Fundamentals of Theoretical Computer Science

S 2023 Exercise Sheet II

Hand in: 23.5.2023, 10:00 a.m.

Note: Only the (partial) tasks marked with ⋆ have to be handed in. A total of 25 points is to be achieved. The tasks will be discussed/solved in the tutorials.

Task 1 — NEA — 2 + 2 points

Give nondeterministic finite automata (NEA), with *as few* states *as possible,* for the following languages over the alphabet {0, 1}. Briefly explain your automaton.

(a) The set of all words whose first and last letters are the same.

(b) The set of all words that contain exactly three 1s.

(c) The set of all words that contain 0101 as a partial word. (⋆)

(d) The set of all words that contain even many 1s *or* exactly two 0s. (⋆)

(e) The set of all words that do not contain a pair of 1s separated by an odd number of symbols.

Task — 2NEA (⋆) — 3 + 3 + 4 points

Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NEA that accepts the language $L = L(M)$.

(a) Prove or disprove that the NEA $M' = (Q, \Sigma, \delta, q_0, Q \setminus F)$ accepts the complement language $\Sigma^* \setminus L$. (⋆)

(b) Prove or disprove that if $L = L(M)$ is nonempty, then there exists an NEA M' with exactly one accepting state such that $L(M') = L$. (⋆)

(c) Specify NEAs that accept the following regular expression-defined languages over the alphabet {0, 1}. (⋆)

$L_1 = L(0(011)^* \cup 1)$,

$L_2 = L((0 \cup 11^*)00^*11^*)$.

Task 3 Power set construction (⋆) 2 + 3 points

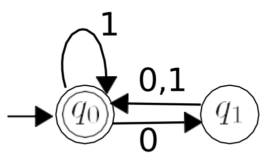
Consider the NEA $M = (Q, \Sigma, \delta, q_0, F)$, with $\Sigma = \{0, 1\}$, $Q = \{a, b, c\}$, $q_0 = a$, $F = \{a\}$, and $\delta(a, 0) = \{a, b\}$, $\delta(b, 1) = \{b, c\}$, and $\delta(c, 0) = \{a, c\}$, $\delta(a, 1) = \delta(b, 0) = \delta(c, 1) = \emptyset$.

(a) Draw the state diagram for M. (⋆)

(b) Use power set construction to find a DEA equivalent to M. Eliminate all redundant states. (⋆)

Task 4 Kleene's algorithm

Construct a regular expression to follow- the finite automaton using Kleene's algorithm.



In doing so, you may simplify intermediate expressions for $R^k_{i,j}$. Describe the accept-ted language in one sentence.

Task 5 — Pumping lemma — 2 × 3 points

Show that the following languages are not regular.

(a) $L = \{0^n\, 10^n \mid n \geq 0\}$ via $\Sigma = \{0, 1\}$

(b) $L = \{1^2\, n \mid n \geq 0\}$ via $\Sigma = \{0, 1\}$ ($\star$)

(c) $L = \{1^p \mid p$ is a prime number$\}$ via $\Sigma = \{1\}$

(d) $L = \{0\ 1^{mn} \mid m, n \in \mathbb{N}, m \neq n\}$ via $\Sigma = \{0, 1\}$

(e) $L = \{0\ 1\ 2^{nnn} \mid n \geq 0\}$ via $\Sigma = \{0, 1, 2\}$

(f) $L = \{0\ 1\ 0^{nmn+m} \mid n, m \in \mathbb{N}\}$ via $\Sigma = \{0, 1\}$ ($\star$)

(g) L = the set of all valid bracket expressions over $\Sigma = \{(,)\}$

(h) L = the set of all words in which the number of 0s and the number of 1s differ by at most 5, via $\Sigma = \{0, 1\}$

(i) $L = \{w1^{|w|} \mid w \in \Sigma^*\}$ via $\Sigma = \{0, 1\}$

(j) $L = \{0\ 1^{ij} \mid i, j \in \mathbb{N}, i > j \geq 1\}$ via $\Sigma = \{0, 1\}$

(k) $L = \{w \mid w = w^R\}$ via $\Sigma = \{0, 1\}$ where w^R equals w, read backward

(l) L = the set of all valid regular expressions over $\Sigma = \{0, 1, (,), \emptyset, \varepsilon, *, \circ, \cup\}$

Task 6 — Pumping lemma

Let $\Sigma = \{a, b, c\}$ and $L = \{a\, b\, c^{ijk} \mid i, j, k \geq 0$ and $j = k$ if $i = 1\}$.

(a) Show that L satisfies the inference of the pumping lemma.

(b) Using the closure properties of regular languages, show that if L is regular, then $L' = \{ab\, c^{jj} \mid j \geq 0\}$ must also be regular.

(c) Infer that L is not regular using the pumping lemma.

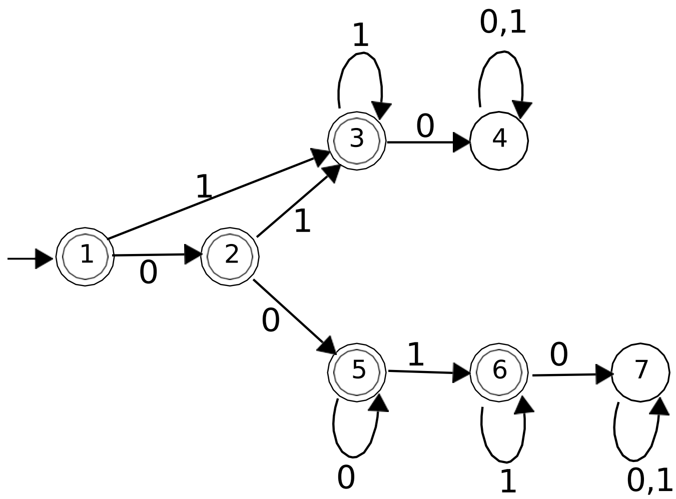
Task 7Myhill-Nerode (MN) Relation

(a) Let $L = L(0\ 1\ 0^{***})$. Give three words that are pairwise equivalent under the MN relation for L, and three words that are pairwise non-equivalent under the MN relation for L.

(b) Let L be the language defined in Task 5(k). Give three words that are pairwise non-equivalent under the MN relation for L.

(c) Determine the equivalence classes of the MN relation for the language $L = \{w \mid$ the fourth last character of w is a 1$\}$ over $\Sigma = \{0, 1\}$.

(d) Minimize the following DEA with the table filling algorithm. Which language is accepted?



Task 8Reset words

Let M be a DEA with state set Q and input alphabet Σ. A word $w \in \Sigma^*$ is called a *reset word for* M if the following holds:
There is a state $h \in Q$ such that the input w in M transfers any state to h.
M is called *resettable* if a reset word exists for M.

(a) Specify a DEA that is not resettable.

(b) Specify a DEA with at least 4 states that is resettable and specify an appropriate reset word.

(c) Show that for each resettable DEA with n states, there is a reset word of length at most n .[3]