Università degli studi di Padova

Stochastic Methods for Finance

Second report

Pricing Call and Put options with binomial model

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Introduction

Goal: This report is divided in two parts and the main goal is to compute the option price of Pfizer Inc. near ATM for a maturity of about three months. This will be done in two ways:

- through a one-step binomial tree;
- through a general binomial tree using VBA.

1.1 About Pfizer Inc.

Pfizer Inc. is an american multinational that works in the pharmaceutical industry. It was founded in 1849 by Charles Pfizer and Charles F. Erhart. Its actual Chariman & CEO is Albert Bourla. It is listed on the New York Stock Exchange and belongs to the Dow Jones index. Some important data provided by the website are shown in the following table (currency in USD):

Market Cap (intraday)	**201.59B**
EBITDA	**14.89B**
Gross Profit (ttm)	**33.33B**
Last split factor	**1054:1000**
Last split date	**Nov 16, 2020**
Closing price (March, 23, 2021)	**35.36**
Forward Annual Dividend Rate	**1.56**
Ex-Dividend Date	**Jan 27, 2021**

All the price values present in this report are expressed in **USD**. For simplicity, in the calculations we won't repeat the currency.

One-step binomial model

2.1 Procedure

In order to achieve our goal, and to construct the binomial model we need:

- a maturity of about three months, $T =$ July, 16, 2021;
- the annual volatility σ_y;
- u and d factors;
- the discount factor $D(0, T)$;
- the strike price $K = 37$ and the value $S = 36.11$ of the underlying, which is the closing price of 30, March, 2021.

2.1.1 Estimation of u and d

From the `Historical data` section in the `https://finance.yahoo.com` website, one can download the data concerning the stock prices in a given period. This, as we're going to show, is useful in order to estimate the annual volatility σ_y. The time interval considered for the Historical Data is:

- starting date: March, 30, 2020;
- end date: March 31, 2021.

This period covers exactly 87 market days. At this point, for each day, we compute the price variation respect to the previous day. Obviously, this is done for every day except for the starting one: this leads to a list of $87 - 1$ variations. Now, computing the standard deviation of the set of those values, we obtain the daily volatility, from which we can easily compute the annual one. All the pervious operations are done with a simple code in Python where we considered as prices the closing ones. For the standard deviation of the variations, and therefore the daily volatility, we get:

$$\sigma_{\text{daily}} = 0.00911.$$

Thus, the annual volatility can be computed as:

$$\sigma_y = \sigma_{\text{daily}}\sqrt{252} = 0.145,$$

where 252 is the number of opening market days in one year.
Finally, for a binomial model, u and d are:

$$u = e^{\sigma_y\sqrt{T}},$$
$$d = e^{-\sigma_y\sqrt{T}},$$

where T is the fraction of year till the expiration date. Since we considered $T =$ July, 16, there are 77 days till the maturity and so $T = 77/252 = 0.306$. Thus:

$$u = 1.09$$
$$d = 0.918.$$

2.1.2 Discount factor $D(0,T)$ and q

The next step is to estimate the discount factor $D(0,T)$. In order to do this we use the box-spread strategy:

- buy a Call with strike K_-, sell a Call with strike K_+;
- buy a put with stike K_+, sell a put with strike K_-,

where K_- and K_+ are two strike prices such that they are about 20 percent different from the ATM price and $K_- < K_+$. In this way, we produce a deterministic payoff payoff at the maturity T equal to $K_+ - K_-$. Since the following holds:

$$\text{Call}_{K_-} - \text{Call}_{K_+} + \text{Put}_{K_+} - \text{Put}_{K_-} = (K_+ - K_-)D(0,T),$$

we can deduce the discount factor $D(0,T)$ from the prices of the Call and Put options provided by the website. Choosing $K_- = 27$ and $K_+ = 45$ we find:

$$D(0,T) = 1.01.$$

What remains is to calculate q which is given by:

$$q = \frac{e^{rt} - d}{u - d}, \tag{2.1}$$

where the factor e^{rt} is simply the inverse of the discount factor $D(0,T)$. The result is;

$$q = 0.442.$$

Now that we have all the ingredients needed, we can finally estimate the price of Call and Put options.

