Transition Risk Add-on: a Simulation Approach

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Anno Accademico: 2024/2025

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1 Introduction

The speech by Mark Carney in 2015 at the Lloyd's of London conference is often cited as a pivotal moment in highlighting climate risk as one of the major threats to the global financial system (Carney, 2015). In his address, Carney, the Governor of the Bank of England, introduced the concept of the "tragedy of the horizon", emphasizing that climate change poses significant risks to financial stability over long-term horizons through its impact on the drivers of financial risks. Carney stressed that these risks represent a threat to current financial system because markets often fail to account for the long-term impacts intrinsic to climate risk, which requires an evaluation that incorporates a forward-looking analysis, which often misaligns with the models currently used by most financial institutions.

Since Carney's address, a growing body of academic literature now focuses on strategies for managing climate risk and several international institutions have recognized the importance of integrating gathered evidence into existing financial risk management practices. Among these, the Network for Greening the Financial System (NGFS), an organization founded in 2018 (Network for Greening the Financial System, 2018) by central banks and regulatory authorities to strengthen the global response to climate-related financial risks, is actively working to develop recommendations and best practices to help central banks incorporate climate risks into financial oversight and ensure systemic stability. These guidelines also advocate for sustainable investments and encourage financial institutions to integrate climate evaluation into their portfolio management activities through various tools and measures, according to (Basel Committee on Banking Supervision, 2021a).

To translate climate change into the context of traditional financial risk management, the concept of climate risk drivers has been introduced. The academic literature broadly agrees that climate risk drivers can be categorized into two main types:

- 1. Physical risks: arise directly from climate-related changes in weather and climate systems, which impact both natural and human-made environments. These risks arise as consequences of global warming, driven primarily by the accumulation of CO₂ and other greenhouse gases in the atmosphere (Intergovernmental Panel on Climate Change (IPCC), 2014). The increase in atmospheric greenhouse gases, resulting from industrial activities and fossil fuel emissions, has been shown to intensify extreme weather events and lead to long-term shifts in climate patterns. Although human actions and planning can reduce exposure to certain physical risks, factors such as location, timing, and magnitude of specific events cannot be fully controlled, adding a layer of uncertainty that complicates risk management strategies that can be useful in preventing induced damages (Risky Business Project, 2014).
- 2. Transition risks: this risk arises from the need to reduce CO₂ emissions through coordinated policies, such as a carbon tax or revisions to emission trading schemes, to facilitate the transition to a low-carbon economy and mitigate the physical risks associated with rising global temperatures. This transition would imply significant economic adjustments, potentially leading to abrupt changes in asset values and repricing as market participants adapt to evolving policy expectations. Transition risks are closely linked to regulatory changes, technological advancements, and shifting market preferences, all driven by the need to achieve international climate

goals as those established in (United Nations Framework Convention on Climate Change (UNFCCC), 2015) and (International Energy Agency (IEA), 2021).

Both types of risk can significantly undermine the financial positions of households and firms by affecting their cash flows and asset values, ultimately impacting their credit risk (Monnin, 2018). These two risks are deeply connected, with the difference that transition risk is often defined on a shorter time horizon compared to physical risk, which is driven by long-term changes in temperature. The timing of climate transition depends on the successful implementation of policies aimed at reducing greenhouse gases, which require complex negotiations and agreements between different actors, including governments, businesses, and civil society.

Concerning the economic impacts caused by such risks, it is crucial to emphasize their potential impact on firms' balance sheets, driven by the evolving expectations of financial actors. Specifically, this article focuses on disorderly transition risk, which arises when shifts toward a low-carbon economy occur suddenly or unpredictably. This can be triggered by factors such as delayed regulatory action, abrupt climate policies, or rapid technological advancements. These unanticipated changes can significantly disrupt financial markets as investors and companies struggle to adapt their expectations and asset allocations. This can lead to sharp revaluations of assets, particularly within carbon-intensive sectors, where the risk of stranded assets becomes particularly acute ((Rozenberg et al., 2020). Stranded assets are investments or resources that can prematurely lose economic value due to external factors, often transforming from revenue sources into liabilities. In the context of climate finance, stranded assets are most commonly associated with fossil fuel reserves and related infrastructures, whose value can significantly vanish as the transition to a low-carbon economy accelerates (Battiston and Monasterolo, 2023). Therefore, the implementation of climate policies, advances in low carbon technologies, and shifting market preferences can make assets reliant on carbon-intensive practices, such as coal mines or oil fields, obsolete or unprofitable. The direct impact on affected firms can be significant. Reduced revenue from stranded assets and increasing compliance costs can strain cash flows, making it more challenging for firms to meet debt obligations and increasing their probability of default (PD) (Agliardi and Agliardi, 2021). This weakens firms' financial position, limiting their access to capital and potentially driving up their borrowing costs. Moreover, the loss in value of stranded assets directly diminishes the quality and quantity of available collateral, undermining firms' balance sheets and increasing lender exposure. Consequently, the cumulative effects of stranded assets, reduced cash flows, and poor collateral quality can lead to market instability and potential financial contagion, both between financial actors (Roncoroni et al., 2021) and between countries, as demonstrated by Silvapulle et al. (2016), due to concentrated value adjustments between portfolios. This context represents the potential for systemic risks that regulators must manage.

Verily, from the standpoint of prudential regulation, the Basel Committee on Banking Supervision (BCBS), had recognized the need for a regulatory action to manage this source of systemic risk, and has increasingly focused on the implications of climate-related risks within the banking sector. Regulators had increasingly acknowledged that climate risks, both physical and transition, can significantly impact banks' credit and market risks, according to (Basel Committee on Banking Supervision, 2021b), ultimately affecting their capital adequacy. Recognizing this, many papers have been published in

recent years to assess the extent to which climate risks should be integrated into the Pillar 1 (Minimum Capital Requirements) and Pillar 2 (Supervisory Review) of the Basel III framework. Regulatory modifications might involve adjustments to existing credit, market, and operational risk frameworks, and may even require the establishment of dedicated capital buffers specifically for climate risk exposures, according to (Baranović et al., 2021). Such changes would ensure that banks have sufficient capital to withstand potential losses arising from climate risks, thereby strengthening the resilience of the financial system (Basel Committee on Banking Supervision, 2011).

Drawing inspiration from Gordy (2003), whom introduced the concept of granularity add-on, which is an additional capital charge to account for the concentration risk within a portfolio of (n) credit exposures determined by a Value at Risk measure, we have built upon Battiston et al. (2022) to integrate transition risk into the determination of economic capital. While Gordy's model did not explicitly consider climate-related exposures, which have only recently become a focus of attention, the underlying principle of applying a risk measure to determine the capital buffers needed to cover different risks remains relevant. Consequently, we have adopted the model developed by Battiston et al. (2022), which incorporates a climate shock $(B \to P)$ by CPRS methodology into a closed-form expression that links financial valuation with transition risk to quantify the potential impact on economic capital in the different scenarios $\{B, P_1, ..., P_L, P_{nscen}\}$ defined within the LIMITS database (for Applied Systems Analysis, 2015).

We applied our approach to a portfolio of (n) Zero Coupon Bond to ensure that transition loss, denoted as $\Delta v_i(BP)$, is defined within the interval [-1,1] for each credit exposure, consistent with the framework of Gordy (2003). Recognizing the challenges associated with obtaining precise sectorial information for available issuers and consequently the difficulties in executing CPRS mapping, we take inspiration from Desnos et al. (2023) and we opt for a Monte Carlo simulation approach, in which we randomly model the technology mixes of the (n) issuers across (m) simulations. Our methodology, which will be further explored in section 3, generates an empirical distribution of m transition risk losses for a portfolio of n issuers. This distribution is determined by aggregating individual transition losses $\Delta v_i(BP)$ of single ZCBs. We then applied a Value-at-Risk measure to this empirical distribution to determine the capital buffer required for transition risk. Given the forward-looking nature of transition risk, our research relies on projections from climate models rather than relying on historical trends. Our analysis requires integrating forward-looking projections from the LIMITS database to incorporate the potential impacts of policy adoption and allows the assessment of risks arising from anticipated policy shifts. The results obtained through this approach are particularly valuable for comparing available Integrated Assessment Model (IAM) scenarios and regions. However, the same methodology can also be adapted to incorporate real-world sectoral data by applying a VaR measure to empirical loss distributions generated across a comprehensive range of existing climate models and scenarios.

Assuming the independence between the single systematic risk factor X included in Gordy (2003) and the introduction of a climate policy, we define total economic capital as the sum of the granularity add-on and the transition risk add-on. We have then applied our model to various technology mix and we have reported the results obtained across different grades of greenness and regions. In section 3 we present the economic capital a financial institution should hold to cover its risks under 2 circumstances:

- 1. A scenario without climate policy introduction, where the economic capital coincides with the granularity add-on.
- 2. A scenario in which transition risk is introduced according to an IAM scenario P.

Our results will demonstrate that for most regions the introduction of a climate policy, especially those targeting significant CO₂ emission reductions, will result in a substantial adjustment in required economic capital. We demonstrate that transition risk add-on decreases as the "greenness" of the technology mix increases. Consequently, investing in renewable-based companies can lead to a reduction in economic capital requirements.

2 Literature Review

The concept of climate finance emerged in the early 1990s, gaining prominence with key international policy milestones. The 1992 United Nations Framework Convention on Climate Change (UNFCCC), adopted at the Rio Earth Summit, established an initial framework for international cooperation on climate change to stabilize greenhouse gas concentration, including the provision of financial resources for mitigation and adaptation efforts, particularly in developing countries, as per (Nations, 1992). The Kyoto Protocol in 1997 further advanced this cause by establishing binding emission reduction targets for industrialized countries and introducing market mechanisms, such as the Clean Development Mechanism (CDM). The CDM, which enabled carbon credit trading, can be considered a pioneering effort in the development of concrete de-carbonization measures (Brack et al., 1999). These developments represent some of the first practical applications of climate finance, bringing the issue of climate-related risks to the forefront of discussions on global financial stability and market operations. The establishment of the Green Climate Fund (GCF) in 2010 (UNFCCC, 2010) further emphasized the critical need for a systematic approach to climate finance in supporting developing countries' transition to sustainable energy and their adaptation to climate impacts.

