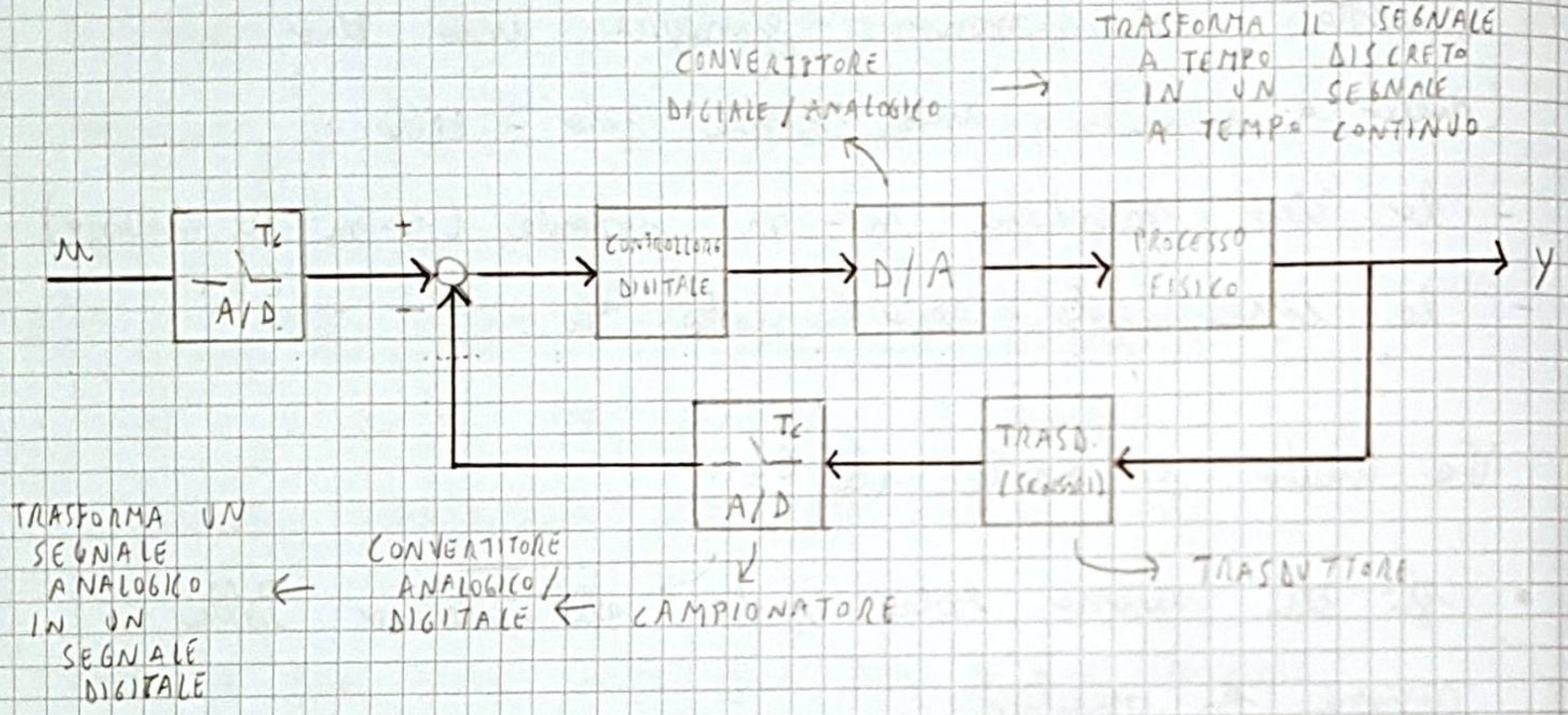
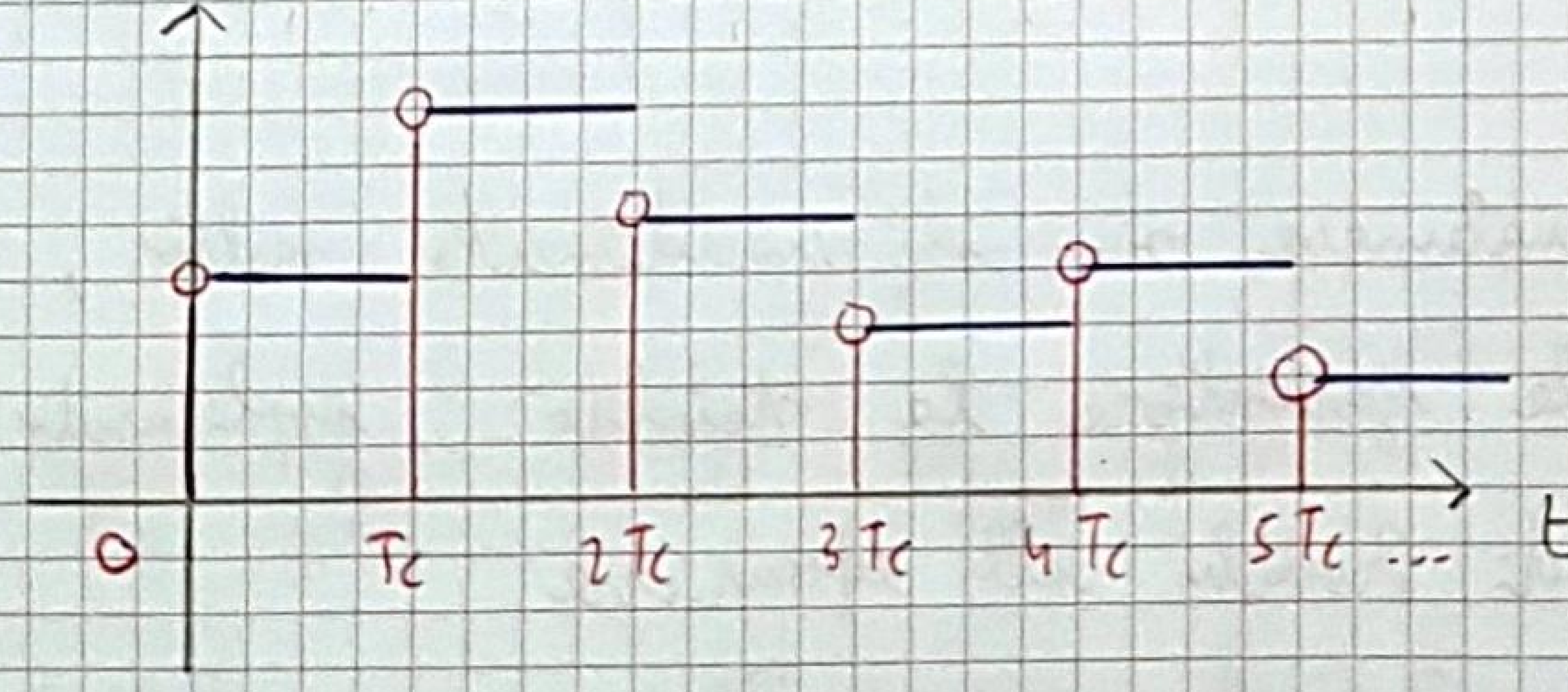
