Three-Body Interaction in Young and Nuclear Star Clusters

Laboratory of Computational Physics mod. B

Francesco Fontana   Maryam Hashemi   Lorenzo Mancini   Giulio Vicentini

University of Padua

Table of contents

Introduction

In this work we're going to study the results of N-Body simulations [2] concerning three-body encounters for two types of star cluster.

Goal of the project: study the statistics of two type of star cluster:

- Young Star Cluster (YSC);
- Nuclear Star Cluster (NSC).

Star clusters

A star cluster is a group of stars that share a common origin and are gravitationally bound for some length of time. Studying star clusters is important because:

- they give us important information about stellar evolutions;
- star cluster dynamics affects the formation of BBH systems with very peculiar properties.

As anticipated, in this work we focus on Young and Nuclear star clusters.

Young (open) clusters are usually found within the spiral arms (of the galaxy), and are generally young objects, although some exceptions might exist, like the one of M67 which is an old, but open cluster YSC **main features**:

- Short lived: age ≤ 0.1 Gyr;
- Less massive: $M_{cl} \geq 10^4 \, M_\odot$;
- Less dense: $\rho_{core} \geq 10^4 \, M_\odot pc^{-3}$;
- Less fast: $v_{esc} \sim 10 km \, s^{-1}$.

Open clusters usually consist of young blue stars.

Nuclear Star Cluster (NSC)

Globular clusters consist of old stars (probably just a few hundred million years younger than the universe itself) and they are very similar to Nuclear clusters.

NSC **main features**:

- Long lived: age $\leq$ 10 Gyr;
- More massive: $M_{cl} \geq 10^6 \ M_\odot$;
- More dense: $\rho_{core} \geq 10^6 \ M_\odot pc^{-3}$;
- Faster: $v_{esc} \sim 100 km \ s^{-1}$.

–

Gravitational waves

Invisible ripple in space-time, predicted by Einstein, that travel at the speed of light.
Some examples of events that could cause gravitational waves are:

- when a star explodes asymmetrically (called a supernova)
- when two big stars orbit each other
- when two black holes orbit each other and merge

GWs were detected for the first time in 2015 thanks to the G-W observatory "**LIGO**" [1] [2]

Three-Body Interactions

During a three-body interaction, i.e. an interaction between a binary and a single star, different events can happen:

- Fly-bies;
- Exchanges;
- Ionizations;
- Mergers.

We are provided with **two separated data-sets**, one for YSC and one for NSC, with 10^4 simulations for each of them.
For each step ($\sim 10^3$) of the simulations we have:

- **masses**, **positions** and **velocities** for each of the three black holes.

All the quantities in the file are expressed in N-Body units (we'll discuss this later).

Analysis

Two main steps (for both YSC and NSC):

1. Determine the type of event for each simulation (differentiate statistics);
2. Compare the properties of the final population of binary black holes and identify the main differences (e.g. semi-major axis and total mass) between the two star cluster.

Differentiate Statistics

We can exploit the binding energy between couples of BHs in order to understand the class of event [3].

$$E_{m_1 m_2}^{int} = \frac{1}{2}\mu v^2 - \frac{G m_1 m_2}{r}$$

where μ is the **reduced mass** of the binary system, while v and r are, respectively, the **relative velocity** and the **relative position** of the two masses.
If $E_{m_1 m_2}^{int}$ is negative then the couple of BHs form a binary system.

Classes of events

Given the binding energy, we can identify the various events in the following way:

- Fly-by $\rightarrow E_{0-1} < 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$;
- Exchange0 $\rightarrow E_{0-1} > 0, \quad E_{1-2} < 0, \quad E_{2-0} < 0$;
- Exchange1 $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$;
- Ionization $\rightarrow E_{0-1} > 0, \quad E_{1-2} > 0, \quad E_{2-0} < 0$;

If an event does not follow any of these cases, then we label it as Unclassified.

Mergers are the most interesting class of event:

- we can observe them with gravitational waves, which can tell us a lot of information on the properties of black holes.

In the simulation those are individuated when one of the masses is equal to zero $m_0 = 0 | m_1 = 0 | m_2 = 0$.
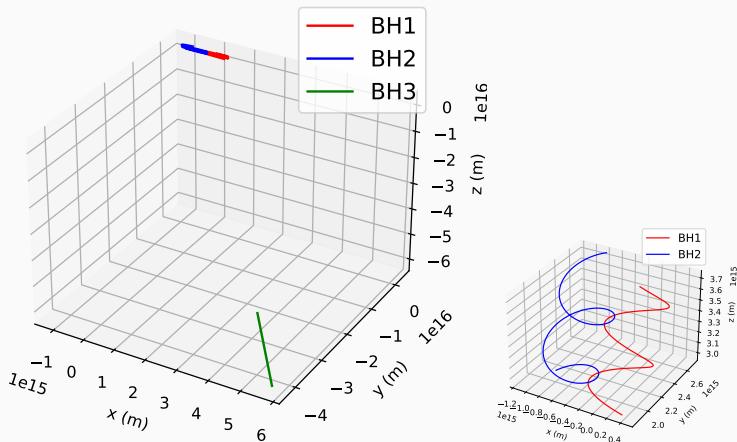
For some events more simulation time is required.

