Three-Body Interaction in Young and Nuclear Star Clusters

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Introduction

Overview

In this work we're going to study the results of N-Body simulations [2] concerning three-body encounters for two types of star cluster.

Goal of the project: study the statistics of two type of star cluster:

- Young Star Cluster (YSC);
- Nuclear Star Cluster (NSC).

Star clusters

A star cluster is a group of stars that share a common origin and are gravitationally bound for some length of time. Studying star clusters is important because:

- they give us important information about stellar evolutions;
- star cluster dynamics affects the formation of BBH systems with very peculiar properties [2].

Young Star Cluster (YSC)

Young (open) clusters are usually found within the spiral arms (of the galaxy), and are generally young objects
YSC **main features**:

- Short lived: age ≤ 0.1 Gyr;
- Less massive: $M_{cl} \geq 10^4\ M_{\odot}$;
- Less dense: $\rho_{core} \geq 10^4\ M_{\odot} pc^{-3}$;
- Less fast: $v_{esc} \sim 10 km\ s^{-1}$.

Open clusters usually consist of young blue stars.

Nuclear Star Cluster (NSC)

Globular clusters consist of old stars (probably just a few hundred million years younger than the universe itself) and they are very similar to Nuclear clusters.
NSC **main features**:

- Long lived: age ≤ 10 Gyr;
- More massive: $M_{cl} \geq 10^6\ M_\odot$;
- More dense: $\rho_{core} \geq 10^6\ M_\odot pc^{-3}$;
- Faster: $v_{esc} \sim 100 km\ s^{-1}$.

Gravitational waves

Invisible ripple in space-time, predicted by Einstein, that travel at the speed of light.
Some examples of events that could cause gravitational waves are:

- when a star explodes asymmetrically (called a supernova);
- when two big stars orbit each other;
- when two black holes orbit each other and merge.

GWs were detected for the first time in 2015 thanks to the GW observatory **LIGO** [1] [2]

Three-Body Interactions

During a three-body interaction, i.e. an interaction between a binary and a single star, different events can happen:

- Fly-bies;
- Exchanges;
- Ionizations;
- Mergers.

Simulation

We are provided with **two separated data-sets**, one for YSC and one for NSC, with 10^4 simulations for each of them.
For each step ($\sim 10^3$) of the simulations we have:

- **masses**, **positions** and **velocities** for each of the three black holes.

All the quantities in the file are expressed in N-Body units (we'll discuss this later).

Analysis

Two main steps (for both YSC and NSC):

1. Determine the type of event for each simulation (differentiated statistics);
2. Compare the properties of the final population of binary black holes and identify the main differences (e.g. semi-major axis and total mass) between the two star cluster.

Differentiated Statistics

Binding Energy

We can exploit the binding energy between couples of BHs in order to understand the class of event [3].

$$E^{int}_{m_1 m_2} = \frac{1}{2}\mu v^2 - \frac{G m_1 m_2}{r}$$

where μ is the **reduced mass** of the two BHs, while v and r are, respectively, the **relative velocity** and the **relative position** of the two masses.
If $E^{int}_{m_1 m_2}$ is negative then the couple of BHs form a binary system.

Classes of events

Given the binding energy, we can identify the various events in the following way:

- Fly-by $\rightarrow E_{0-1} < 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$
- Exchange0 $\rightarrow E_{0-1} > 0, \quad E_{1-2} < 0, \quad E_{2-0} < 0;$
- Exchange1 $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$
- Ionization $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0;$

If an event does not follow any of these cases, then we label it as Unclassified.

Mergers

Mergers are the most interesting class of event:

- we can observe them with gravitational waves, which can tell us a lot of information on the properties of black holes.

In the simulation those are individuated when one of the masses is equal to zero $m_0 = 0 | m_1 = 0 | m_2 = 0$.

Not happened events

For some events more simulation time is required.

