# Notes: AdS4D in Cartesian Coordinates

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**Abstract**

We describe compactified Cartesian coordinates for global $\mathrm{AdS}_4$, and write down the form that the metric takes in these coordinates.

## 1 Preliminaries

The metric of global $\mathrm{AdS}_4$ is

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\chi^2 + r^2 \sin^2\chi d\theta^2$$

where we have defined $f(r) = 1 + r^2/L^2$ for convenience. Here, $L$ is the AdS radius of curvature, related to the cosmological constant by $\Lambda_4 = -(D-1)(D-2)/(2L^2) = -3/L^2$. It is useful to introduce a compactified radial coordinate $\rho$. We choose

$$r = \frac{\rho}{1-\rho/\ell}, \tag{1}$$

where $\ell$ is an arbitrary compactification scale, independent of the AdS length scale $L$, such that the $\mathrm{AdS}_3$ boundary is reached when $\rho = \ell$. In all of the following, we set $\ell = 1$, though note that this scale is implicitly present since $\rho$ has dimensions of length. Transforming to the $\rho$ coordinate, the metric (1) takes the form:

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = \frac{1}{(1-\rho)^2}\left(-\hat{f}(\rho)dt^2 + \frac{1}{\hat{f}(\rho)}d\rho^2 + \rho^2 d\chi^2 + \rho^2\sin^2\chi\right) \tag{2}$$

where $\hat{f}(\rho) = (1-\rho)^2 + \rho^2/L^2$.

## 2 Metric of Pure $\mathrm{AdS}_4$ in Cartesian Coordinates

We now introduce Cartesian coordinates $x = \rho\cos\chi$, $y = \rho\sin\chi\sin\theta$, $z = \rho\sin\chi\cos\theta$ in terms of which the compactified coordinates in the previous section are $\rho = \sqrt{x^2+y^2+z^2}$,

$\chi = \arccos\left(x/\sqrt{x^2+y^2+z^2}\right)$, $\theta = \arctan(y/z)$. The metric of $\mathrm{AdS}_4$ written in these Cartesian coordinates has the following nonzero components[1]

$$
\begin{aligned}
\hat{g}_{tt} &= \frac{1}{(1-\rho)^2}\left[-\hat{f}(\rho)\right] \\
\hat{g}_{xx} &= \frac{1}{(1-\rho)^2}\frac{1}{\rho^2}\left[\frac{1}{\hat{f}(\rho)}x^2+y^2+z^2\right] \\
\hat{g}_{xy} &= \frac{1}{(1-\rho)^2}\frac{xy}{\rho^2}\left[\frac{1}{\hat{f}(\rho)}-1\right] \\
\hat{g}_{xz} &= \frac{1}{(1-\rho)^2}\frac{xz}{\rho^2}\left[\frac{1}{\hat{f}(\rho)}-1\right] \\
\hat{g}_{yy} &= \frac{1}{(1-\rho)^2}\left[\frac{y^2}{\rho^2}\left(\frac{1}{\hat{f}(\rho)}+\frac{x^2}{y^2+z^2}\right)+\frac{z^2}{y^2+z^2}\right] \\
\hat{g}_{yz} &= \frac{1}{(1-\rho)^2}\left[\frac{yz}{\rho^2}\left(\frac{1}{\hat{f}(\rho)}+\frac{x^2}{y^2+z^2}\right)-\frac{yz}{y^2+z^2}\right] \\
\hat{g}_{zz} &= \frac{1}{(1-\rho)^2}\left[\frac{z^2}{\rho^2}\left(\frac{1}{\hat{f}(\rho)}+\frac{x^2}{y^2+z^2}\right)+\frac{y^2}{y^2+z^2}\right].
\end{aligned}
\tag{3}
$$

[1]The elements of the Jacobian of the transformation are $\frac{\partial\rho}{\partial x}=\frac{x}{\rho}$, $\frac{\partial\rho}{\partial y}=\frac{y}{\rho}$, $\frac{\partial\rho}{\partial z}=\frac{z}{\rho}$, $\frac{\partial\chi}{\partial x}=-\frac{\sqrt{y^2+z^2}}{\rho^2}$, $\frac{\partial\chi}{\partial y}=\frac{1}{\rho^2}\frac{xy}{\sqrt{y^2+z^2}}$, $\frac{\partial\chi}{\partial z}=\frac{1}{\rho^2}\frac{xz}{\sqrt{y^2+z^2}}$, $\frac{\partial\theta}{\partial x}=0$, $\frac{\partial\theta}{\partial y}=\frac{z}{y^2+z^2}$, $\frac{\partial\theta}{\partial z}=-\frac{y}{y^2+z^2}$.