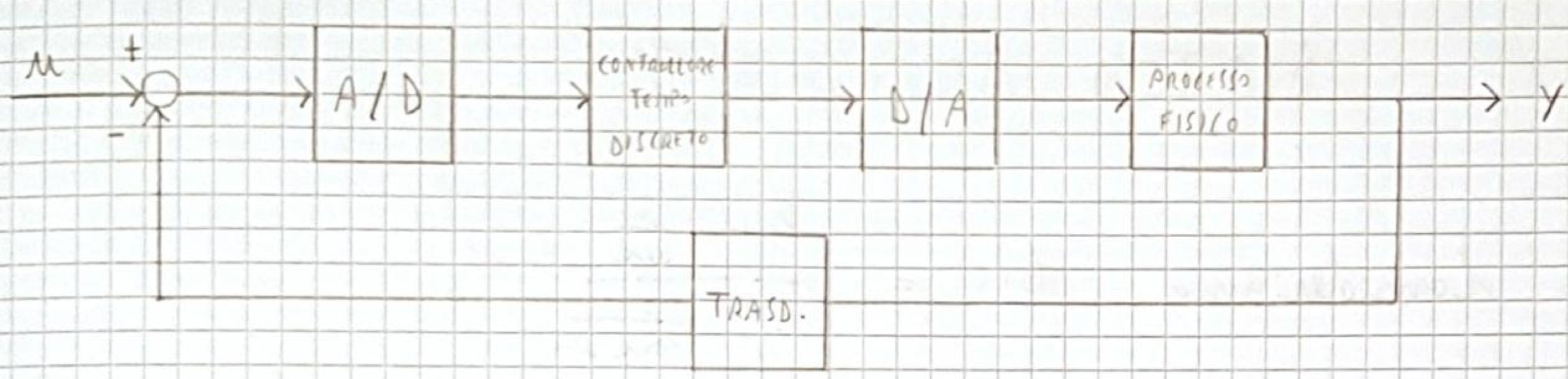
SISTEMI DI CONTROLLO DIGITALI (A TEMPO DISCRETO)