The academic field of climate finance began to emerge in the 2000s, as economists explored the links between climate risks and financial stability, and investigated strategies for integrating climate considerations into financial markets. A seminal work in this field, (Stern, 2006) provided one of the earliest comprehensive economic assessments of climate change impacts, concluding that the long-term costs of inaction would significantly overweight the costs of proactive mitigation. This influential report drew significant attention to the economic risks posed by climate change, stimulating further research on the topic. Since then, numerous researchers have developed various models to measure transition risk. Notably, (Monnin, 2018) developed a framework for managing the evolution of climate risk into credit risk. This framework incorporates the concept of stranded assets and utilizes the Merton model Merton (1974) to assess the impact of a climate scenario on a company's probability of default (PD), suggesting that climate shocks can lead to an increase in issuer's PD. Numerous models have been developed to assess the potential impact of transition risk. (Agliardi and Agliardi, 2021) enhanced a structural bond pricing model that incorporates both uncertainty about corporate earnings and climate-related risks. This model is based on a double Poisson diffusion process to capture potential downward jump in firm value. In 2018, (Monasterolo and Raberto, 2017) realized the first macro-economic framework that integrated various financial variables and climate risk concepts. This pioneering work laid the foundation for subsequent research aimed at

refining and expanding similar models. (Rozenberg et al., 2020) conducted a comparative analysis of the impact of different climate mitigation policies, including mandates, feebates and carbon pricing, utilizing a Ramsey model with distinct capital stocks for clean and polluting technologies. Their work demonstrated that each of these instruments can effectively reduce CO₂ emissions. (Reboredo and Otero, 2021) found that mutual fund investors consider climate-related transition risk to be an undesirable fund characteristic and tend to allocate more money to funds with lower climate-related transition risk, as measured by a basic carbon metric. In the same year (Beirne et al., 2021) conducted an empirical study examining the link between climate risk and sovereign borrowing costs for a range of emerging economies. Their findings revealed a direct and proportional relationship between these two variables, highlighting a significant regional disparity in climate risk exposure. We will compare these regional discrepancies with the results obtained through our model. In 2022 (Fried et al., 2022) developed a dynamic, general equilibrium model to quantify the macroeconomic impacts of a climate policy through the hypothetical adoption of a carbon tax in the US. The findings of this study are significant not only for demonstrating the effectiveness of a carbon tax, but also for being the first to utilize a general equilibrium model with a Leontief production that incorporates both "green" and "fossil" input factors. (Blasberg et al., 2022) realized a model that quantifies transition risk across different sectors and markets by a Wasserstein distance. Their analysis demonstrated that realized measure is a key determinant of Credit Default Swap spreads, highlighting the significant impact of transition risk on derivative markets. (Fried et al., 2022) provide evidence that many firms anticipate the future introduction of climate policies, even with uncertain timing. This anticipation can influence current investment decisions, potentially leading to capital misallocation as investors shift towards renewable sectors. Such capital flows can create significant risks for financial stability. To effectively address these challenges, regulators can integrate climate stress testing practices with explained climate policy measures and approaches, as highlighted by Havne et al. (2020). The results obtained in these researches can be integrated into a simulation approach similar to the one described in our model to align with the evolving regulatory framework.

3 Model set up

This section outlines the theoretical framework for determining the total economic capital that a financial institution should hold to cover the risks arising from un-diversified systematic risk and transition risk. We build upon on the framework developed by Gordy (2003) to calculate the granularity add-on and incorporate climate-related risks by introducing a potential transition effect according to a set of pre-defined IAM scenarios.

In his model, Gordy (2003) considered losses U_i as the primary outcome variable, rather than default events, and generalized the standard assumption of conditional independence of defaults to conditional independence of U_i . Furthermore, exposure to issuer i, represented by A_i , is assumed to be known and non-stochastic. For a portfolio composed of (n) issuers the author defined the variable "Portfolio Loss Ratio" as

$$L \equiv \frac{\sum_{i=1}^{n} U_i A_i}{\sum_{i=1}^{n} A_i} \tag{1}$$

To introduce transition risk, we consider an additional portfolio loss ratio generated by the

introduction of a climate policy, where U_i is replaced by $\Delta v_j(BP)$, the value adjustment caused by a climate transition Battiston and Monasterolo (2023), and the exposure A_i is represented by the face value of a ZCB, which is equal to 1. We define "Transition Risk Portfolio Loss Ratio" for a portfolio of n issuers as

$$TRL \equiv \frac{\sum_{j=1}^{n} 1 \cdot -\Delta v_j(P)}{\sum_{j=1}^{n} 1}$$

$$= \frac{\sum_{j=1}^{n} -\Delta v_j(P)}{n}, \quad \text{since } A_j = 1$$
(2)

The choice to consider - $\Delta v(BP)$ is inferred from loss definition in Hull (2006), where worst cases losses are included in the right tail of distribution, which is consistent with the approach in Gordy (2003).

We begin by identifying the assumptions required to compute the granularity add-on and proceed to develop a climate measure that aligns with these previsions. This analysis considers the actuarial framework implemented by Gordy (2003), where credit loss arises solely in the event of obligor default.

A fundamental concept in rating-based capital rules introduced by Gordy (2003) is portfolio invariance, which ensures that the capital charge for a given instrument is solely determined by its individual characteristics and is independent of the overall portfolio composition. Portfolio invariance is a crucial property for systematic factor models, which rely on a single systematic risk factor X to capture the dependence structure between issuers and define a portfolio-invariant capital charge. However, real-world portfolios typically consist of a finite number of issuers with uneven exposures, deviating from the assumption of perfect granularity. This leads to a discrepancy, as capital charges calibrated under the assumption of perfect granularity (where idiosyncratic risk is fully diversified away) tend to underestimate actual capital requirements. This discrepancy, known as the granularity add-on, is inversely proportional to (n), the size of issuers in the portfolio.

$$\alpha_q(L_n) = \mu(\alpha_q(X)) + O(n^{-1}) \tag{3}$$

with

$$\mu(\alpha_q(X)) = \mathbb{E}[LGD] \cdot \overline{p} \cdot (1 + w(\alpha_q(X) - 1)) \tag{4}$$

where $\mu(\alpha_q(X))$ represents the VaR for a finite homogeneous portfolio, $O(n^{-1})$ the granularity add-on and $\alpha_q(L_n)$ the capital charge for a perfect granular portfolio, therefore being an asymptotic approximation.

Wilde and Martin (2002) have demonstrated that

$$O(n^{-1}) = \frac{\beta}{n}.$$

with

$$\beta = \frac{1}{2\lambda} \left(\lambda^2 + \eta^2 \right) \left(\frac{1}{\sigma^2} \left(1 + \frac{\sigma^2 - 1}{\alpha_q(X)} \right) \left(\alpha_q(X) + \frac{1 - w}{w} \right) - 1 \right) \tag{5}$$

as per Wilde (2001), where:

- $\alpha_q(X)$: qth percentile of systemic risk factor $X \sim \text{Gamma}(0.25, 4)$.
- σ^2 : variance of systematic risk X, assumed to be equal to 4.
- λ and η^2 : mean and variance of LGD_j ~ Gamma(4, 0.125).
- 'w': factor loading, calibrated for each rating grade to match the within-grade default correlation.
- \bar{p} : unconditional default probabilities, corresponding to stylized values depending on the issuers' rating classes.

Portfolio invariance within a Value-at-Risk framework relies on two key conditions: (1) the portfolio must be asymptotically fine-grained, where no single exposure dominates overall portfolio risk, (2) the model must depend on at most one systematic risk factor. These assumptions simplify the joint distribution of credit losses across instruments, focusing on sectoral and macroeconomic factors as drivers of issuers' correlation structure. Recognizing that real-world portfolios deviate from these ideal conditions, Gordy (2003) proposes a practical methodology to account for residual idiosyncratic risk and ensure adequate capital reserves. The great advantage of this approach is that it considers the q-th quantile of systematic risk factor to directly compute the VaR for a finite homogeneous portfolio.

We now examine how the aforementioned assumptions underlying Gordy (2003) model should be adapted to develop a coherent climate framework.

(1st assumption): losses $\{U_i\}$ are bounded in the interval [-1,1] and, conditional on X, are mutually independent.

For TRL, if we choose U_i to be represented by Δv , that for a Zero-Coupon Bond (ZCB) is surely included in the same interval [-1,1], 1st assumption is respected if we assume that, conditional on the climate shock $u_j(P)$, the losses Δv are independent. Furthermore, we can consider the single factor X to coincide with the idiosyncratic shock to the asset values $\eta_j(P)$ introduced by Battiston and Monasterolo (2021) and note that the independence between idiosyncratic shocks and climate shocks present in Battiston and Monasterolo (2020) is a key element for the respect of 1st assumption if no additional element is introduced in the model. This implies that idiosyncratic shocks of different issuers are related through a common dependence on (X), but this aspect does not pose a problem for the treatment of the η 's distribution, particularly when incorporating regional dependence in the derivation of climate-related PD (Roncoroni et al., 2021).

According to Gordy (2003), to guarantee that idiosyncratic risk vanishes as more assets are added to the portfolio, the sequence of exposure sizes must neither blow up nor shrink to zero too quickly. This is translated into

(2nd assumption):

- (a) $\sum_{i=1}^{n} A_i \to \infty$,
- (b) there exists a $\zeta > 0$ such that $\frac{A_n}{\sum_{i=1}^n A_i} = O(n^{-(1/2+\zeta)})$.

The restrictions in (A-2) guarantee that the share of the largest single exposure in total portfolio exposure vanishes to zero as the number of exposures in the portfolio increases. Accordingly to Gordy (2003) both conditions are satisfied if all A_j are bounded from below by a positive minimum size and from above by a finite maximum size. Given the fact that in transition risk portfolio loss ratio all exposures are assumed equal to 1, both points (a) and (b) are surely respected.

