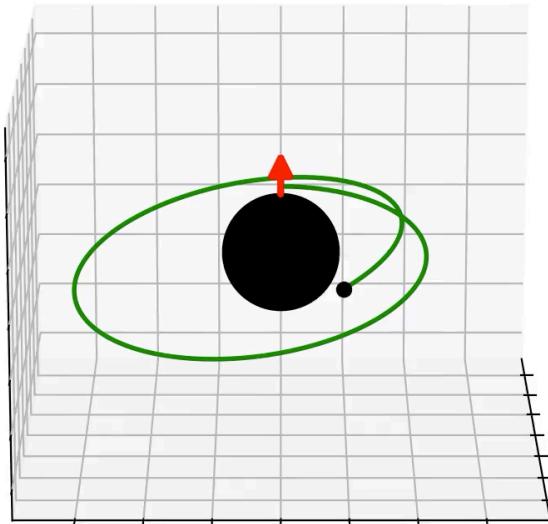


Fast EMRI Waveforms (FEW)



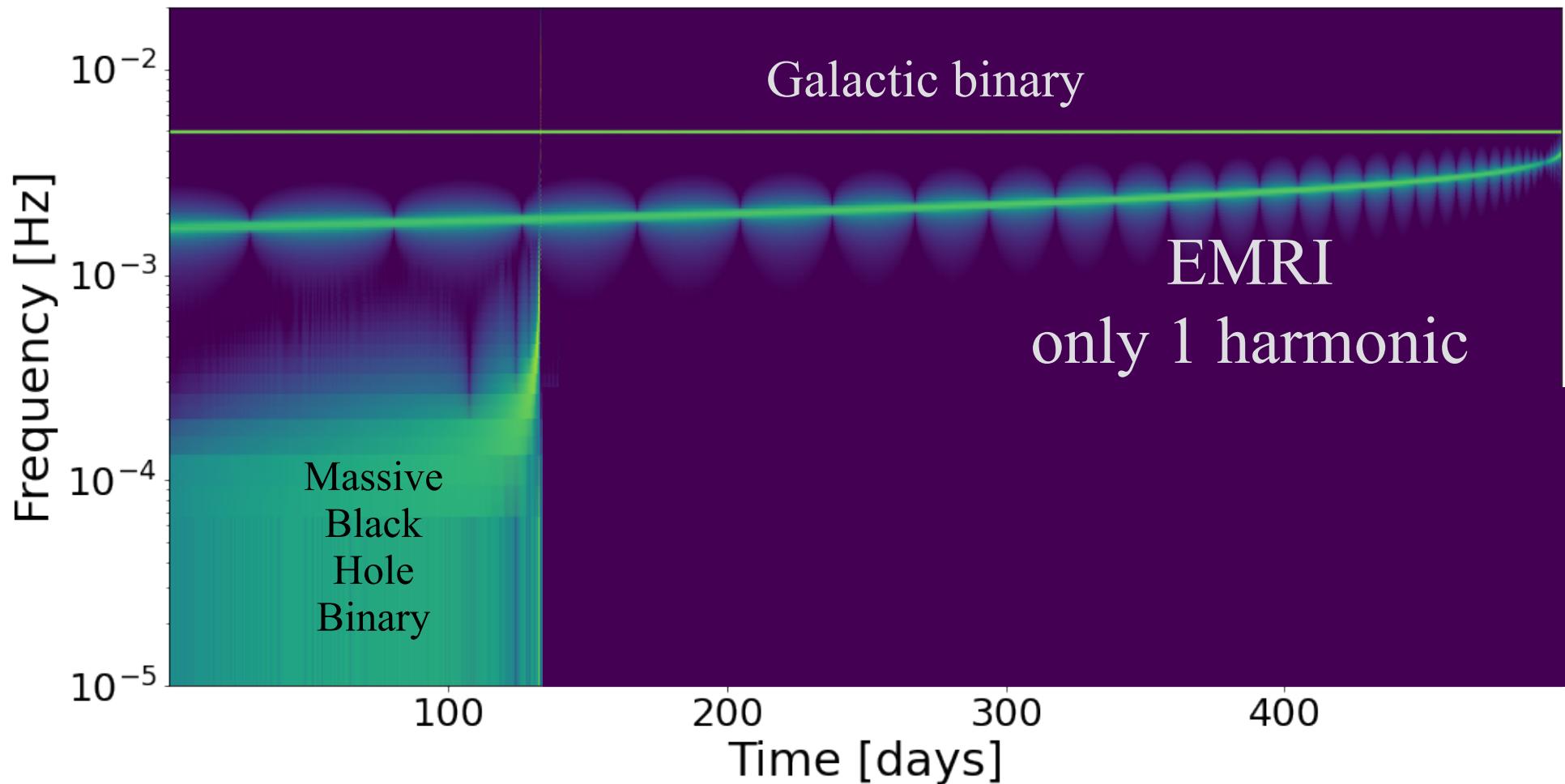
Lorenzo Speri

C. Chapman-Bird, M. Katz,
N. Warburton, S. Hughes, A. Chua,
J. Thompson, H. Khalvati, P. Lynch,
S. Kejriwal, S. Isoyama, O. Burke, +



