lisabeta: LISA response implementation and waveform interface

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lisabeta: overview

Objectives and scope

- Science prospective, Figures of Merit
- Prototyping real analysis (LDC)
- Source types: MBHB, SBHB, GB
- Consortium-available lisabeta_release (full members)

https://gitlab.in2p3.fr/marsat/lisabeta
https://gitlab.in2p3.fr/marsat/lisabeta_release

Tools implemented

- SNR computations
- MCMC: ensemble sampler with parallel tempering (`ptemcee`)
- Informed proposals to deal with degeneracies
- Fast likelihoods
- Waveforms: PhenomD, PhenomHM, PhenomXHM, SEOBNRv4HM_ROM, SEOBNRv5HM_ROM
- Limited so far to **spin-aligned**

Data analysis: levels of approximation

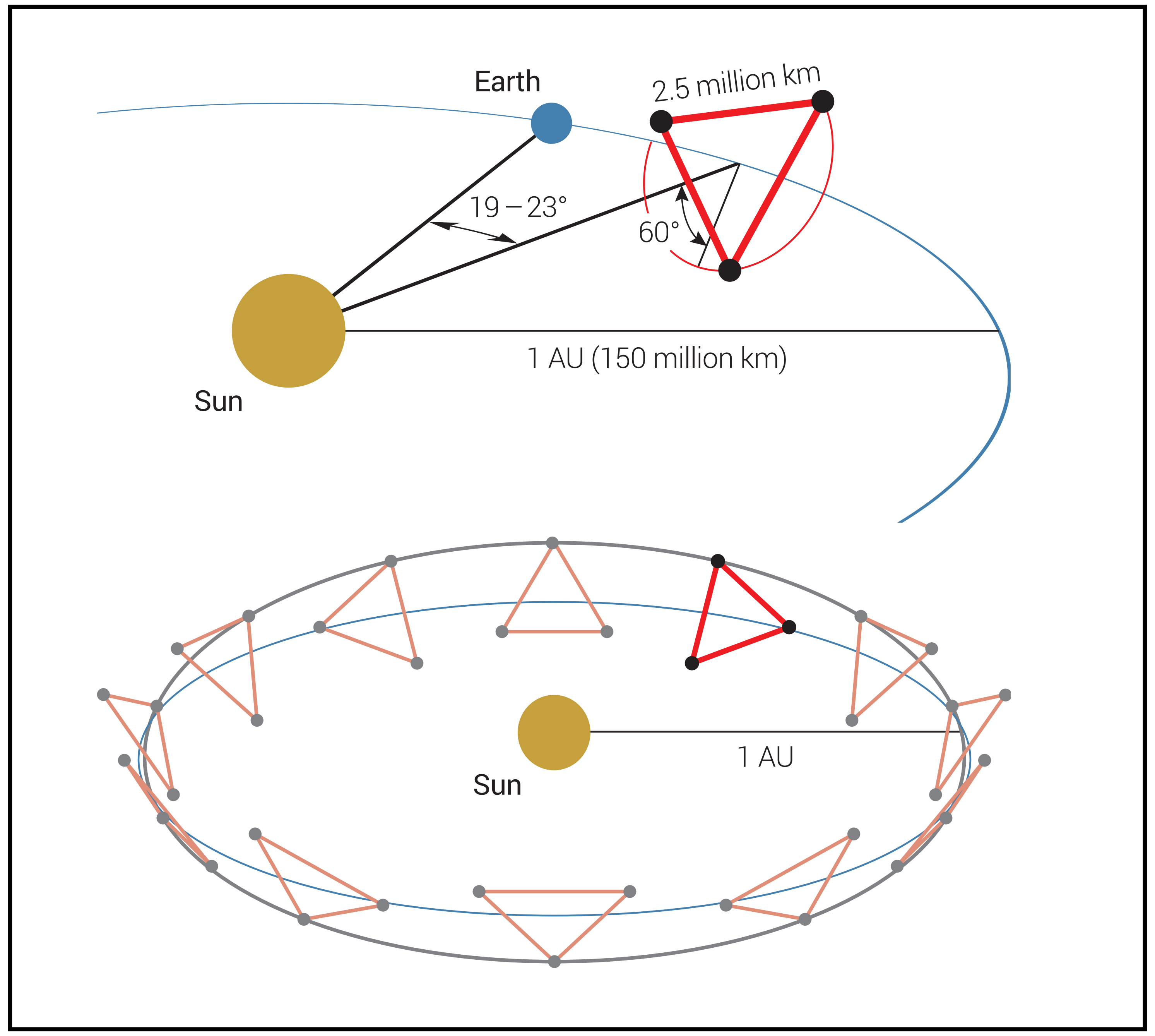
- Fisher matrix: for high SNR limit (depends on signal !) ✓
- Idealized PE: zero-noise realization ✓
- Bayesian PE initialized from Fisher ✓
- Bayesian PE from priors ✓
- Bayesian PE with noise ✓
- Superposition of sources, unknown noise, noise artifacts… in progress

Computational cost

- SNR: few ms
- Fisher: ~200ms at high-M, worse at low-M
- Likelihood: MBHB 3-4ms, SBHB 3-5ms
- Bayesian PE with Fisher init. (best case): ~1CPUh
- Bayesian PE in simulated noise (LDC): ~8 CPUh

Caveat: `lisabeta` built for fast PE in simple cases — data model might not work in general

LISAinstrumental response: time-domain and Fourier-domain



Response

From spacecraft s to spacecraft r through link l: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

Levels of approximations:

- **Equal-armlength response** (lisabeta)
- Realistic orbits (lisabeta, in dev.)
- Pointing-ahead and other effects

Fourier-domain, at leading order in the separation of timescales:

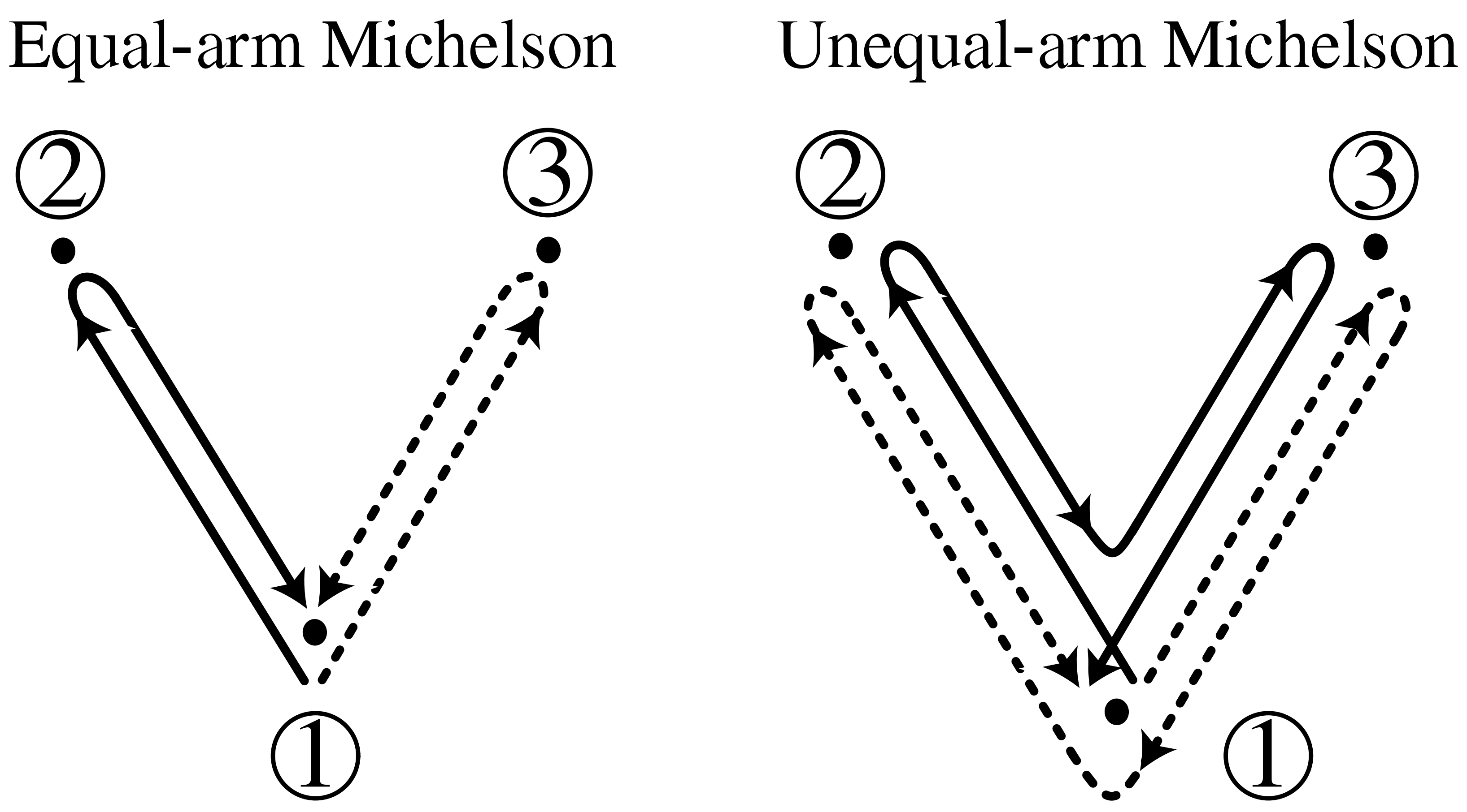
$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - k\cdot n_l\right)\right] \exp\left[i\pi f \left(L + k\cdot\left(p_r + p_s\right)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

Time and frequency-dependency

Time-Delay interferometry

1st and 2nd generation TDI:

$$X_1^{\rm GW} = \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]}_{X^{\rm GW}(t)}$$
$$- \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]_{,2233}}_{X^{\rm GW}(t-2L_2-2L_3)\simeq X^{\rm GW}(t-4L)}.$$



Perturbative formalism for the Fourier-domain response

Expansion of f-dependent convolution

Input: $\tilde{h}(f) = A(f)e^{-i\Psi(f)}$

Modulation + delays: convolution with f-dependent kernel

$$s(t) = F(t)h(t + d(t))$$

$$\tilde{s}(f) = \int df' \, \tilde{h}(f - f')\tilde{G}(f - f', f')$$

$$G(f, t) \equiv e^{-2i\pi f d(t)} F(t)$$

[Marsat-Baker 2018]

Leading order: $\mathcal{T}(f) = G(f, t_f)$

time-of-frequency $t_f \equiv -\frac{1}{2\pi}\frac{\mathrm{d}\Psi}{\mathrm{d}f}$

Example correction, Quadratic phase term: $\mathcal{T}(f) = \sum \frac{1}{p!}\left(\frac{i}{8\pi^2}\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}\right)^p \partial_t^{2p} G(f, t_f)$ → Evaluation on a stencil cf SUA [Klein&al 2014]

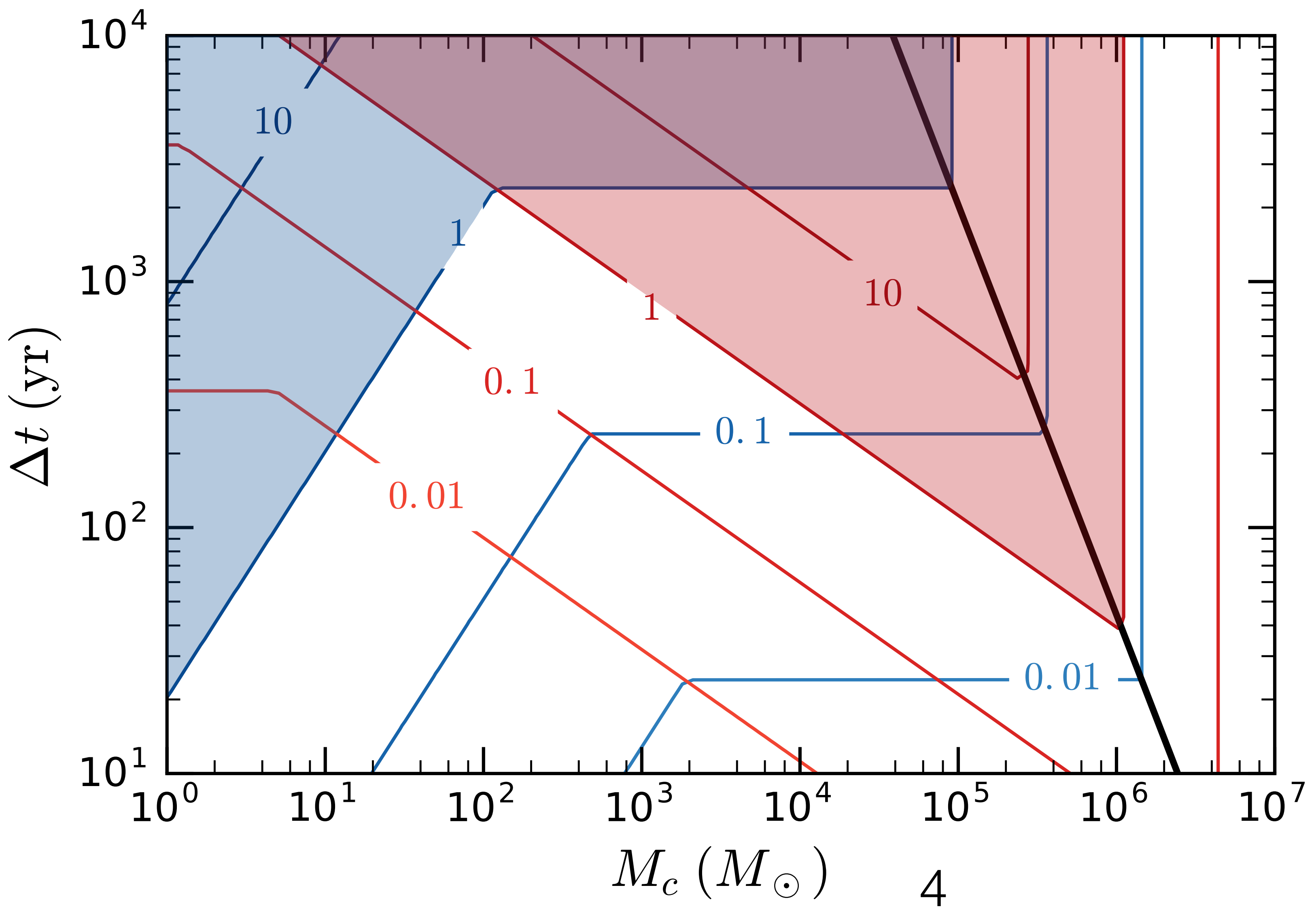
For the **spin-aligned case**: leading-order sufficient

