Three-Body Interaction in Young and Nuclear Star Clusters

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Introduction

In this work we're going to investigate some simulations concerning three-body encounters.

Goal of the project: study the statistics of two type of star cluster:

- Young Star Cluster (YSC)
- Nuclear Star Cluster (NSC)

A star cluster is a group of stars that share a common origin and are gravitationally bound for some length of time. Studying star clusters is important, as they give us important information about stellar evolutions

As anticipated, in this work we focus on Young and Nuclear star clusters

Young Star Cluster (YSC)

Young (open) clusters are usually found within the spiral arms (of the galaxy), and are generally young objects, although some exceptions might exist, like the one of M67 which is an old, but open cluster YSC **main features**:

- Short lived: age ≤ 0.1 Gyr
- Less massive: $M_{cl} \geq 10^4 \, M_\odot$
- Less dense: $\rho_{core} \geq 10^4 \, M_\odot pc^{-3}$
- Less fast: $v_{esc} \sim 10 km \, s^{-1}$

Open clusters usually consist of young blue stars

Nuclear Star Cluster (NSC)

Globular clusters consist of old stars (probably just a few hundred million years younger than the universe itself) and they are very similar to Nuclear clusters

NSC **main features**:

- Long lived: age $\leq$ 10 Gyr
- More massive: $M_{cl} \geq 10^6 \ M_\odot$
- More dense: $\rho_{core} \geq 10^6 \ M_\odot pc^{-3}$
- Faster: $v_{esc} \sim 100 km \ s^{-1}$

Three-Body Interactions

During a three-body interaction, i.e. an interaction between a binary and a single star, different events can happen:

- fly-by
- exchange
- ionization
- merging

We are provided with **two separated data-sets**, one for YSC and one for NSC, with 10^4 simulations for each of them.
For each step of the simulations (referring to the `file`) we have:

- **masses**, **positions** and **velocities** for each of the three black holes

All the quantities in the file are expressed in N-Body units (we'll discuss this later)

Two main steps (for both YSC and NSC):

1. Determine the type of event for each simulation (differentiate statistics)
2. Compare the properties of the final population of binary black holes and identify the main differences (e.g. semi-major axis and total mass) between the two star cluster

In the following slides we're going to show the results of the previous steps

Differentiate Statistics

Binding Energy

Discriminative features to understand the class of the event.

$$E_{m_1 m_2}^{int} = \frac{1}{2}\mu v^2 - \frac{Gm_1 m_2}{r}$$

where μ is the **reduced mass** of the binary system, while v and r are, respectively, the **relative velocity** and the **relative position** of the two masses.

If $E_{m_1 m_2}^{int}$ is negative then the couple of BHs form a binary system.

Given the binding energy, we can identify the various events in the following way:

- Fly-by $\rightarrow E_{0-1} < 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$
- Exchange0 $\rightarrow E_{0-1} > 0, \quad E_{1-2} < 0, \quad E_{2-0} < 0$
- Exchange1 $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$
- Ionization $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$

If an event does not follow any of these cases, then we label it as Unclassified

The most interesting class of event:

- mergers

In the simulation those are individuated when one of the masses is equal to zero $m_0 = 0 | m_1 = 0 | m_2 = 0$

For some events more simulation time is required

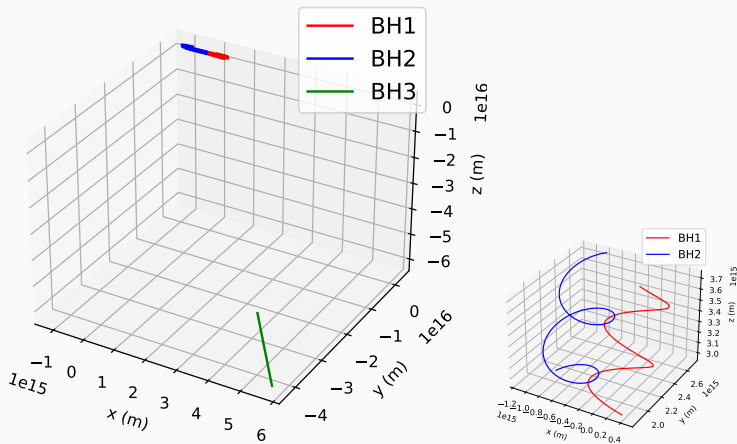**Figure 1:** 3-dimensional plot of trajectories

To differentiate from fly-by we check if the third mass is approaching or moving away from the binary system.
Giving a look to the distance between the third BH and the binary, at the end of the simulation and 10 instants before it.