Figure 1: 3-dimensional plot of trajectories. On the right a zoom on the binary system.

To differentiate from fly-by we check if the third mass is approaching or moving away from the binary system.
Giving a look to the distance between the third BH and the binary, at the end of the simulation and 10 instants before it.

- $d_{1-0} < d_0 \longrightarrow$ fly-by;
- $d_{1-0} > d_0 \longrightarrow$ not happened.

event	#simulations	event	#simulations
Exchange0	443	Exchange0	313
Exchange1	1678	Exchange1	727
Fly_by	7228	Fly_by	7927
Ionization	63	Ionization	1009
Merger	11	Merger	22
Unclassified	188	Unclassified	1
Not Happened	389	Not Happened	1

Table 1: Tables with the results of the event identification between YSC (left table) and NSC (right table).

Comparing Properties

In order to compare the properties of the final population of the black holes and identify the main differences between the two star clusters we plot distributions of two main physical properties:

- masses distributions
- semi-major axes distributions

Moreover, we can give a quantitative value of similarity between the distributions performing a **Kolmogorov-Smirnov test**.

From N-body to physical units

Before presenting the results, it is important to convert the data in physical units. Remember that:

- $L_{code} = L_{phys}/L_{scale}$
- $M_{code} = M_{phys}/M_{scale}$
- $T_{code} = T_{phys}/T_{scale}$

with:

$$L_{scale} = 1\,\text{pc} \quad M_{scale} = 1M_\odot \quad T_{scale} = \sqrt{\frac{(L_{scale})^3}{G * M_{scale}}}$$

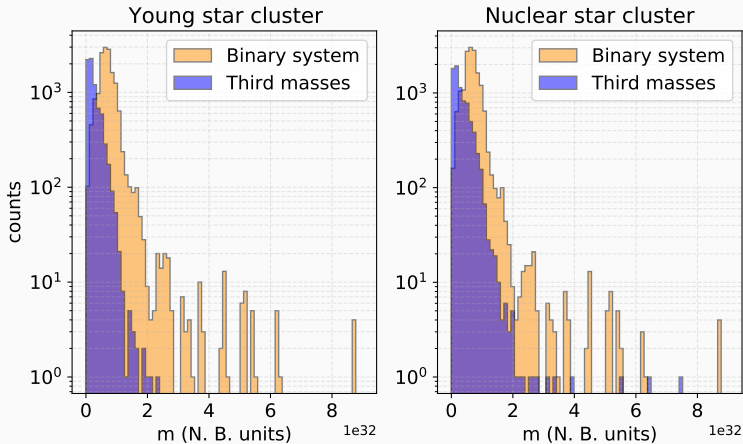In the following slides we're going to show some plots for the mass and semi-major axis distributions

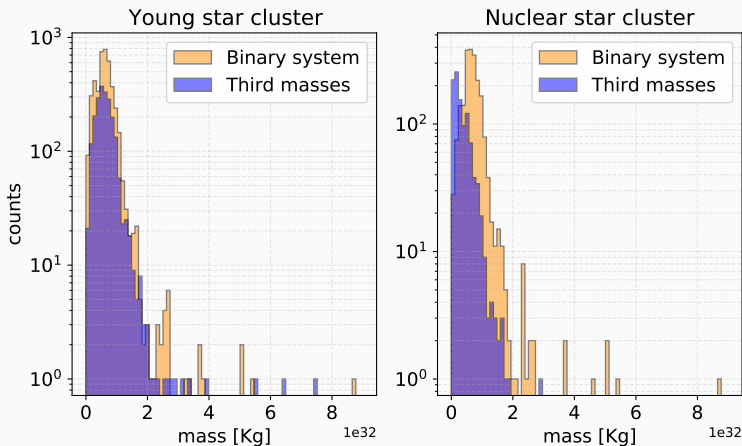**Figure 2:** Mass distribution for "fly-by" events.
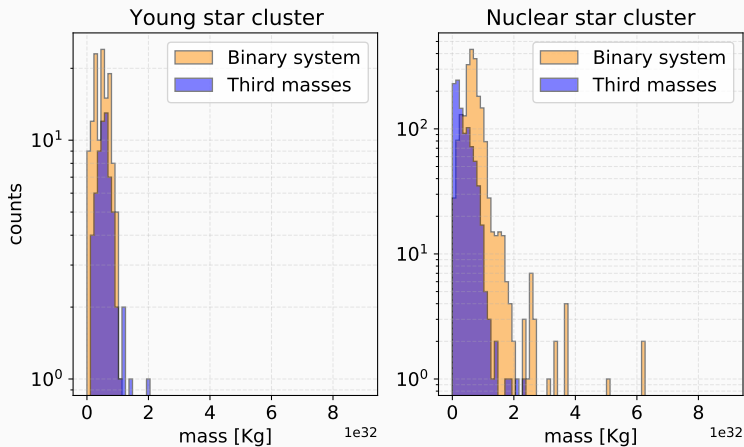
Figure 3: Mass distribution for "exchange" events.

