

Three-Body Interaction in Young and Nuclear Star Clusters

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Introduction

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In this work we're going to study the results of **N-Body simulations** [2] concerning three-body encounters for two types of star cluster.

Goal of the project: study the statistics of two type of star cluster:

- **Young Star Cluster** (YSC);
- **Nuclear Star Cluster** (NSC).

A **star cluster** is a group of stars that share a common origin and are gravitationally bound for some length of time. Studying star clusters is important because:

- they give us important information about stellar evolutions;
- star cluster dynamics affects the formation of BBH systems with very peculiar properties [2].

Young Star Cluster (YSC)

Young (open) clusters are usually found within the spiral arms (of the galaxy), and are generally young objects

YSC main features:

- Short lived: $\text{age} \leq 0.1 \text{ Gyr}$;
- Less massive: $M_{cl} \geq 10^4 M_{\odot}$;
- Less dense: $\rho_{core} \geq 10^4 M_{\odot} pc^{-3}$;
- Less fast: $v_{esc} \sim 10 km s^{-1}$.

Open clusters usually consist of young blue stars.

Nuclear Star Cluster (NSC)

Globular clusters consist of old stars (probably just a few hundred million years younger than the universe itself) and they are very similar to Nuclear clusters.

NSC **main features:**

- Long lived: $\text{age} \leq 10 \text{ Gyr}$;
- More massive: $M_{cl} \geq 10^6 M_{\odot}$;
- More dense: $\rho_{core} \geq 10^6 M_{\odot} pc^{-3}$;
- Faster: $V_{esc} \sim 100 km s^{-1}$.

Three-Body Interactions

During a three-body interaction, i.e. an interaction between a binary and a single star, different events can happen:

- Fly-bies;
- Exchanges;
- Ionizations;
- Mergers (of relevant importance because we observe them with gravitational waves).

Gravitational waves

Invisible ripple in space-time, predicted by Einstein, that travel at the speed of light.

Some examples of events that could cause gravitational waves are:

- when a star explodes asymmetrically (called a supernova);
- when two big stars orbit each other;
- when two black holes orbit each other and **merge**.

GWs were detected for the first time in 2015 thanks to the GW observatory **LIGO** [1] [2]

We are provided with **two separated data-sets**, one for YSC and one for NSC, with 10^4 simulations for each of them.

For each step ($\sim 10^3$) of the simulations we have:

- **masses, positions and velocities** for each of the three black holes.

All the quantities in the file are expressed in N-Body units (we'll discuss this later).

Two main steps (for both **YSC** and **NSC**):

1. Determine the type of event for each simulation (differentiated statistics);
2. Compare the properties of the final population of binary black holes and identify the main differences (e.g. semi-major axis and total mass) between the two star cluster.

Differentiated Statistics

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We can exploit the binding energy between couples of BHs in order to understand the class of event [4].

$$E_{m_1 m_2}^{int} = \frac{1}{2} \mu v^2 - \frac{G m_1 m_2}{r}$$

where μ is the **reduced mass** of the two BHs, while v and r are, respectively, the **relative velocity** and the **relative position** of the two masses.

If $E_{m_1 m_2}^{int}$ is negative then the couple of BHs form a binary system.

Classes of events

Given the binding energy, we can identify the various events in the following way:

- **Fly-by** $\rightarrow E_{0-1} < 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$
- **Exchange0** $\rightarrow E_{0-1} > 0, \quad E_{1-2} < 0, \quad E_{2-0} < 0;$
- **Exchange1** $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$
- **Ionization** $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$

If an event does not follow any of these cases, then we label it as **Unclassified**.

Mergers are the most interesting class of event:

- we can observe them with gravitational waves, which can tell us a lot of information on the properties of black holes.

In the simulation those are individuated when one of the masses is equal to zero $m_0 = 0 | m_1 = 0 | m_2 = 0$.

Not happened events

For some events more simulation time is required.