Precessing case: see [Pratten&al 2023] (PhenomXPHM)

Breakdown: monochromatic limit

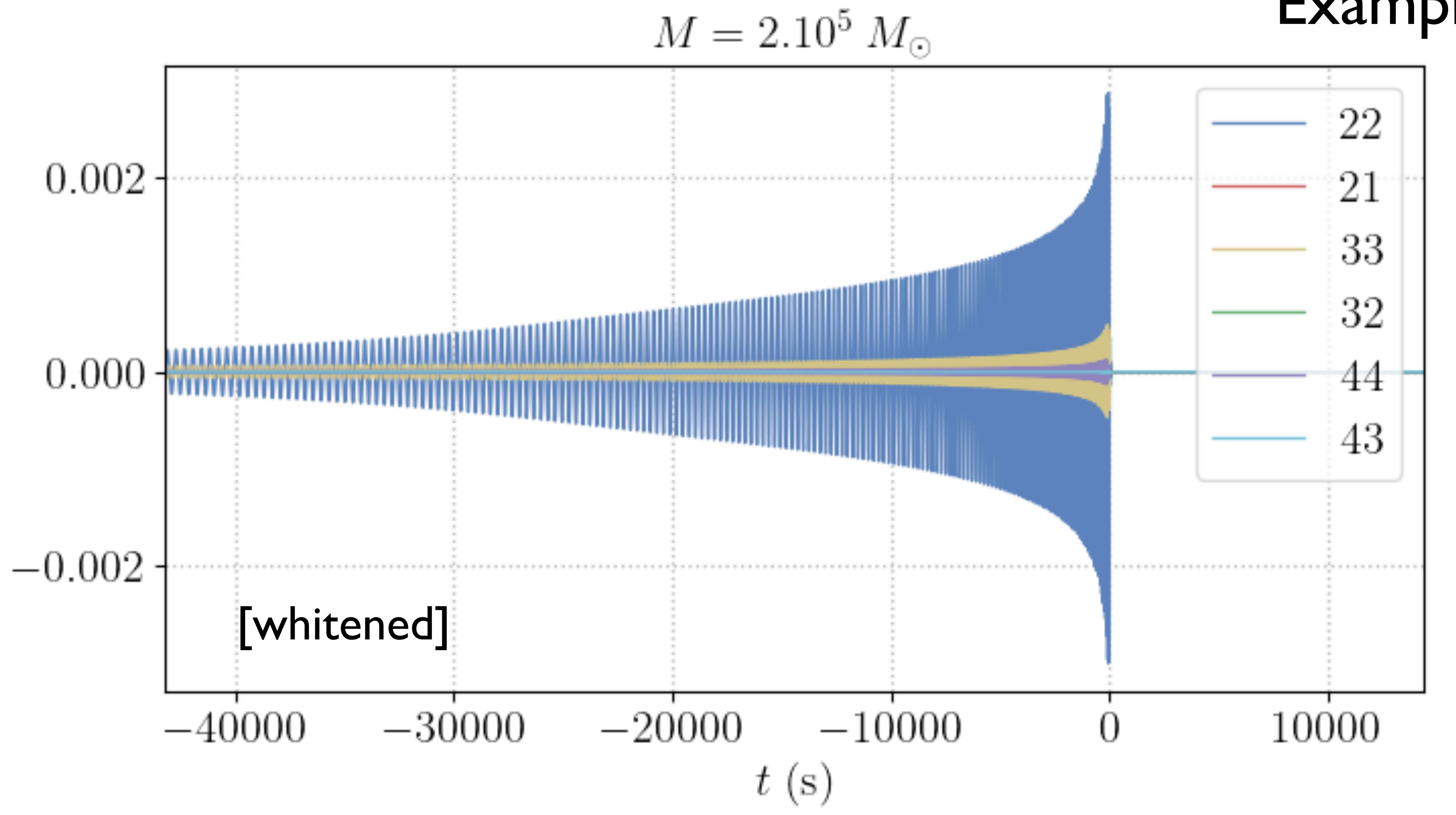
Perturbative scheme breaks down when radiation reaction timescale is longer than LISA orbital timescale