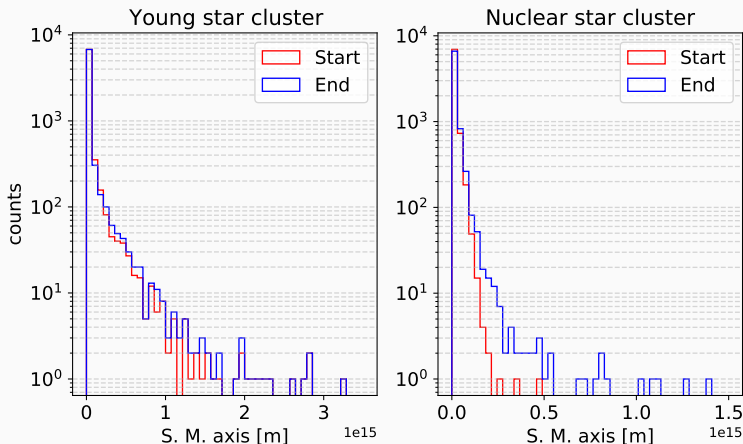# Ionization

Figure 4: Mass distribution for "ionization" events.

Fly-by

Figure 5: Semi major axis distribution for "fly-by" events.

Stat.	pvalue	Stat.	pvalue	Stat.	pvalue
-	-	-	-	–	-

Table 2: Results of Kolmogorov test for: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).
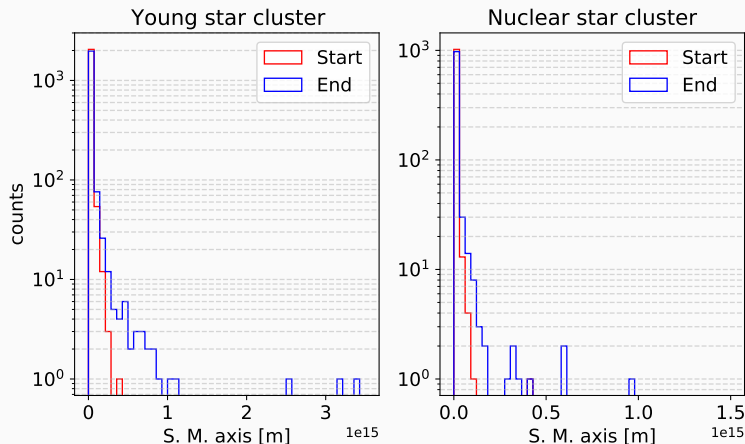
Figure 6: Semi major axis distribution for "exchange" events.

Stat.	pvalue
0.0363	0.122

Stat.	pvalue
0.0413	0.336

Stat.	pvalue
0.277	1.04e-47

Table 3: Results of Kolmogorov test for: initial_young vs. final_young (left), initial_nuclear vs. final_nuclear (middle), final_young vs. final_nuclear (right).

Mergers

Introduction

Differentiate Statistics

Comparing Properties

Mergers

Conclusion

- Mergers are the most interesting and exotic phenomena that happened in a dynamical simulation of star cluster
- Given the poor quantity of data that correspond to such events, it is meaningless to study the statistic distributions of their properties
- Instead, we can give a look to their features singularly, for example checking from which type of event they occur.
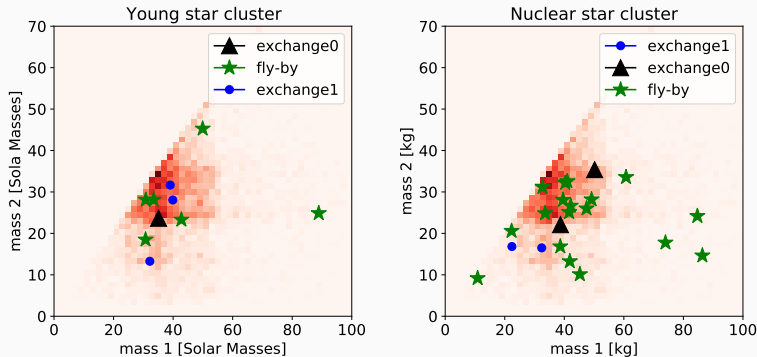
Figure 7: Scatter plot of the masses of binaries for merge event, subdivided from which type of event they occurred. On the background (red), a scatter plot of all masses for all the event