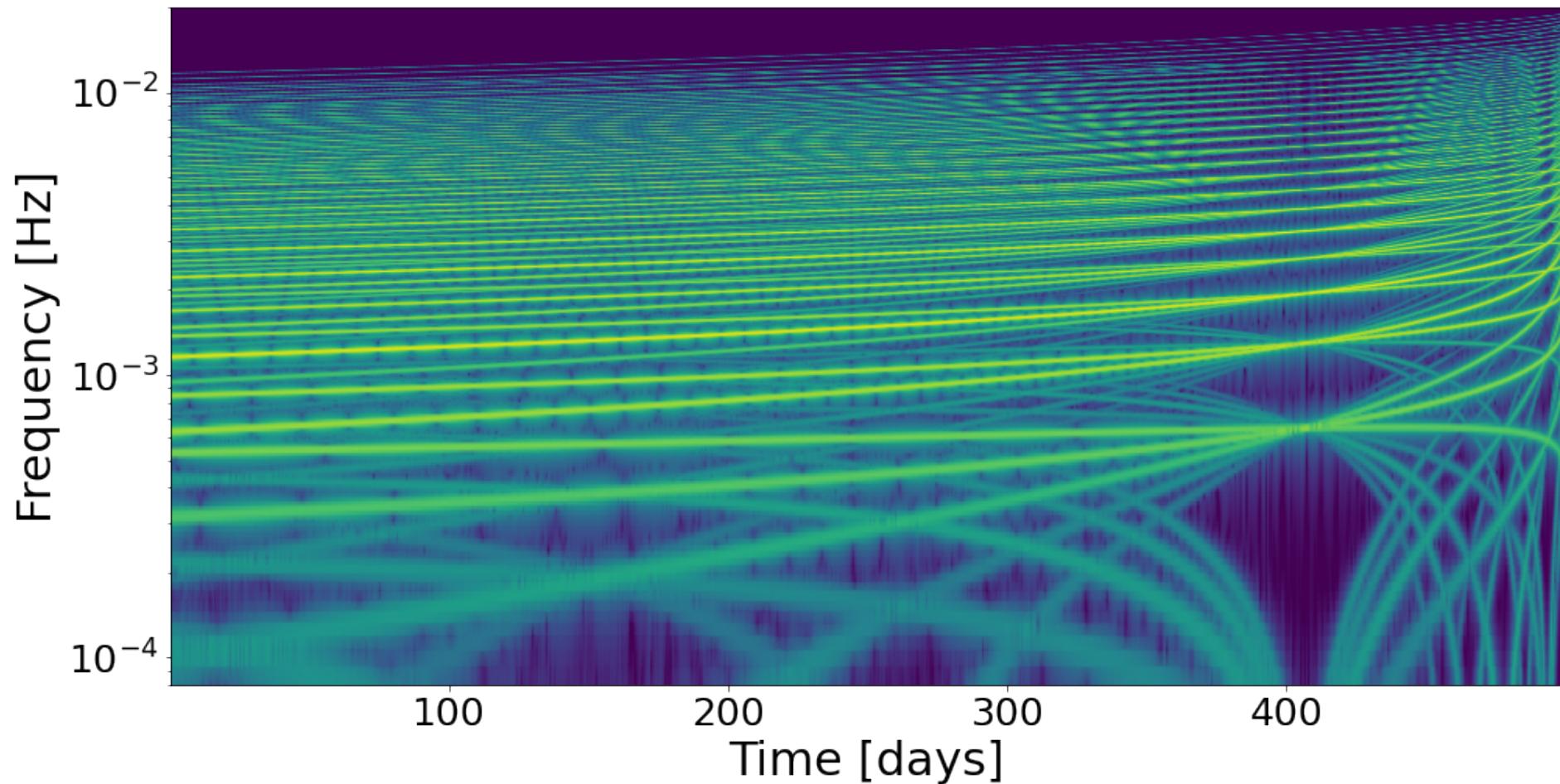
Max-Planck-Institut
für Gravitationsphysik
ALBERT-EINSTEIN-INSTITUT

LISA Sources



EMRI Evolution Time Scale $T_{\text{ev}} \sim 1/\eta$

EMRI Spectrogram



$$h = \sum_V A_V(t) \exp \left[-i\Phi_V(t) \right] \quad \text{\# Harmonics} \sim \text{orbit complexity}$$

Fast EMRI Waveforms

$$h = \sum_{lmnk} \left\{ Y_{lm}(\theta, \phi) \right.$$

$$A_{lmnk}(p(t), e(t), x_I(t))$$

$$\left. e^{-i\Phi_{mnk}(t)} \right\}$$

Trajectory

Amplitudes

Waveform Implementation: Fast EMRI Waveforms

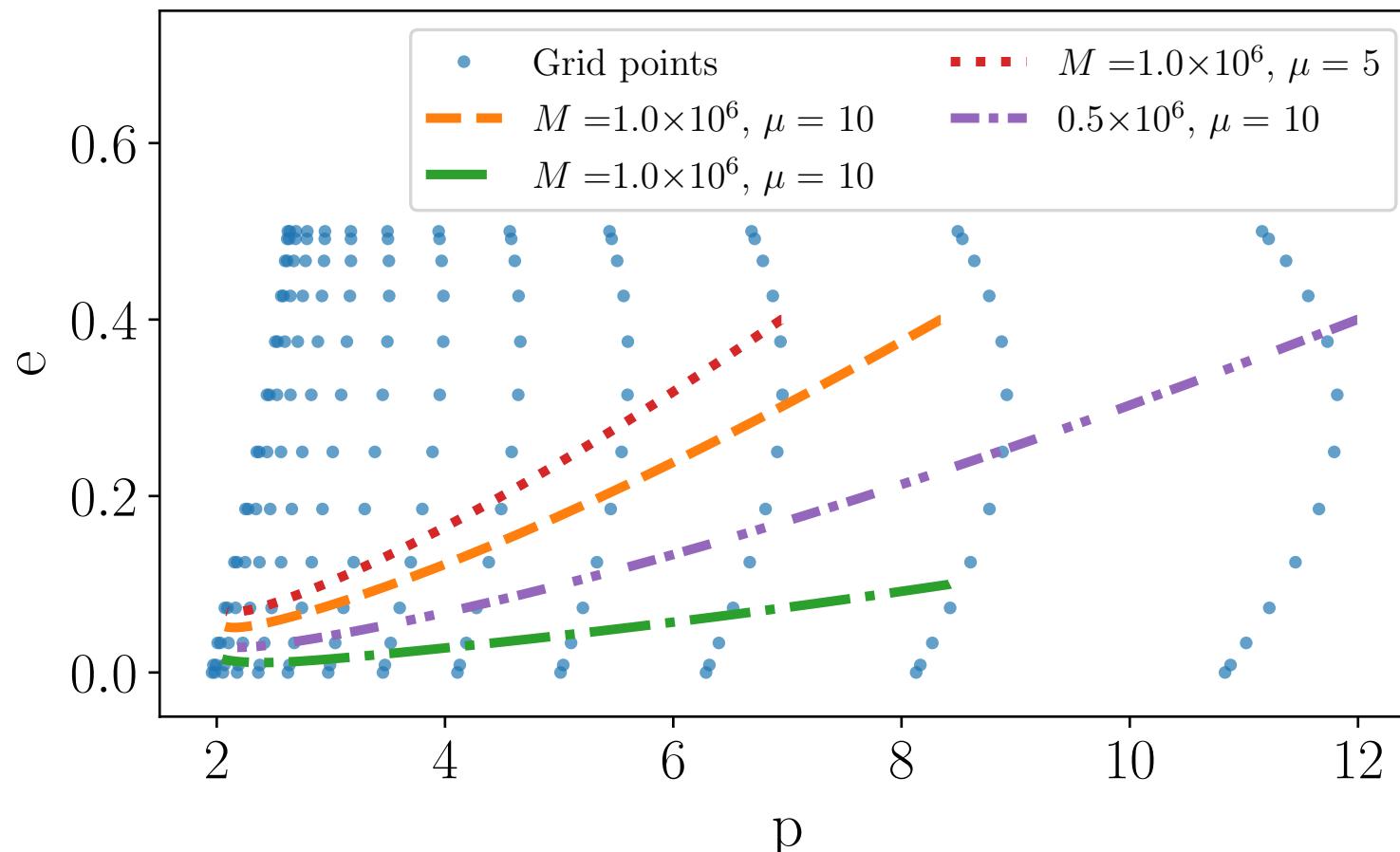
Trajectory

$$\frac{d}{dt}(p, e, x_I) = \eta f_{p,e,x_I}(a, p, e, x_I)$$

$$\frac{d}{dt}\Phi_{\varphi,\theta,r} = \Omega_{\varphi,\theta,r}(a, p, e, x_I)$$

