Università degli studi di Padova

Stochastic Methods for Finance

Third report

Option Greeks with VBA

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1 Goal

The purpose of this report is to compute the value of the Greeks for an European Option as function of the underlying T and the time to maturity T. The results will be shown in 3D graphs.

2 Greeks

In order to achieve our goal, we consider (**suppose USD as currency**):

- a range for the underlying price $S = \{20, 21, ..., 200\}$;
- the strike price $K = 100$;
- a range for the time to maturity $T = \{0.1, 0.2, ..., 1\}$, in years;
- the free-risk interest rate $r = 0.01$;
- the volatility $\sigma = 0.2$.

Finally, we repeat the computation with a schock of volatility of $+50\%$: the new volatility will be $\sigma_{new} = 0.3$. In the following section we're going to show the various surfaces obtained by VBA computations for both σ and σ_{new} and for both types Call and Put.

2.1 Delta

The Delta Greek is the rate of change between the option's price and the underlying asset's price:

$$\Delta = \frac{\partial V}{\partial S}, \tag{1}$$

where V is the option's price, computed in according to the B-S formula (formulas are shown in the appendix).
Delta assumesthe values between 0 and 1 for a long Call and values between -1 and 0 for long Put.

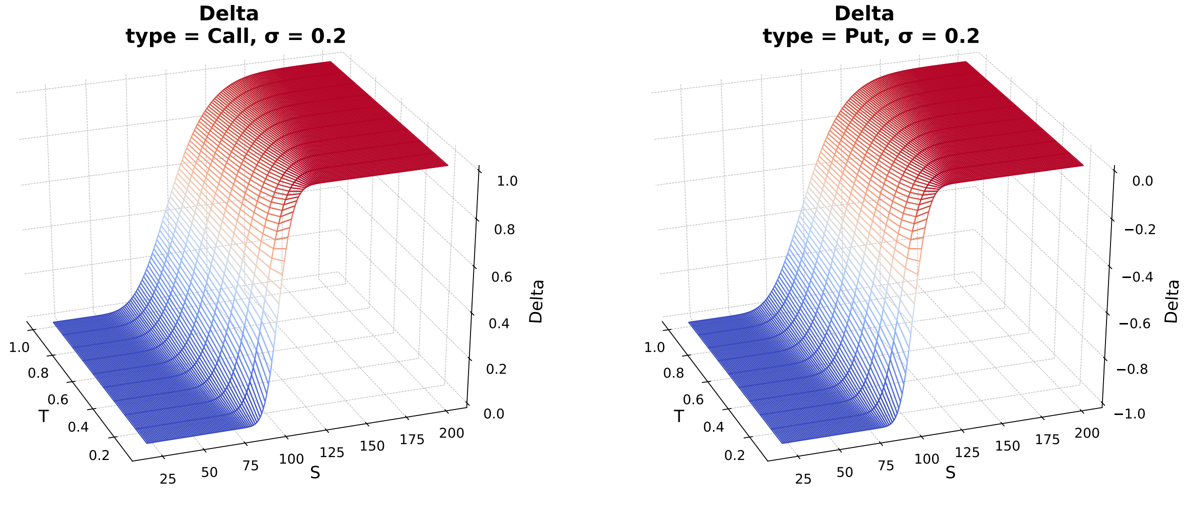

Figure 1: Surface of Delta for Call (on the left) and Put (on the right), with a volatility $\sigma = 0.2$.

