Fundamentals of Theoretical Computer Science S 2023

Exercise Sheet III Delivery: Friday, 9.6.2023, 10:00 a.m.

Note: Only the (partial) tasks marked with marked (sub-)tasks have to be handed in. A total of 25 points is to be achieved. The tasks will be discussed/solved in the tutorials.

Task 1 Grammars 1 2 + 2 -t- 1 points

Give context-free grammars for the following languages. In each case, justify the correctness of your grammar.

(a) The set of all palindromes over the alphabet Y (0, 1}.

(b) The language (1'01' | n o) over the alphabet Z = {0, 1}.

(c) The language (n'h' | o o m} over the alphabet L (o, fi). (+)

(d) The language (0- i 2- | *p,* qmo < q or)}over the alphabet Z (0, 1, 2}. (+)

(e) The set of all words containing at least three len over the alphabet Z (0, 1}. (+)

Task 2 Derivatives and syntax trees

Give a derivation chain and syntax tree for each of the following words and grammars:

(a) = 01100110 and your grammar from task 1(a).

(b) = 00111222 and your grammar from task 1(d).

Task 3 Uniqueness

Is your solution for task 1(c) unique? If no, find an unambiguous gram- matic for the same language. Justify the uniqueness.

Task 4 Grammars II (+) Prove 5Points

that the grammar G with the productions

generates exactly the words over Z (n, b} which contains the same number of a's as b's. Use double inclusion and induction for both directions.

Task 5 CNF and the CYK algorithm 2 points

(a) Let G be the grammar from task 4. Give a grammar G' in CNF such that L(G) L(G'). Use the algorithm from the lecture for this. You may make simplifications between the steps of the algorithm, but you must justify them. (+)

(b) Run the CYK algorithm with the word = nnnbahbhb and your gram- matic.

Task 6 Blank word

Let G' be a context-free grammar. Describe a procedure, as simple as possible, to determine whether e C *L(G)*.

Task 7 Closing properties 3Points

(a) Let Vj, L2 be context-free languages. Prove that then +i L2 is also context free.

(b) *Let Li , L2* be context-free languages. Prove that then also *L* o *L2* is context free.

(c) Let £ be a context-free language. Prove that then fi* is also context-free. (+)

(d) Find two context-free languages *L , L2* such that the cut *L L2* is n3t context-free.

(e) Find a context-free language *L* over an alphabet X such that the comple- ment Y* nicfit is context-free.

Task 8 Context-free and non-context-free languages 5 x 2 points

Are the following languages context-free? For each language, either give a context-free grammar, or show context-free using the closure properties, or prove that the language is not context-free.

(a) $L = \{1^{2^n} \mid n \in \mathbb{N}_0\}$ über $\Sigma = \{1\}$.

(b) *L* {1* | *p* is a prime number} over L (1}.

(c) $L = \{0\ 1\ 0^{pqr} \mid p, q, r \in \mathrm{N}_0\ , p \leq q \leq r\}$ via $\Sigma = \{0, 1\}$.

(d) $L = \{0\ 1\ 0^{pqr} \mid p, q, r \in \mathrm{N}_0\ , p \leq q \geq r\}$ u¨ber $\Sigma = \{0, 1\}$. (?)

(e) $L = \{0\ 1\ 0^{mm+nn} \mid m, n \in \mathrm{N}_0\ \}$ u¨ber $\Sigma = \{0, 1\}$.

(f) $L = \{0\ 1\ 0^{pqrp+q+r}\ \ 1 \mid p, q, r \in \mathrm{N}_0\ \}$ u¨ber $\Sigma = \{0, 1\}$.

(g) $L = \{0\ 1\ 0^{pqr} \mid p, q, r \in \mathrm{N}_0\ , q \leq p + r\}$ via $\Sigma = \{0, 1\}$. (?)

(h) $L = \{0\ 1\ 0^{pqr} \mid p, q, r \in \mathrm{N}_0\ , q \geq p + r\}$ via $\Sigma = \{0, 1\}$. (?)

(i) $L = \{\mathrm{w}1^{|w|} \mid w \in \Sigma^{\star}\ \}$ u¨ber $\Sigma = \{0, 1\}$. (?)

(j) $L = \{w \circ w \mid w \in \Sigma^{\star}\ \}$ u¨ber $\Sigma = \{0, 1\}$. (?)