- $d_{1-0} < d_0 \longrightarrow$ fly-by
- $d_{1-0} > d_0 \longrightarrow$ not happened

Outcome statistics

event	#simulations
exchange0	443
exchange1	1678
fly_by	7228
ionization	63
merge	11
unclassified	188
not happened	389

event	#simulations
exchange0	313
exchange1	727
fly_by	7927
ionization	1009
merge	22
unclassified	1
not happened	1

Table 1: Tables with the results of the event identification between YSC (left table) and NSC (right table).

Comparing Properties

Properties distribution

In order to compare the properties of the final population of the black holes and identify the main differences between the two star clusters we plot distributions of two main physical properties:

- masses distributions
- semi-major axes distributions

Moreover, we can give a quantitative value of similarity between the distributions performing a **Kolmogorov-Smirnov test**.

Before presenting the results, it is important to convert the data in physical units. Remember that:

- $L_{code} = L_{phys}/L_{scale}$
- $M_{code} = M_{phys}/M_{scale}$
- $T_{code} = T_{phys}/T_{scale}$

with:

$$L_{scale} = 1 \text{ pc} \quad M_{scale} = 1 M_{\odot} \quad T_{scale} = \sqrt{\frac{(L_{scale})^3}{G * M_{scale}}}$$

In the following slides we're going to show some plots for the mass and semi-major axis distributions
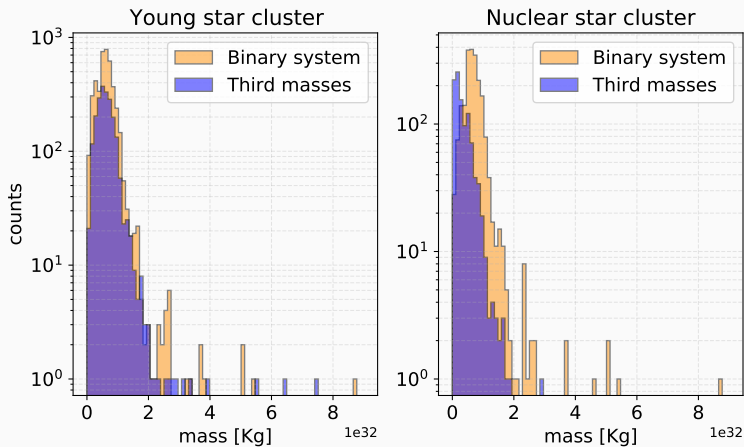
Fly-by
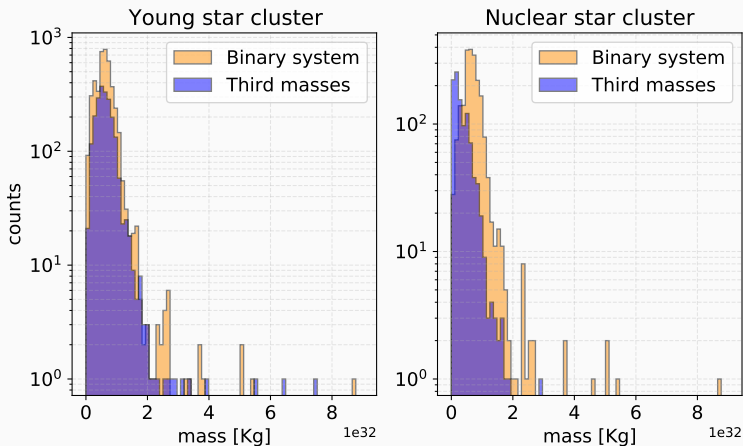