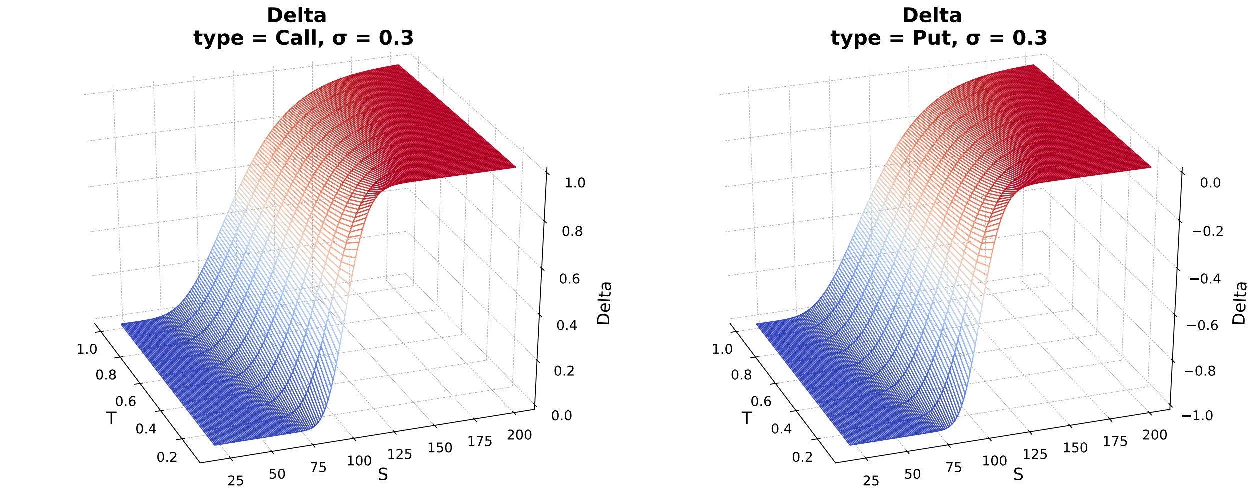

Figure 2: New surfaces of Delta after +50% change in volatility.

2.2 Theta

Theta can be expressed as time decay of an option. It is:

$$\Theta = -\frac{\partial V}{\partial T},$$

and it is almost always a negative number.

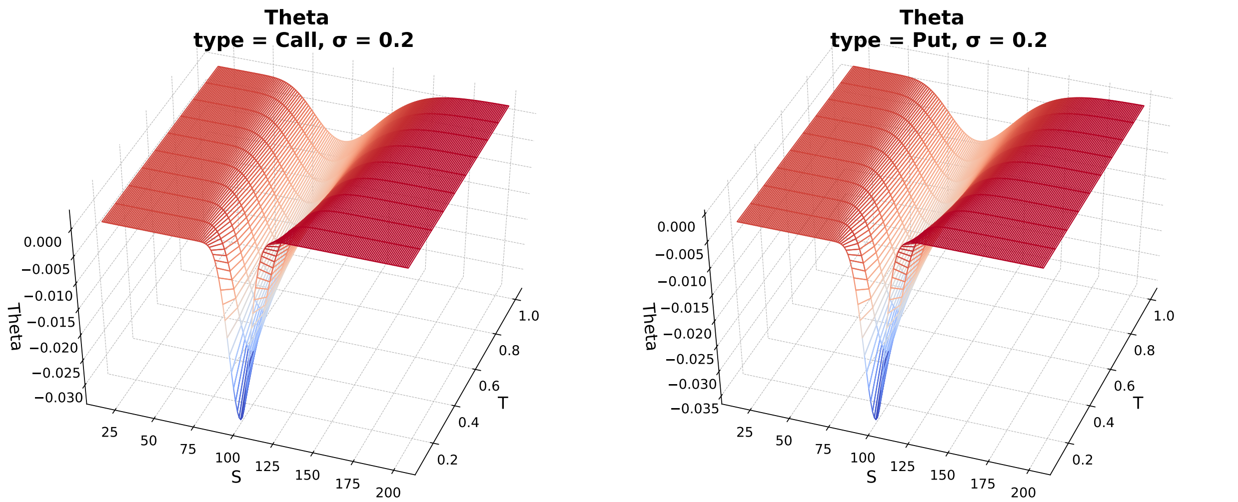

Figure 3: Surface of Theta for Call (on the left) and Put (on the right), with a volatility $\sigma = 0.2$.