$$m\Phi_\varphi + k\Phi_\theta + n\Phi_r$$

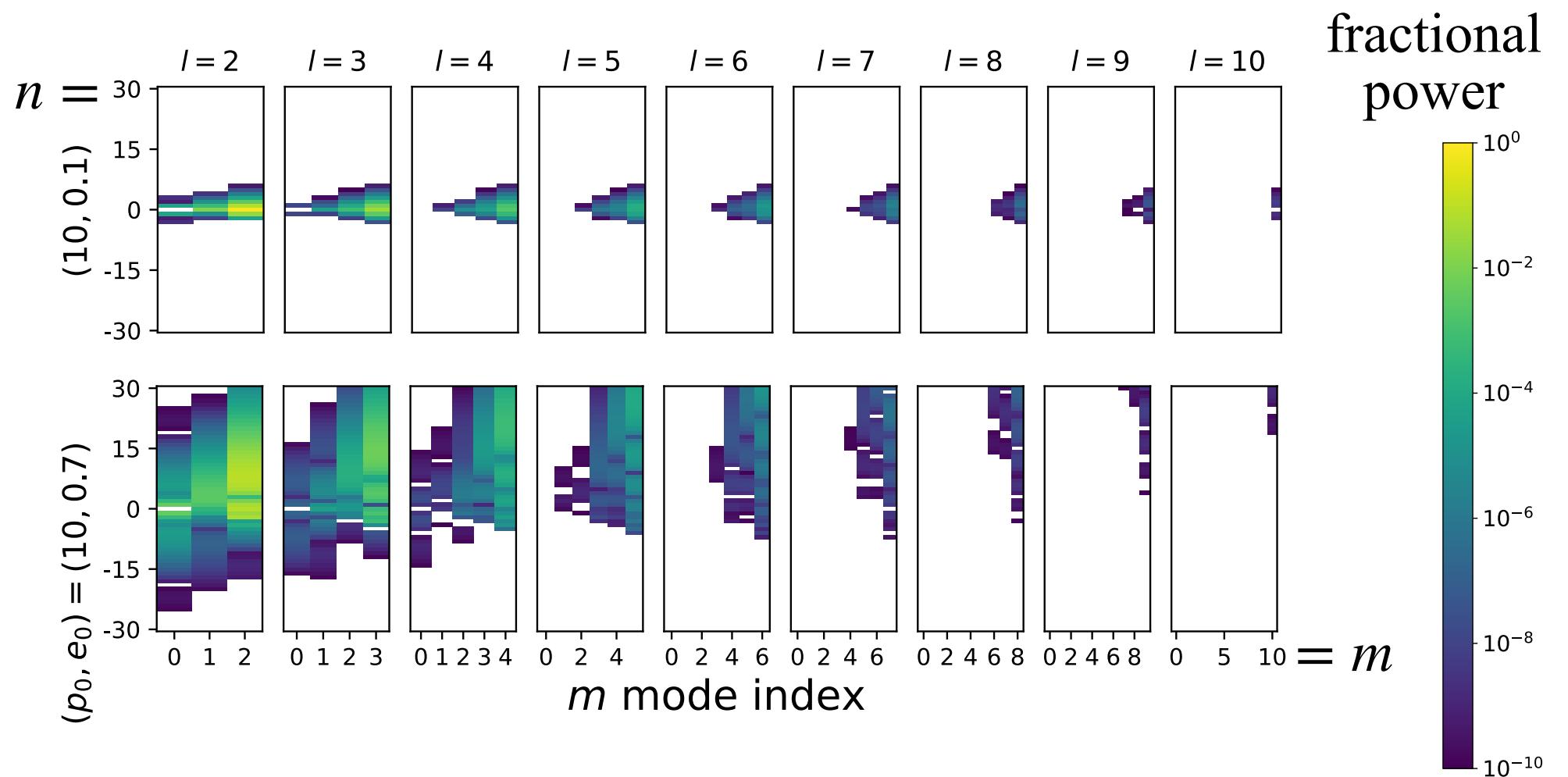
$$e^{-i\Phi_{mnk}(t)}$$



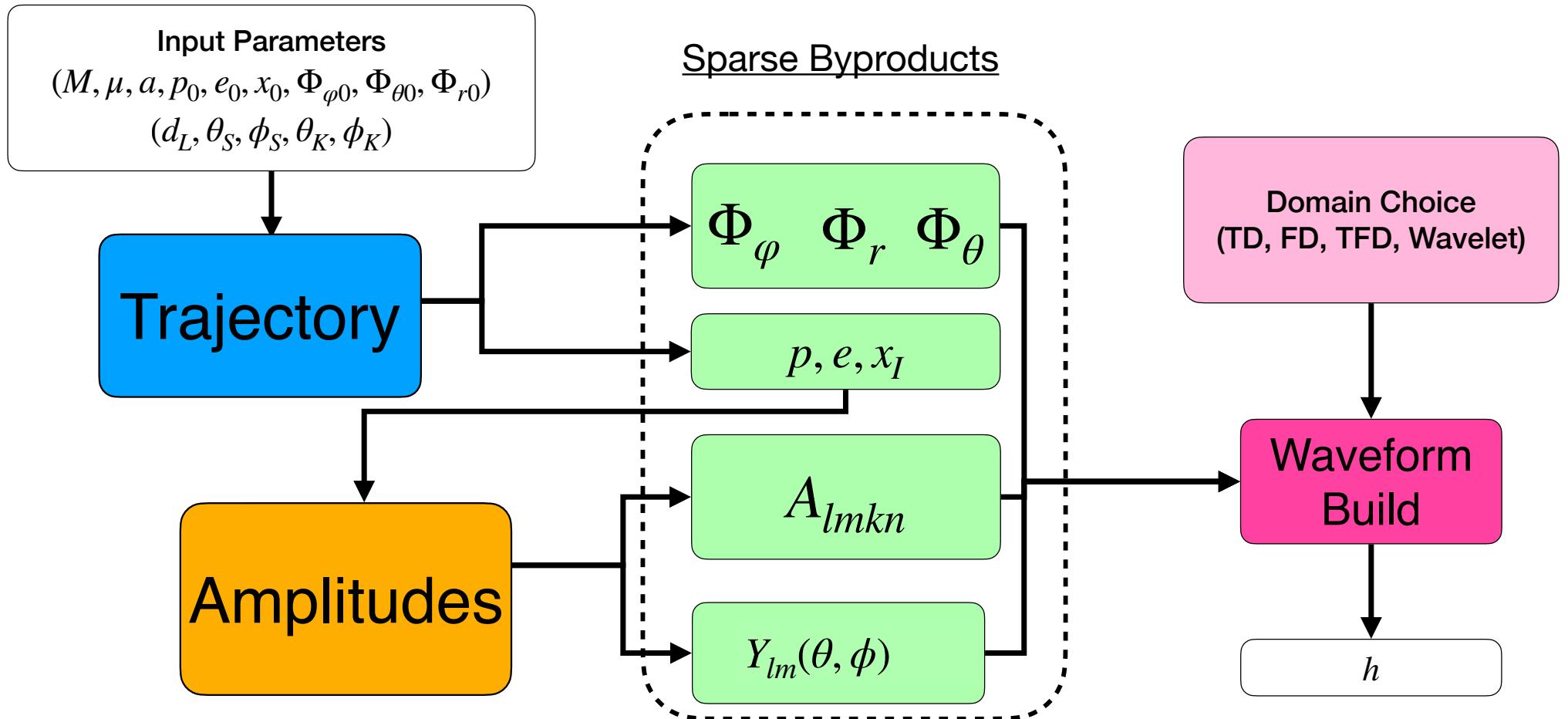
Waveform Implementation: Fast EMRI Waveforms

Amplitudes

$A_{lmnk}(a, p(t), e(t), x_I(t))$



Fast EMRI Waveforms (FEW)

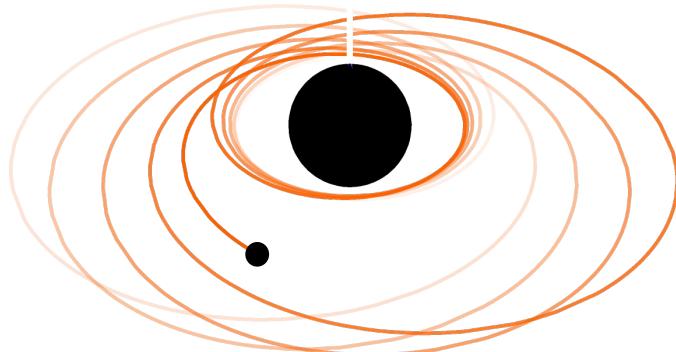


Chua+ 2020, Katz+ 2021, Speri+ 2023

bhptoolkit.org/FastEMRIWaveforms

FEW Status

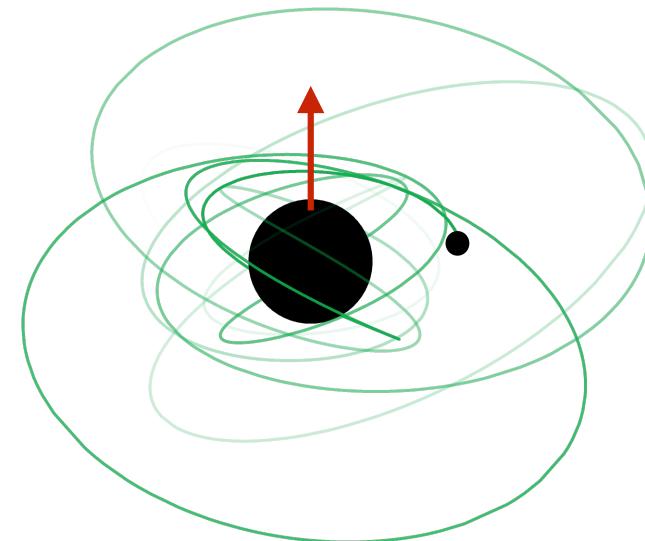
Relativistic
Schwarzschild
Eccentric



Time Domain



Augmented Analytical Kludge
5PN trajectory (AAK)