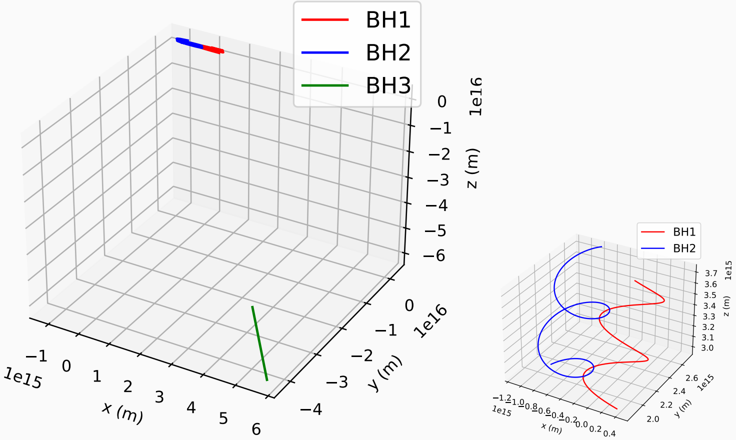

Figure 1: 3-dimensional plot of trajectories. On the right a zoom on the binary system.

Not happened events

To differentiate from fly-by we check if the third mass is approaching or moving away from the binary system.
Giving a look to the distance between the third BH and the binary, at the end of the simulation and 10 instants before it.

- $d_{1-0} < d_0 \longrightarrow$ fly-by;
- $d_{1-0} > d_0 \longrightarrow$ not happened.

Outcome statistics

event	#simulations
Exchange0	443
Exchange1	1678
Fly_by	7228
Ionization	63
Merger	11
Unclassified	188
Not Happened	389

event	#simulations
Exchange0	313
Exchange1	727
Fly_by	7927
Ionization	1009
Merger	22
Unclassified	1
Not Happened	1

Table 1: Tables with the results of the event identification between YSC (left table) and NSC (right table).

Comparing Properties

Properties distribution

In order to compare the properties of the final population of the black holes and identify the main differences between the two star clusters we plot distributions of two main physical properties:

- masses distributions
- semi-major axes distributions

Moreover, we can give a quantitative value of similarity between the distributions performing a **Kolmogorov-Smirnov test**.

From N-body to physical units

Before presenting the results, it is important to convert the data in physical units. Remember that:

- $L_{code} = L_{phys}/L_{scale}$
- $M_{code} = M_{phys}/M_{scale}$
- $T_{code} = T_{phys}/T_{scale}$

with:

$$L_{scale} = 1 \text{ pc} \quad M_{scale} = 1M_{\odot} \quad T_{scale} = \sqrt{\frac{(L_{scale})^3}{G * M_{scale}}}$$

In the following slides we're going to show some plots for the mass and semi-major axis distributions

Fly-by

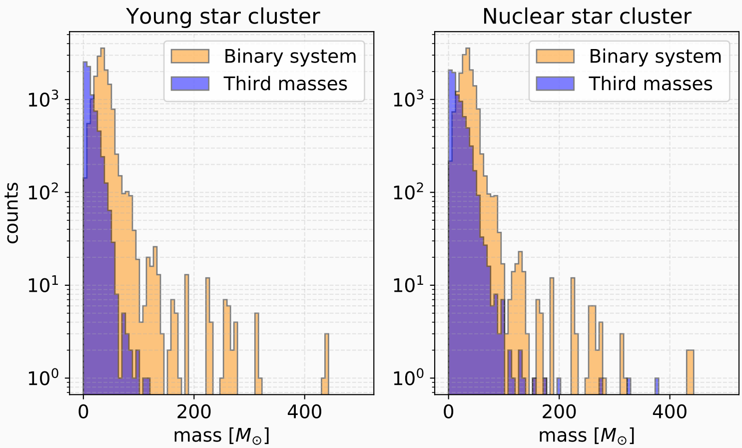

Figure 2: Mass distribution for "fly-by" events.