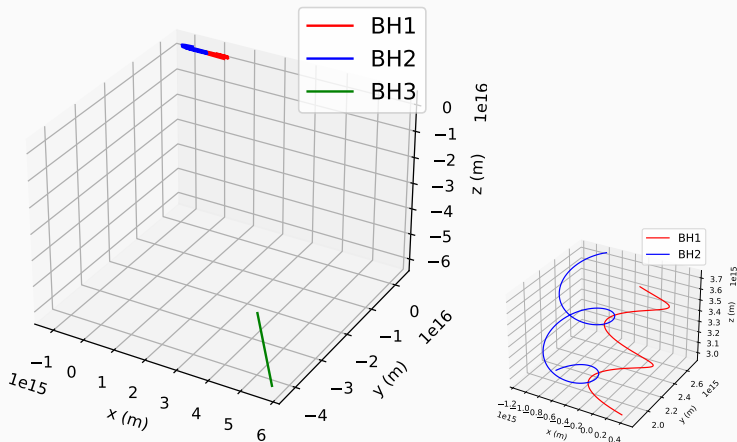


Figure 1: 3-dimensional plot of trajectories. On the right a zoom on the binary system.

Not happened events

To differentiate from fly-by we check if the third mass is approaching or moving away from the binary system.

Giving a look to the distance between the third BH and the binary, at the end of the simulation and 10 instants before it.

- $d_{1-0} < d_0 \longrightarrow$ fly-by;
- $d_{1-0} > d_0 \longrightarrow$ not happened.

Outcome statistics

(YSC)		-	(NSC)		-
event	#simulations		event	#simulations	
Exchanges0	443		Exchanges0	313	
Exchanges1	1678		Exchanges1	727	
Fly_bies	7228		Fly_bies	7927	
Ionizations	63		Ionizations	1009	
Mergers	11		Mergers	22	
Unclassified	188		Unclassified	1	
Not Happened	389		Not Happened	1	

Table 1: Tables with the results of the event identification between YSC (left table) and NSC (right table).

Comparing Properties

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In order to compare the properties of the final population of the black holes and identify the main differences between the two star clusters we plot distributions of two main physical properties:

- masses distributions
- semi-major axes distributions

Moreover, we can give a quantitative value of similarity between the distributions performing a **Kolmogorov-Smirnov test**.

From N-body to physical units

Before presenting the results, it is important to convert the data in physical units. Remember that:

- $L_{code} = L_{phys}/L_{scale}$
- $M_{code} = M_{phys}/M_{scale}$
- $T_{code} = T_{phys}/T_{scale}$

with:

$$L_{scale} = 1 \text{ pc} \quad M_{scale} = 1M_{\odot} \quad T_{scale} = \sqrt{\frac{(L_{scale})^3}{G * M_{scale}}}$$

In the following slides we're going to show some plots for the mass and semi-major axis distributions

We introduce the so called **Kolmogorov-Smirnov** test [3], useful in order to check if two distributions are the same or not:

- Non-parametric statistical test for one dimensional distributions;
- Based on the comparison of empirical CDFs;
- In *Python* it is implemented in the *scipy* library;
- It returns the statistic $D_n = \sup_{-\infty < x < +\infty} |F_n^1(x) - F_0^2(x)|$.

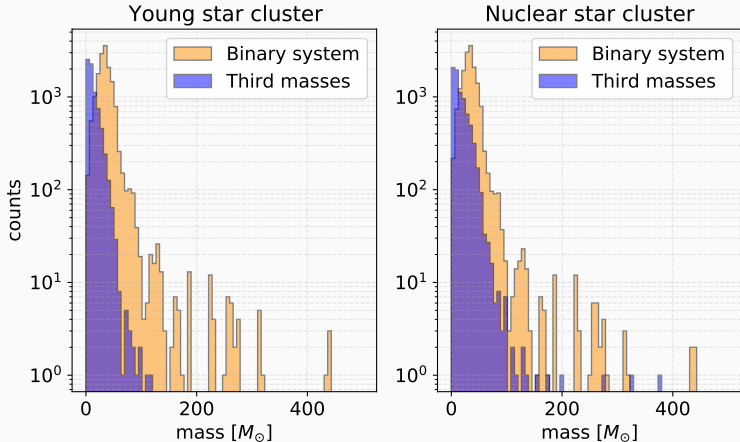


Figure 2: Comparison between the mass distribution of the binary system and the mass of the third black holes for both kind of star clusters (Fly-By event).