Figure 2: Mass distribution for "fly-by" events.
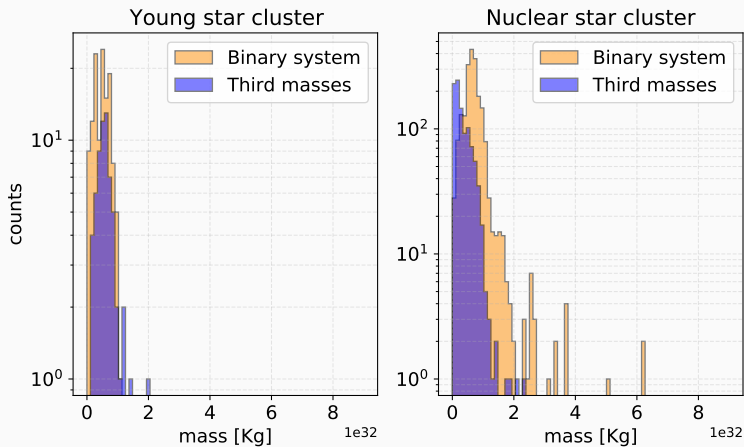
Figure 3: Mass distribution for "exchange" events.

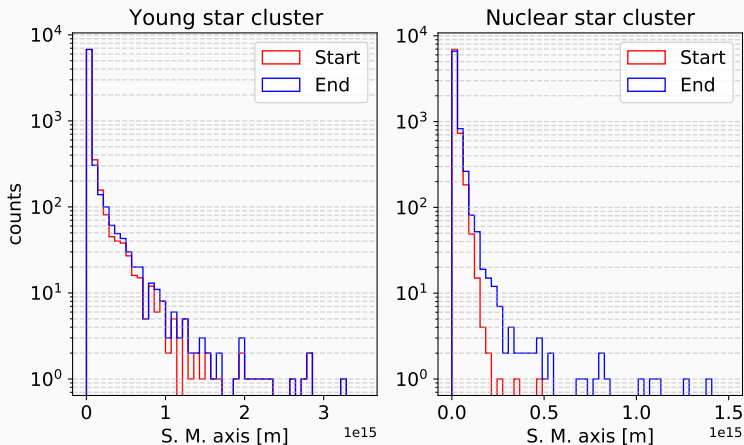**Figure 4:** Mass distribution for "ionization" events.

Fly-by

Figure 5: Semi major axis distribution for "fly-by" events.

Stat.	pvalue	Stat.	pvalue	Stat.	pvalue
-	-	-	-	–	-

Table 2: Results of Kolmogorov test for: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).

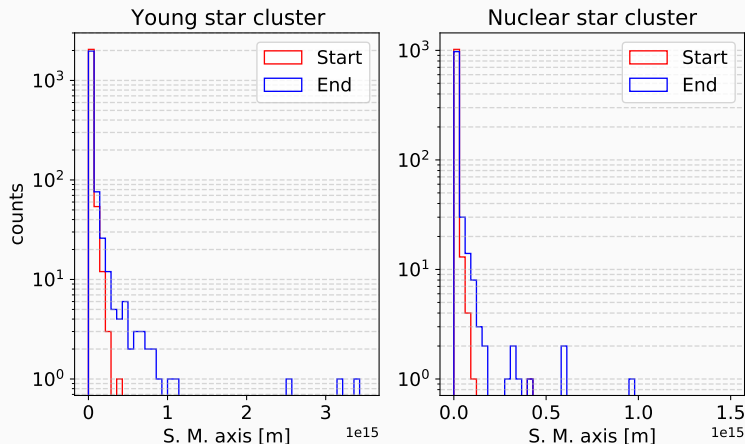**Figure 6:** Semi major axis distribution for "exchange" events.

Stat.	pvalue
0.0363	0.122

Stat.	pvalue
0.0413	0.336

Stat.	pvalue
0.277	1.04e-47

Table 3: Results of Kolmogorov test for: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).