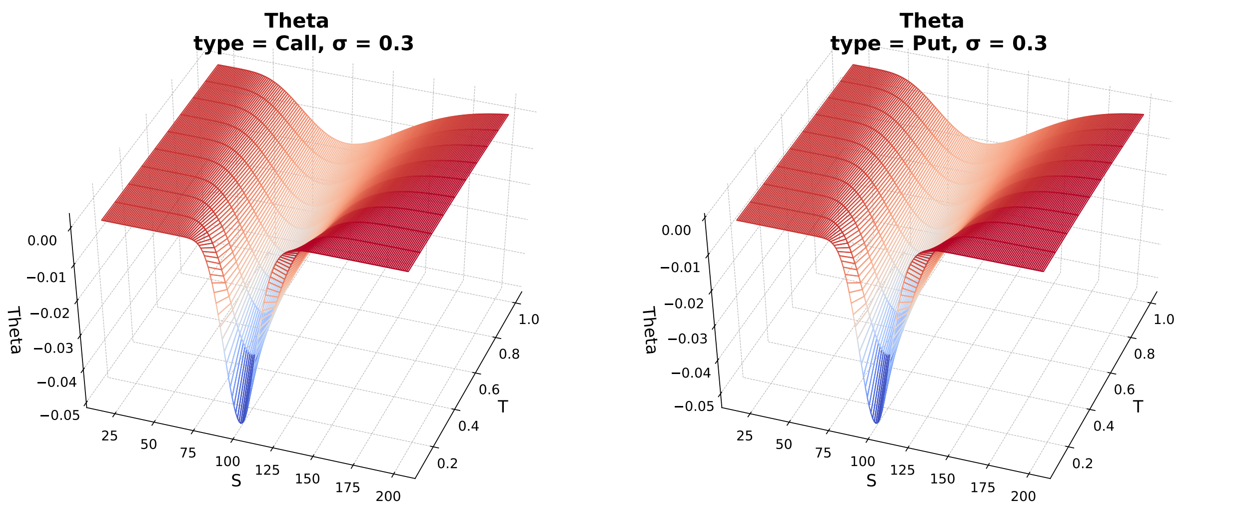

Figure 4: New surfaces of Theta after +50% change in volatility.

2.3 Rho

Rho is the derative of the option price with respect to the free risk interest rate:

$$\rho = \frac{\partial V}{\partial r}$$

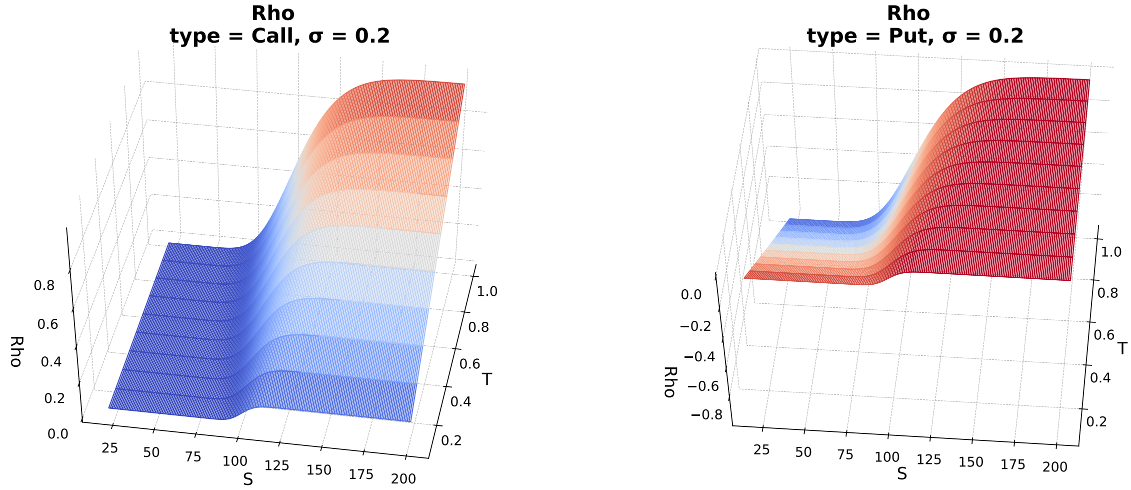

Figure 5: Surface of Rho for Call (on the left) and Put (on the right), with a volatility $\sigma = 0.2$.

