Università degli studi di Padova

Stochastic Methods for Finance

Eigth report

Heston model calibration

Author:
Lorenzo Mancini *2019098*

Professor:
prof. Martino Grasselli

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1 Goal

The purpose of this report is to calibrate the Heston model using market data.

2 Introduction: the Heston Model

The Heston model is a stochastic volatility model: the volatility is assumed to be not constant, as it is in the Black-Scholes formula. The underlying obeys to the following dynamics:

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S, \tag{1}$$

$$d\nu_t = \kappa \left(\theta - \nu_t\right) dt + \xi \sqrt{\nu_t} dW_t^\nu, \tag{2}$$

where ν_t is the istantaneous variance. As usual, the other parameters are:

- μ is the rate of return of the underlyinh;
- θ is the long variance;
- σ is the volatility of the volatility;
- κ is the rate at which ν_t reverts to θ.

We want to develop a method in order to estimate the value of the parameters describing equations (1) and (2). In order words, we would like to use market data in order to calibrate the Heston model such that the differences between the model estimation and real market values are minimized. To this end, we need a cost function to be minimized. As market data, here we refer to Call Option prices for a given underlying and for different strikes and maturities. For the computation we used Tesla Inc. stock options

2.1 Tesla Inc.

Having determined a set of strikes K_i and maturities T_i, the cost function can be defined as:

$$error = \sum_{i,j} \left| \text{imp\_vol}^*(K_i, T_j) - \text{imp\_vol}^{\text{data}}(K_i, T_j) \right|, \tag{3}$$

where imp_vol* refers to the implied volatility of the model.
Here, we could have used the square distance, but the algorithm shows better performance with the error defined as in (3).
Note that also prices can be used for the cost function but in that case one shoul also consider the fact that prices are higher for far maturities. Hence, here we convert prices to implied volatility. So, the algorithm takes as input a matrix with implied volatilities, where each row refers to a certain maturity date and each column represets a strike. The expiration dates and the strikes used in the computation are:

- **Maturities**:
  - June, 11, 2021;
  - July, 16, 2021;
  - October, 15, 2021;
  - December, 17, 2021;
  - January, 21, 2022.

- **Strikes**:
  520\$, 530\$, 540\$, 550\$, 560\$, 580\$, 590\$, 600\$, 610\$, 620\$, 630\$.

Furthermore, the trading date considered is May, 14, 2021 and the underlying price is 586.17\$. The minimization of the error is performed with the Differential Evolution method, available in the `scipy` library in Python. We want to determine the parameters θ, κ, ξ, ρ, ν. In order to minimize the error, one must introduce an initial condition and some bounds for those parameters (see the code). With these constraints, we obtain:

$$\theta = 0.444294 \quad \kappa = 2.979744 \quad \xi = 0.999859 \quad \rho = -0.775651 \quad \nu_0 = 0.326034,$$

with an average error of:

$$\text{Average error} = 0.734389394\%$$

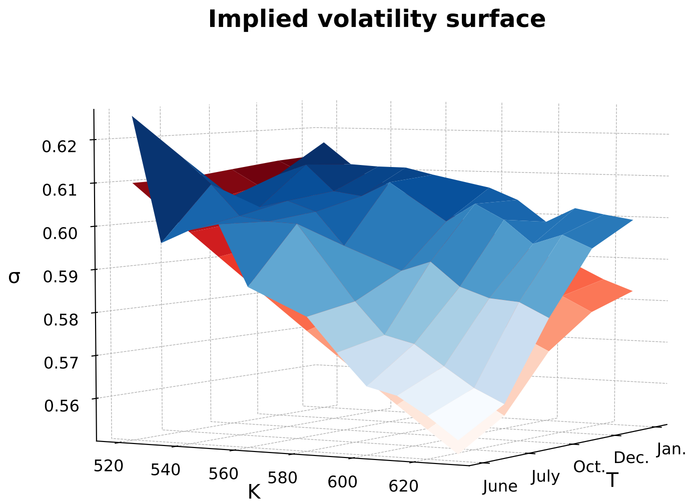

Figure 1: Comparison between the implied volatility of the Market (in blue) and the implied volatility of the model (in red)

3 Conclusion

In the present work we calibrated the Heston model using the implied volatility as a reference. The algorithm reached satisfying results with a small error and a reduced computational time.