Exchange

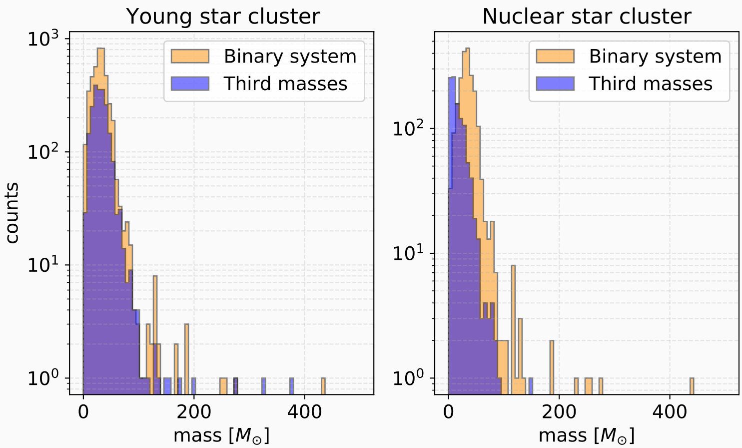

Figure 3: Mass distribution for "exchange" events.

Ionization

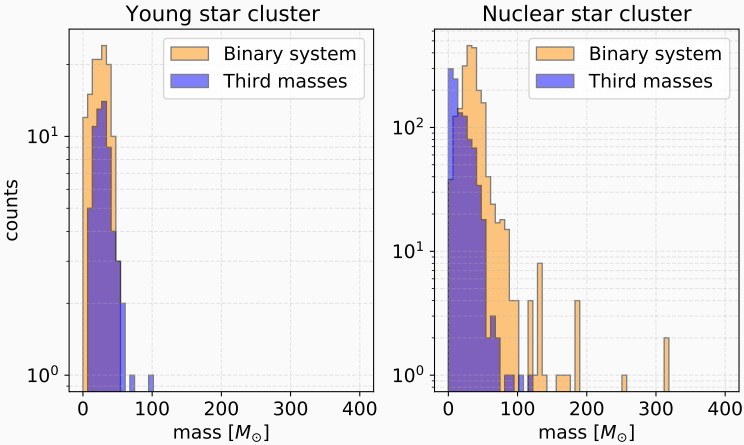

Figure 4: Mass distribution for "ionization" events.

Fly-by

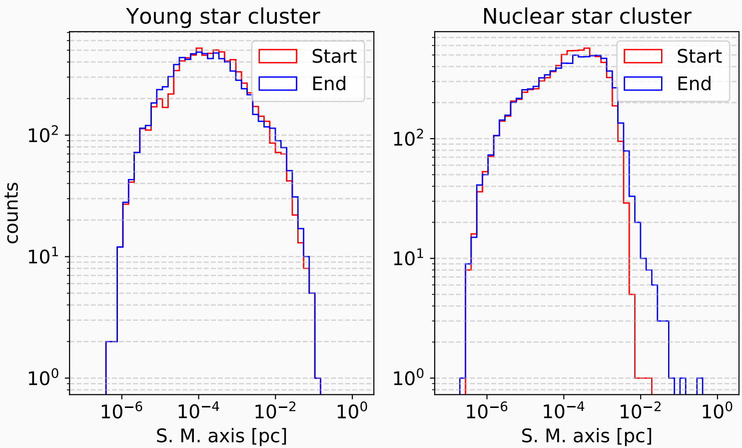

Figure 5: Semi major axis distribution for “fly-by” events.

Kolmogorov-Smirnov test

Useful for comparing two distributions:

- Non-parametric test for 1-dim probability distributions
- It's based on the distance between the empirical CDFs
- In *`Python`* it is implemented in the *`scipy`* library

Kolmogorov result for “fly-by”

Stat.	pvalue
0.0412	4.77e-6

Stat.	pvalue
0.0438	4.63e-7

Stat.	pvalue
0.06	5.03e-16

Table 2: Results of Kolmogorov test for **fly-bies**: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).