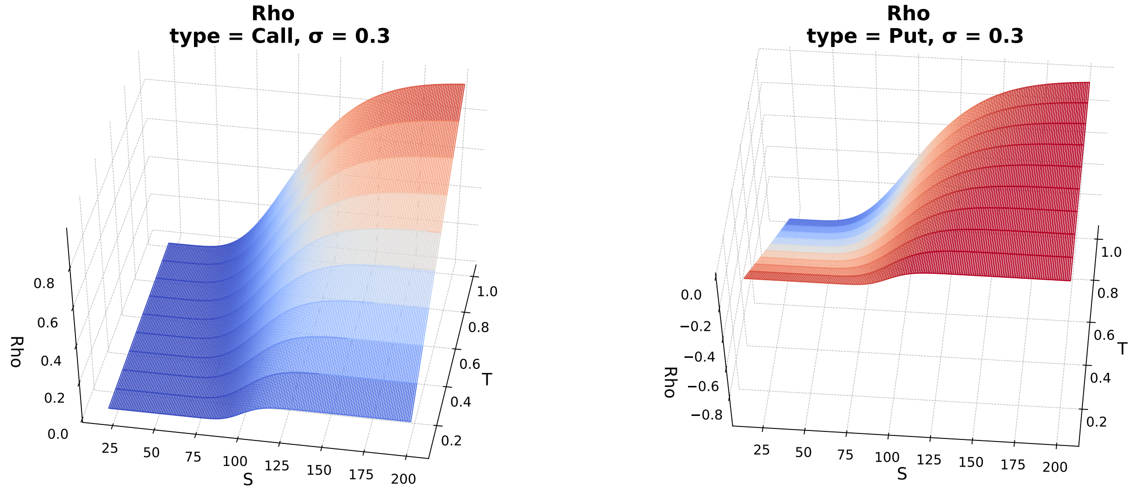

Figure 6: New surfaces of Rho after +50% change in volatility.

2.4 Vega and Gamma

Vega represents a measure of the sensitivity to volatility. Indeed, it is:

$$\mathcal{V} = \frac{\partial V}{\partial \sigma}.$$

On the other hand, Gamma is the derivative of Δ with respect to the underlying price.

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}.$$

Since an Option can be very sensitive to volatility, Vega represent a very important Greek.

