Hebbian Neural Cellular Automaton

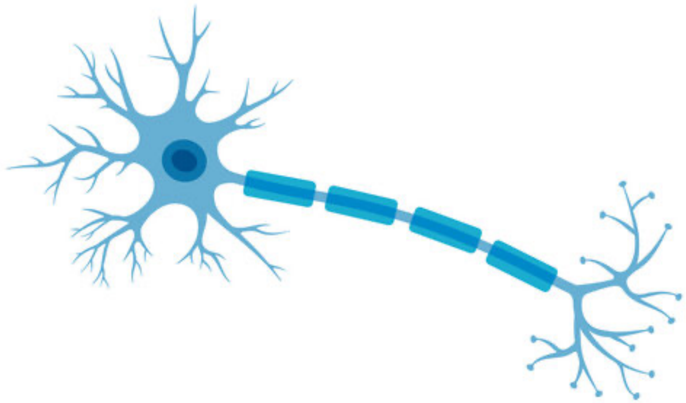
Francesca Perlini, Lorenzo Tecchia,

Philippe Skarbon, Theyn Kan

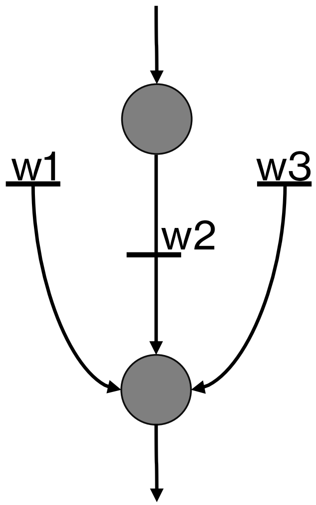
Critical brain theory

- The brain is constantly moving towards a (sub)critical state.
- Critical states are great for information transfer.
- Epilepsy, dementia, alzheimers.
- What keeps the brain in the critical state?

Oversimplified neuron dynamics

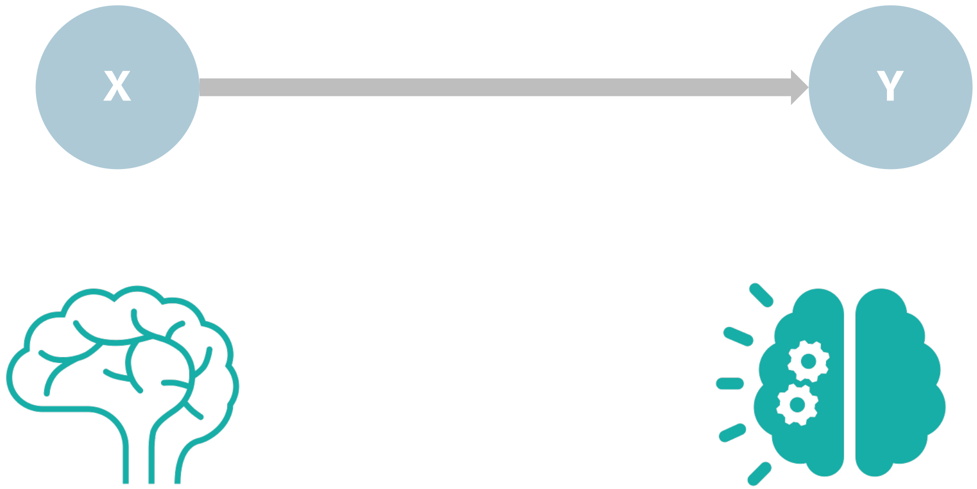


- Dendrites (in-degree), axons (out-degree)
- Sum of weights > Threshold
- Spike/firing



Neural plasticity

Spike-Time-Dependent-Plasticity (SDTP)



X spikes before **Y**

Stronger weight

Learning constant

Y spikes before **X**

Weaker weight

Forgetting constant

Cellular automaton based on STDP

- Network model
- Nodes have two states: firing or not firing
- STDP rules
- Give stimulus
- Calculate avalanches
- Synaptic weights move the model between order and chaos
- Neuroplasticity keeps synaptic weights at the critical state between order and chaos

Research question

Can spike-time-dependent-plasticity rules in a network model create phase transitions from an ordered to a chaotic phase?

Hypotheses

We expect that STDP rules can create a phase transition from an ordered to a chaotic phase.