Frequency Domain

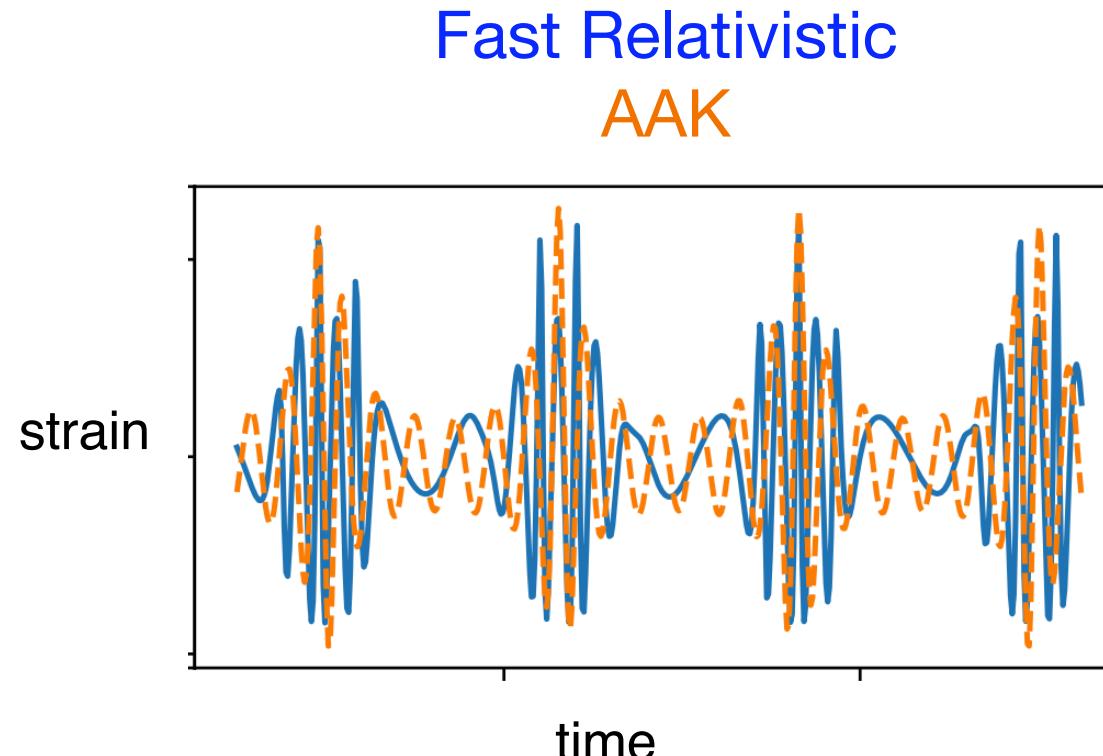
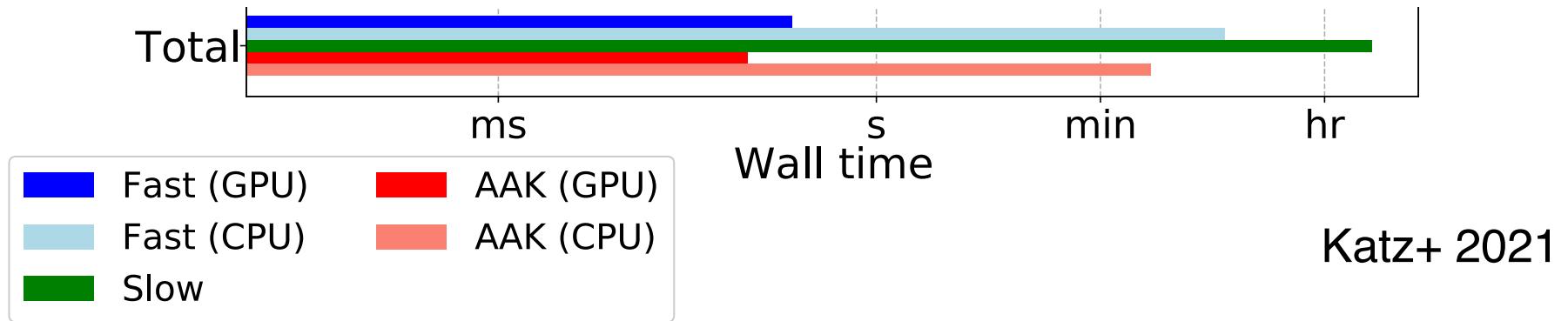


Time-frequency



Waveform Speed

- Time domain model, slowest point



Waveform Speed

- Frequency domain model

Speri+ 2023, Hughes+ 2021

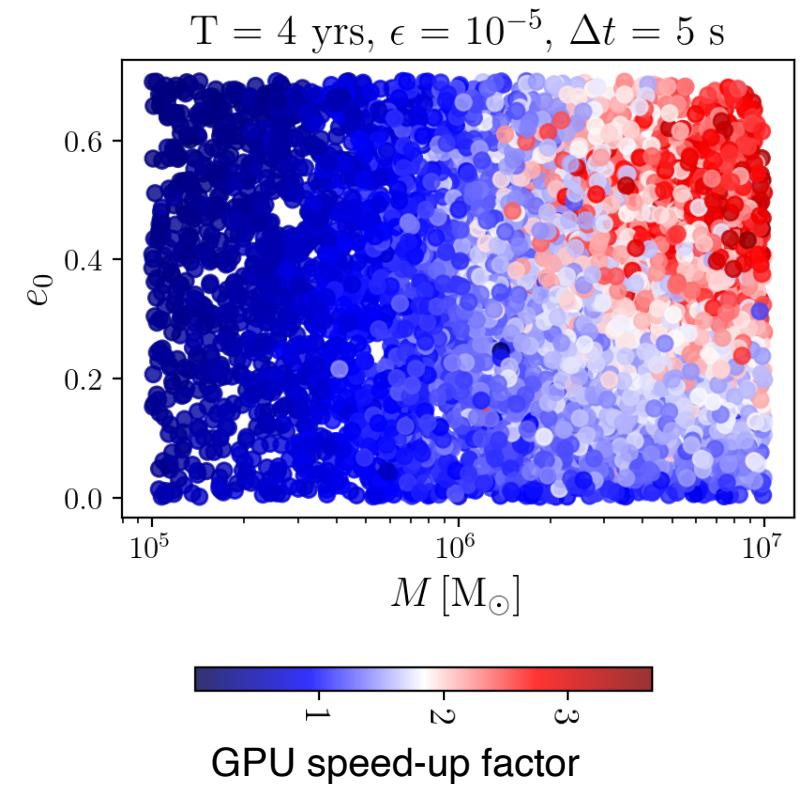
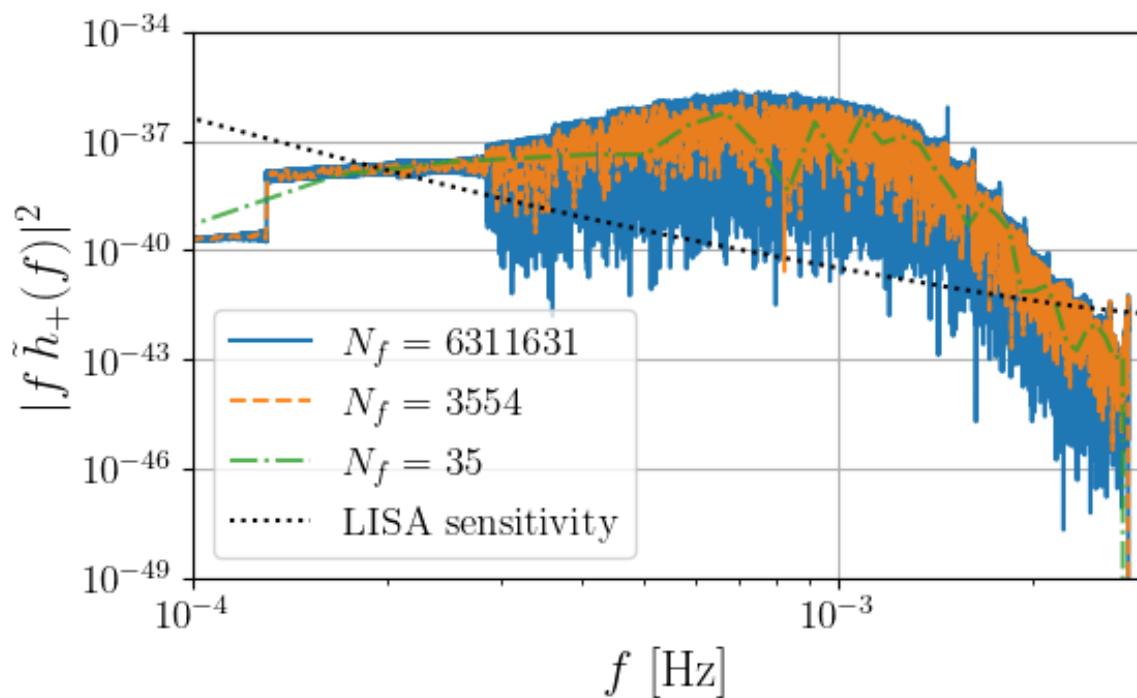
4 years, $\Delta t = 10 s$