Exchange (initial distribution)

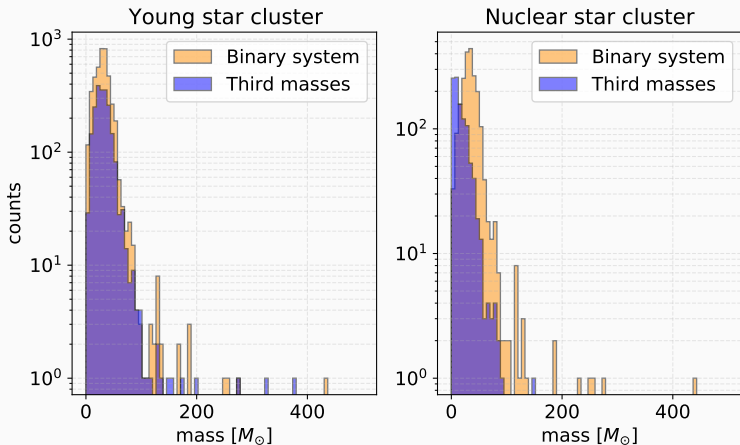


Figure 3: Comparison between the mass distribution of the binary system and the mass of the third black holes **at the beginning of the simulation** for both kind of star clusters (**Exchange event**).

Exchange (final distribution)

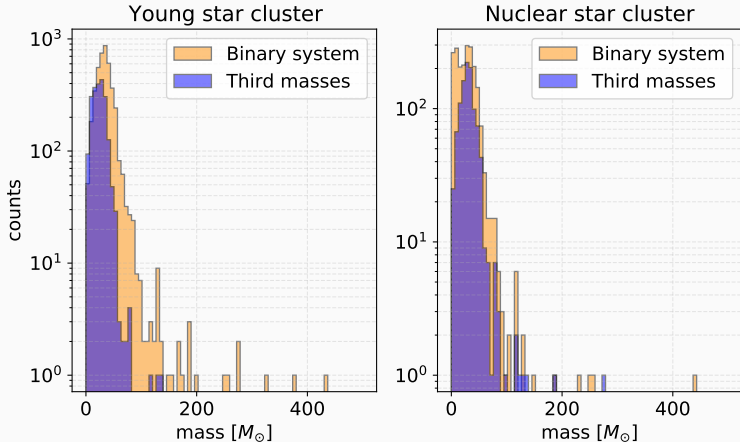


Figure 4: Comparison between the mass distribution of the binary system and the mass of the third black holes **at the end of the simulation** for both kind of star clusters (Exchange event).

We can exploit the K-S test in order to perform some checks:

- Compare the **initial total mass distribution** vs. the **final total mass distribution** of the binary system for both kind of clusters;
- Compare the **final total mass distribution** of the YSCs and the **final total mass distribution** of the NSCs.

Initial total mass vs. final total mass:

Stat.	pvalue	Stat.	pvalue
0.149	6.07e-21	0.340	8.64e-54

Table 2: Comparison between the total mass distributions of the binary system before and after the exchange event for both **YSCs** (left) and **NSCs** (on the right).

Since for both kind of clusters we have high *statistic* and small *pvalue*, then we can qualitatively conclude that the two distributions are not the same.

Final total mass of YSCs vs. final total mass of NSCs:

Stat.	pvalue
0.316	1.05e-62

Table 3: Comparison between the total mass distribution for YSCs and the total mass distribution of NSCs of the binary system after the exchange event.

Also in this case, distributions are quite different.

It is now interesting to see how the results change if we distinguish between the two types of exchange: `exchange0` and `exchange1`. Thus, we compare the initial total distribution and the final total distributions for both kind of exchanges and for both kind of star clusters.