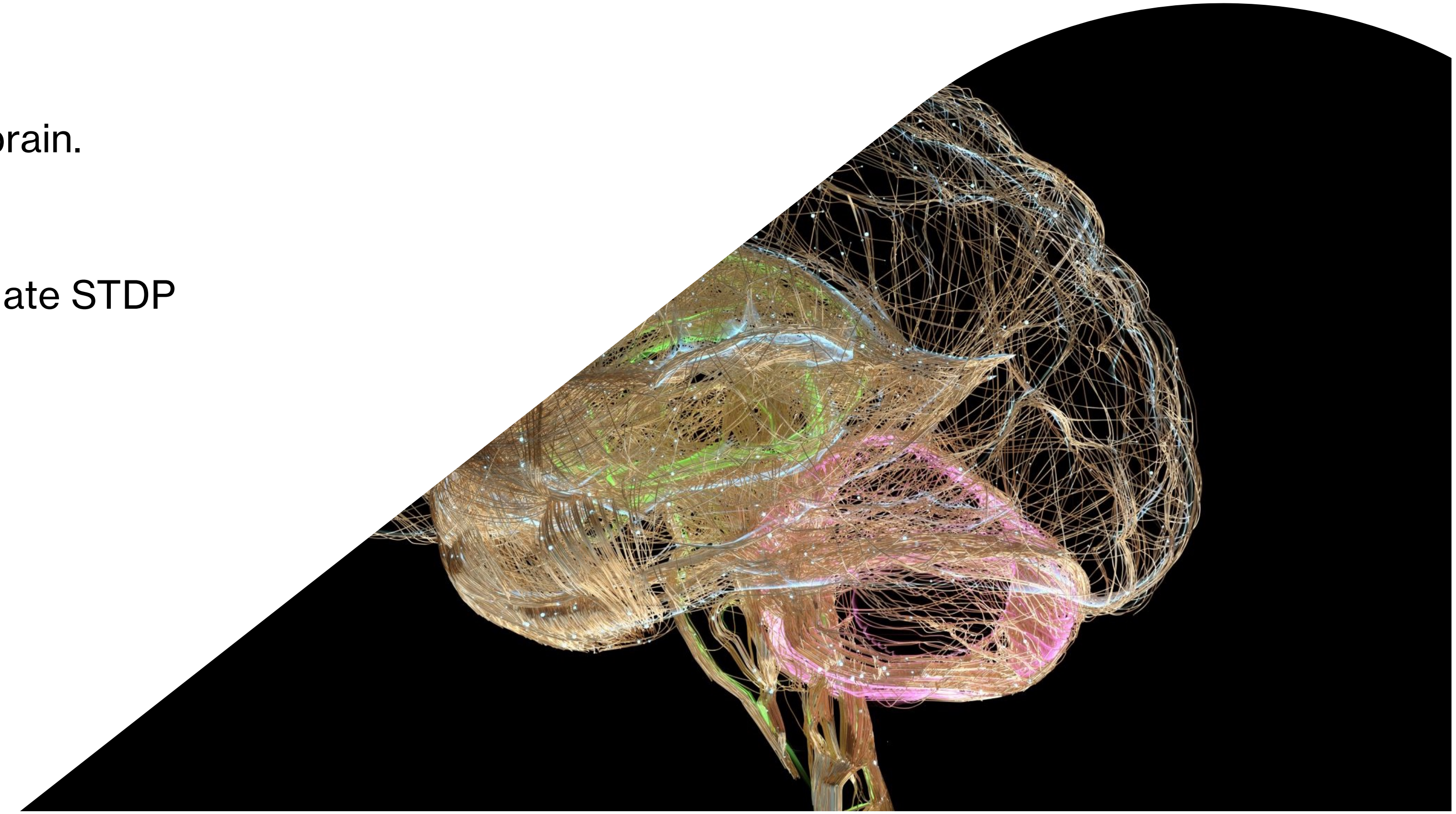
Number and size of avalanches as a metric

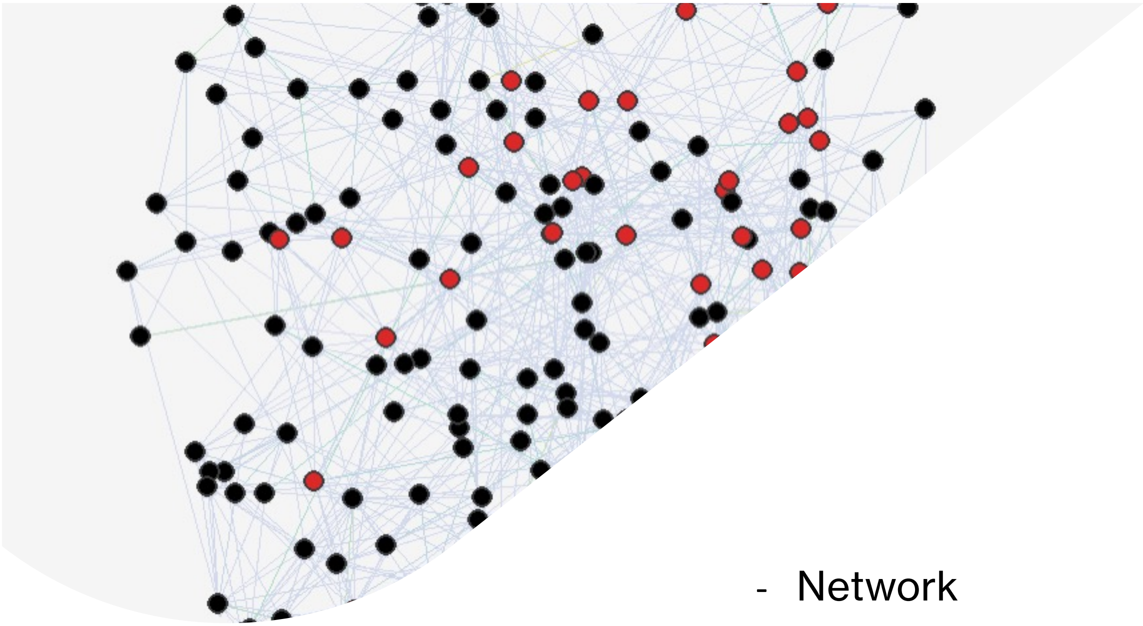
- Ordered: no avalanches / small avalanches

- Chaotic: infinite avalanches / huge avalanches

Clarification

- We do not model the brain.
- We model and investigate STDP behaviour.