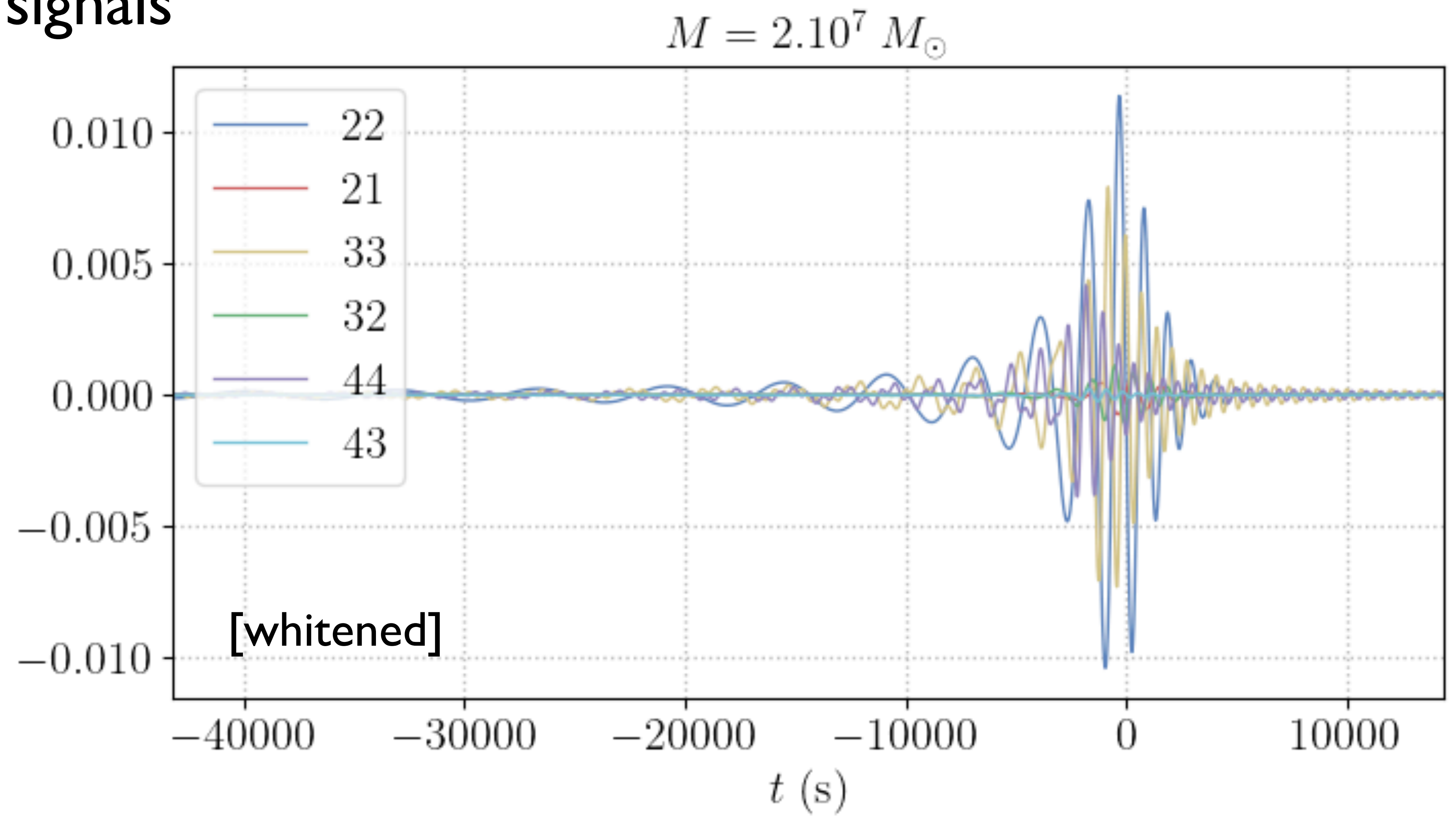
→ Quasi-monochromatic signals (GB, early SBHB) need other treatment (heterodyne+FFT)



Waveform structure: spin-aligned case with HM

Example MBHB signals





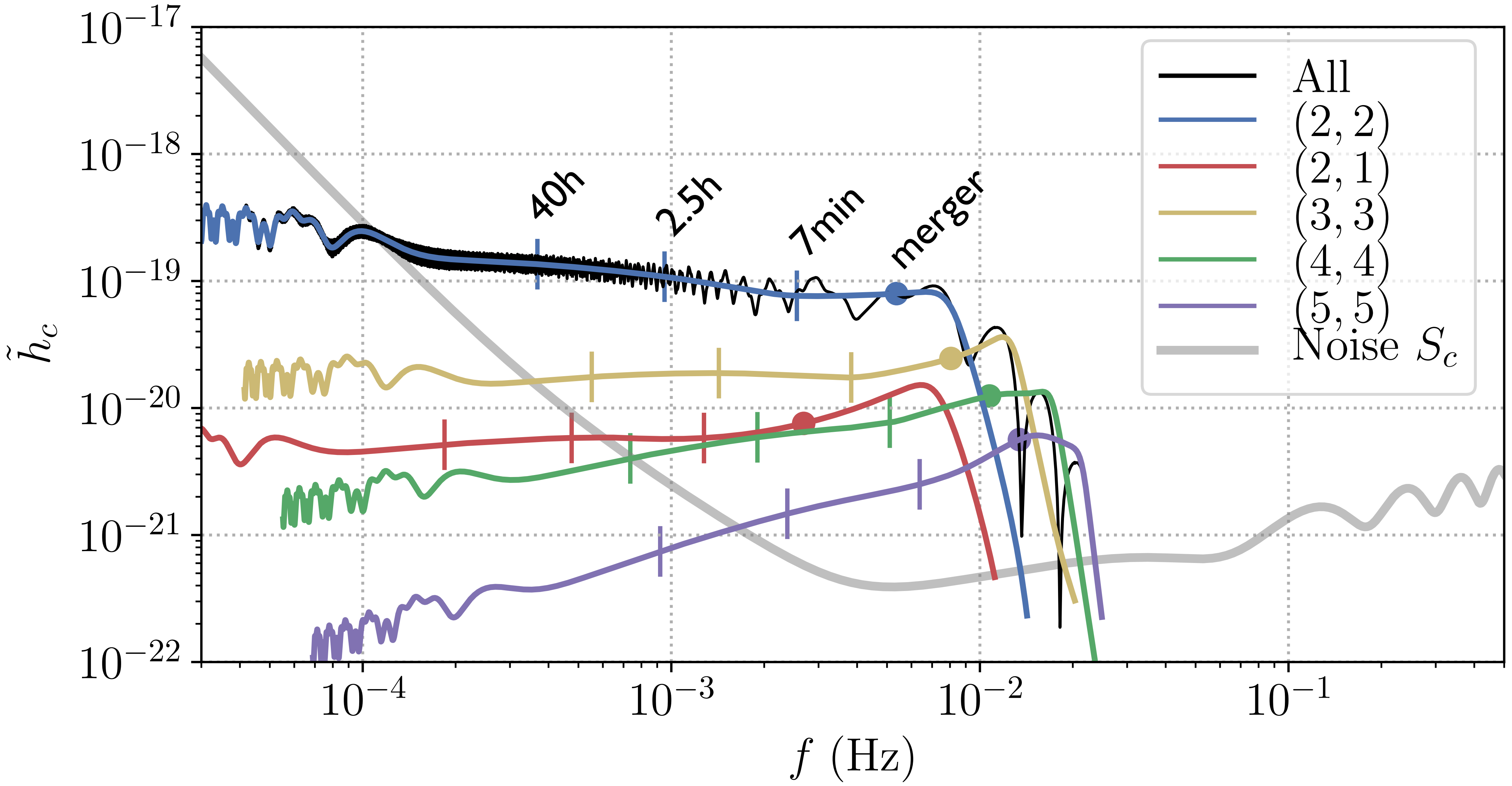
Waveform decomposition:

$$h_+ - ih_\times = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}$$

For planar orbits (**no precession**):