Exchange0

Stat.	pvalue	Stat.	pvalue
0.0609	0.383	0.591	4.59e-51

Table 4: Comparison between initial and final total mass distribution of the binary system for **YSC** (left) and **NSC**.

Note that in this case, for YSCs, the total mass distribution does not change during the **exchange0** event.

Exchange1

Stat.	pvalue	Stat.	pvalue
0.218	2.29e-35	0.568	1.41e-108

Table 5: Comparison between initial and final total mass distribution of the binary system for **YSC** (left) and **NSC**.

Here, in both cases, the distributions cannot be considered equal.

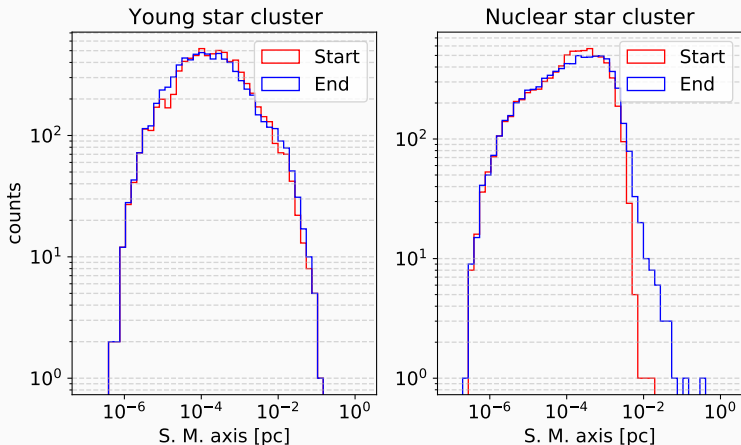


Figure 5: Semi major axis distribution for “fly-by” events. The plot shows a comparison between the initial (red) and the final (blue) distributions in a double log scale.

Kolmogorov results for “fly-by”

Stat.	pvalue	Stat.	pvalue	Stat.	pvalue
0.0412	4.77e-6	0.0438	4.63e-7	0.06	5.03e-16

Table 6: Results of Kolmogorov test for fly-bies: `initial_young` vs. `final_young` (left), `initial_nuclear` vs. `final_nuclear` (middle), `final_young` vs. `final_nuclear` (right).

In all the cases, the statistic and the pvalue are small: we have a small difference between empirical CDFs.

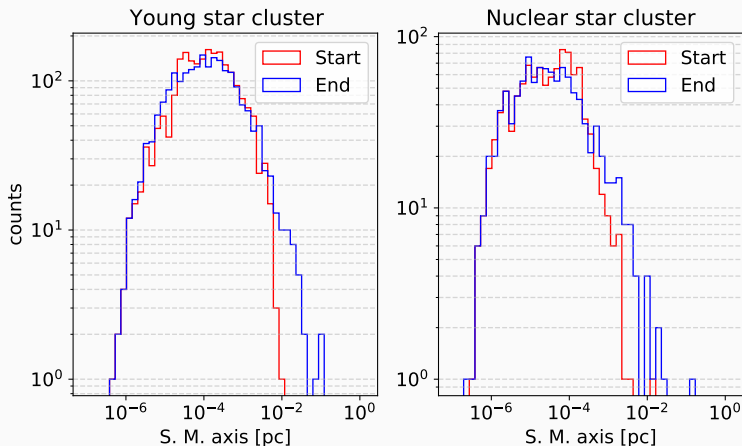


Figure 6: Semi major axis distribution for “exchange” events. The plot shows a comparison between the initial (red) and the final (blue) distributions in a double log scale.

Kolmogorov result for “exchanges”

Stat.	pvalue	Stat.	pvalue	Stat.	pvalue
0.0594	0.00112	0.0663	0.0205	0.265	6.00e-44

Table 7: Results of Kolmogorov test for **exchanges**: **initial_young** vs. **final_young** (left), **initial_nuclear** vs. **final_nuclear** (middle), **final_young** vs. **final_nuclear** (right).