Thus, under quite general conditions, also the conditional distribution of the transition portfolio loss ratio TRL_n surely degenerates to its conditional expectation as $n \to \infty$ and we have a portfolio-invariance capital charge linked to transition risk. More formally, we can show that in our model:

If (1st assumption) and (2nd assumption) hold, then, conditional on X = x, U = u and on independence between idiosyncratic and climate shocks, both

$$TRL_n - \mathbb{E}[TRL_n|x, u] \to 0$$
, almost surely. $L_n - \mathbb{E}[L_n|x] \to 0$, almost surely.

(Gordy, 2003), these conditions are sufficient to affirm that for any $\varepsilon > 0$,

$$F_n(\alpha_q \mathbb{E}[L_n|X] + \varepsilon) \to [q,1],$$

$$F_n(\alpha_q \mathbb{E}[L_n|X] - \varepsilon) \to [0,q].$$

which ensures

$$\alpha_q(L_n) - \alpha_q(\mathbb{E}[L_n|X]) \to 0.$$

The interpretation of this result is that if capital is strictly greater than the qth quantile of $\mathbb{E}[L_n|X]$, then it is guaranteed, in the limit, to cover (or, at least, to come arbitrarily close to covering) q or more of the distribution of both loss by Gordy (2003). Similarly, if capital is strictly less than the qth quantile of $\mathbb{E}[L_n|X]$, then it is guaranteed, in the limit, to fail to cover the qth quantile of the distribution of loss (or, at least, to come arbitrarily close to).

Given the additional assumption imposed in our model and the respect of previous conditions, we can affirm that the same essential outcome is valid also for TRL_n :

$$\alpha_q(TRL_n) - \alpha_q(\mathbb{E}[TRL_n|X]) \to 0.$$

Additional assumptions on systemic risk factor (X) had been introduced by Gordy (2003) to derive the expression used to calculate the granularity add-on. Importantly, none of these assumptions on X are affected by the introduction of a transition policy, nor do they influence the climate component of our model, given the assumption of independence between climate shocks and systemic/idiosyncratic shock.

To calculate the transition add-on we apply a VaR measure into a Monte Carlo simulation, in which we generate an empirical distribution of TRL_n by randomly assigning a technology mix (e) to each issuer (j) across a large number (m) of simulations. Specifically, we developed a code that by parallel computing generates an empirical distribution of portfolio climate losses for a given fossil/ renewable mix. This is achieved by aggregating the climate losses $\Delta v_{j,m}(BP)$ of each exposure, where $\Delta v_{j,m}(BP)$ is determined by the impact of a climate shock $u_{j,m}(BP)$. This shock is associated with each issuer (j) using the CPRS methodology Battiston (2022). Finally, we determine the transition risk capital requirement by applying $VaR(\alpha)$ to the empirical distribution of TRL_m .

The convergence of the empirical q-th quantile to the population quantile is ensured by the application of the Law of Large Numbers (LLN), which establishes the convergence of expectiles for i.i.d. data with a large number of simulations Collin (2022).

$$\alpha_q(\widehat{TRL}_m) \xrightarrow[m \to \infty]{} \alpha_q(TRL)$$

3.1 Climate valuation

Accurately pricing forward-looking transition risk within investors' portfolios is crucial for enhancing correct risk management practices and enabling financial regulators to maintain financial stability, as highlighted by map (2021). This is particularly important because traditional risk management methods could significantly underestimate climate risks due to their long-term horizons and non-linear characteristics Ackerman (2017). This analysis focuses on the effects of climate policies needed to realize the transition to a low-carbon economy when these are implemented in a disorderly manner, which occurs when the timing and magnitude of climate policies in terms of reallocation of capital into low carbon activities Weitzman (2009) is sudden and not fully anticipated by market players and investors (Roncoroni et al., 2021), which cannot adequately evaluate this risk in their investment strategies. Arising uncertainty could result in unexpected losses and in the formation of stranded assets, which can have a significant impact on lending, investment and trading portfolios, ultimately contributing to capital depletion, as outlined in Elderson and Heemskerk (2020).

Transition risk can significantly impact companies' performances, with a magnitude that is linked to the pre-shock technology mix of the issuers Battiston and Monasterolo (2023). Particularly, carbon-intensive firms would be vulnerable to the introduction of climate policies, which could lead to a decline in their performance and the formation of carbon stranded assets. Indeed, Battiston et al. (2017) found that contracts at risk of stranding represent a significant portion of actual portfolios, reaching 43% of equity holdings within investments and pension funds in US and EU. These risks arise because, in the absence of mitigation measures, climate risk can trigger tipping points, leading to potentially irreversible changes within the economic system and cascading effects capable of triggering a serious financial crisis, as highlighted by (Battiston and Monasterolo, 2020).

We consider a set of risk-adverse investors operating within the regions defined in LIMITS database, under classical conditions of incomplete information, incomplete markets and deep uncertainty. For each region (r) included in our analysis, we consider a set of issuers {1, ..., j, ..., n} that are represented by a portfolio of fossil and renewable cash flows. Transition risks can affect cash flows through various channels, including increased research and development expenditures for new and alternative technologies, higher costs associated with the adoption of new practices and processes, declining demand for carbon-intensive products and services, rising production costs due to changes in input prices (e.g. for energy and water) and stricter output requirements (e.g. for carbon emissions and waste treatment), as outlined by (Monnin, 2018). If not properly assessed, these changes can lead to lower-than-expected cash flows, potentially impairing a firm's ability to service and repay its debt, depending on its financial condition and its technology

mixes. We model the transmission channel that links the transition shocks with the probability of default (PD) for each issuer (j) using the CLIMAFIN tool (Battiston and Monasterolo, 2020). Transition risk impacts issuers' economic equilibrium through three primary channels:

- 1. Changes in market share: a shift alters the macroeconomic trajectory of different sectors.
- 2. Impact on issuer cash flows: a decline in market share within energy market directly impact issuer sales, assuming market conditions remain constant, thus affecting existing cash flows (Battiston et al., 2019b).
- 3. Profitability changes: the effect on profitability varies significantly depending on the issuers' pre-shock energy mix. Fossil fuel-intensive companies will experience a decline in their profits due to reduced sales and potential cost increases, while renewable energy companies may benefit from a more favorable market environment.

These three channels ultimately impact the Gross Value Added (GVA) of affected sectors, with the direction of this impact determined by specific changes in profitability. For any issuer j, the GVA within a region (r) is assumed to be equal to the sum of the regional GVA of the sectors (s) in which it operates.

$$GVA_{j,r} = \sum_{s} GVA_{j,s,r}$$
 (6)

We model our climate valuation approach based on the work of Battiston and Monasterolo (2020). From an accounting standpoint, we consider the issuers' balance sheet to consist solely of assets and liabilities. Liabilities are assumed to be constant and equal to $L_j(T)$, while assets are modeled as follows:

$$A_j(T) = A_j(0) (1 + \eta_j(T) + \xi_j(BP))$$
 (7)

where

- $A_i(0)$: asset's value at T=0.
- $\eta_j(T) \in \mathbb{R}$: idiosyncratic shock reflecting variations in asset value caused by shocks to productivity, management, and non-systematic sources (Roncoroni et al., 2021), with $\phi(\eta_1, \ldots, \eta_j, \ldots, \eta_n)$ denoting the joint probability distribution (accounting for potentially correlated shocks).
- $\xi_j(BP)$: climate policy shock.

$$\xi_i(BP) = \chi_i u_i(BP) > -1 \text{ by construction.}$$
 (8)

The elasticity of assets with respect to revenues is assumed to be one. Typical empirical values for χ_j usually range from 0.2 to 0.6 (Battiston et al., 2019b). Nevertheless, we utilized a value of 1, as we deemed maximizing the impact of transition risk on companies' revenues the optimal choice, given our objective of providing an estimation of portfolio losses under a worst-case scenario. Consequently, the impact of a climate policy on asset A_j is considered equal to $u_j(BP)$.

 $u_j(S, BP, M)$ represent climate transition shocks computed according to a climate model M for a transition scenario $B \to P$, that is defined as

$$u_{j}^{\text{GVA}}(BP) = \text{GVA}_{j}(P) - \text{GVA}_{j}(B)$$

$$= \sum_{s} \frac{\text{GVA}_{j,s}(P) - \text{GVA}_{j,s}(B)}{\text{GVA}_{j}(B)} \cdot \frac{\text{GVA}_{j,s}(B)}{\text{GVA}_{j}(B)}$$

$$u_{j}^{\text{GVA}}(BP) = \sum_{s} \left(u_{j,s}^{\text{GVA}}(BP) \cdot w_{j,s}^{\text{GVA}}(B) \right)$$
(9)

Climate shocks are computed as the longitudinal variations in sector outputs across different IAM trajectories with regards to the LIMITSBaseline scenario B. In our model we assume a proportional relationship between the % change in the GVA of issuer j and the % change in the output of the corresponding IAM variables (which are provided by the climate economic models included in LIMITS database), which can be identified by CPRS mapping according to Battiston (2022). Climate shocks typically exhibit values between -1 and 0 for fossil fuel variables. This reflects the expected decline in GVA for related sectors following the implementation of a climate policy P. In contrast, climate shocks associated with renewable energy sectors are often positive. Equation (9) posits that sector weights w(B) are invariant across all time periods. This assumption effectively prevents issuers from modifying their pre-shock technology mix in reaction to a climate shock (B \rightarrow P).

$$\Rightarrow A_j(T) = A_j(0) \left(1 + \eta_j(T) + u_j(BP) \right) \tag{10}$$

Following several approaches in the modern literature, we introduce an adjunctive key assumption (Battiston and Monasterolo, 2023).

(3rd assumption): $\eta_j(T)$ and $u_j(BP)$ are independent.

This assumption is justified, given that frequent, minor productivity shocks can arise across time and among companies independently of climate policy implementation (Roncoroni et al., 2021). For example, two fossil-fuel operating companies may exhibit distinct idiosyncratic shocks, largely attributable to differences in management quality or productivity discrepancies, as discussed in (Battiston and Monasterolo, 2021).