2.1.3 Call

For the Call, the price is:

$$\begin{aligned} p_0(\text{Call}) &= D(0,T)\mathbb{E}^{\mathbb{Q}}\left[(S_T - k)^+\right] \\ &= D(0,T)\left[q(Su - k)^+ + (1-q)(Sd - k)^+\right] \\ &= 1.03 \end{aligned} \tag{2.2}$$

2.1.4 Put

On the other hand, for the put, the relation is:

$$\begin{aligned} p_0(\text{Put}) &= D(0,T)\mathbb{E}^{\mathbb{Q}}\left[(k - S_T)^+\right] \\ &= D(0,T)\left[q(Su - k)^+ + (1-q)(k - Sd)^+\right] \\ &= 2.15 \end{aligned} \tag{2.3}$$

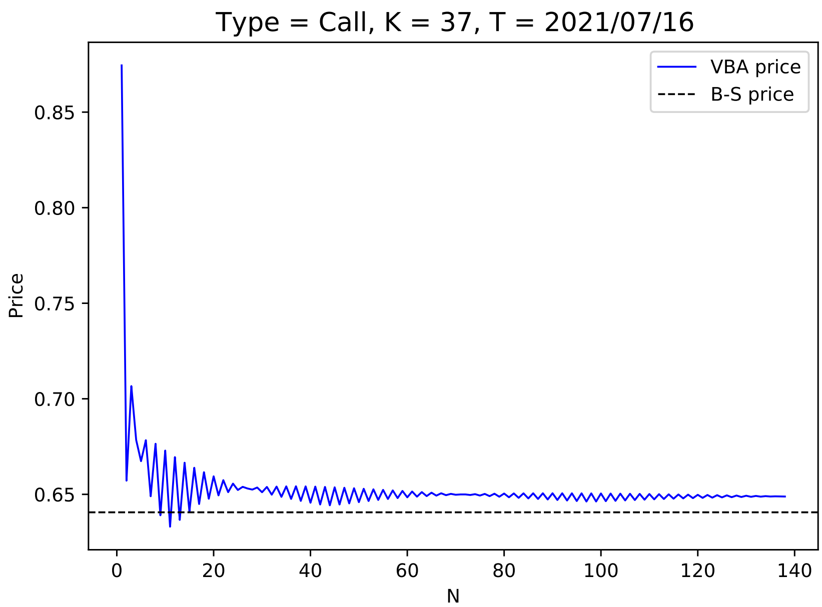
Binomial model whith VBA

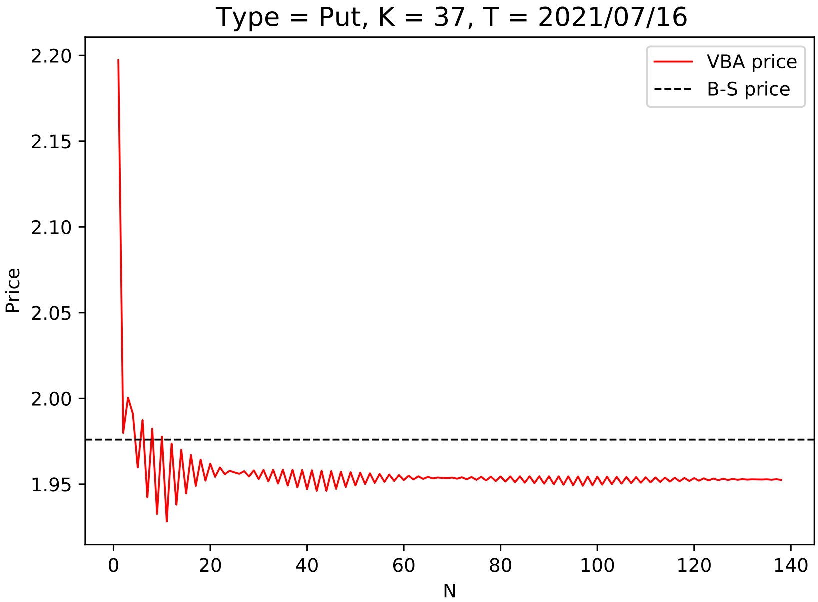
3.1 Procedure

In this second part we want to build a N steps binomial model. For this purpose we wrote a VBA function with the following parameters:

- the underlying price $S = 36.11$;
- the strike price $K = 37$;
- the time to maturity expressed as fraction of years, in this case $T = 77/252 = 0.306$;
- the risk free rate $r = -0.038$, obtained from the discount factor;
- the annual volatility $\sigma_y = 0.145$;
- the number N of steps, which we want to vary;
- the option type (Call or Put);
- the option style (American or European), in our case we used American.

Note the fact that we are specifying the option style, because, for American options, there is the possibility to exercise before the expiration date.
Putting all those parameters in excel and using the VBA function we can obtain the price in relation to the number of steps. The results are shown in the following figures.





In the previous plots we also showed the price of the option obtained with the B-S formula (black line). It is important to note that the latter works for the European style, whereas in the VBA code we considered the option as American.

3.2 Conclusion

In the following table we compare the results obtained. For the N-step binomial tree we take te mean over the various steps.

Option	**One-step bin. tree**	**N-step bin. tree (VBA)**	**B-S formula**
Call	1.03	0.653	0.641
Put	2.15	1.96	1.98

As can be seen, with a N- step binomial tree we are able to obtain values that are close to the prediction of the B-S formula. The small difference may be due to the fact that, as we told, the B-S formula is for European options, whereas we treated them as American. Furthermore, we can note that a simple one-step model is not enough to get very good results.