In the first two cases, the statistic and the pvalue are small: we have a small difference between empirical CDFs. In the last case the statistic is quite high and thus the two distribution cannot be considered similar.

Mergers

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Merger events

- Mergers are the most interesting and **exotic phenomena** that happened in a dynamical simulation of star cluster
- Given the poor quantity of data that correspond to such events, it is meaningless to study the statistics distributions of their properties
- Instead, we can give a look to their features singularly, for example checking from which type of event they occur.

Mass distribution

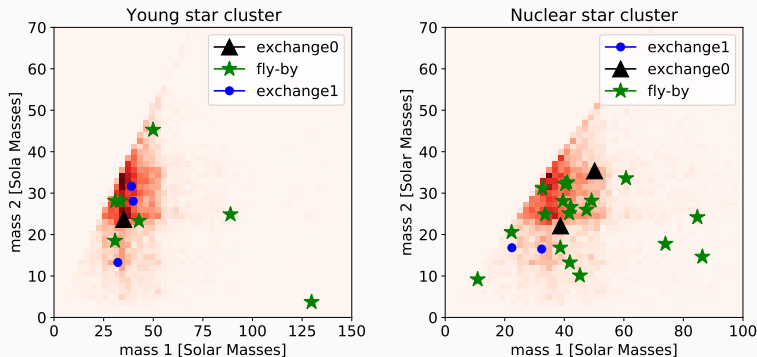


Figure 7: Scatter plot of the masses of binaries for merge event, subdivided from which type of event they occurred. On the background (red), a scatter plot of all masses for all the events that can lead to a merger (Fly-bies and Exchanges)

Some trajectories of Mergers

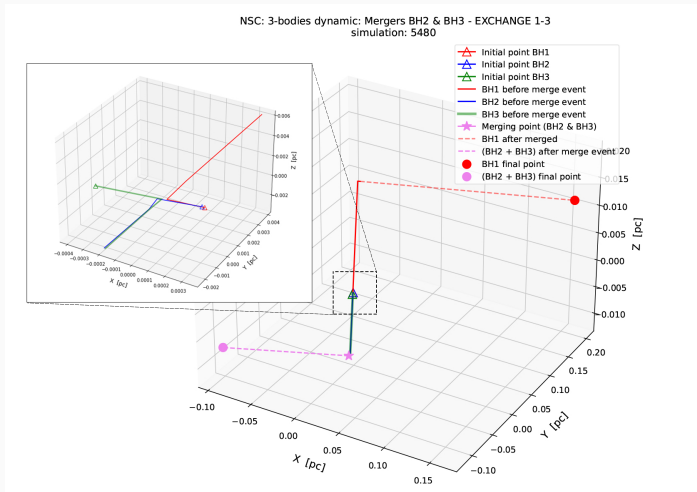


Figure 8: Plot of the trajectory of a merge event on NSC, after an *exchange0* event of 1st BH with the 3rd one

Some trajectories of Mergers

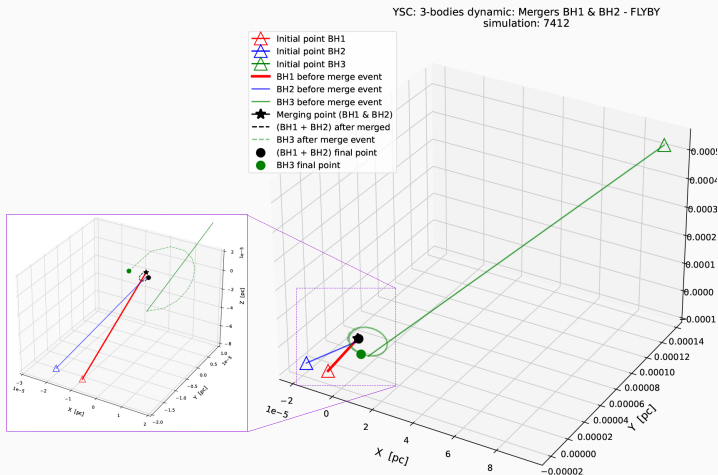


Figure 9: Plot of the trajectory of a merge event on YSC, after a *flyby* event with another binary formation.