Model Components

- Network
  - 3D neuron positions
  - Directed edges, distance-based connectivity
  - Weighted synapses (beta-distributed at initialization)

Excitatory: $0 \leq w \leq w_{max}$
Inhibitory: $w_{min}^{inh} \leq w \leq 0$

- Neurons
  - Firing/Non-firing
  - Inhibitory/Excitatory
  - Previous firing (STDP)
  - LIF Potential Membrane

Model Dynamics

At each time step, synaptic input is computed as: $v = W^T \cdot s$

s : vector of all the states of the neurons (firing/non-firing)

W : synaptic weight matrix

$\mathbf{W_{i,j}}$: synaptic strength from neuron I to neuron J

v : total synaptic input current

Element Wise : $v_j = \sum_i W_{ij} \, s_i$

This computes synaptic input only; neuron dynamics and learning are applied afterward.

Leaky Integrate-and-Fire (LIF) Model

Each neuron integrates synaptic input with leak as follows

$$V_i(t+1) = (1-\lambda)V_i(t) + v_i(t) - V_{reset} \cdot fired_i(t)$$

λ: Leak rate

$s_j(t)$: Firing state of neuron j at time t

V_{reset}: Reset amount after firing

$v_i(t)$: Synaptic input computed as $W^T s$

A neuron then fires when its membrane potential exceeds a threshold

$$s_i(t) = \begin{cases} 1 & \text{if } V_i(t) \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

STDP and Weight Stabalization

For synapse A→B:

- **LTP:** A fires before B → weight increases
- **LTD:** B fires before A → weight decreases

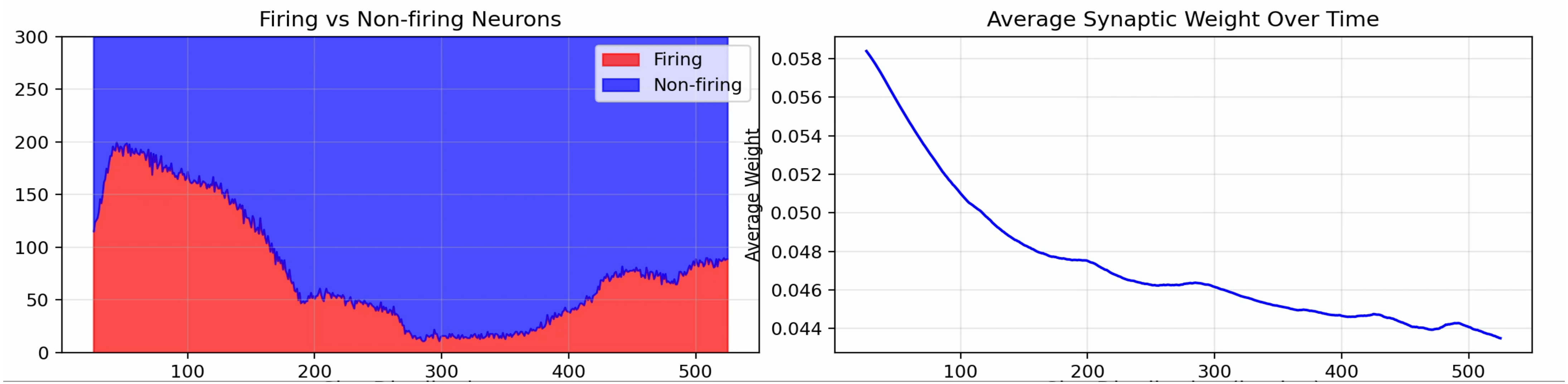
To prevent runaway growth:

- Baseline alpha decay: global decay
- Oja rule: activity-dependent normalization

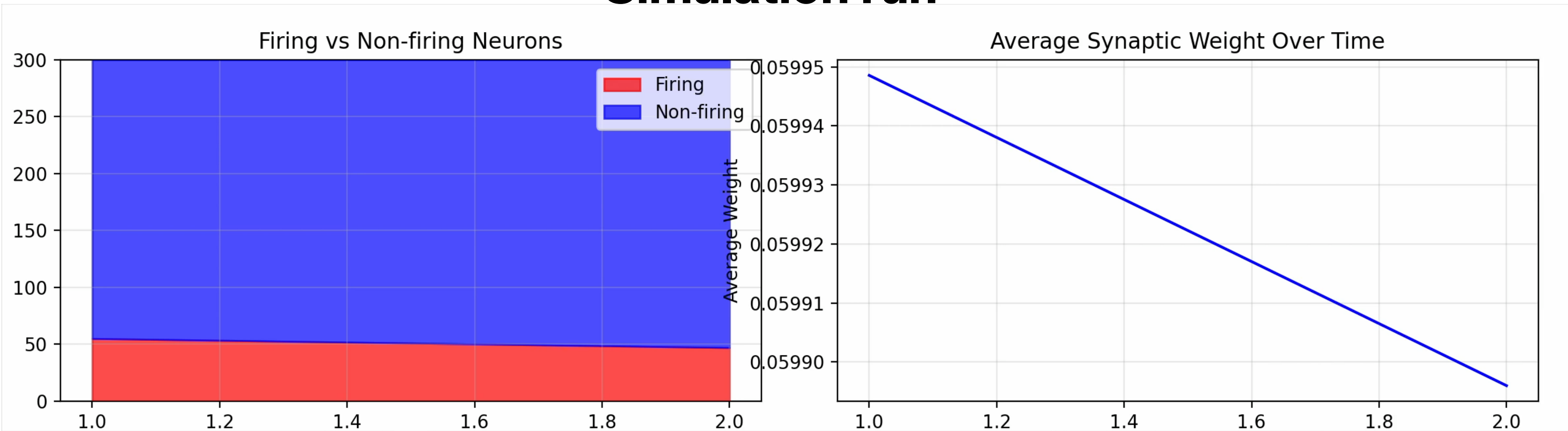
DEMO RUN

CASE A:

- STDP
- Leaky-Integrated-Firing potential
- Oja Rule
- Alpha decay



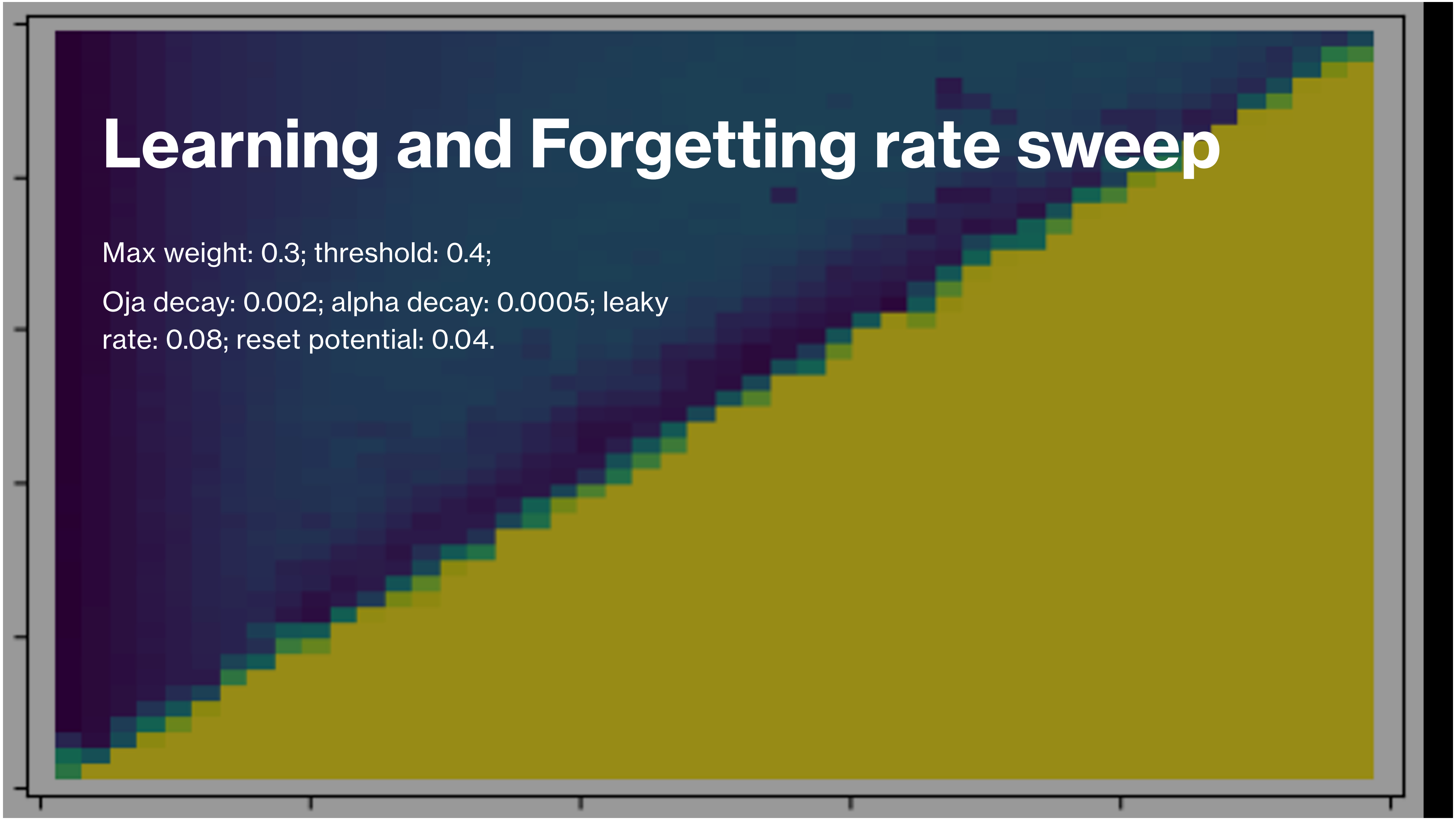
Simulation run