Following Merton model Merton (1974), we utilize defined values for assets and liabilities to construct a threshold model for default prediction:

$$j$$
's default $\iff \eta_j(T) \le \theta_j(BP) = \frac{L_j(T)}{A_j(0)} - 1 - u_j(T, BP)$ (11)

where:

- $\theta_j(BP)$: default threshold when transition risk is introduced by a transition shock $B \to P$.
- $u_j(T, BP)$: climate shocks defined on the interval $\{-1, +\infty\}$ according to (9). These shocks can be correlated among issuers.

Given the distribution support of climate shocks, their impact is to shift (left/right) the distribution of the idiosyncratic shock and consequently (up/down) the default threshold (and the PD). The direction of these shifts is contingent upon the relative weight of low-carbon sectors within the current revenue composition (Battiston and Monasterolo, 2020).

We can derive the PD $q_i(BP)$ of an issuer j by (7), as the probability that $\eta_i(T) \leq \theta_i(BP)$:

$$q_j(BP) = P(\eta_j < \theta_j(BP)) = \int_{\eta_{\text{inf}}}^{\theta_j(BP)} \phi(\eta_j) \, d\eta_j$$
 (12)

where

- $\phi(\eta_i)$: probability distribution of the idiosyncratic shocks η_i .
- η_{inf} : lower bound of the $\phi(\eta_i)$ distribution support.

Given the assumption of independence between idiosyncratic and climate shocks, we can derive the PD adjustment under the transition BP (Battiston and Monasterolo, 2020) as:

$$\Delta q_j(BP) = q_j(P) - q_j(B) = \int_{\theta_j(B)}^{\theta_j(P)} \phi(\eta_j) \, d\eta_j \tag{13}$$

where
$$\theta_i(P) = \theta_i(B) - \xi_i(P) = \theta_i(B) - u_i(P)$$
.

Although this methodology is theoretically solvable for any distribution of stochastic shocks, we leverage a key results from Bolt and Tieman (2004) to simplify the analysis through the use of a linear model.

(4th assumption): $\phi(\eta_j) \sim U(a_j, b_j) \ \forall j \in \mathcal{J}$, where \mathcal{J} denotes the set of all issuers.

The assumption that idiosyncratic shocks on asset values are drawn from the same uniform distribution across firms is reasonable since it is a Beta distribution, being bounded from both extremes, and does not excessively underestimate the distribution of tail events, as per Roncoroni et al. (2021). Moreover, this assumption facilitates the solution of our evaluation problem through a linear model. Indeed, we can demonstrate how the assumption of a uniform distribution for $\phi(\eta_i)$ results in a linear expression for $q_i(P)$.

Probability density function for a uniform distribution is given by:

$$\phi_j(x) = \begin{cases} \frac{1}{b_j - a_j}, & \text{if } x \in [a_j, b_j], \\ 0, & \text{otherwise.} \end{cases}$$

If we insert PDF $f(x) = \frac{1}{b_j - a_j}$ for $x \in [a_j, b_j]$ in (11):

$$\Delta q_j(BP) = \int_{\theta_j(B)}^{\theta_j(P)} \frac{1}{b_j - a_j} dx$$

This integral can be further reduced to the following linear combination of climate shocks:

$$\Delta q_{j}(BP) = \int_{\theta_{B}}^{\theta_{P}} \frac{1}{b_{j} - a_{j}} dx = \frac{\theta_{P} - \theta_{B}}{b_{j} - a_{j}}$$

$$= \frac{\theta_{B} - \xi_{j} - \theta_{B}}{b_{j} - a_{j}} \quad \text{(substituting } \theta_{P} = \theta_{B} - \xi_{j}\text{)}$$

$$= \frac{-\xi_{j}}{b_{j} - a_{j}} \quad \text{(simplifying terms)}$$

$$= \frac{-u_{j}(P)}{b_{j} - a_{j}} \quad \text{(since } \xi_{j} = u_{j}(P), \text{ assuming elasticity} = 1)$$

$$(14)$$

From (12) we can observe that the denominator (b - a) is always positive, as b > a by the definition of a uniform distribution. Therefore, the sign of the numerator depends on the value of climate shocks:

- $\Delta q(P)$ increases with the magnitude of climate shocks $|u^{\text{GVA}}(P)|$ if $u^{\text{GVA}}(P) < 0$, which are usually observed for fossil-based companies.
- $\Delta q(P)$ decreases for climate shock values $u^{\text{GVA}}(P) > 0$, that are typically observed for renewable-based companies.

These results are in accordance with expectations regarding the impact of transition risk on the probability of default.

The key result to introduce climate risk into portfolio evaluation have been developed in Battiston et al. (2017), Battiston and Monasterolo (2020). The authors have linked $\Delta q_j(BP)$ to the adjustment in the discounted expected value of the issuer's bond resulting from the transition $B \to P$ Battiston and Monasterolo (2021):

The value at T of a ZCB emitted by issuer j will be given by:

$$v_j(T) = \begin{cases} R_j = (1 - \text{LGD}_j), & \text{if } j \text{ defaults (with prob. } q_j), \\ 1, & \text{else (with prob. } 1 - q_j). \end{cases}$$

where:

- R_j : % of notional recovered upon default, equivalent to 1 LGD_j . Given that $v_j(T)$ is an element of $\{0,1\}$, R_j is defined within the same interval.
- LGD_j: % loss in the event of default. In this model, we assume this variable to be sampled from a Gamma distribution, in accordance with Gordy (2003).
- q_i : probability of default.

The expected value of a ZCB payoff is given by:

$$\mathbb{E}[v_i] = (1 - q_i) + q_i R_i = 1 - q_i (1 - R_i) = 1 - q_i \text{LGD}_i$$
(15)

Current bond price v_i^* is derived by discounting $\mathbb{E}[v_i]$:

$$v_j^* = e^{-y_f T} \mathbb{E}[v_j] = e^{-y_f T} (1 - q_j LGD_j)$$
 (16)

where y_f : risk-free rate.

We can finally define the value adjustment caused by a transition (BP):

$$\Delta v_j(BP) = v_j^*(q_j(BP)) - v_j^*(q_j(B)) = -e^{-y_f T} LGD_j \Delta q_j(BP)$$
(17)

This result can be demonstrated starting by (16):

$$\Delta v_j(BP) = v_j^*(q_j(BP)) - v_j^*(q_j(B))$$

$$= e^{-y_f T} (1 - q_j(BP) LGD_j) - e^{-y_f T} (1 - q_j(B) LGD_j)$$

$$= e^{-y_f T} [(1 - q_j(BP) LGD_j) - (1 - q_j(B) LGD_j)]$$

$$= e^{-y_f T} [-q_j(BP) LGD_j + q_j(B) LGD_j]$$

$$= e^{-y_f T} LGD_j [-(q_j(BP) - q_j(B))]$$

$$= -e^{-y_f T} LGD_j \Delta q_j(BP).$$

Given the 4th assumption we can replace $\Delta q_i(BP)$ with (12):

$$\Delta v_j(BP) = e^{-y_f T} LGD_j \frac{u_j(BP)}{b_j - a_j}$$
(18)

Given that both the discounting term e^{-y_fT} and LGD_j are invariably positive definite, an inverse relationship between $v_j(BP)$ and $\Delta q_j(BP)$ is observed. Consequently, the dependence on climate shock values demonstrated a contrary behavior to the earlier observations for $\Delta q_j(P)$:

- $\Delta v_j^*(BP) < 0$ and increases with the magnitude of the climate shock $|u_j(BP)|$ if $u_j(BP) < 0$;
- $\Delta v_j^*(BP) > 0$ and increases with the magnitude of the policy shock if $u_j(BP) > 0$, with the constraint $v_j^* \leq 1$.

3.2 Total add-on determination

Following the establishment of the assumptions underlying our model and the implementation of the methodology for pricing climate transition risk, we can now define the capital requirements necessary to cover both undiversified systemic risk and transition risk.

In light of the independence assumption between the systemic risk factor (X) and the introduction of a climate policy (P), we can proceed with the application of equation (3)

to determine the granularity add-on. Subsequently, a transition risk term is added, which will be non-zero solely in the event of a specific climate policy (P) being implemented. Based on this approach, the total economic capital is expressed as:

$$EC_{\alpha}(n, e) = \alpha_{q}(L_{n}) + TR_{\text{add-on}(\alpha)}(e)$$
(19)

where

- $EC_{\alpha}(n,e)$ represents the economic capital required for a financial institution to mitigate un-diversified idiosyncratic risk and transition risk. This variable aligns with the granularity add-on under the condition of no de-carbonization measures being implemented within our model.
- TR_{add-on}(e) is contingent upon the energy mix (e) of the portfolio, which is simulated through the random assignment of weights to various energy sources (and consequently, to distinct CPRS).

To find TRadd-on, we utilize $TRL_m(n)$ definition in equation (2) and we classify the IAM energy variables into three main categories:

- 1. Primary Energy Fossil
- 2. Secondary energy Electricity Fossil
- 3. Secondary energy Electricity Renewable

Consequently, climate shocks will be represented for any given issuer (j) as:

$$u_{j}^{\text{GVA}}(BP) = u_{j,\text{PrFos}}^{\text{GVA}}(BP)w_{j,\text{PrFos}}^{\text{GVA}}(B) + u_{j,\text{ElFos}}^{\text{GVA}}(BP)w_{j,\text{ElFos}}^{\text{GVA}}(B) + u_{j,\text{ElRen}}^{\text{GVA}}(BP)w_{j,\text{ElRen}}^{\text{GVA}}(BP)$$

We simulate the association of energy sources (e) and their respective weights (w) for each issuer (j) within our portfolio. This procedure is iterated across each simulation (m), yielding a distribution of climate shocks and, subsequently, a distribution of associated portfolio losses. To quantify the discrepancy between investments in fossil-fuel-based and renewable-based issuers, we employ VaR on the empirical distribution of portfolio losses arising from varying degrees of portfolio greenness. Portfolio greenness is quantified through the values assigned to the weights corresponding to the energy source categories $w_{i,\mathrm{ElFes}}^{\mathrm{GVA}}(B), w_{i,\mathrm{PrFos}}^{\mathrm{GVA}}(B)$ and $w_{i,\mathrm{ElRen}}^{\mathrm{GVA}}(B)$.