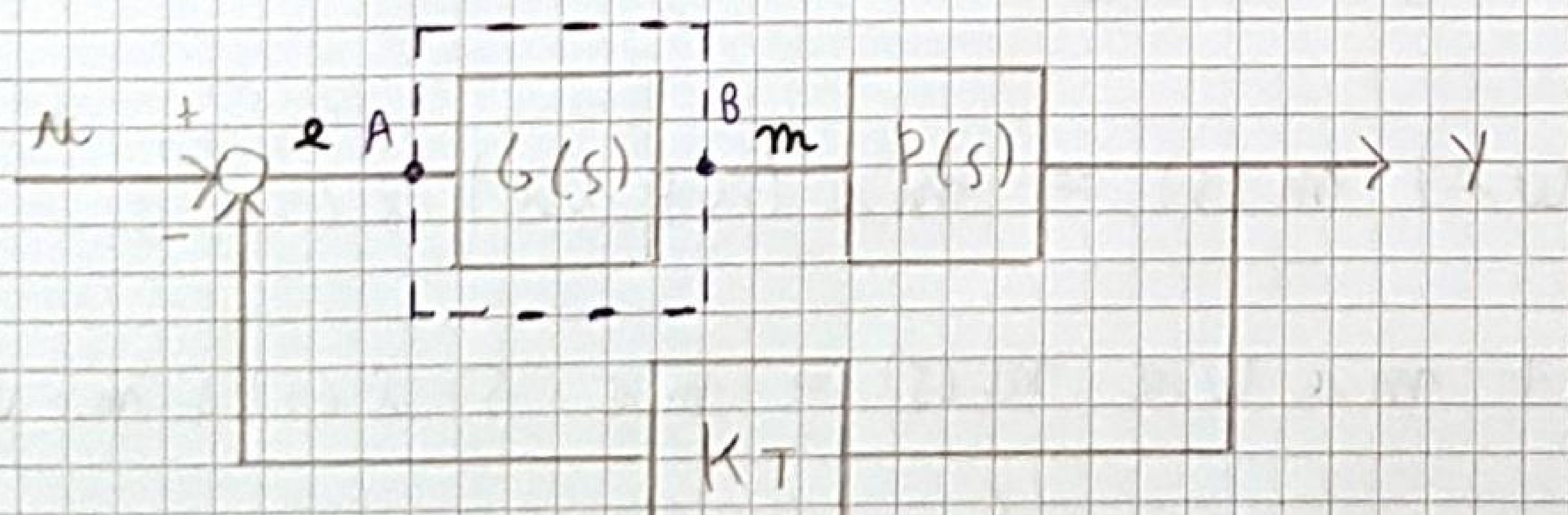
Il convertitore digitale / analogico viene anche detto ORGANO DI TENUTA DI ORDINE 0, in quanto viene utilizzato un segnale costante per passare da un istante di campionamento all'altro.