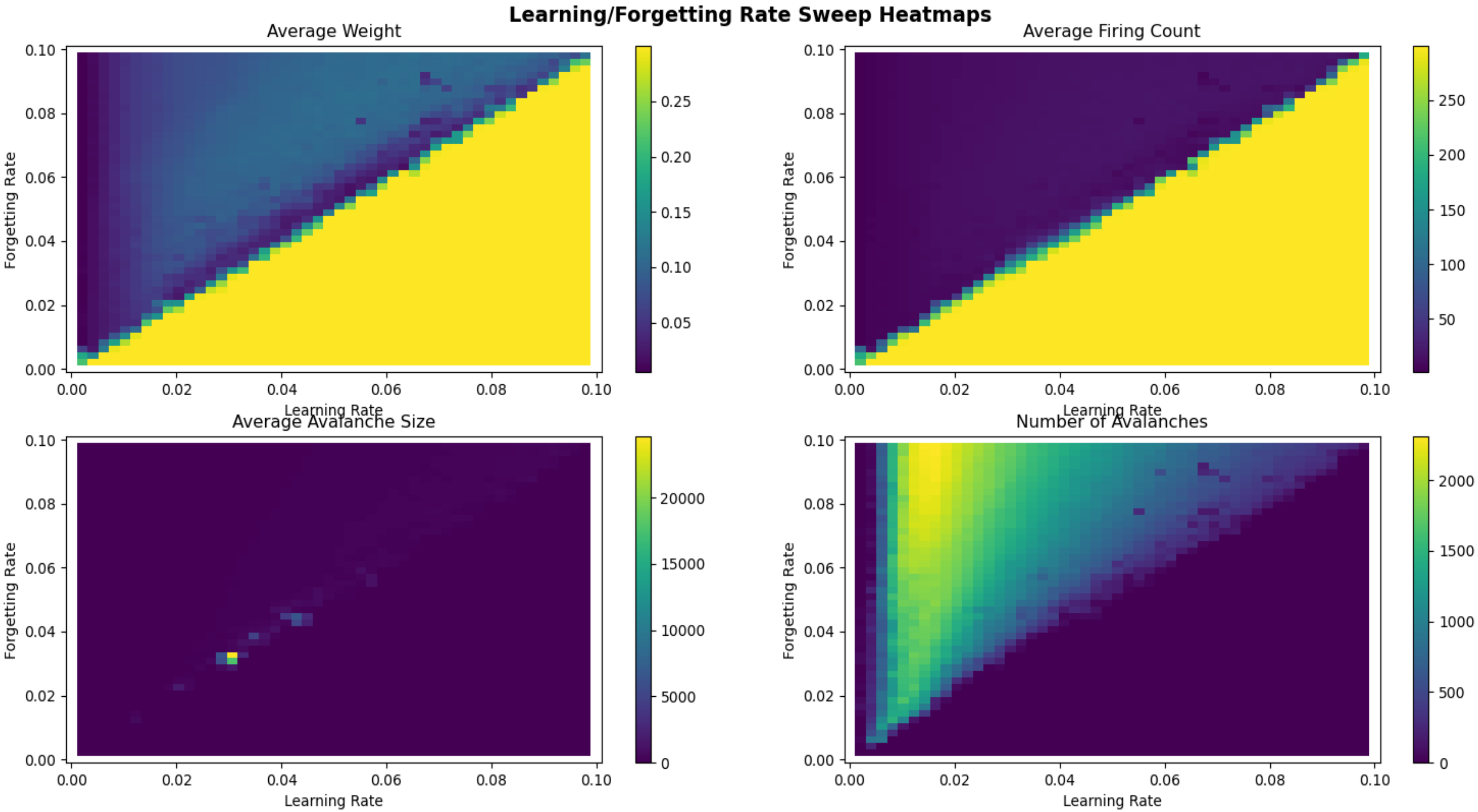


Learning and Forgetting rate sweep

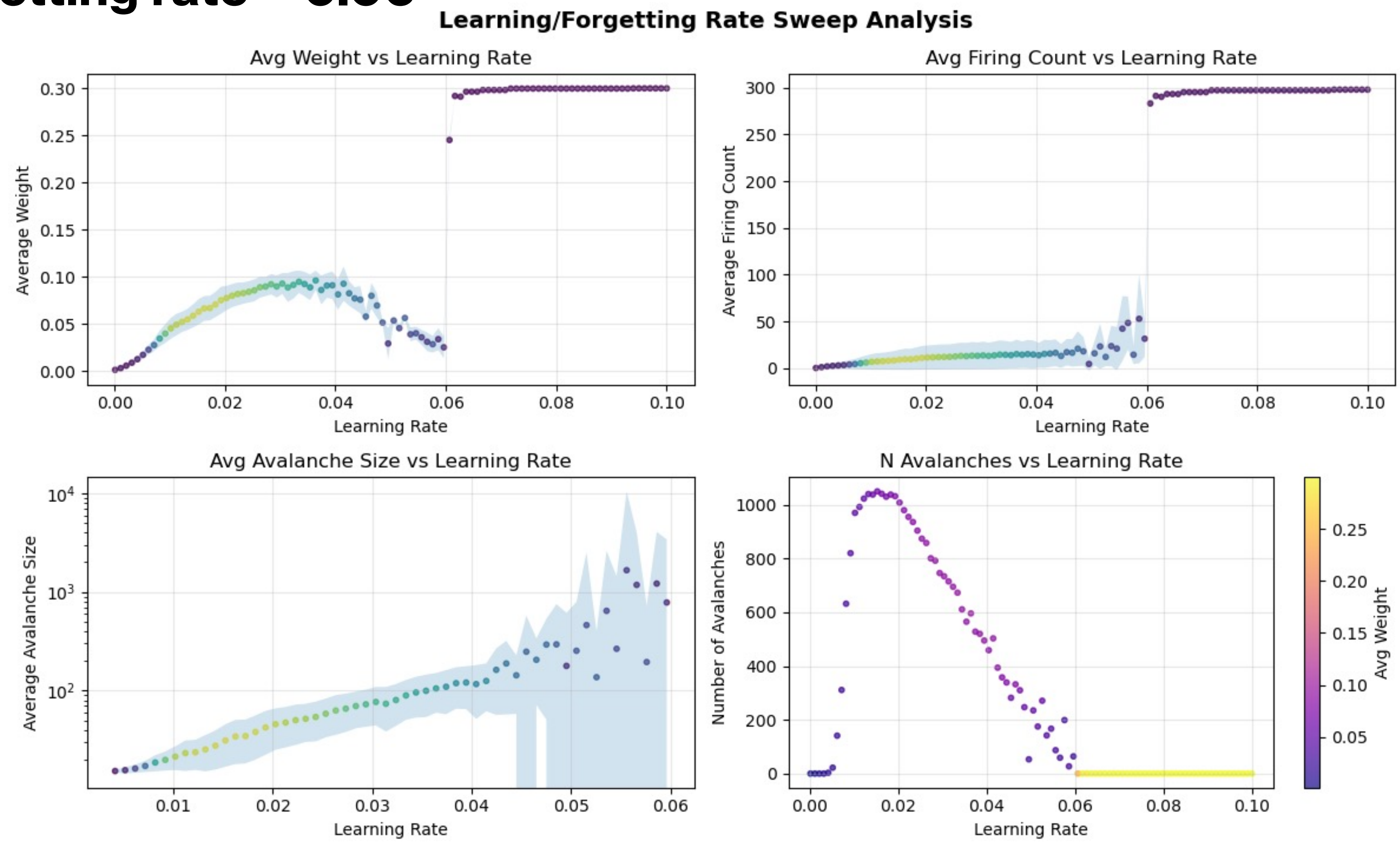
Max weight: 0.3; threshold: 0.4;