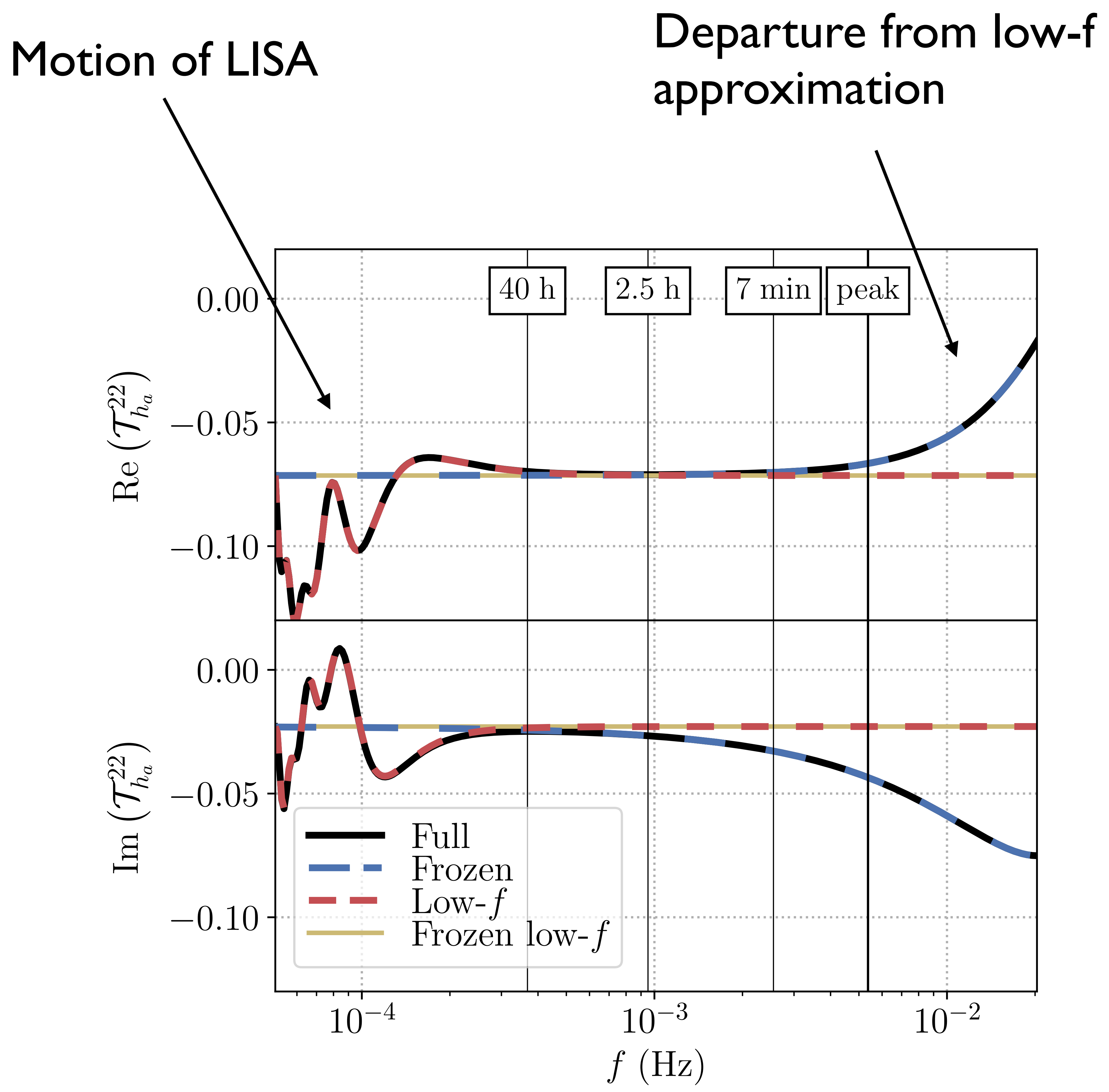
$$h_{\ell,-m} = (-1)^\ell h^*_{\ell m}$$

