Università degli studi di Padova

Stochastic Methods for Finance

Fourth report

Implied volatility and Greeks of Amazon's call options

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1 Goal

The purpose of this report is to compute the implied volatility of Amazon's call options for different strikes and different maturities. Then, the results are used to compute the Greeks. **Currency: USD.**

2 About Amazon.com Inc.

Amazon.com Inc. is an American company based in Seattle. It was founded by Jeff Bezos in 1994. It started as an online marketplace for books and now it focuses on e-commerce, cloud computing and artificial intelligence. In the following table we show some important market data:

Market Cap (intraday)	**1.7T**
EBITDA	**48.15B**
Gross Profit (ttm)	**152.76B**
Closing price (April, 14, 2021)	**3333**

3 Implied volatility

First, we compute the historical volatility of Amazon. In order to achieve this, we download the historical data from the `yahoo.com` website. From this dataset, we can easily compute the returns over time (with a very simple script in Python). In the following figure we show the graph.

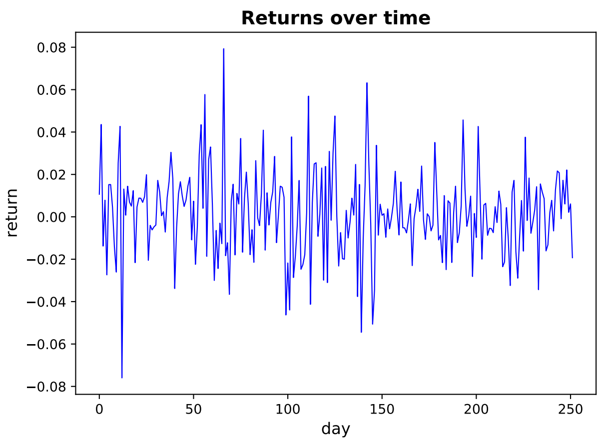

Figure 1: Returns in one year.

Now, the daily historical volatility is just the standard deviation of the returns, $\sigma_{\text{daily}} = 0.0207$. Then, the annual one is:

$$\sigma_y = \sigma_{\text{daily}}\sqrt{252} = 0.328, \tag{1}$$

where 252 is the number of opening market days in one year.
For the future computations, we consider:

- underlying price $S = 3333$, equal to the closing price of April, 14, 2021;
- dividend rate, $d = 0$, since Amazon does not provide any dividends;

- "Call" as option type;
- the free risk interest rate is computed for each maturity with the strategy of the box-spread (see maturities);
- volatility σ equal to the historical volatility;
- a list of maturities:
  - T_1 = April, 23, 2021; $r_1 = 0.08$;
  - T_2 = April 30, 2021; $r_2 = 0.08$;
  - T_3 = June, 18, 2021; $r_3 = 0.113$;
  - T_4 = Sept, 17, 2021; $r_4 = 0.092$;
  - T_5 = March, 18, 2022; $r_5 = 0.02$;
- a range of strikes that depends on the market availability for each maturity.

It is important to note the fact that we are supposing the options to be European style (also if in this case it is not true).
Now, for each maturity, we compute the Black-Scholes price with different strikes K:

$$\text{Call } = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \tag{3}$$

The result is compared with the "Last Price" present on the website. At this point we can calculate the implied volatility with a function written in VBA. This is an example:

strike K	**Last price**	**Black-Scholes price**	**Implied volatility**
3120	234	229	0,374
3125	292	225	0,754
3130	213	220	0,233
...	...	...	...
3520	7	17	0,245
Average volatility			0,277

Table 1: Black-Scholes price and implied volatilities for T = April, 23, 2021.

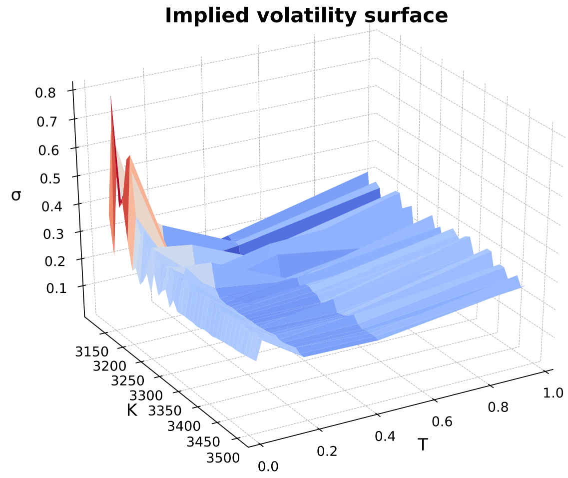

Figure 2: Implied volatility surface.

We end up with five mean volatility values, one for each maturity, and we compute the mean over those values:

σ_{T_1}	0.277
σ_{T_2}	0.364
σ_{T_3}	0.216
σ_{T_4}	0.201
σ_{T_5}	0.251

Thus, we obtain:

$$\bar{\sigma}_{\text{imp\_vol}} = 0.262 \tag{4}$$

In the following section we are going to compute the greeks using the volatility obtained.

4 Greeks

For simplicity, we considered the average of the free-risk interest rates written before:

$$\bar{r} = 0.08 \tag{5}$$

4.1 Delta

The Delta Greek is the rate of change between the option's price and the underlying asset's price:

$$\Delta = \frac{\partial V}{\partial S}, \tag{6}$$

where V is the option's price.

