PERTURBATIONS OF KERR-AdS

Let $\hat{g}_{\mu\nu}$ be the pure AdS metric in spherical coords. Kerr-AdS can be written as

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \begin{pmatrix} \bar{g}_{tt} & & & \\ \bar{g}_{t\rho} & \bar{g}_{\rho\rho} & & \\ \bar{g}_{ty} & \bar{g}_{\rho y} & \bar{g}_{yy} & \\ \bar{g}_{t\xi} & \bar{g}_{\rho\xi} & \bar{g}_{y\xi} & \bar{g}_{\xi\xi} \end{pmatrix} = \hat{g}_{\mu\nu} + h_{\mu\nu}$$

in quasi-spherical coords $x^\mu = \{t, \rho, y, \xi\}$ (i.e. $g_{\mu\nu} \to \hat{g}_{\mu\nu}$ at the bdy but $h_{\rho\mu}, \mu \neq \rho$, do not have the required fall-offs),

where $y = \frac{\theta}{\pi}$, $\xi = \frac{\varphi}{2\pi}$.

$\bar{g}_{\mu\nu}$ are linear in $(1-\rho)$ near the bdy.

$$T_{tt} = \frac{1}{64\pi^3}\left(12\bar{g}_{(1)yy} + 8\pi^2 \bar{g}_{(1)\rho\rho} + \frac{3\,\bar{g}_{(1)\xi\xi}}{\sin^2(\pi y)}\right)$$

$$T_{ty} = \frac{3}{16\pi} \bar{g}_{(1)ty}$$

$$T_{t\zeta} = \frac{3}{16\pi} \bar{g}_{(1)t\zeta}$$

To have

$$T_{ty} = T_{ty(\text{Kerr-AdS})} + \varepsilon_\nu \left(\nu_y^{s=1,22} + \nu_y^{s=2,22} \right),$$

we use

$$\bar{g}_{ty} = \bar{g}_{ty(\text{Kerr-AdS})} + \frac{16\pi}{3} q\, \varepsilon_\nu \left(\nu_y^{s=1,22} + \nu_y^{s=2,22} \right)$$

To have

$$T_{t\xi} = T_{t\xi(\text{kerr-AdS})} + \varepsilon_v \left(V_\xi^{s=1,22} + V_\xi^{s=2,22} \right)$$

we use

$$\bar{g}_{t\xi} = \bar{g}_{t\xi(\text{kerr-AdS})} + \frac{16\pi}{3} q \, \varepsilon_v \left(V_\xi^{s=1,22} + V_\xi^{s=1,22} \right).$$

To have

$$T_{tt} = T_{tt(\text{kerr-AdS})} + \varepsilon_Y Y^{22}$$

we use

$$\bar{g}_{yy} = \bar{g}_{yy(\text{kerr-AdS})} + \frac{1}{3} \frac{64\pi^3}{22} q \, \varepsilon_Y Y^{22}$$

$$\bar{g}_{\rho\rho} = \bar{g}_{\rho\rho(\text{kerr-AdS})} + \frac{1}{3} \frac{64\pi^3}{8\pi^2} q \, \varepsilon_Y Y^{22}$$

$$\bar{g}_{\xi\xi} = \bar{g}_{\xi\xi(\text{kerr-AdS})} + \frac{1}{3} \frac{64\pi^3 \sin(\pi y)^2}{3} q \, \varepsilon_Y Y^{22}$$

(recall $\bar{g}_{yy} = \pi^2 \bar{g}_{\theta\theta}$ and $\bar{g}_{\xi\xi} = (2\pi)^2 \bar{g}_{\varphi\varphi}$)