Response: treat each harmonic separately



Structure of FD response and interpolation

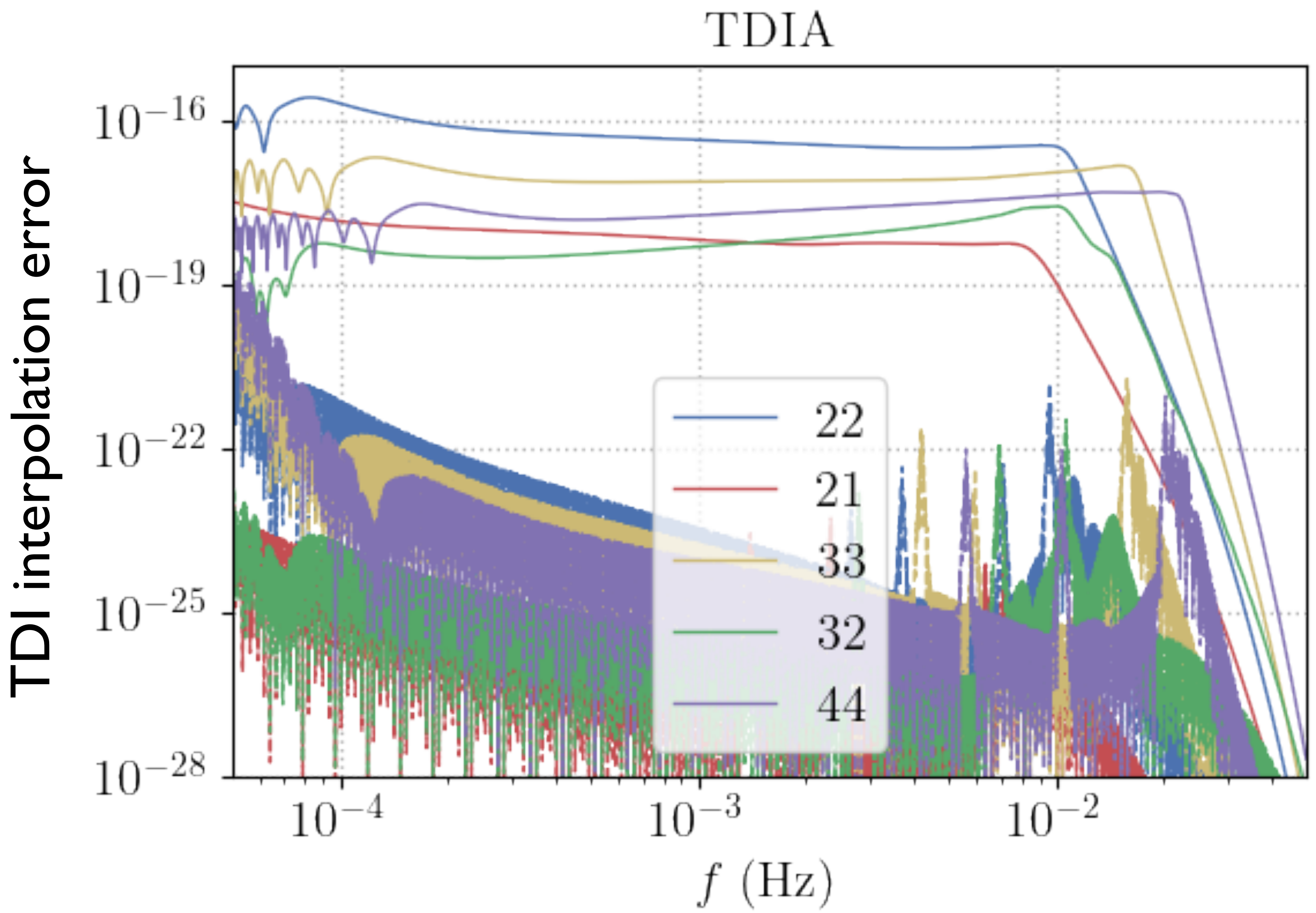
Struture of FD transfer functions



Waveform/Response interpolation errors

Input: for each mode $A(f),\ \Phi(f),\ t_f$ Note: amplitude/phase, not h

on sparse frequency grid (200-300) + spline interpolation



Sparse spline representation with few hundred points (Spin-aligned case)

Same representation for short MBHBs/very long SBHB signals

Inner products and likelihoods

$$(h_1|h_2) = 4\mathrm{Re} \int df \, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$

Discretization on a grid

Adjust frequency resolution for inspiral/merger

Similar to multi-banding [Vinciguerra+ 2017]

$$(h|d) = 4\mathrm{Re} \sum_i \Delta f_i \frac{\tilde{h}(f_i)\tilde{d}^*(f_i)}{S_n^i}$$

Safe, used e.g. for Fisher matrices Cost: ~50ms

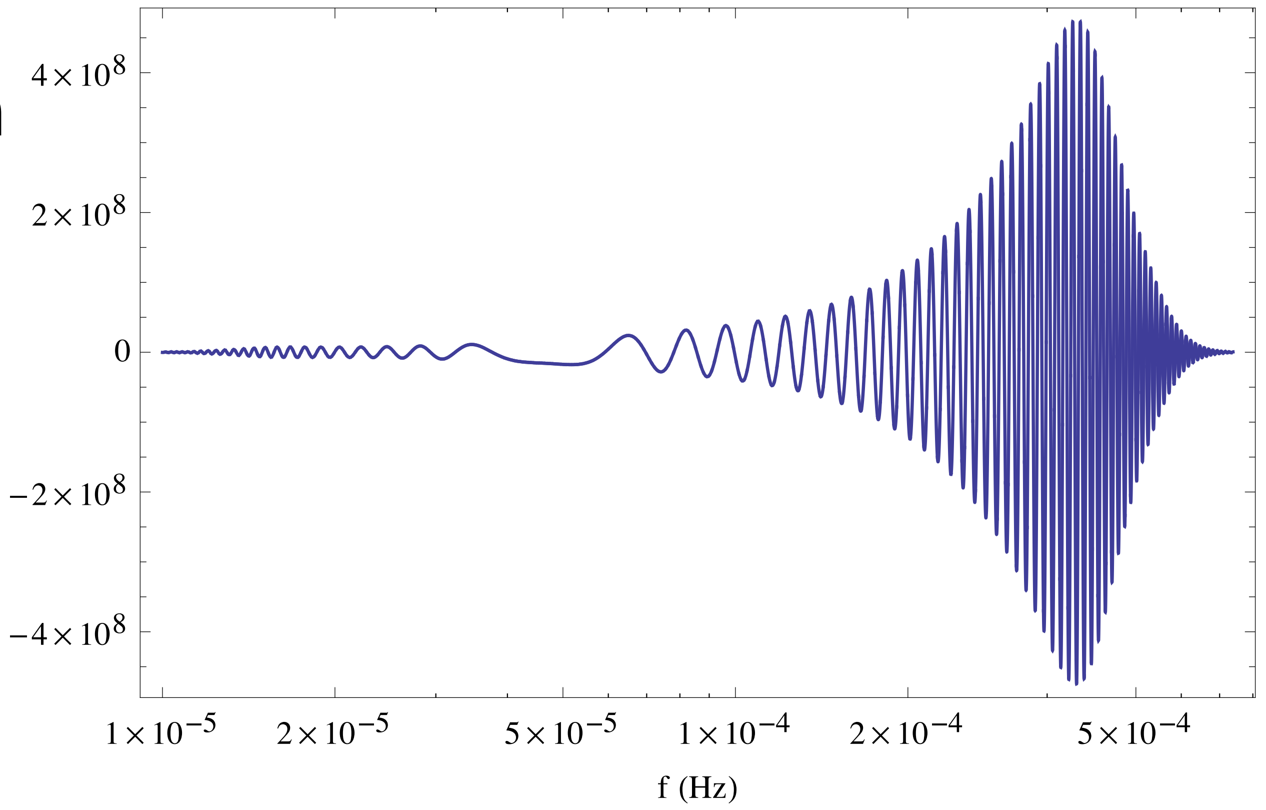
Flexibility in choice of freqs. required