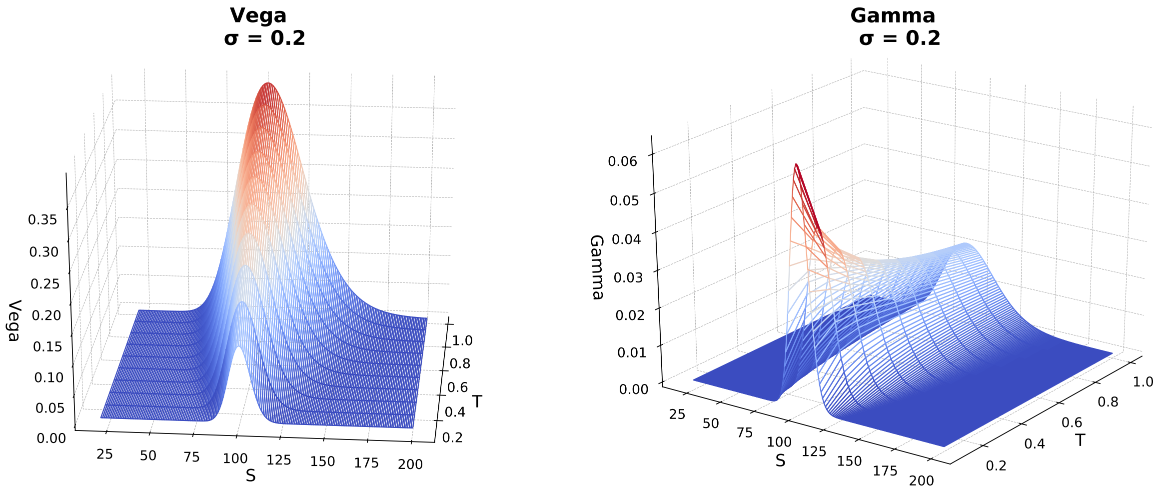

Figure 7: Surface of Vega (on the left) and Gamma (on the right) with a volatility $\sigma = 0.2$.

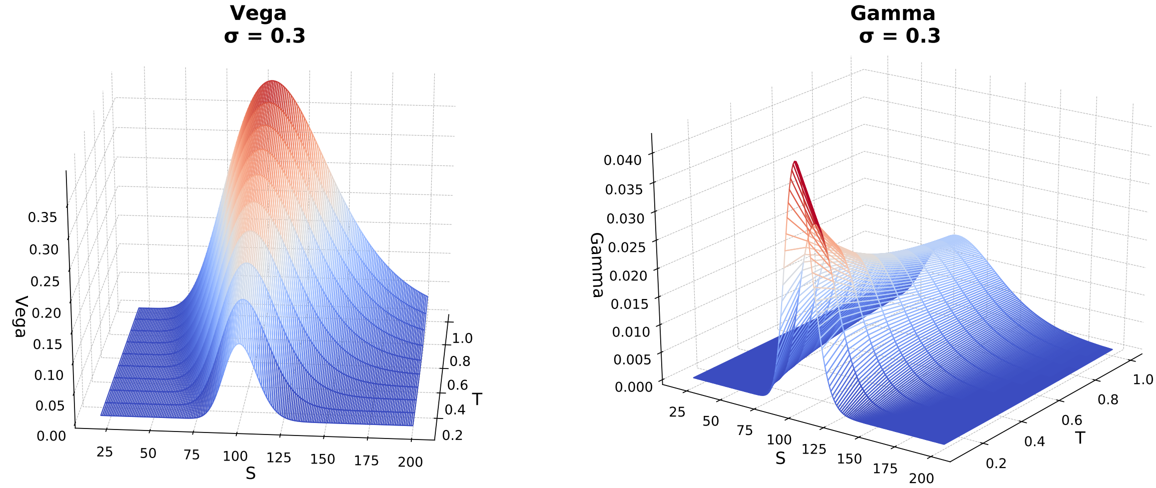

Figure 8: New surfaces of Vega and Gamma after +50% change in volatility.

3 Conclusions

As can be seen from the plots above, the various surfaces appear as expected from theory: in particular, we can observe that, after the sock in volatility, all the surfaces are less steep than before.

A Formulas

Assuming an European Option for an asset with strike K, underlying price S_0 and no dividends we have the following formulas.

A.1 Black-Scholes pricing formula

$$\text{Call} = S_0 N(d_1) - Ke^{-rT} N(d_2),$$

$$\text{Put} = Ke^{-rT} N(-d_2) - S_0 N(-d_1).$$

where

$$d_1 = \frac{\ln(S_0/K) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln(S_0/K) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

A.2 Delta

$$\Delta(\text{Call}) = N(d_1), \tag{2}$$

$$\Delta(\text{Put}) = N(d_1) - 1. \tag{3}$$

A.3 Theta

$$\Theta(\text{ Call }) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT} N(d_2), \tag{4}$$

$$\Theta(\text{ Put }) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT} N(-d_2), \tag{5}$$

where

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}. \tag{6}$$

A.4 Rho

$$\rho(\text{Call}) = KTe^{-rT} N(d_2), \tag{7}$$

$$\rho(\text{Put}) = -KTe^{-rT} N(-d_2). \tag{8}$$

A.5 Vega

$$\mathcal{V} = S_0 \sqrt{T} N'(d_1). \tag{9}$$

A.6 Gamma

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}. \tag{10}$$