Mergers

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Mergers

Conclusion

- Mergers are the most interesting and exotic phenomena that happened in a dynamical simulation of star cluster
- Given the poor quantity of data that correspond to such events, it is meaningless to study the statistic distributions of their properties
- Instead, we can give a look to their features singularly, for example checking from which type of event they occur.
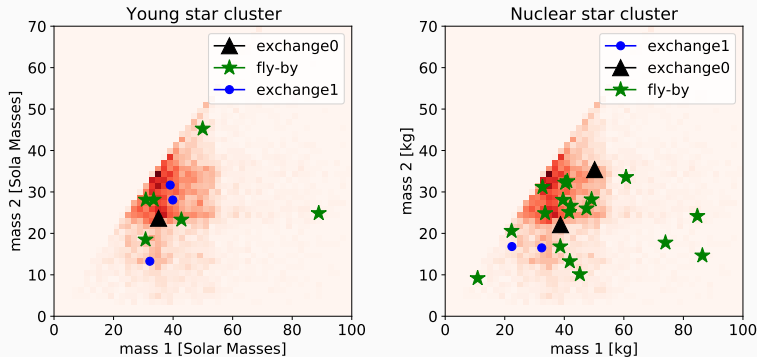
Figure 7: Scatter plot of the masses of binaries for merge event, subdivided from which type of event they occurred. On the background (red), a scatter plot of all masses for all the event

In the end, it is interesting to understand how the systems will evolve in the universe life-time. Indeed, it is possible to estimate which event did not merge during the simulation but it is supposed to merge during the lifetime of the universe.

2 cases: (Keplerian mechanics)

- Peters equation [1964]:
  (dependence on both e and a)

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4(1 - e^2)^{7/2}}{m_1 m_2(m_1 + m_2)}$$

- Approximated Peters equation :
  (only dependence on $a \rightarrow$ circular orbit, $e = 0$)

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4}{m_1 m_2(m_1 + m_2)}$$
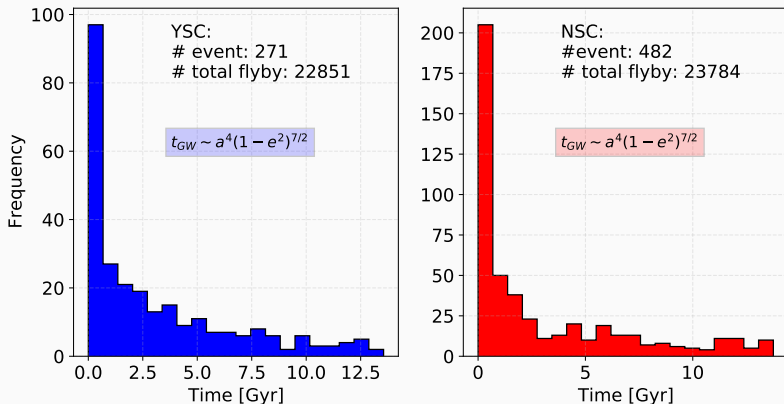
Figure 8: Distribution of fly-by events that will merge in a Hubble time **after the simulation**
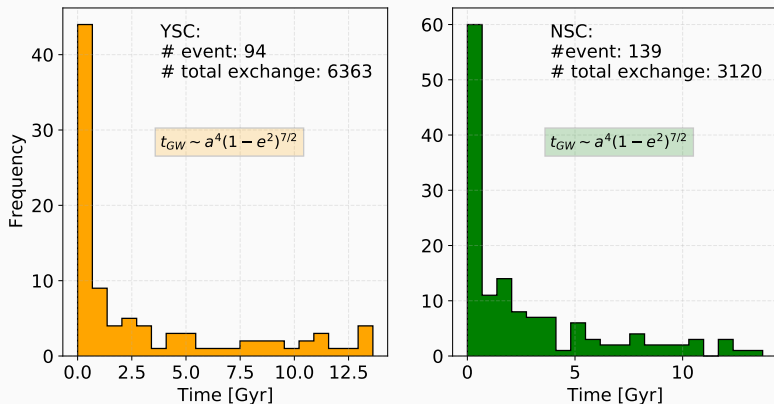
Figure 9: Distribution of exchange events that will merge in a Hubble time **after the simulation**

Conclusion

Introduction

Differentiate Statistics

Comparing Properties

Mergers

Conclusion

In the present work, we study the outcomes of simulation concerning three-body interactions. In particular, we:

- exploit some property (e.g. binding energy) in order to distinguish different events
- study the statistics of different events for both Young and Nuclear star clusters
- perform a quick statistical test in order to verify our results (e.g. Kolmogorov test)

Here we discuss how this work can be developed further:

- perform more advanced statistical tests (e.g. Bayes Factor) in order to compare distributions and thus verify our results
- perform a different (and more advanced) estimation of future mergers

Thanks for your attention!