Fresnel overlaps

0-noise overlaps
Oscillatory integrals with quadratic spline phase:
Fresnel integrals

$$\int_{f_i}^{f_{i+1}} P(f) e^{i[af+bf^2]}$$

$$\longrightarrow \int_{f_i}^{f_{i+1}} e^{i[af+bf^2]}$$



Cost: <15ms

Heterodyne likelihood

[Cornish 2012] [Cornish 2021] [Zackay+ 2018]

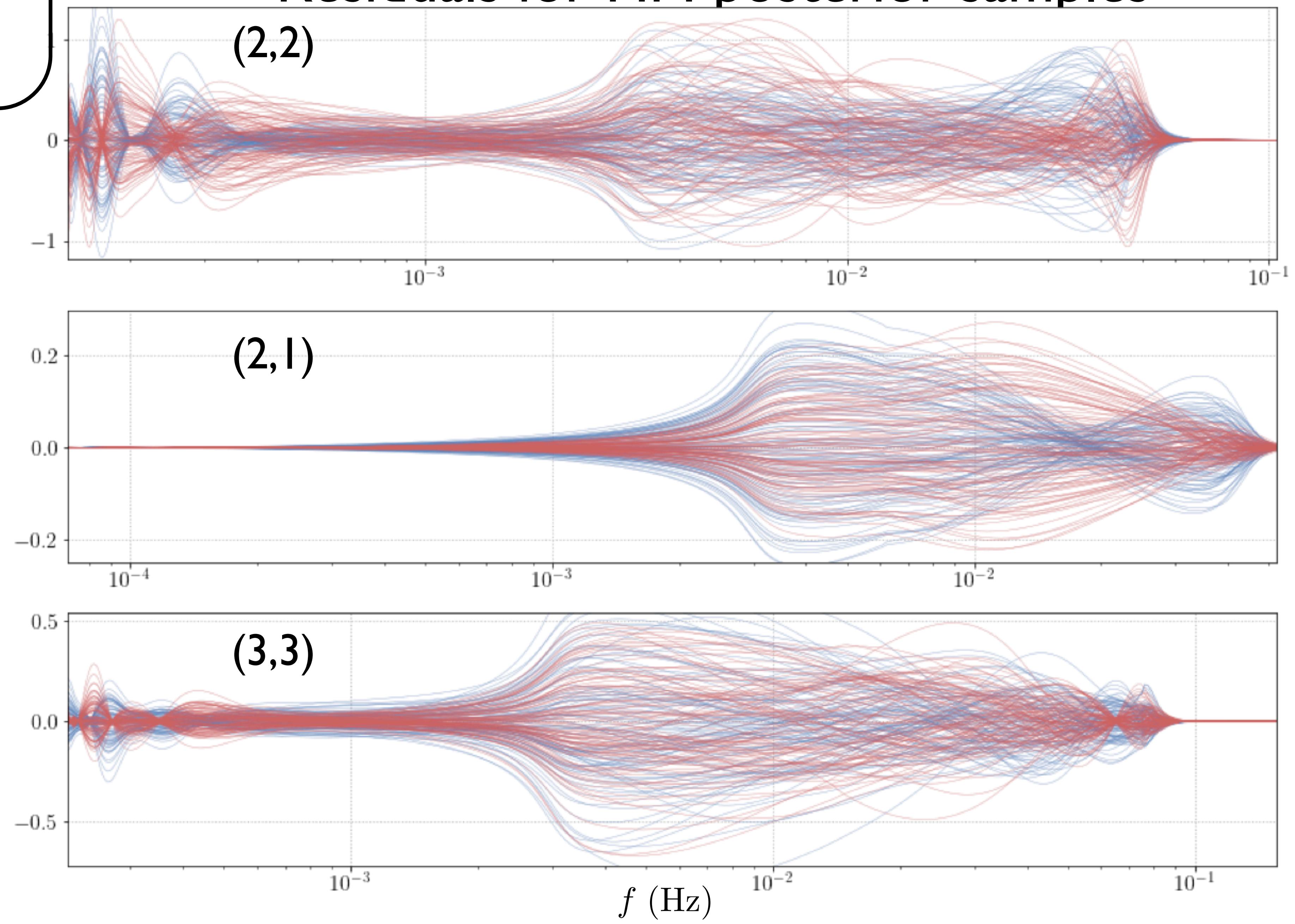
$$\zeta(f) = h(f)/\overline{h}(f)$$

$$(d|h) = \int df \, \frac{\overline{h}d^*}{S_n}(f)\zeta(f)$$

$$(h|h) = \int df \, \frac{\overline{h}\overline{h}^*}{S_n}(f)\zeta\zeta^*(f)$$

Introduce a reference waveform, decompose overlaps in fast-varying (precomputed) and slow varying parts (coarsely resolved)

Residuals for HM posterior samples



Cost: <5ms

lisabeta: waveform interface

Input: for each mode
$A(f),\ \Phi(f),\ t_f$
on user-specified frequencies

Stand-alone reimplementation

- (EOBNRv2HM_ROM)
- PhenomD
- PhenomHM
- PhenomXHM

→

- Historically, hard to work with lalsuite…
- Provide stand-alone lightweight package without the whole lalsuite
- PhenomHM refactored for speed
- Access to analytical t(f), amplitude/phase output
- Wasteful in programming time !

LAL interface modified

- SEOBNRv4HM_ROM
- SEOBNRv5HM_ROM

→

- Use lalsimulation
- Temporary tweak to interface for amplitude/phase output

NR surrogate (injections)

- NRHybSur3dq8

→

- Use gwsurrogate time-domain waveforms
- Fast Fourier transforms with SPA+FFT

Time-domain LISA response

Time-domain response:

$$y_{slr} = \frac{1}{2}\frac{1}{1 - k \cdot n_l} n_l \cdot [H(t - (k \cdot p_s + L)) - H(t - k \cdot p_r)] \cdot n_l$$

$$\begin{aligned} X_1^{\mathrm{GW}} = & \underbrace{\left[(y_{31}^{\mathrm{GW}} + y_{13,2}^{\mathrm{GW}}) + (y_{21}^{\mathrm{GW}} + y_{12,3}^{\mathrm{GW}})_{,22} - (y_{21}^{\mathrm{GW}} + y_{12,3}^{\mathrm{GW}}) - (y_{31}^{\mathrm{GW}} + y_{13,2}^{\mathrm{GW}})_{,33}\right]}_{X^{\mathrm{GW}}(t)} \\ & - \underbrace{\left[(y_{31}^{\mathrm{GW}} + y_{13,2}^{\mathrm{GW}}) + (y_{21}^{\mathrm{GW}} + y_{12,3}^{\mathrm{GW}})_{,22} - (y_{21}^{\mathrm{GW}} + y_{12,3}^{\mathrm{GW}}) - (y_{31}^{\mathrm{GW}} + y_{13,2}^{\mathrm{GW}})_{,33}\right]_{,2233}}_{X^{\mathrm{GW}}(t - 2L_2 - 2L_3) \simeq X^{\mathrm{GW}}(t - 4L)} . \end{aligned}$$