Exchanges

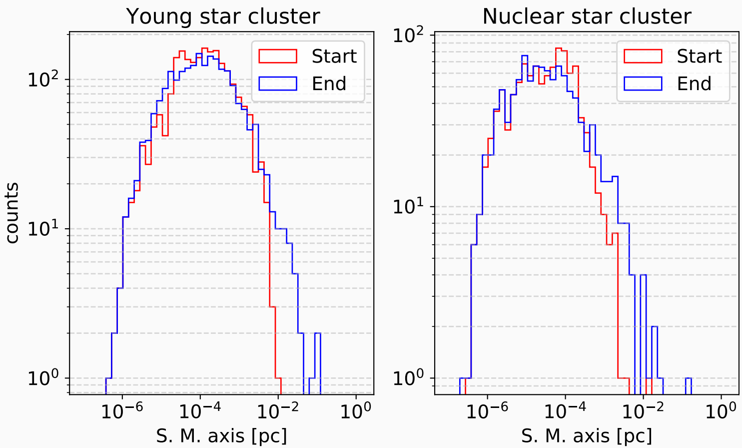

Figure 6: Semi major axis distribution for "exchange" events.

Kolmogorov result for “exchanges”

Stat.	pvalue
0.0594	0.00112

Stat.	pvalue
0.0663	0.0205

Stat.	pvalue
0.265	6.00e-44

Table 3: Results of Kolmogorov test for **exchanges**: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).

Mergers

Merger events

- Mergers are the most interesting and exotic phenomena that happened in a dynamical simulation of star cluster
- Given the poor quantity of data that correspond to such events, it is meaningless to study the statistic distributions of their properties
- Instead, we can give a look to their features singularly, for example checking from which type of event they occur.

Mass distribution

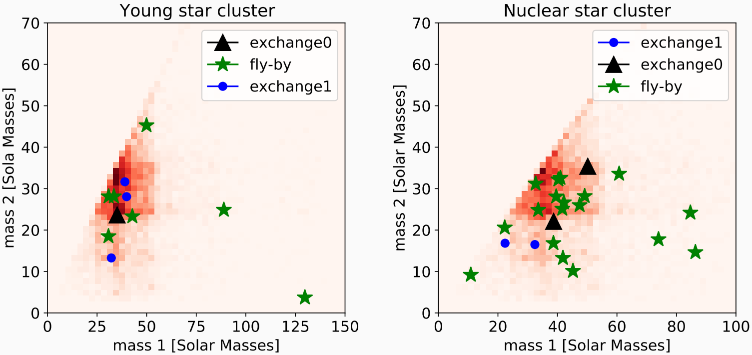

Figure 7: Scatter plot of the masses of binaries for merge event, subdivided from which type of event they occurred. On the background (red), a scatter plot of all masses for all the events that can lead to a merger (Fly-bies and Exchanges)

Some trajectories of Mergers

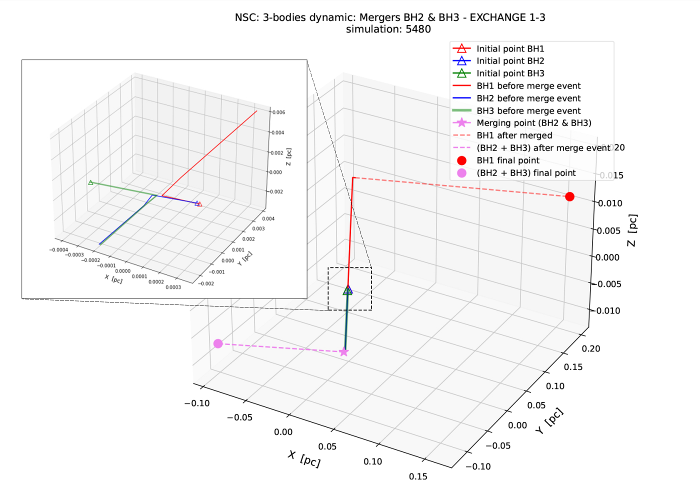

Figure 8: Plot of the trajectory of a merge event on NSC, after an *exchange0* event of 1st BH with the 3rd one

