This is the sixth part of a series of notes in which we compute expressions relevant for the modified cartoon reduction of the equations of motion.

In this note we calculate the expression of the Einstein equations in generalised harmonic (GH) form on the hypersurface S' at $z \geq \epsilon, w^p = 0\ \forall p$.

The necessary expressions for scalar, vector and $\binom{0}{2}$ tensor densities at S' can be obtained from the expressions of APPENDIX A of 1603.00362 or eq. (27)-(31) of 1004.4970, by treating the t coord just like one of the x^i coords.

Using these results and the ones from previous notes, we now find the GH form of the Einstein equations at S'.

In general, the Einstein eqs. in GH form with negative cosmological constant Λ read

$$R^{(D)}_{AB} = \bar{T}_{AB}, \text{ where } \bar{T}_{AB} = \frac{2}{D-2}\Lambda g_{AB} + 8\pi\left(T_{AB} - \frac{1}{D-2} T g_{AB}\right)$$

(here $\Lambda = -\frac{D(D+1)}{2L^2}$, where L is the AdS radius, and T_{AB} is the energy-momentum tensor and $T := g^{AB} T_{AB}$ is its trace) and $R^{(D)}_{AB}$ is the Ricci tensor of the D-dimensional spacetime (M, g):

$$R^{(D)}_{AB} = -\frac{1}{2} g^{CD} g_{AB,CD} - g^{CD}{}_{,(A} g_{B)C,D} - H^{(D)}_{(A,B)}$$
$$+ H^{(D)}_C \Gamma^{(D)C}_{AB} - \Gamma^{(D)C}_{DA} \Gamma^{(D)D}_{CB},$$

where $\Gamma^{(D)C}_{AB} = \frac{1}{2} g^{CD} \left(g_{AD,B} - g_{AB,D} + g_{BD,A} \right)$ are the Christoffel symbols of the metric g_{AB} and $H^{(D)A} = - g^{CD} \Gamma^{(D)A}_{CD}$ are the source functions associated with the metric g_{AB}.