Specifically, we initially compute $\alpha_q(TRL_m)$ for a distribution of portfolios built with $w_{j,\text{ElRen}}^{\text{GVA}}(B)=1$. This represents the climate capital requirement associated with a 100% green portfolio. Subsequently, we determine the potential transition risk add-on for portfolios exhibiting varying degrees of greenness (reflected by differing values assigned to fossil weights) by comparing their $\alpha_q(TRL_m)$ to that of the 100% green portfolio.

Through this approach, we derive:

$$TR_{\text{add-on}(\alpha)}(e) = \alpha_g(TRL_m(e)) - \alpha_g(TRL_m(100\%\text{ElRen}))$$
(20)

If we integrate (18) in equation (20) we derive:

$$\operatorname{TR}_{\operatorname{add-on}(\alpha)}(e) = \alpha_{q} \left(-\frac{e^{-y_{f}T}}{n} \sum_{j=1}^{n} \frac{u_{j,m}(e) \operatorname{LGD}_{j,m}}{b_{j} - a_{j}} \right) - \alpha_{q} \left(-\frac{e^{-y_{f}T}}{n} \sum_{j=1}^{n} \frac{u_{j,m}(\operatorname{ElRen}) \operatorname{LGD}_{j,m}}{b_{j} - a_{j}} \right)$$

$$= -\frac{e^{-y_{f}T}}{n} \left[\alpha_{q} \left(\sum_{j=1}^{n} \frac{u_{j,m}(e) \operatorname{LGD}_{j,m}}{b_{j} - a_{j}} \right) - \alpha_{q} \left(\sum_{j=1}^{n} \frac{u_{j,m}(\operatorname{ElRen}) \operatorname{LGD}_{j,m}}{b_{j} - a_{j}} \right) \right]$$

$$(21)$$

where

- b_j and a_j : bounds of the uniform distribution for the idiosyncratic shocks, determined by the regional provenance of the economic activity. We set a_j =-1 and $b_j = \max_R u_{R,P}(P)$ to ensure that both $\Delta q_j(BP)$ and $\Delta v_j(BP)$ satisfy the fundamental conditions of $q_j(BP)$ and $v_j(BP)$ (which are constrained within the interval of 0 to 1).
- LGD_{j,m} is generated from the Gamma distribution specified in Gordy (2003), with parameters mean $\lambda = 0.5$ and standard deviation $\eta = 0.25$. As emphasized in (Gordy, 2003), this specification permits $LGD_{j,m} > 1$; however, provided η remains sufficiently small, the aggregate losses within the portfolio will exhibit stable behavior, rendering the concern negligible.

Based on expression (21), it is evident that the values of A_j and L_j do not impact the results. Therefore, our analysis does not necessitate granular accounting information of individual issuers. The sole data relevant for our analysis pertains to the Climate Policy Relevant Sectors of the investments, specifically the energy sources corresponding to their revenues. We are now in a position to present the definitive formula employed in our code to calculate the total economic capital:

$$EC_{\alpha}(n, e) = \mathbb{E}[LGD] \cdot \overline{p} \cdot (1 + w(\alpha_{q}(X) - 1)) +$$

$$+ \frac{1}{n} \cdot \frac{1}{2\lambda} \left(\lambda^{2} + \eta^{2}\right) \left(\frac{1}{\sigma^{2}} \left(1 + \frac{\sigma^{2} - 1}{\alpha_{q}(X)}\right) \left(\alpha_{q}(X) + \frac{1 - w}{w}\right) - 1\right) +$$

$$+ \frac{e^{-y_{f}T}}{n} \left[\alpha_{q} \left(\sum_{j=1}^{n} \frac{u_{j,m}(e)LGD_{j,m}}{b_{j} - a_{j}}\right) - \alpha_{q} \left(\sum_{j=1}^{n} \frac{u_{j,m}(ElRen)LGD_{j,m}}{b_{j} - a_{j}}\right)\right]$$
(22)

In the absence of a climate transition, this formula reduces to equation (3), where adjunctive economic capital corresponds to the capital charge for undiversified idiosyncratic risk. This is because climate shocks $u_j(BP)$ are equal to 0 in this context, and the third term in the equation vanishes. Consequently, if a climate transition occurs, it will have an impact on economic capital, that we expect to worsen proportionally with the portfolio's exposure to fossil related investments.

In the following section, we compare the results obtained from our simulation across different regions, models and scenarios included in the LIMITS database.

4 Results

The objective of this section is to present the results obtained by applying our simulation approach to expression (22).

With regards to the terms elaborated by Gordy (2003), our simulations utilize the same distributions and stylized values for key parameters. Specifically, the Loss Given Default $LGD \sim \text{Gamma}(0.5, 0.25)$ and the systemic risk factor $X \sim \text{Gamma}(0.25, 4)$. Factor loadings (w) and unconditional default probabilities are also assigned the same stylized values as in Gordy (2003).

We first compute the granularity add-on based on one million simulations, assuming the absence of any climatic transition scenario. In this context, the transition risk add-on is zero, as the variable $u_j(BP)$ has a numerator equal to zero. Consequently, economic capital is equal to $\alpha_q(\text{Ln})$, which depends solely on the credit rating of the companies included in the portfolio.

The related results, presented in percentage points in table 1, demonstrate that the granularity add-on increases proportionally with the credit rating of the considered issuer and decreases with the number of exposures comprised in the portfolio.

Rating				VaR				Granularity Add-on
Hatting	100	200	500	1000	2000	5000	∞	Granularity Add-on
A	1.228	0.797	0.538	0.452	0.408	0.383	0.364	0.864
BBB	1.907	1.466	1.201	1.113	1.068	1.042	1.020	0.887
BB	5.713	5.249	4.971	4.878	4.831	4.804	4.764	0.949
В	18.465	17.964	17.664	17.564	17.514	17.484	17.385	1.080
CCC	38.386	37.838	37.509	37.399	37.344	37.311	37.117	1.269

Table 1: Granularity Add-on for n = 100 based on Gordy (2003) framework

We decided to present the granularity add-on for a portfolio composed of 100 issuers in the last column of the table. This choice reflects the number of ZCBs included in our portfolio during the climate simulations, where we substitute 100 for (n) in equation (21).

We then introduce transition effects based on existing NGFS scenarios and assign climate shocks to each issuer based on the simulated technology mix. Once the energy sources, weights, sectors and $LGD_{j,m}$ values have been simulated for each issuer, the climate shocks are assigned according to the operating region using equation (9). Subsequently, we proceed to compute the transition risk add-on for each model and scenario by applying equation (21).

In our simulation, we have worked with the LIMITS WP4 database, with a focus specifically on two IAMs: WITCH and REMIND. These models can be considered as general equilibrium models based on the inter-temporal optimization of a welfare function, as described in for Applied Systems Analysis (2015). The primary difference between these

models lies in the form of their maximized utility functions, which represents the transmission of climate policies to economic variables, and in the inclusion of regional spill-over effects, which are a prominent feature of the WITCH model.

For both IAMs, in our analysis, we consider all five scenarios included in the LIMITS WP4 database:

1. Energy independence: involves restrictions on overall primary energy imports depending on the state of art of considered region, as shown in figure 2. These restrictions are to be implemented before 2030 and maintained throughout the remained of the century Jewell et al. (2016).

Region-type	Includes	Target
Energy importing developed regions with low energy demand growth	Europe and OECD regions other than North America	Cut their 2010 net import dependence in half by 2030 and maintain that relative level through the 21st century.
Energy importing emerging economies with high energy demand growth	China, India and other non- OECD Asian economies	Maintain their current level of net- energy imports throughout the century
3. Energy exporters	Middle East, Former Soviet Union, Africa, Latin America, and other regions which are net-exporters	Never become energy importers
4. The U.S./North American region	U.S. and North America	Becomes energy independent by 2030 and maintains energy independence, consistent with political debates ^{29,30} and recent modeling results ^{8,63-65} .

Figure 1: Regional import reduction targets in the Energy independence scenario.

- 2. Oil independence: assumes that all oil-importing countries reduce their net oil import dependence by 50% by 2030 and maintain this level of reduction until the end of the century. The United States and North America are excluded from this target, as they are assumed to achieve oil independence before 2030 Jewell et al. (2016).
- 3. Pledges: reflects the implementation by all regions of climate mitigation consistent with the pledges made during the Copenhagen Accord. Output trajectories are extrapolated beyond 2020 by projecting the achieved rate of GHG emission reduction, based on specific technology assumptions and GHG targets, from 2020 to 2100.
- 4. Climate stabilization 450: involves the introduction of climate policies beginning in 2020 with the goal of stabilizing the atmospheric concentration of Greenhouse Gases (GHGs) at 450 ppm CO2-equivalent (CO2e) before 2100,
- 5. Climate stabilization 500: less stringent climate policy compared to LIMITS450. The objective is to stabilize the atmospheric concentration of GHGs at 500 parts per million CO2-equivalent before 2100.

These scenarios represent potential pathways identified by the scientific community for reducing GHG emissions and limiting global temperature rise to below 2 degrees Celsius compared to pre-industrial levels. The first two scenarios, Energy Independence and Oil Independence, focus on energy trade restrictions between regions. LIMITS450 and

LIMITS500 represent climate policies that set specific emission targets for countries to be achieved through coordinated action, as outlined in Battiston et al. (2019a). For any of these five scenarios the LIMITS database reports trajectories of IAM variables' output that are consistent with WITCH and REMIND assumptions.