Some trajectories of Mergers

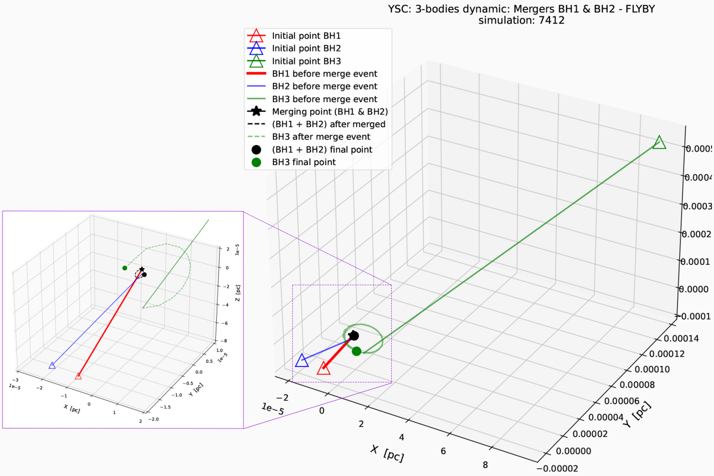

Figure 9: Plot of the trajectory of a merge event on YSC, after a *flyby* event with another binary formation.

Future Mergers

In the end, it is interesting to understand how the systems will evolve in the universe life-time. Indeed, it is possible to estimate which event did not merge during the simulation but it is supposed to merge during the lifetime of the universe.

Computation of t_{GW}

Coalescence time by gravitational wave emission

2 cases:

- Peters equation (1964) [2]:
  (dependence on both e and a)

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4(1-e^2)^{7/2}}{m_1 m_2 (m_1 + m_2)}$$

- Approximated Peters equation :
  (only dependence on $a \to$ quasi-circular orbit, $e = 0$)

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1 m_2 (m_1 + m_2)}$$

ExampleComparison Peters formulas

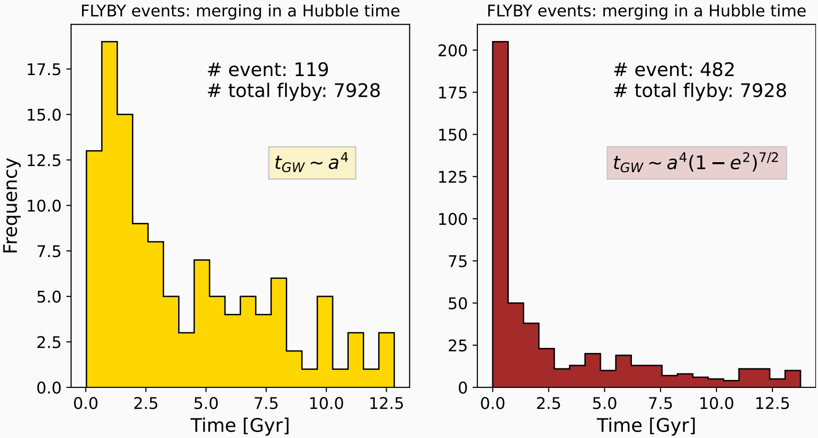

Figure 10: Histogram plot of the number of FLYBY events in NSC, that will merge in a Hubble time.

Comparison Peters formulas

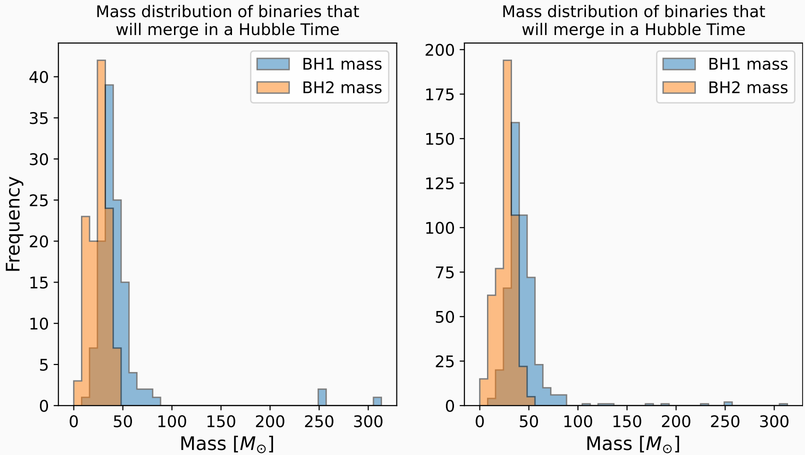

Figure 11: Histogram plot of mass distribution of FLYBY events in NSC, that will merge in a Hubble time.

