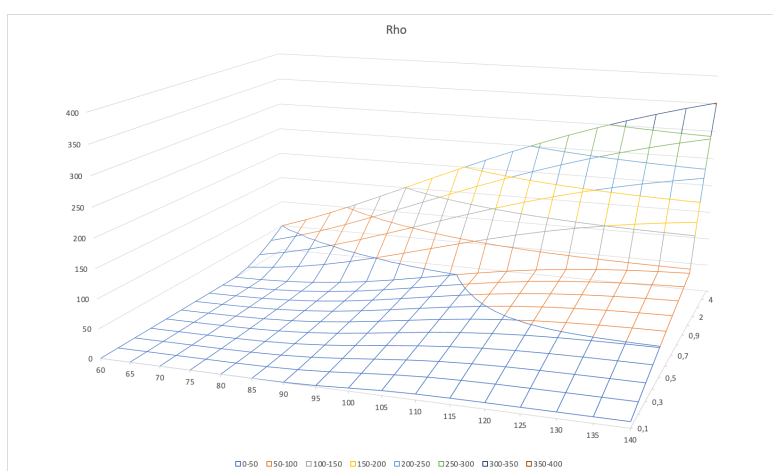
Gamma:





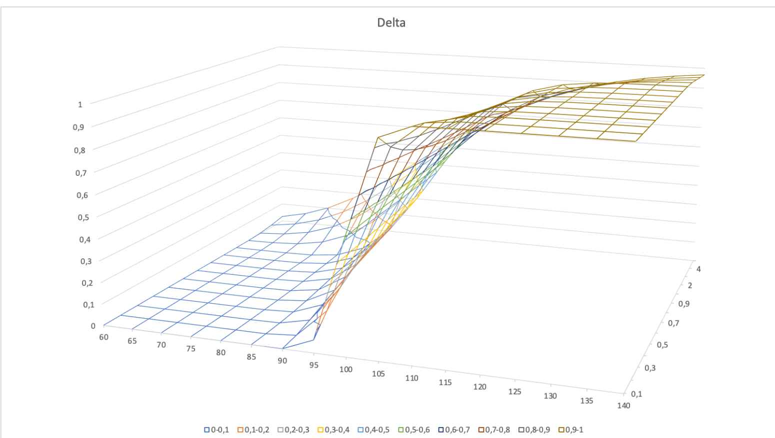
Rho:

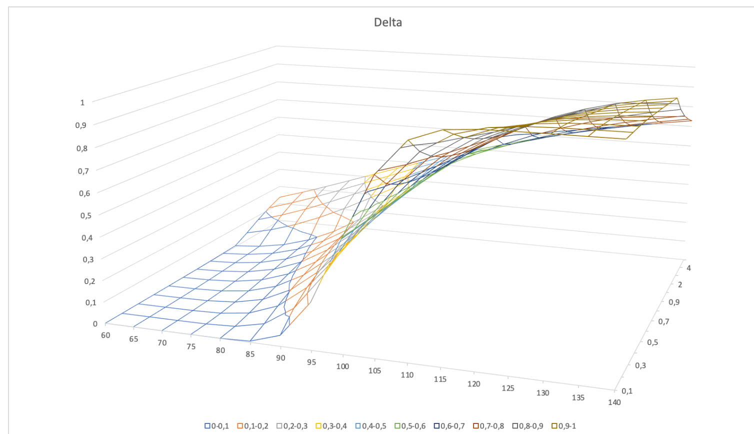


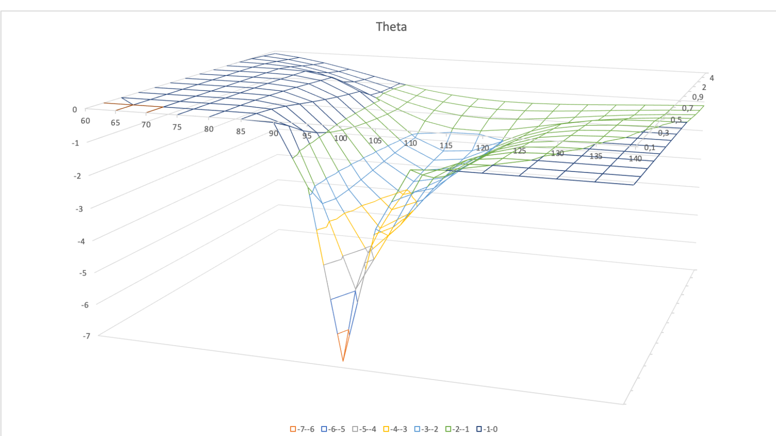


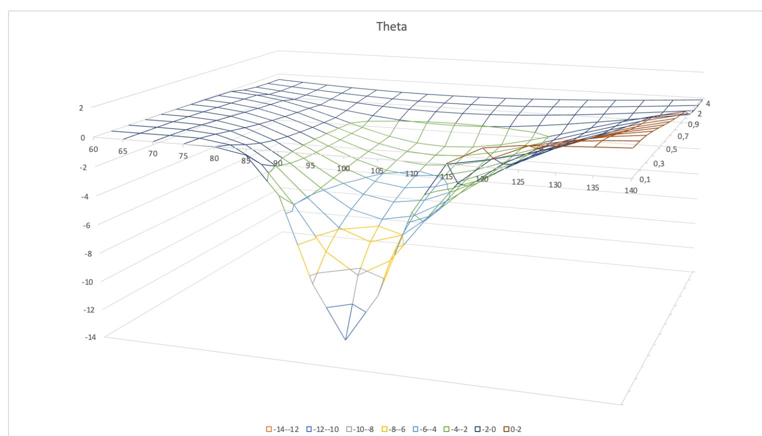
- **Sensitivity analysis with respect to dividend yield q and volatility sigma:**
We will now first change the volatility to a value of 0.1 in our data set and recalculate the greeks. Secondly, using the same initial volatility we would change the dividend yield to 0.02 (and thus consider a dividend-paying asset). We show the surfaces obtained.