Mode-by-mode TDI transfer functions (~derivatives):

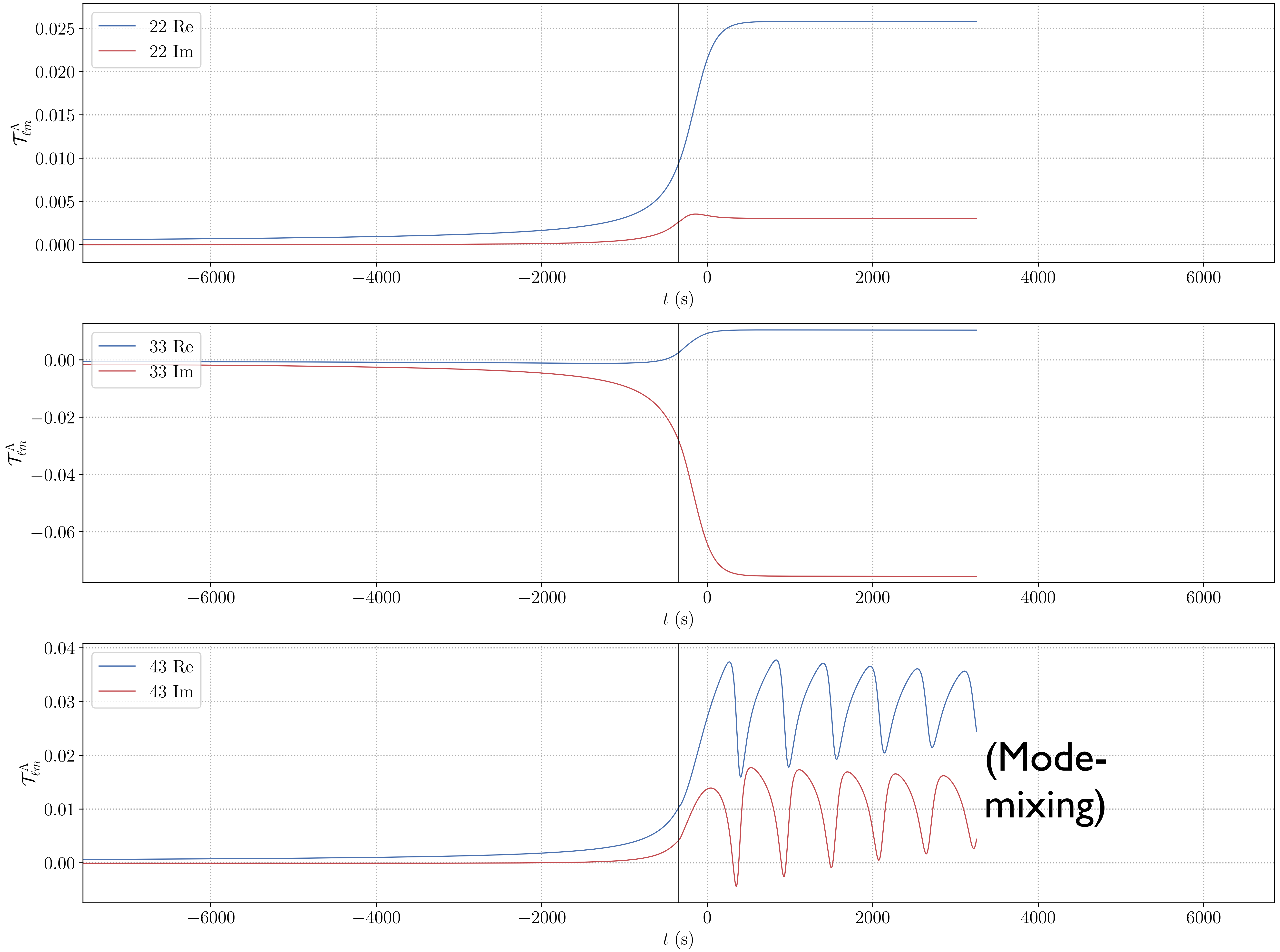
$$\mathrm{TDI} = \sum_{\ell, m>0} \left[\mathcal{T}_{\ell m}^{\mathrm{TDI}} h_{\ell m} + \left(\mathcal{T}_{\ell m}^{\mathrm{TDI}} h_{\ell m} \right)^* \right]$$

Time delays: need to interpolate

$$h_{\ell m}(t - nL - p_A), \; A = 1, 2, 3, \; n = 0, .., 4$$ (TDII)

+ geometric prefactors, linear combinations to rebuild TDI

Sparse spline representation for transfer function (<1000 pts)
(Spin-aligned case)



(Mode-mixing)

Interface of waveforms and LISA response

Waveform modelling

HM: $h_{\ell m} = A_{\ell m} \exp i\Phi_{\ell m}$

Prec.: $h^P_{\ell m'},\ D^\ell_{mm'}(\alpha, \beta, \gamma)$

EMRIs: A_{lmnk}, Φ_{mnk}

TDI response pieces

$\mathcal{T}_{\ell m}$

$\mathcal{T}_{\ell m'},\ \mathcal{T}_{\ell m m'}$?

$\mathcal{T}_{\ell mnk}$?

Waveform

$h_+,\ h_\times$

TDI

TDI A, E, T

Likelihood

$$\ln \mathcal{L} = -\frac{1}{2}(s - d | s - d)$$

sparse: reduced set of freqs/times (splines), multiple harmonics

dense: full set of freqs/times

Trade-off: keep sparse representation vs keep multiple waveform/response pieces (EMRIs: too many harmonics)

lisabeta: outlook and perspectives

- `lisabeta`: take as input amplitude/phase/tf for separate harmonics on sparse frequencies, compute sparse FD response, TDI FD given by spline interpolation - can we always keep the same "Fast-PE" data model ?

- Precession: important missing part, work in progress

- Should move on from reimplementation to using modern and generic waveform interface

- Current developments: waveform systematics, inspiral tests of GR, extra polarizations, future instruments (GWspace2050)

- Fourier domain may not be best given instrumental non-stationarities (Wavelet domain)