Fly-By

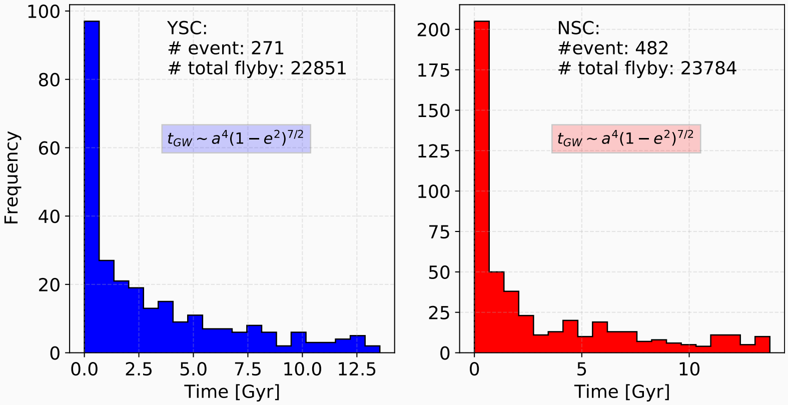

Figure 12: Histogram plots of **FLY-BY** events that will merge in a Hubble time (starting from the end of the simulation) for both YSC and NSC

Exchanges

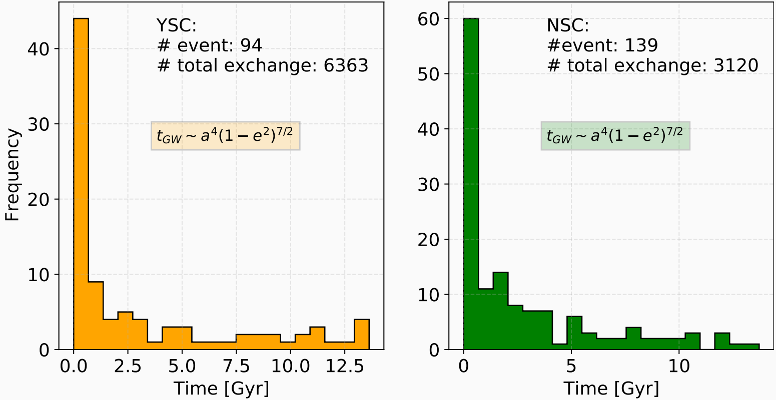

Figure 13: Histogram plots of **EXCHANGE** events that will merge in a Hubble time (starting from the end of the simulation) for both YSC and NSC

Summary of future mergers

Simulation that will merge in the universe life-time by GW emissions:

NSC

- flyby : 482 events
- exchanges : 139 events

YSC

- flyby : 271 events
- exchanges : 94 events

Conclusion

Summary and results

In the present work, we study the outcomes of simulation concerning three-body interactions. In particular, we:

- exploit some properties (e.g. binding energy) in order to distinguish different events
- study the statistics of different events for both Young and Nuclear star clusters
- perform a quick statistical test in order to verify our results (e.g. Kolmogorov test)

Future developments

Here we discuss how this work can be developed further:

- perform more advanced statistical tests (e.g. Bayes Factor) in order to compare distributions and thus verify our results
- perform a different (and more advanced) estimation of future mergers

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Individual contributions

- Francesco Fontana: Pre-processing of data, differentiated statistics, trajectories plots for mergers during simulations, estimations of coalescence time for future Mergers
- Maryam Hashemi: Theoretical introduction, Odds Ratio and conclusion part.
- Lorenzo Mancini: Analysis of the statistics between YSCs and NSCs (mass and semi-major axis distributions), statistical test (Kolmogorov-Smirnov) over S. M. axis distributions.
- Giulio Vicentini: Classification of the events, differentiated statistics, analysis of events that didn't happen within the simulation time and analysis of the Mergers.

Thanks for your attention!