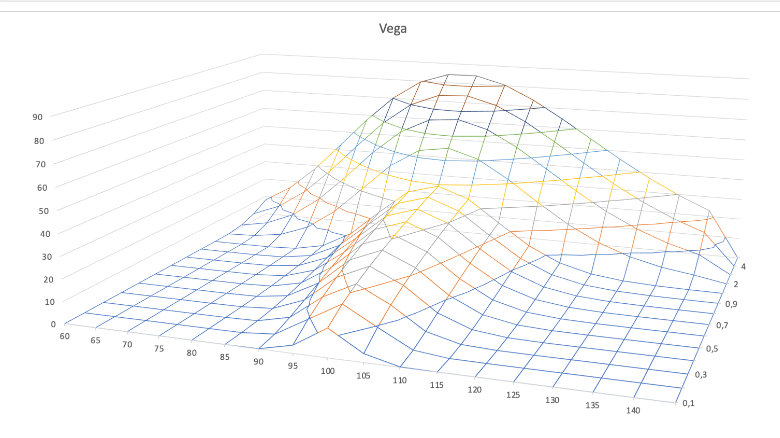
Volatility to 0,1	Dividend yield 2%

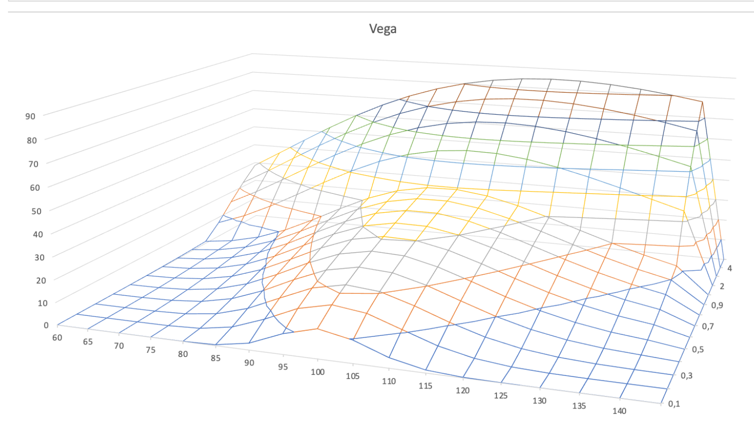


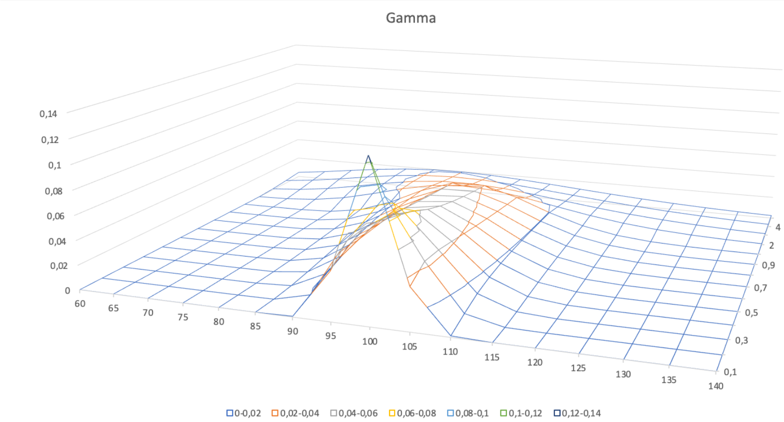


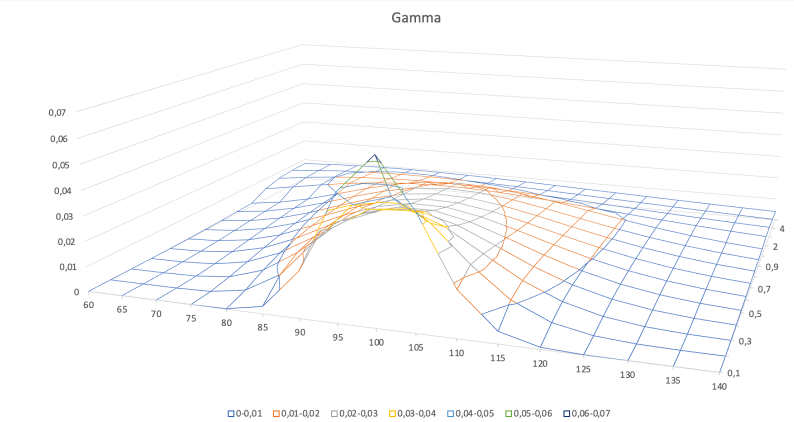


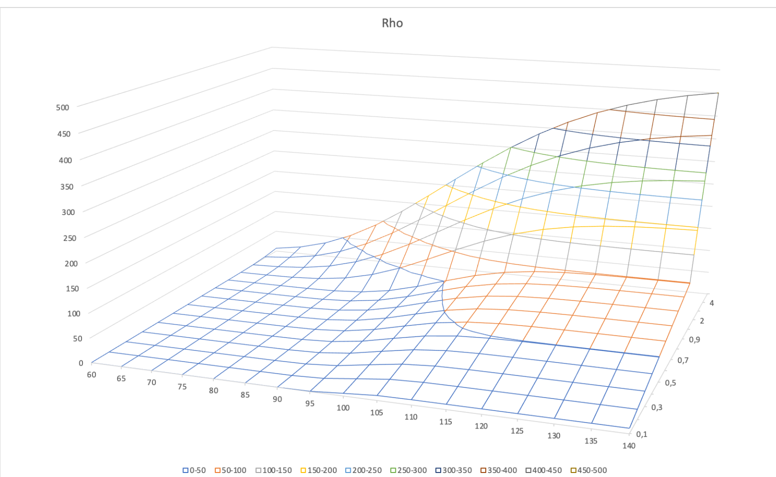


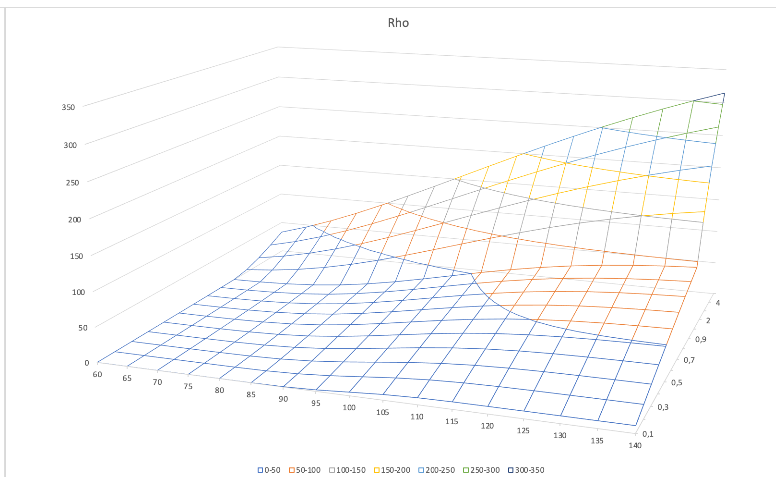





Gamma
0-0,02 0,02-0,04 0,04-0,06 0,06-0,08 0,08-0,1 0,1-0,12 0,12-0,14


Gamma
0-0,01 0,01-0,02 0,02-0,03 0,03-0,04 0,04-0,05 0,05-0,06 0,06-0,07


Rho
0-50 50-100 100-150 150-200 200-250 250-300 300-350 350-400 400-450 450-500


Rho
0-50 50-100 100-150 150-200 200-250 250-300 300-350

Conclusions:

As a first analysis, we can see from the first graphs that for Delta, Theta, and Gamma, as maturity increases, there is a smoothing effect, in which the singular points (the peaks of the surfaces/differences) tend to decrease.
On the contrary, for Rho and Vega, as maturity increases, there is a greater propensity to have peaks at the asset value.

Lastly, we can see how the variation (a decrease) in volatility tends to highlight the peaks while the presence of dividends has little effect on the Greeks even though we note in the theta the presence of positive values when we have high strike prices.