Sintesi: calcolare $G(z)$ per soddisfare le specifiche

I segnali che entrano ed escono dal riquadro blu sono segnali a tempo continuo.



SINTESI APPROSSIMATA: Progettare il controllore a tempo continuo $G(s)$ e poi trovare il $G(z)$ corrispondente in modo che tutto il blocco riquadrato in blu si comporti il più possibile in modo simile a $G(s)$

SINTESI APPROSSIMATA

Consideriamo $G(s) = \dfrac{1 + \frac{s}{\omega_a}}{1 + \frac{s}{m_a \omega_a}}$

$$G(s) = \frac{1 + \frac{s}{\omega_a}}{1 + \frac{s}{m_a \omega_a}} = \frac{\omega_a m_a \left(1 + \frac{s}{\omega_a}\right)}{\omega_a m_a \left(1 + \frac{s}{m_a \omega_a}\right)} =$$

$$= \frac{\omega_a m_a + m_a s}{\omega_a m_a + s} = \frac{m_a (s + \omega_a)}{s + \omega_a m_a}$$

$$\frac{m(s)}{e(s)} = G(s) = \frac{m_a (s + \omega_a)}{s + m_a \omega_a}$$

$$m(s) = \frac{m_a (s + \omega_a)}{s + m_a \omega_a} e(s)$$

$$(s + m_a \omega_a)\, m(s) = m_a (s + \omega_a)\, e(s)$$

$$s\, m(s) + m_a \omega_a\, m(s) = m_a \cdot s \cdot e(s) + m_a \omega_a\, e(s)$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\frac{dm}{dt} + m_a \omega_a\, m(t) = m_a \frac{de(t)}{dt} + m_a \omega_a\, e(t)$$

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$$\left.\frac{dm}{dt}\right|_{t=hT_C} \simeq \frac{m(hT_C) - m[(h-1)T_C]}{T_C}$$

$$\left.\frac{de}{dt}\right|_{t=hT_C} \simeq \frac{e(hT_C) - e[(h-1)T_C]}{T_C}$$

METODO DELLE DIFFERENZE ALL'INDIETRO

$h = 0, 1, \dots$

$$\frac{m(hT_C) - m[(h-1)T_C]}{T_C} + m_a w_a \cdot m(hT_C) \simeq$$

$$\simeq m_a \frac{e(hT_C) - e[(h-1)T_C]}{T_C} + m_a w_a \cdot e(hT_C)$$

Per alleggerire la scrittura, scrivo $m(hT_C)$ e $m[(h-1)T_C]$ come $m(h)$ e $m(h-1)$ in quanto, fissato un intervallo di campionamento T_C, m dipenderà da h.

$$\frac{m(h) - m(h-1)}{T_C} + m_a w_a\, m(h) =$$

$$= m_a \frac{e(h) - e(h-1)}{T_C} + m_a w_a\, e(h)$$

$$m(h) - m(h-1) + T_C\, m_a w_a\, m(h) =$$

$$= m_a [e(h) - e(h-1)] + T_C\, m_a w_a\, e(h)$$

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$$m(h)\,[1 + T_c\, m_e\, w_e] =$$

$$= m(h-1) + m_e\,[e(h) - e(h-1)] + T_c\, m_e\, w_e\, e(h)$$

$$m(h) = \frac{m(h-1) + m_e\, e(h)\,[1 + T_c\, w_e] - m_e\, e(h-1)}{1 + T_c\, m_e\, w_e}$$

$$m(h) - m(h-1) + T_c\, m_e\, w_e\, m(h) =$$

$$= m_e\,[e(h) - e(h-1)] + T_c\, m_e\, w_e\, e(h)$$

$$\Big\downarrow\ Z$$

$$m(z) - z^{-1}\, m(z) + T_c\, m_e\, w_e\, m(z) =$$

$$= m_e\, e(z) - m_e\, z^{-1}\, e(z) + T_c\, m_e\, w_e\, e(z)$$

$$m(z)\,[1 - z^{-1} + T_c\, m_e\, w_e] =$$

$$= e(z)\,[m_e\,(1 - z^{-1}) + T_c\, m_e\, w_e]$$

$$G(z) = \frac{m(z)}{e(z)} = \frac{m_e\,[(1 - z^{-1}) + T_c\, w_e]}{(1 - z^{-1}) + T_c\, m_e\, w_e} =$$

$$= \frac{m_e\left[\dfrac{1 - z^{-1}}{T_c} + w_e\right]}{\dfrac{1 - z^{-1}}{T_c} + m_e\, w_e}$$

$$G(z) = \frac{m_e\left[\frac{1-z^{-1}}{T_c} + w_e\right]}{\frac{1-z^{-1}}{T_c} + m_e w_e}$$

$$G(s) = \frac{m_e(s+w_e)}{s+m_e w_e}$$

$$s = \frac{1-z^{-1}}{T_c}$$