Operation = FD waveform (Downsampled) TD waveform

Speed CPU [s] = 13.7 (0.4) 90.6

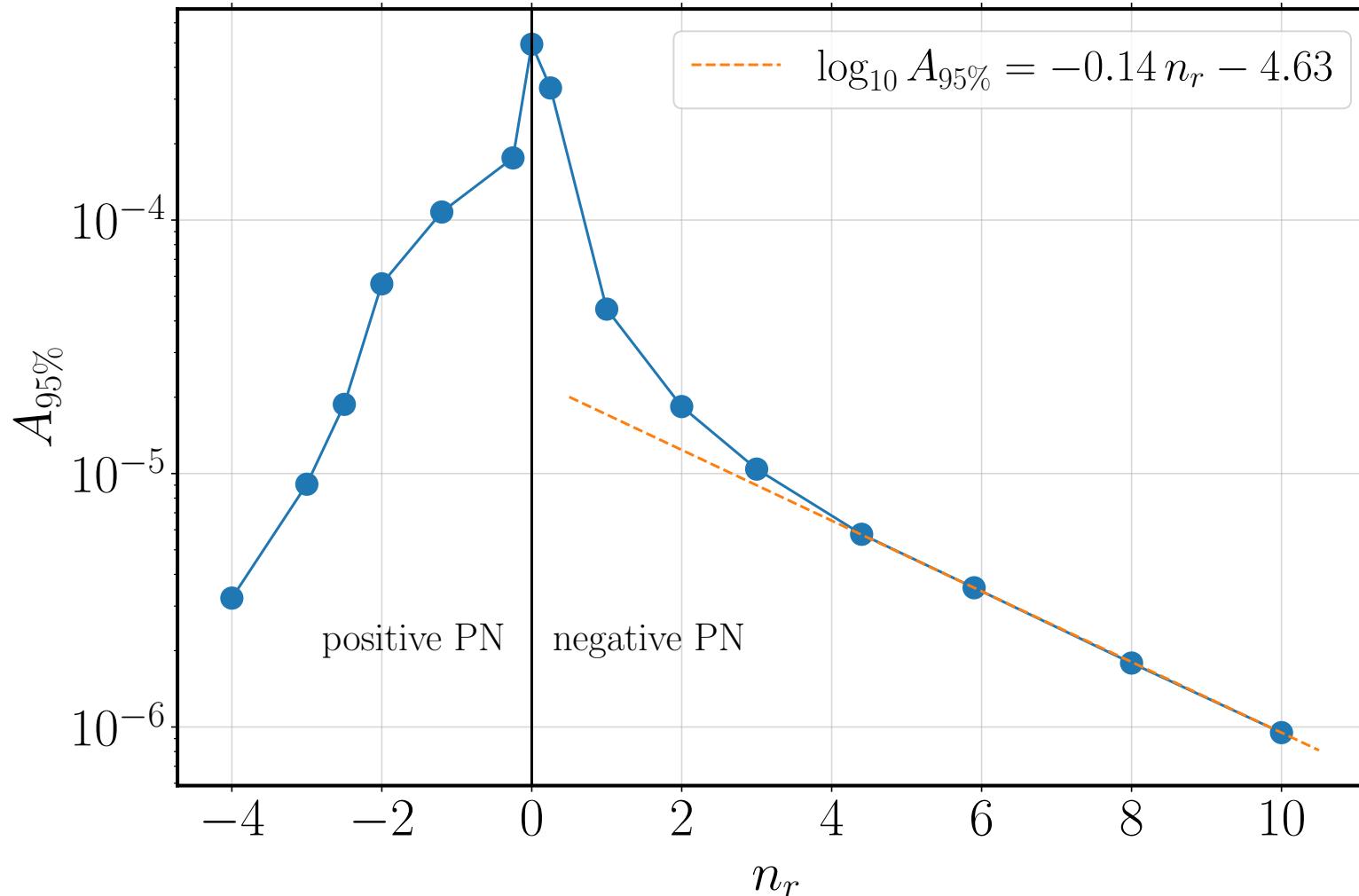
Speed GPU [s] = 0.048 (0.035) 0.064

frequency bins N_f = 6311631 (7864) 6311631



EMRI Beyond vacuum

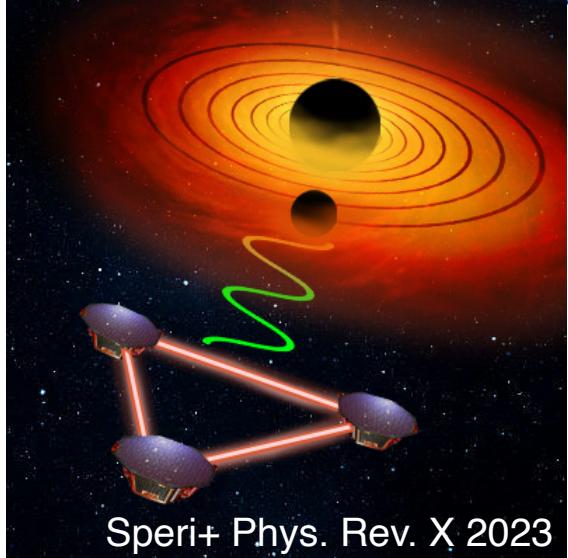
$M_1 = 10^6 M_\odot$, $M_2 = 50 M_\odot$, $a = 0.9$, SNR = 50, $T_{\text{inspiral}} = 4 \text{ yr}$