In the end, it is interesting to understand how the systems will evolve in the universe life-time. Indeed, it is possible to estimate which event did not merge during the simulation but it is supposed to merge during the lifetime of the universe.

Coalescence time by gravitational wave emission

2 cases:

- Peters equation (1964) [2]:
(dependence on both e and a)

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4 (1 - e^2)^{7/2}}{m_1 m_2 (m_1 + m_2)}$$

- Approximated Peters equation :
(only dependence on $a \rightarrow$ quasi-circular orbit, $e = 0$)

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4}{m_1 m_2 (m_1 + m_2)}$$

Comparison example: Fly-by events in NSC

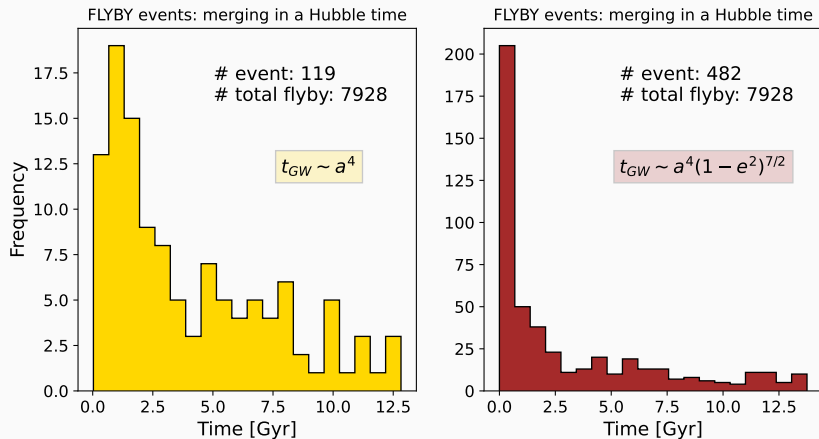


Figure 10: Histogram plot of the number of FLYBY events in NSC, that will merge in a Hubble time.

Comparison example: Fly-by events in NSC

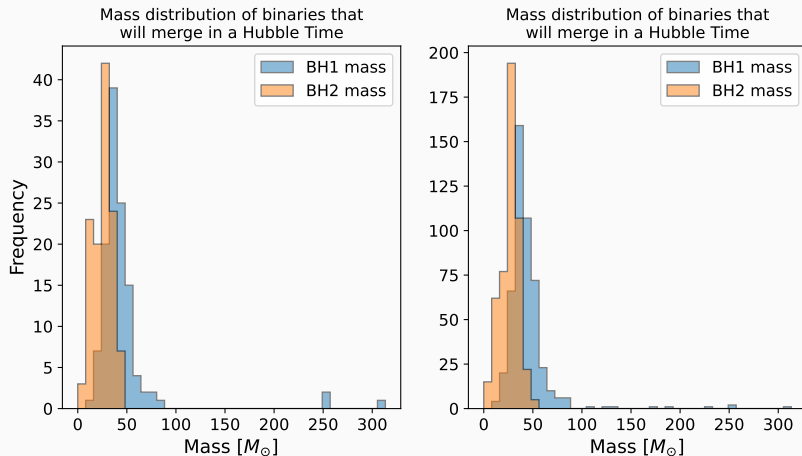


Figure 11: Histogram plot of mass distribution of FLYBY events in NSC, that will merge in a Hubble time.