Scenario name	Scenario type	Near-term Target/Fragmented Action	Fragmented Action until	Long-term Target in 2100	Energy security objective
Baseline	Baseline	None	-	None	None
Energy Independence	Independence	None	-	None	Energy independence
Oil independence	Independence	None	-	None	Oil independence
Pledges*	Reference	Stringent	2100	None	None
450	Climate Policy	Weak	2020	450 ppm CO ₂ e (2.8 W/m ²)	None
500*	Climate Policy	Weak	2020	450 ppm CO ₂ e (3.2 W/m ²)	None

Figure 2: Scenario design and naming convention of the LIMITS study.

4.1 Comparison between Models and Scenarios

To reflect typical investment choices and the global reach of financial markets, we initially allocate the number (n) of ZCB within our portfolio according to the following percentages: 35% North America, 20% Europe, 20% Asia, 10% South America, 10% Africa, 5% Rest of the world.

We run our model on three different technology mixes to reflect possible investment choices available to financial actors:

- 1. Green: the portfolio consists solely of issuers operating exclusively with renewable energy sources. This is defined as the basis to compute the add-on in section 3.2.
- 2. Fossil: the portfolio includes issuers operating exclusively with fossil energy sources.
- 3. Mix: the portfolio comprises issuers operating with a balanced energy mix, utilizing 50% renewable energy sources and 50% fossil energy sources (25% primary energy and 25% secondary energy).

For all technology mix we assign a maximum of 3 IAM energy variables for category (primary energy — fossil, secondary energy — electricity — fossil, secondary energy — electricity — renewable).

To obtain the transition risk add-on according to equation (21), we conduct (m) = 10.000 simulations under the following assumptions:

- The risk free-rate y_f is set at 3%.
- The portfolio comprises 100 issuers (n = 100).
- The loss given default $LGD_{j,m}$ for each issuer (j) and simulation (m) is drawn from a Gamma(0.5, 0.25) distribution.
- Climate shocks are randomly assigned based on the simulated energy sources for each issuer according to operating regions.

- The sum of sectorial weights for each issuer is constrained to equal one: $\sum_{s=1}^{\infty} (w_s) = 1$.
- Value-at-Risk is computed at a 99.5 confidence interval.

Figure 3 presents the transition risk add-on for the different technology mixes, expressed in percentage points.

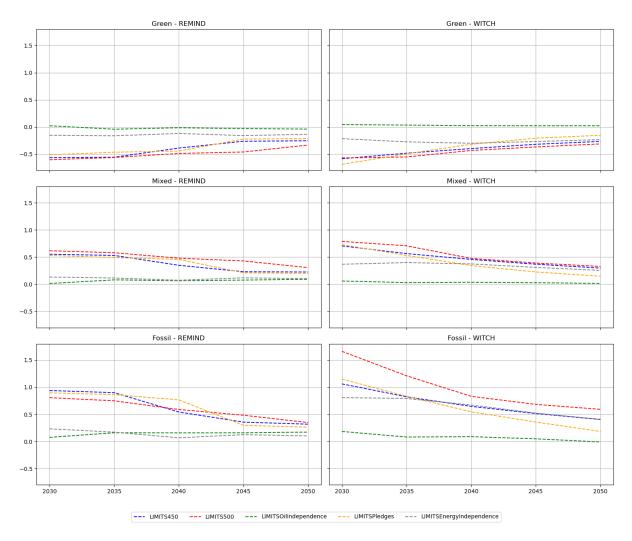


Figure 3: Transition risk add-on, expressed in percentage points, for different technology mixes, where mixed portfolio includes issuers operating both in fossil and renewable sectors.

For a green portfolio, as expected, the add-on exhibits negative values across most scenarios. This suggests that, even in the 99.5 percentile worst-case scenario, investing in a renewable-based portfolio may reduce the economic capital requirement for a financial institution. Conversely, both mixed and fossil fuel-based portfolios demonstrate positive transition risk add-ons, implying an increase in economic capital requirements. In the most extreme scenario, the increase in economic capital for a fossil fuel-based portfolio can reach 1.661% by 2030 under LIMITS500 simulated by WITCH model. Furthermore, we observe that investing in a "mixed" portfolio results in an increase in economic capital that is generally less pronounced than that associated with fossil fuel-based portfolios.

For green portfolios, similar results are observed across both the REMIND and the WITCH models for all scenarios. Notably, in the LIMITSOilIndependence, the addon approaches zero for both models, suggesting that climate actions under this scenario would have minimal impact on transition risk for green investments. Similar observations can be made for LIMITSEnergyIndependence, another scenario that does not explicitly target emission reduction, although the impact on economic capital is slightly more pronounced, particularly in the WITCH model, where the add-on reaches a peak of 30.2 bp in 2040. Conversely, the remaining three scenarios all demonstrate a more significant impact, especially in the short term. The maximum reduction in economic capital reaches approximately 60 bp in LIMITS500 under the REMIND model and 70 bp in LIMITSPledges under WITCH model. However, the positive effect on economic capital diminishes significantly over time, particularly under the LIMITSPledges, where the add-on increases notably between 2030 and 2050: 30.2 bp in REMIND and by 53.6 bp in WITCH. Importantly, the transition risk add-on is consistely higher under LIMITS500 compared to LIMITS450 across all time frames. This finding is consistent with the expectation that more stringent emission reduction policies would generally lead to higher transition risk.

For a fossil-fuel based portfolio we can notice significant discrepancies between the two IAMs. While the add-on for the LIMITSOilIndependence remains close to zero for both models, the WITCH model consistently predicts substantially higher add-ons across other scenarios, particularly for LIMITS500 and LIMITSEnergyIndependence scenarios. For instance, under the LIMITSEnergyIndependence the WITCH model predicts an impact ranging from 80.8 bp in 2030 and 40.5 bp in 2050, compared to 23.3 bp in 2030 and 10.2 bp in 2050 for REMIND model, representing approximately a four-fold difference. Similarly, for LIMITS500 the WITCH model predicts an add-on ranging from 166.1 bps in 2030 to 59.2 bps in 2050, compared to 80.7 bps in 2030 and 34.8 bps in 2050 for the REMIND model, indicating approximately a two-fold difference. For emission reduction target scenarios (LIMITSPledges, LIMITS450, and LIMITS500), the REMIND model generally predicts higher add-ons, even surpassing the WITCH model for the LIMITS450 scenario until 2040. In contrast, the WITCH model exhibits a more pronounced increase in economic capital for the LIMITS500 scenario compared to the LIMITS450 scenario in 2030 (a difference of 60 basis points), although this difference diminishes to 18.7 basis points by 2050.

In intermediate line we represent the results of mixed portfolio. This portfolio consistently exhibits positive transition risk add-ons across all model-scenario combinations and timeframes, implying an increase in economic capital for financial institution. However, these results are always lower than those observed for the fossil portfolio. Notably, both IAM predicts the highest impact under LIMITS500, while the introduction of a climate policy according to LIMITSOilIndependence has a minimal impacts on economic capital. A comparison with fossil portfolio reveals that results for the mixed portfolio are qualitatively similar although the magnitude of the add-on is lower. This observation suggests that the influence of fossil fuel-based CPSR is predominant over the renewable CPRS in the determination of climate shocks $u_i^{\rm GVA}$ of company (j).

In conclusion, the implementation of a climate policy alligned with the Pledges established by the Copenhaghen there have a significant impact on economic capital for all considered energy mixes across all model-scenario combination. Furthermore, a notable

observation across all three portfolios is a general decline in the transition risk add-on over time. This decline can be attributed to the decrease in the discounting factors and the increasing influence of idiosyncratic shocks over time, as reflected by the growing values of denominators in equation (21) from 2030 to 2050.

Assuming the independence between undiversified idiosyncratic risk and transition risk, we simply sum the values presented in table 1 with the values presented in figure 3 to obtain the total economic capital for each combination of rating class, technology mix, model and scenario. The resulting total adjunctive capital requirements are presented in Appendix B.

These results could be further refined by incorporating a factor that reflects climate risk based on the issuer's credit rating profile or by integrating a climate measure into the simulation of $LGD_{j,m}$ values. Given our choice to run the climatic model on a portfolio comprising 100 ZCB (n=100), the impact of the granularity add-on is significant in determining the overall economic capital, particularly for low-rated issuers. It is important to note that for this "worldwide portfolio", based on chosen parameters and stylized values, total economic capital is always positive definite for any combination of rating class, model and scenario. However, this result may not hold true for different values of n.

Precise numerical results for economic capital are presented in appendix B for each credit rating and each technology mix.

4.2 Regional Analysis

In this section, we present the results obtained for the transition risk add-on for different regions included in the LIMITS WP4 database, as detailed in appendix C. For each region, we applied the same methodology and values described in the previous section, assuming that all the issuers within the portfolio belong exclusively to a specific region. The primary objective of this further application is to compare the potential impacts of specific climate policy introductions across different geographical areas and to identify the discrepancies in outcomes across different model-scenario combinations for each region.

In figure 4 we present the different results obtained for each IAM model-scenario combination across the six different regions included in our analysis. The results for a fossil-fuel based portfolio are represented by a continuous line, while the results related to a renewable-based portfolio are represented through a dashed line. The colors for analyzed region are represented in the legend below the comprehensive figure.

If we consider LIMITS450, REMIND model predicts significant transition risk add-on for Europe and Rest of the world, both for fossil and renewable portfolios, while the results for Asia and North America are similar and generally less significant. Conversely, the WITCH model significantly mitigates the transition impact on Europe while projecting more substantial impacts for Asia and Africa. Furthermore, both models indicate negligible add-ons for North America, with similar minimal impacts prospected also for Latin America (between approximately 100 and 30 bp). Regarding renewable portfolios, the WITCH model consistently produces similar results across all region, except for Asia, where a more significant positive impact is observed.