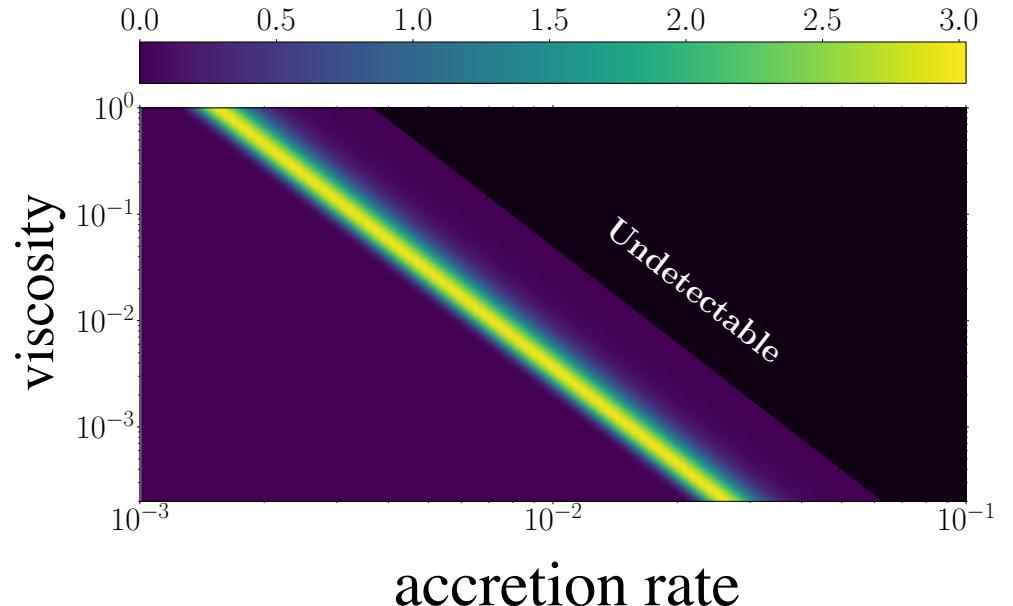
$$\frac{dr}{dt} = \eta f_{GW}(a, r)(1 + Ar^{\textcolor{red}{n}_r})$$