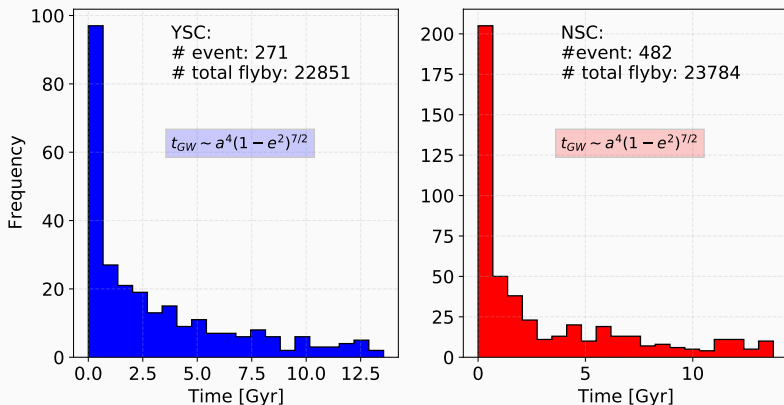


Figure 12: Histogram plots of FLY-BY events that will merge in a Hubble time (starting from the end of the simulation) for both YSC and NSC

Exchanges

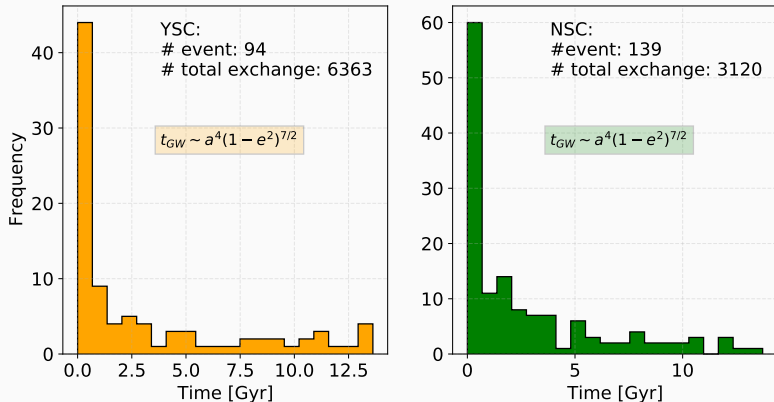


Figure 13: Histogram plots of **EXCHANGE** events that will merge in a Hubble time (starting from the end of the simulation) for both YSC and NSC

Summary of future mergers

Simulation that will merge in the universe life-time by GW emissions:

NSC

- Fly-by : **482** events $\sim 2.0\%$ of total fly-by events
- exchanges : **139** events $\sim 4.5\%$ of total exchange events

YSC

- Fly-by : **271** events $\sim 1.2\%$ of total fly-by events
- exchanges : **94** events $\sim 1.4\%$ of total exchange events

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In the present work, we study the outcomes of simulation concerning three-body interactions. In particular, we:

- exploit some properties (e.g. binding energy) in order to distinguish different events
- study the statistics of different events for both **Young** and **Nuclear** star clusters
- perform a quick statistical test in order to verify our results (e.g. Kolmogorov test)

Main results and observations:

- During exchange events, **NSCs** tend to arrive in a situation where the single BH is more massive than before the event;
- In the case of **YSCs** we have the reversed situation: after the exchange event, the masses of the single BHs seem to be slightly smaller than before the event¹;
- For the **future mergers** it seems that for both kind of clusters and independently of the event class, most mergers will happen in a short time.

¹It is important to remark that in this case, if we consider only the **exchange0** event, the mass distribution remains the same during the evolution.

Here we discuss how this work can be developed further:

- Perform more advanced statistical tests (e.g. **Bayes Factor**) in order to compare distributions with a different method and thus verify our results;
- Perform a different (and more advanced) estimation of future mergers.

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Individual contributions

- **Francesco Fontana**: Pre-processing of data, differentiated statistics, trajectories plots for mergers during simulations, estimations of coalescence time for future Mergers
- **Maryam Hashemi**: Theoretical introduction, Odds Ratio and conclusion part.
- **Lorenzo Mancini**: Analysis of the statistics between YSCs and NSCs (mass and semi-major axis distributions), statistical test (Kolmogorov-Smirnov) over S. M. axis distributions.
- **Giulio Vicentini**: Classification of the events, differentiated statistics, analysis of events that didn't happen within the simulation time and analysis of the Mergers.

Thanks for your attention!