Assuming the introduction of a climate policy consistent with the LIMITS500 scenario, we observe that the REMIND model predicts a significant transition risk add-on for

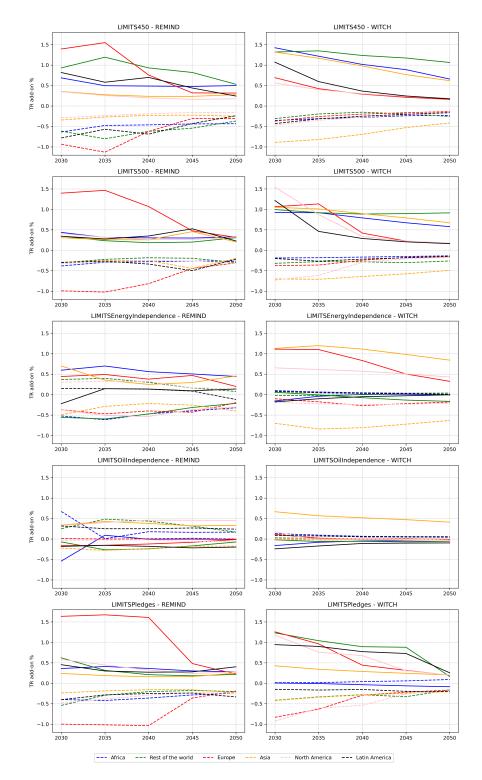


Figure 4: Comparison of transition risk add-on in percentage points across LIMITS scenarios, computed for different regions according to (17) for fossil (solid lines) and green portfolios (dashed lines) composed of 100 ZCB.

European issuers, while other regions exhibit similar results. In contrast, the WITCH model reduces the impact on European issuers while projecting substantial losses for other regions, particularly in the early years (between 2030 and 2040). Furthermore, WITCH indicates a more positive impact for renewable-based issuers operating in Asia

and North America. For renewable portfolios, both models result in a transition impact that is below 50 bp for Africa, Europe, Latin America and Rest of the World.

Under the LIMITSPledges scenario, the REMIND model again predicts the most significant impact in case of European issuers, with similar results for remaining regions. Instead, for this scenario, WITCH forecasts more substantial impacts for fossil-based issuers in Latin America, North America and Rest of the World, while the impact for European issuers is reduced by at least 50 bp across all time periods. Notably, under WITCH our analysis highlights the particular reduction effect on economic capital associated with obligations from African issuers operating with fossil technology mixes.

Finally, under LIMITSOilIndependence the WITCH IAM forecasts relatively consistent results across all regions and technology mixes, with the exception of Asia, where significant losses are observed. REMIND forecasts impacts below 50 bp for all regions with the treat that fossil-based portfolios outperform renewable-based ones in most regions (Africa, Rest of the world, Europe, Latin America). These results are clearly attributable to the nature of this scenario, which does not prioritize the reduction of global emissions but instead focuses on restricting international trade in fossil fuels. The same observation can be made also for LIMITSEnergyIndependence: REMIND forecasts the most significant impacts for Africa, Asia and Europe issuers, both for fossil and renewable technology mix, and a significant positive effect for fossil-based North American and "Rest of the world" issuers. Conversely, WITCH predicts significant impacts primarily for issuers in Asia, Europe and North America for all technology mixes. Furthermore, the WITCH IAM predicts results similar to those observed under emission reduction scenarios, in which fossil-fuel technology mixes generally induce a negative impact on economic capital across all regions, while renewable-based issuers generate positive effects across most regions.

Precise numerical results are presented in appendix A, in tables 2, 3 and 4.

The objective of figure 5 is to compare the effects of different models and scenarios for issuers within single regions. While discussing these results, we should note that our approach does not account for potential spillover effects within the worldwide financial network.

- Africa: the WITCH model predicts negligible positive effects (¡50 bp) for renewable-based portfolios, with significant negative impacts exclusively under LIMITS450 and LIMITS500 for fossil-fuel based issuers. The REMIND model forecasts similar results for emission reduction scenarios and a significant impact under LIMITSEnergyIndependence. A comparison of the two models reveals that REMIND predicts more significant impacts for renewable-based investments compared to WITCH IAM, with the exception of LIMITSOilIndependence.
- Rest of the world: most significant impacts are predicted for LIMITS450 in both models. Furthermore, a positive impact on economic capital arises for fossil-based portfolios under LIMITSEnergyIndependence and LIMITSOilIndependence by RE-MIND. WITCH predicts negligible effects for green portfolios across all scenarios, while the most significant results for fossil technology mixes are shown under LIM-ITS500 and LIMITSPledges.
- Asia: REMIND predicts similar results across all scenarios and technology mixes, with very small add-on observed under LIMITSPledges. In contrast, the WITCH

model forecasts significant impacts, particularly under emission reduction scenarios, with the exception of renewable-based portfolios under LIMITSOilIndependence.

- Europe: highlights higher add-on values compared to other regions. Both models exhibit similar results, with the most pronounced impacts observed under Copenhaghen Pledges scenario in the initial years, that reflect the significant commitment assumed by European countries in that accord. The greatest discrepancy between the two models is obtained under WITCH LIMITSEnergyIndependence, that predicts an impact exceeding 100 bp for fossil-fuel technology mix.
- North America: the WITCH model forecasts higher impacts across all technology mixes and scenarios, with the exception of LIMITSOilIndependence. This is particularly evident under emission reduction scenarios, particularly under LIMITS500, where the predicted fossil add-on exceeds 150 bp by 2030. It is noteworthy that under LIMITSEnergyIndependence fossil add-on reach a positive impact close to 50 bp.
- Latin America: WITCH predicts negligible impacts (smaller than 50 bp) for all renewable-based portfolios. Fossil add-on results are negligible for trading schemes scenarios, showing significant (and similar) results only under emission reduction scenarios. REMIND forecasts similar conclusions.

A potential enhancement to our model could involve integrating these data with granularity add-on values computed by Gordy (2003) model using parameters and distribution calibrated to the specific financial context of each region. This development would enable a calculation of the total economic capital (22) segmented by the regional origin of their investments.

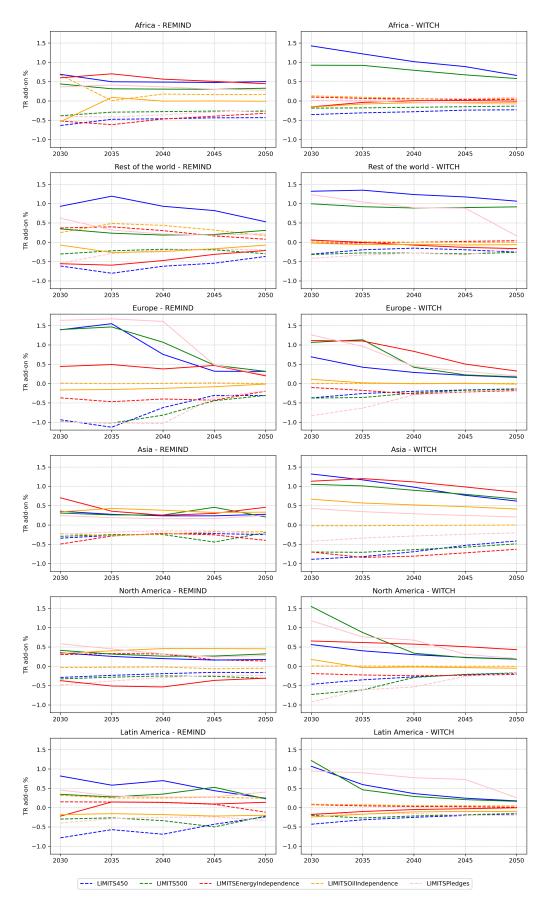


Figure 5: Transition risk add-on in percentage points, computed for different regions according to (17) for fossil (solid lines) and green portfolios (dashed lines) composed of 100 ZCB.

5 Conclusions

We have built a model that adjust the granularity add-on, which is derived from undiversified idiosyncratic risk, by a factor reflecting climate transition risk. We calculate the granularity add-on, $O(n^{-1})$ by equation (5), depending on the variables associated with systemic risk factor X and credit ratings of the issuers involved. Because our model uses the stylized values presented in Gordy (2003), our results are consistent with theirs.

We then introduced a second, independent source of risk, related to climatic transition, according with the CLIMAFIN methodology Battiston et al. (2019b). This risk arises from the potential implementation of climate policies aligned with pre-determined NGFS scenarios, aimed at reducing or shifting global CO2 emissions from fossil-fuel activities. Due to the difficulty in obtaining precise information on the NACE economic sectors in which public companies operate, we employed a Monte Carlo analysis to create a wide-ranging distribution of hypothetical issuers.

We performed m=10.000 simulations, randomly assigning to each issuer a technology mix and the weights of underlying energy sources and linked CPRS. Subsequently, we associated a climate shock to each simulation and issuer, assuming IAM outputs are proportional to the shocks on issuers' GVA. Using this methodology, we constructed an empirical distribution of losses due to transition risk under two IAMs: WITCH and REMIND. These models are used to generate the trajectories of energy sources output, which are used to determine climate shocks, as explained in section 3.1. We then applied VaR to this empirical distribution, defined on the same confidence interval of granularity add-on (i.e. 99.9%). This approach also accounts for uncertainty in IAM trajectories and sectorial weights, which provide the basis for calculating climate shocks. We developed a Python script that adds a transition-related buffer to the granularity add-on when a climate policy is implemented under a given scenario S. The total economic capital is calculated as by the sum of these two components, following the key assumption of independence between the systematic risk factor X and the introduction of a climate policy. This assumption is inherently debatable due to the complex nature of climate risks. However, our approach is partially supported by Ugolini et al. (2024), who demonstrated that financial analysts already incorporate transition risk into credit risk measures, such as CDS spread, and consequently into default probability calculations Beirne et al. (2021). Therefore, the implementation of a climate policy might not introduce a new source of systemic risk, as the market may have already priced in some level of this risk.