Oja decay: 0.002; alpha decay: 0.0005; leaky rate: 0.08; reset potential: 0.04.



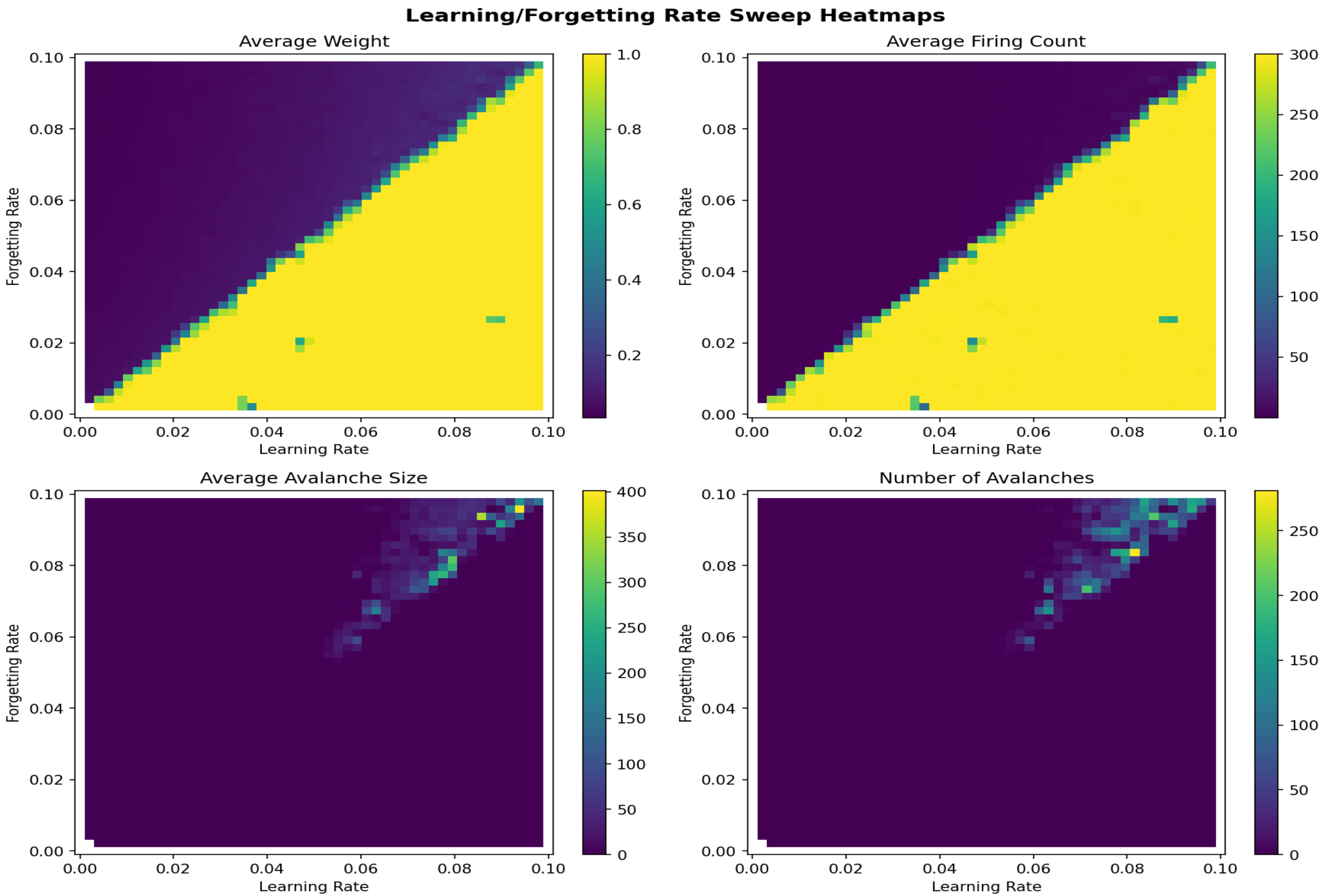

Learning/Forgetting Rate Sweep Heatmaps
Average Weight
Average Firing Count
Average Avalanche Size
Number of Avalanches
Learning Rate
Forgetting Rate
0.00
0.02
0.04
0.06
0.08
0.10
0.05
0.10
0.15
0.20
0.25
50
100
150
200
250
0
5000
10000
15000
20000
500
1000
1500
2000