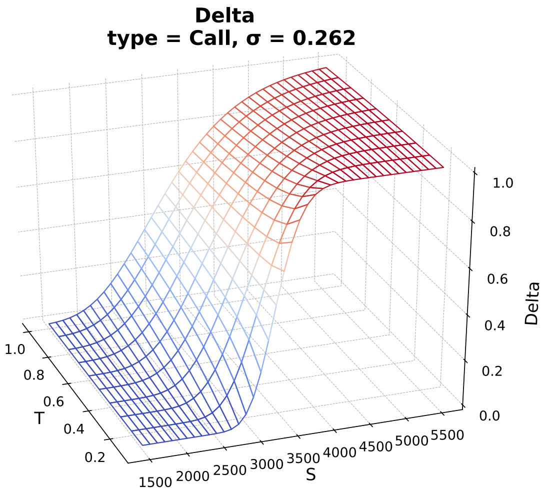

Figure 3: Surface of Delta for Call, with a volatility $\sigma = 0.262$.

4.2 Theta

Theta can be expressed as time decay of an option. It is:

$$\Theta = -\frac{\partial V}{\partial T},$$

In our case, we have the following surface:

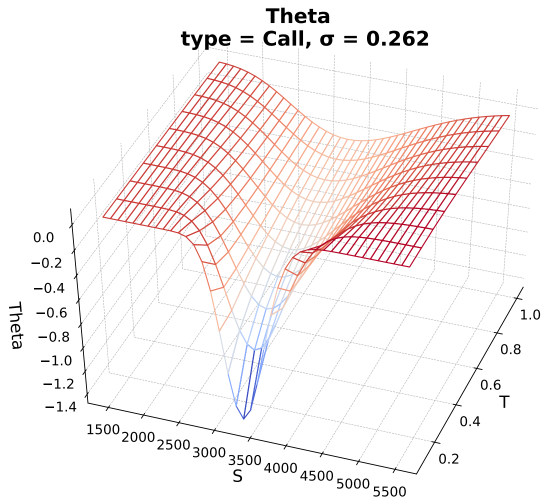

Figure 4: Surface of Theta for Call with a volatility $\sigma = 0.262$.

4.3 Rho

Rho is the derative of the option price with respect to the free risk interest rate:

$$\rho = \frac{\partial V}{\partial r}$$

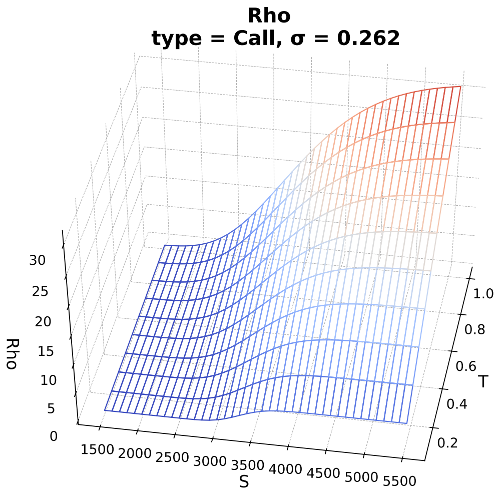

Figure 5: Surface of Rho for Call with a volatility $\sigma = 0.262$.

4.4 Vega and Gamma

Vega represents a measure of the sensitivity to volatility. Indeed, it is:

$$\mathcal{V} = \frac{\partial V}{\partial \sigma}.$$

On the other hand, Gamma is the derivative of Δ with respect to the underlying price.

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}.$$

Here we show the corresponding surfaces.

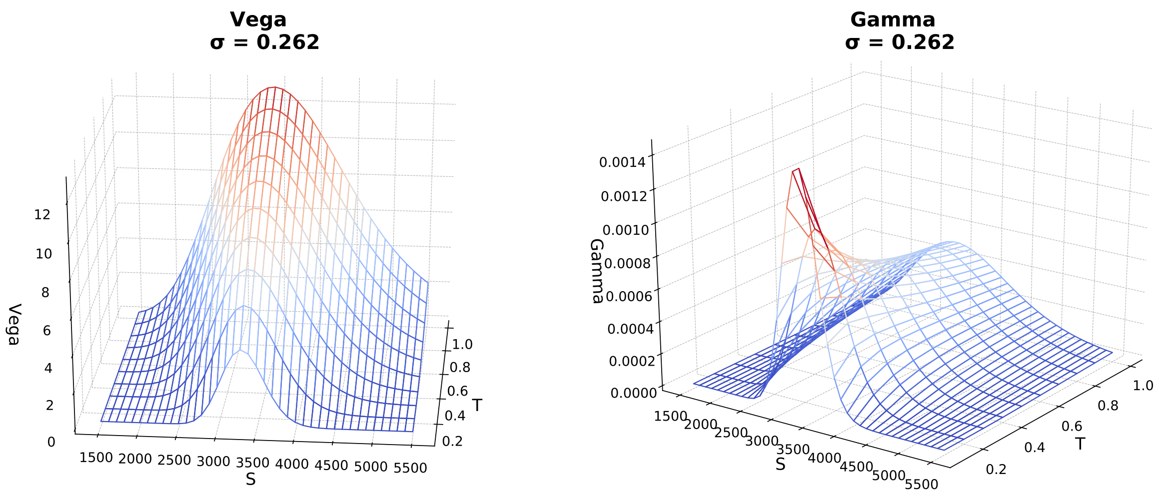

Figure 6: Surface of Vega (on the left) and Gamma (on the right) with a volatility $\sigma = 0.262$.

5 Conclusion

In this report the computed the implied volatility of Amazon stock's options for different strikes and maturities. The average value obtained over the expiration dates is

$$\bar{\sigma}_{\text{imp\_vol}} = 0.262, \tag{7}$$

which is reasonably different from the historical one $\sigma_{\text{historical}} = 0.328$. In the second part we computed the surface of various Greeks. The corresponding graphs respect what is expected from theory.