Future Mergers

In the end, it is interesting to understand how the systems will evolve in the universe life-time. Indeed, it is possible to estimate which event did not merge during the simulation but it is supposed to merge during the lifetime of the universe.

2 cases: (Keplerian mechanics)

- Peters equation (1964) [2]:
  (dependence on both e and a)

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4 red(1 - e^2)^{7/2}}{m_1 m_2 (m_1 + m_2)}$$

- Approximated Peters equation :
  (only dependence on $a \rightarrow$ circular orbit, $e = 0$)

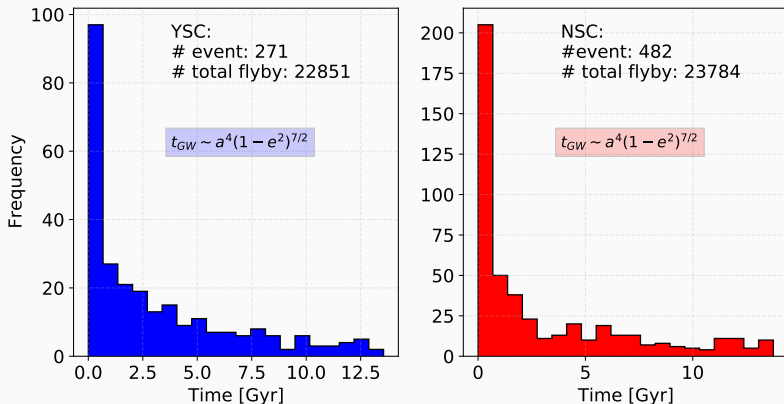$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4}{m_1 m_2 (m_1 + m_2)}$$

Fly-By

Figure 8: Distribution of fly-by events that will merge in a Hubble time **after the simulation**
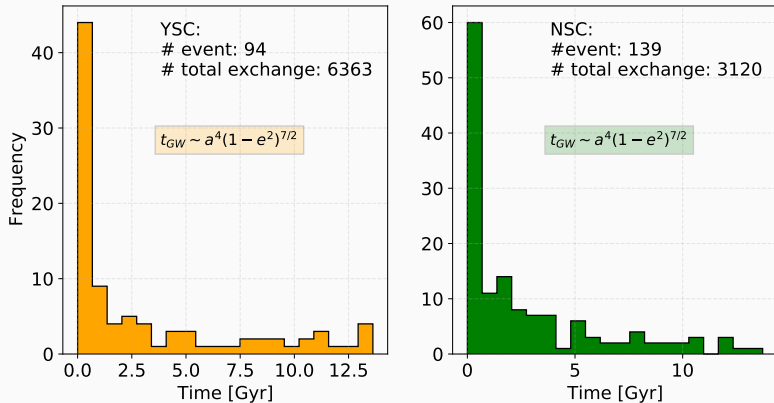
Figure 9: Distribution of exchange events that will merge in a Hubble time after the simulation

Conclusion

Summary

In the present work, we study the outcomes of simulation concerning three-body interactions. In particular, we:

- exploit some property (e.g. binding energy) in order to distinguish different events
- study the statistics of different events for both Young and Nuclear star clusters
- perform a quick statistical test in order to verify our results (e.g. Kolmogorov test)

Here we discuss how this work can be developed further:

- perform more advanced statistical tests (e.g. Bayes Factor) in order to compare distributions and thus verify our results
- perform a different (and more advanced) estimation of future mergers

📄 J. Aasi, B. P. Abbott, R. Abbott, et al.
Advanced LIGO.
Classical and Quantum Gravity, 32(7):074001, mar 2015.

📄 U. N. Di Carlo, N. Giacobbo, M. Mapelli, M. Pasquato, M. Spera,
L. Wang, and F. Haardt.
Merging black holes in young star clusters.
Monthly Notices of the Royal Astronomical Society,
487(2):2947–2960, May 2019.

📄 M. Mapelli.
Astrophysics of stellar black holes, 2018.

Thanks for your attention!