Speri+ Phys. Rev. X 2023

Environmental effects: a **curse** and a **bless**

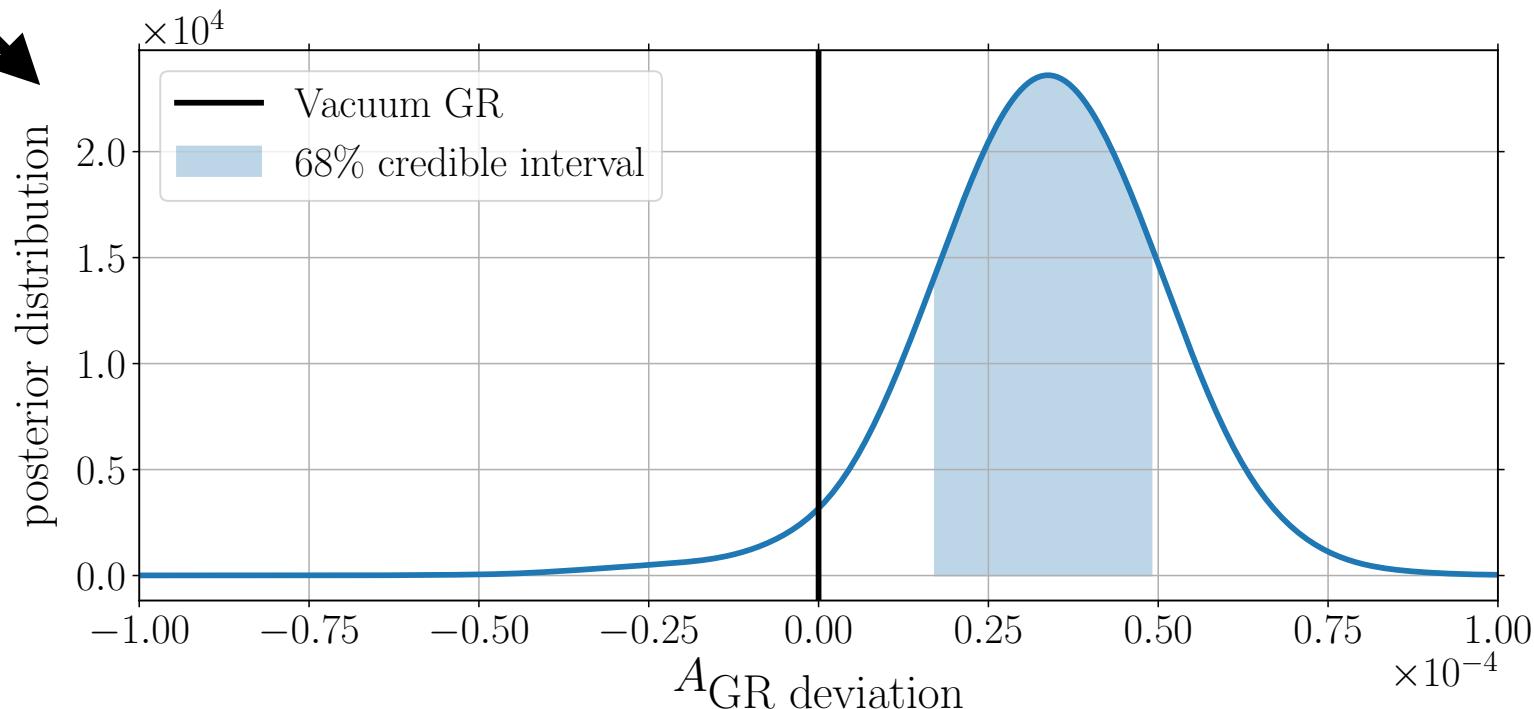


bless



accretion rate

curse

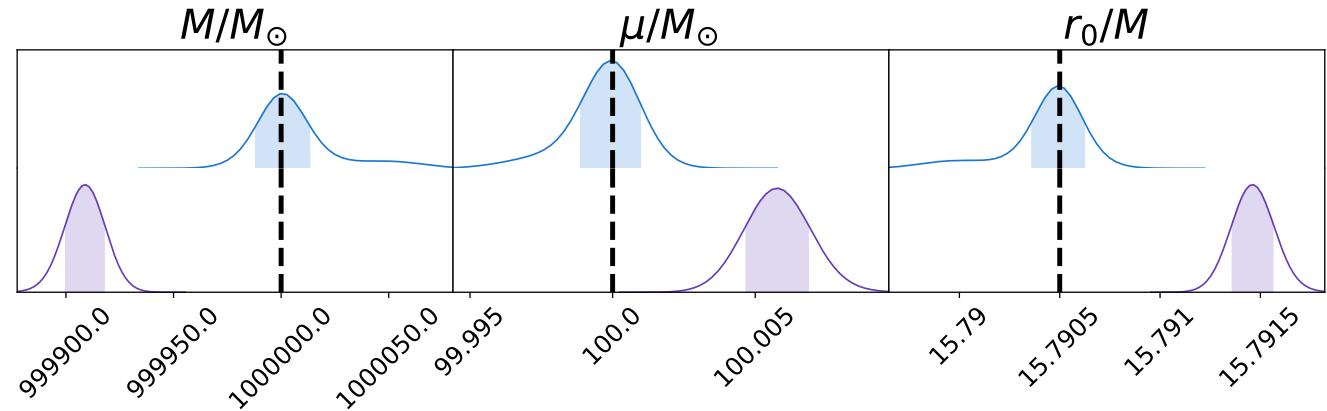


Accuracy requirements

mass ratio $\mu/M = 10^{-4}$

2nd order mass ratio

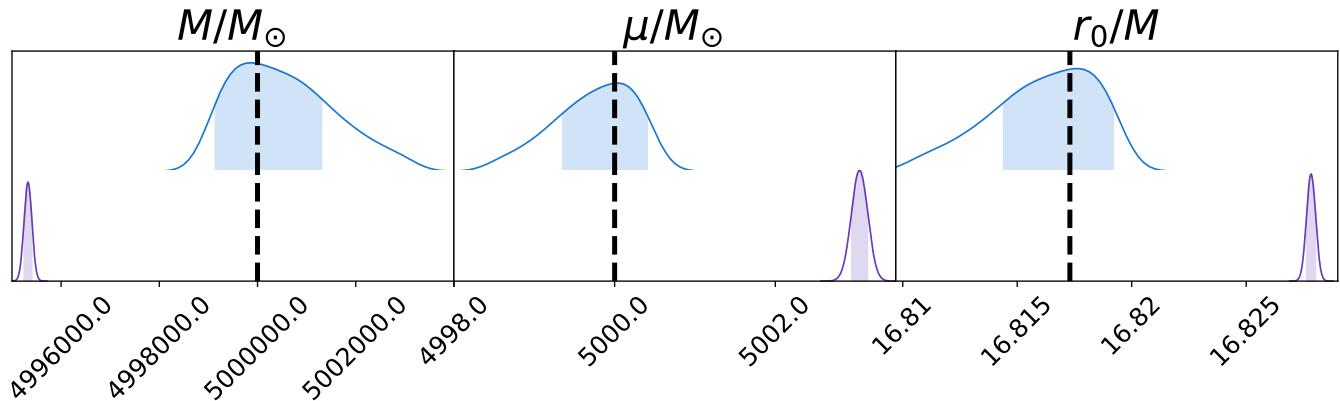
1st order mass ratio



mass ratio $\mu/M = 10^{-3}$

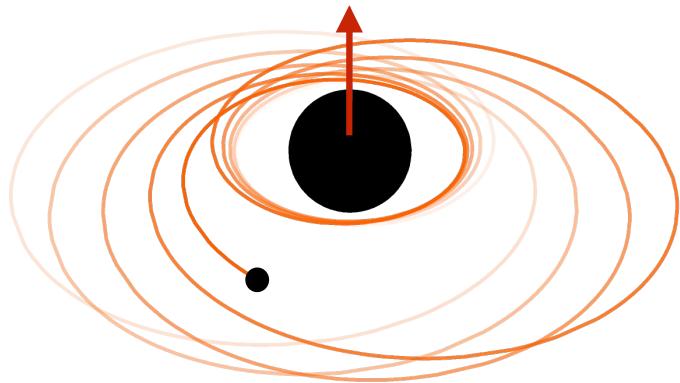
2nd order mass ratio

1st order mass ratio



FEW development

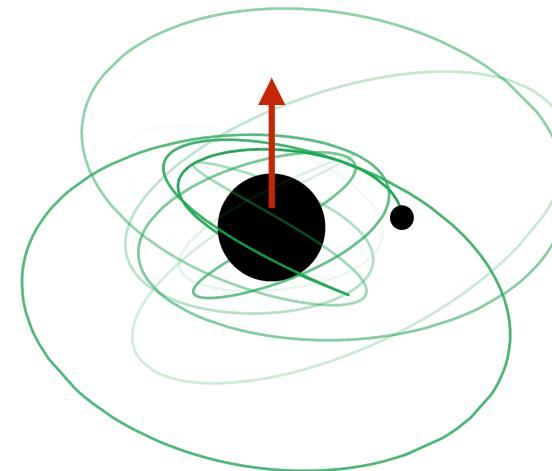
Relativistic Kerr Equatorial



equatorial eccentric orbit

interpolation of $\approx 10^4$ functions
parameter space 12 dimensions

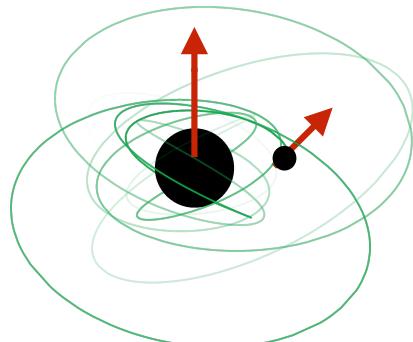
PN Generic



generic orbit

PN amplitudes and fluxes
parameter space 14 dimensions

Our dream: Relativistic Generic Kerr



Conclusions

FEW projects

- Kerr Equatorial
- PN Generic
- Interpolation
- Time-Frequency
- Frequency Domain Response
- Beyond vacuum effects

Long term goal

- Waveforms for LISA DA

Reach out if you want to get involved

bhptoolkit.org/FastEMRIWaveforms

[BeyondVacuum-EMRIs.ipynb](#)

[FD EMRI](#)

DOCUMENTATION:

- Overall Waveform Models
- Trajectory Package
- Amplitude Package
- Summation Package
- Utilities
- Citations

TUTORIAL:

- Fast and Accurate EMRI Waveforms Tutorial
- Full EMRI Waveforms in Schwarzschild Eccentric
- Trajectory Module
- Amplitude Module
- Spin-weighted spherical harmonics
- Mode Selection
- Parallelized Cubic Spline Interpolation
- Mode Summation
- Utility functions
- Creating modules
- Augmented Analytic Kludge with 5PN trajectory
- Citing waveforms and modules
- Single Mode Frequency Domain Waveform Construction
- EMRI Waveforms in frequency domain