Forgetting rate = 0.06

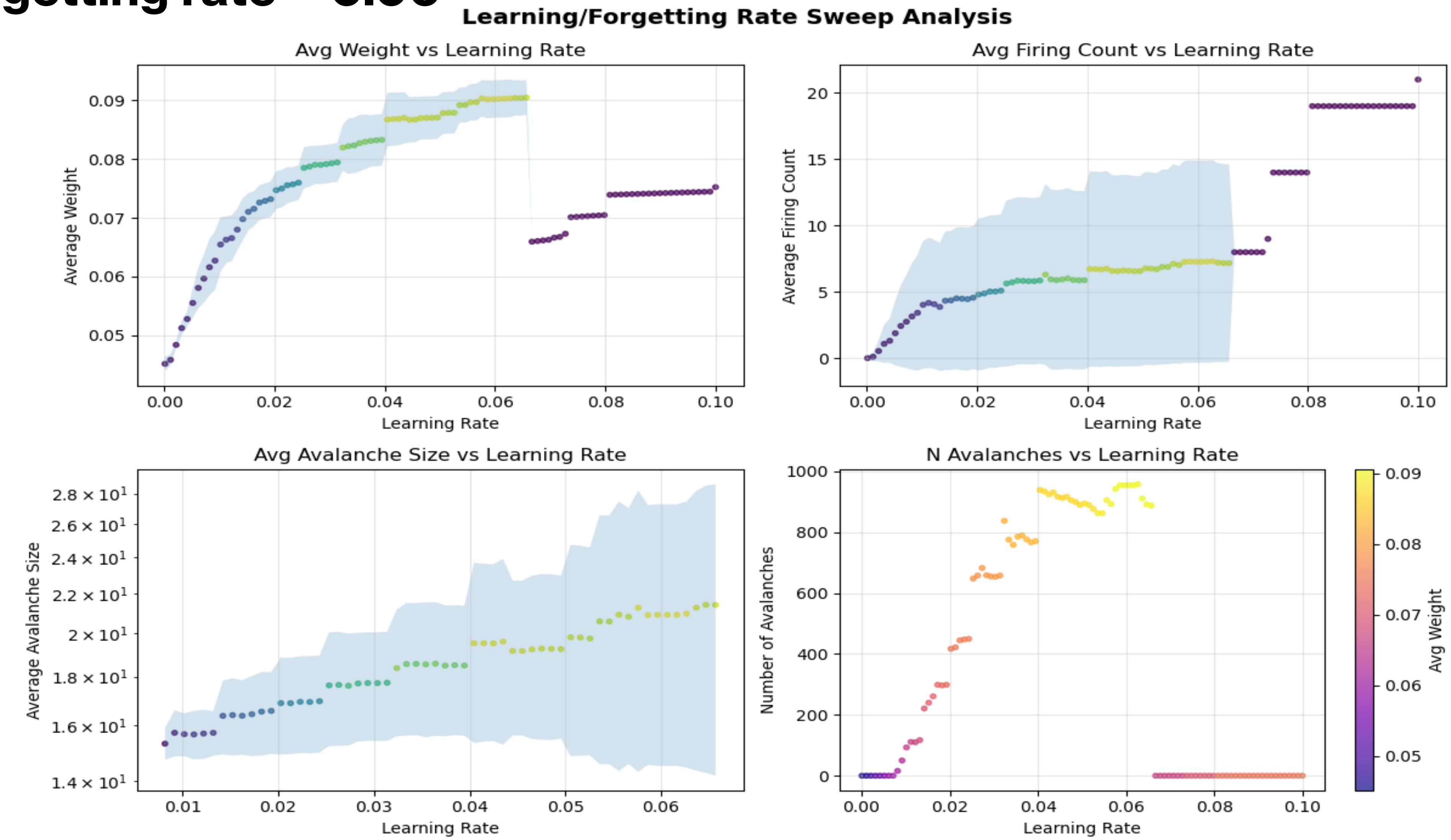


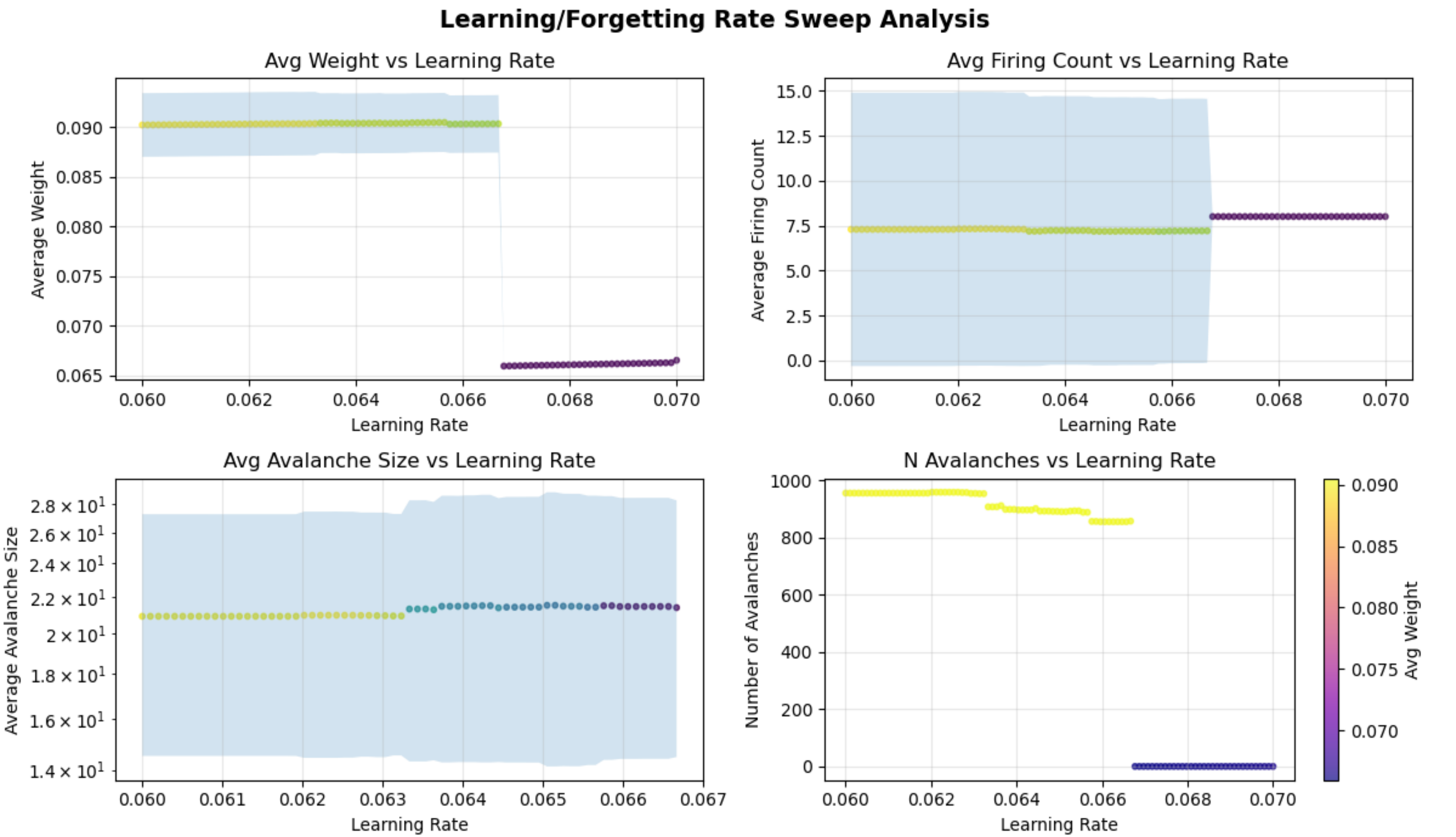
CASE B:

- STDP
- Leaky-Integrated-Firing potential
- Oja Rule
- Alpha decay


Learning/Forgetting Rate Sweep Heatmaps
Average Weight
Average Firing Count
Average Avalanche Size
Number of Avalanches
Forgetting Rate
Learning Rate
0.00
0.02
0.04
0.06
0.08
0.10
0.2
0.4
0.6
0.8
1.0
50
100
150
200
250
300
0
350
400

Forgetting rate = 0.06





Learning/Forgetting Rate Sweep Analysis
Avg Weight vs Learning Rate
Average Weight
Learning Rate
0.065
0.070
0.075
0.080
0.085
0.090
0.060
0.062
0.064
0.066
0.068
0.070
Avg Firing Count vs Learning Rate
Average Firing Count
Learning Rate
0.0
2.5
5.0
7.5
10.0
12.5
15.0
Avg Avalanche Size vs Learning Rate
Average Avalanche Size
1.4×10^1
1.6×10^1
1.8×10^1
2×10^1
2.2×10^1
2.4×10^1
2.6×10^1
2.8×10^1
0.060
0.061
0.062
0.063
0.064
0.065
0.066
0.067
N Avalanches vs Learning Rate
Number of Avalanches
0
200
400
600
800
1000
Avg Weight
0.070
0.075
0.080
0.085
0.090

References

Beggs, J.M. & Plenz, D. (2003). Neuronal avalanches in neocortical circuits. *Journal of Neuroscience*, 23(35), 11167-11177.

Bi, G.Q. & Poo, M.M. (1998). Synaptic modifications in cultured hippocampal neurons. *Journal of Neuroscience*, 18(24), 10464-10472.

Oja, E. (1982). Simplified neuron model as a principal component analyzer. Journal of Mathematical Biology*, 15(3), 267-273.

Scarpetta, S., I. Apicella, L. Minati, and A. de Candia. “Hysteresis, Neural Avalanches and Critical Behaviour near a First-Order Transition of a Spiking Neural Network.” *Physical Review E* 97, no. 6 (2018): 062305. https://doi.org/10.1103/PhysRevE.97.062305.

Q&A Time