Section 4 presented the results, which demonstrated a significant impact of climate transition risk on economic capital for all regions, particularly when an emission-reduction policy is implemented. This impact is observed under both coordinated and non-coordinated policy approaches (correspondent to WITCH and REMIND models). Regarding our results, we emphasize that negative shocks arising from the analyzed portfolio could be partially offset by positive shocks from more climate-aligned positions within the balance sheet of a financial institution. Furthermore, our analysis does not consider macroeconomic reverberations of a climate shock (positive or negative) predicted by Battiston and Monasterolo (2020). Finally, it should be noted that several recent studies, such as McCollum et al. (2018), have indicated that the trajectories included in the LIMITS database may not be sufficient to achieve the emission reduction targes defined by the Paris agreement Intergovernmental Panel on Climate Change (IPCC) (2014). This suggests the potential for even more significant losses and, consequently, larger impacts on

economic capital. From an investment perspective, financial actors have several options to mitigate this risk. A primary approach is to select securities from issuers included in green indices and ETFs (such as the MSCI Europe Climate Change Select Index, Solactive European Low Carbon Index, Solactive European Sustainability Index, NASDAQ OMX GreenE conomy Index and S&P Global Clean Energy Index) as demonstrated by Cepni et al. (2022). Their research shows that green bonds within these indices can mitigate risks associated with climate uncertainty due to their positive correlations with an index representing transition risk.

The stream of research related to the regulation of climate risks has gained growing attention in recent years. However, regulators have yet to implement specific measures to ensure that financial institutions accurately assess and manage these risks. This regulation would safeguard the stability of the financial system and prevent spillover effects to other economic sectors, as advocated by Silvapulle et al. (2016). According to Battiston and Monasterolo (2020), analyses of the impact of potential losses from climate transition on a financial institution's stability should account for the leverage effect. A shock to the values of corporate bonds held on a financial institution's balance sheet affects its capital in proportion to its leverage ratio. For instance, a 1% loss in corporate bond value could result in a 30% reduction in the financial institution's capital if it maintains a leverage ratio equal to 30. This could substantially impair the solvency of the institution holding the portfolio and, consequently, threaten the stability of the financial system.

Given the potential impact of climate risks on the sustainability of the financial system, a coordinated regulatory action is required. The systemic nature of climate risks, which extends beyond the idiosyncratic risks faced by individual investors, is typically not captured by banks' internal risk management or by the backward-looking Basel Pillar I regulatory framework. Therefore, a complementary micro-prudential and macro-prudential approach may be required, encompassing dedicated disclosure requirements, macro-prudential policies (including capital-based measures) and climate stress-testing Baranović et al. (2021). From a micro-prudential perspective, several measures could be introduced to integrate the existing regulatory framework and enhance the monitoring of individual climate exposures and address the inherent complexity of transition risk ECB (2022).

- Integrating direct supervision and reporting of climate risks into existing risk monitoring processes.
- Developing adequate climate stress testing methodologies techniques, according with the framework realized by Hayne et al. (2020).
- Enhancing models to incorporate climate variables within the Basel III Internal Ratings-based Approach (IRB) for International Settlements (2001).

To prevent contagion and preserve financial system stability, the aforementioned microprudential actions can be complemented by measures targeting systemic risk management. These measures could include disclosure, as part of Basel Pillar II, for supervised institutions to report environmental information related to their investments. A further measure could involve increasing the granularity of data collection to enable sector-specific measurement of emissions and climate risk profiles. Dafermos and Nikolaidi (2020) discussed the potential effects of using capital requirements to promote the transition to a low-carbon economy. Specifically, they analyzed the impact on global warming and physical risk of implementing a GSF (Green Supportive Factor), which allows banks to hold less capital for loans supporting activities that can reduce carbon emissions, and a DPF (Dirty Penalizing Factor), which requires banks to hold more capital for loans financing high-carbon activities. They demonstrated that both requirements can slow global warming, especially when adopted simultaneously or in combination with green fiscal policies. Specifically, the DPF reduces banks' credit provision and leverage, making them less fragile, but increases loan defaults due to reduced economic activity. Conversely, the GSF increases bank leverage by boosting green credit. These effects are quantitatively small and are mitigated when there is a simultaneous implementation of both the DPF and the GSF. Finally, their results indicated that the most effective climate policy mix combines a green fiscal policy complemented with the DPF. However, it should be noted that while creating incentives for banks and financial markets to invest in green assets appears advantageous for the green economy, using regulations designed to mitigate risk within the financial system to mobilize investment should be approached with caution Matikainen (2017). Capital buffers are intended to enhance banks resilience by providing a cushion against potential losses from various risks, and a green supporting factor could inadvertently increase their vulnerability by reducing the total amount of capital to be held in the balance sheet. Therefore, a DPF would be preferable, but there is still a debate regarding the appropriate eventual magnitude of such a factor. Banks associations oppose this last option due to its potential negative impact on profit margins. Finally, while capital-based macro-prudential measures could certainly enhance banks' resilience to climate risks by influencing incentives and prices in the allocation of banks funding, they require careful calibration, as adjunctive capital buffer requirements for climate risk may constrain banks' available capital space.

The model developed for this article can be integrated into actual financial risk management frameworks. Particularly, the results for renewable-based portfolios can serve as a basis for calculating additional economic capital to be kept as a buffer against transition risk. Financial institutions can use their actual portfolio data to effectively map the CPRS of their investments and accurately assess the impact of climate shocks on their holdings. Real climate shock data can be used for pricing adjustments of held portfolios and to evaluate the discrepancy with our "green portfolio" benchmark, reflecting the potential impact of investing in a 100% renewable portfolio, as described in equations (20) and (22). Our approach could be further improved by incorporating additional transition risk measures, perhaps derived from real-time data Apel et al. (2022), or by integrating a transition risk dependence into the Loss Given Default calculation to reflect the lower losses associated with green assets after the implementation of a climate policy. Further improvements could also include using alternative risks measures to VaR for forwardlooking climate shocks, or integrating more complex economic models, such as EIRIN Monasterolo and Raberto (2017), to model issuer default conditions. To relax the key assumption of independence between undiversified idiosyncratic risk and transition risk, a correlation factor between an issuer's rating class and its climatic profile could be incorporated. Alternatively, the probability of default and related parameters used in the calculation could be expressed as functions of climatic risk variables.

In conclusion, the framework realized by Gordy (2003) for calculating the total economic capital could be enhanced for each involved region by integrating relevant regional data for the granularity add-on computation. This approach would enable a precise estimation of the total economic for each region similar to the one presented in appendix B.

A Numerical results for different technology mixes

In below tables we show the precise numerical results obtained for the "balanced portfolio" anticipated in section 4.1.

Model	Scenario	2030	2035	2040	2045	2050
REMIND	LIMITS450	-0.566	-0.559	-0.391	-0.265	-0.256
REMIND	LIMITS500	-0.606	-0.564	-0.490	-0.463	-0.335
REMIND	LIMITSOilIndependence	0.021	-0.043	-0.016	-0.032	-0.041
REMIND	LIMITSPledges	-0.515	-0.467	-0.445	-0.225	-0.213
REMIND	LIMITSEnergyIndependence	-0.156	-0.164	-0.121	-0.161	-0.136
WITCH	LIMITS450	-0.588	-0.485	-0.400	-0.321	-0.268
WITCH	LIMITS500	-0.572	-0.555	-0.435	-0.370	-0.314
WITCH	LIMITSOilIndependence	0.042	0.033	0.023	0.021	0.021
WITCH	LIMITSPledges	-0.691	-0.497	-0.321	-0.208	-0.155
WITCH	LIMITSEnergyIndependence	-0.217	-0.276	-0.302	-0.270	-0.235

Table 2: Transition risk add-on for a green portfolio, expressed in percentage points

Model	Scenario	2030	2035	2040	2045	2050
REMIND	LIMITS450	0.939	0.899	0.543	0.356	0.318
REMIND	LIMITS500	0.807	0.749	0.589	0.483	0.348
REMIND	LIMITSOilIndependence	0.075	0.160	0.158	0.159	0.172
REMIND	LIMITSPledges	0.896	0.862	0.767	0.297	0.263
REMIND	LIMITSEnergyIndependence	0.233	0.171	0.067	0.125	0.102
WITCH	LIMITS450	1.061	0.823	0.646	0.514	0.405
WITCH	LIMITS500	1.661	1.210	0.832	0.683	0.592
WITCH	LIMITSOilIndependence	0.184	0.081	0.088	0.048	-0.009
WITCH	LIMITSPledges	1.149	0.833	0.546	0.357	0.184
WITCH	LIMITSEnergyIndependence	0.808	0.792	0.671	0.522	0.405

Table 3: Transition risk add-on for a fossil portfolio, expressed in percentage points

Model	Scenario	2030	2035	2040	2045	2050
REMIND	LIMITS450	0.551	0.530	0.348	0.229	0.224
REMIND	LIMITS500	0.617	0.579	0.479	0.428	0.305
REMIND	LIMITSOilIndependence	0.012	0.079	0.064	0.075	0.089
REMIND	LIMITSPledges	0.529	0.490	0.456	0.209	0.197
REMIND	LIMITSEnergyIndependence	0.131	0.113	0.072	0.117	0.102
WITCH	LIMITS450	0.705	0.564	0.456	0.368	0.296
WITCH	LIMITS500	0.788	0.707	0.475	0.389	0.327
WITCH	LIMITSOilIndependence	0.058	0.029	0.035	0.027	0.013
WITCH	LIMITSPledges	0.728	0.530	0.343	0.225	0.147
WITCH	LIMITSEnergyIndependence	0.367	0.398	0.373	0.309	0.254

Table 4: Transition risk add-on for mixed energy sources, expressed in percentage points

B Total economic capital for different credit rating classes

Model	Scenario			Green					Mix			Fossil					
		2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	
REMIND	LIMITS450	0.298	0.305	0.473	0.599	0.608	1.415	1.394	1.212	1.093	1.088	1.803	1.763	1.407	1.220	1.182	
REMIND	LIMITS500	0.258	0.300	0.374	0.401	0.529	1.481	1.443	1.343	1.292	1.169	1.671	1.613	1.453	1.347	1.212	
REMIND	LIMITSOilIndependence	0.885	0.821	0.848	0.832	0.823	0.876	0.943	0.928	0.939	0.953	0.939	1.024	1.022	1.023	1.036	
REMIND	LIMITSPledges	0.349	0.397	0.419	0.639	0.651	1.393	1.354	1.320	1.073	1.061	1.760	1.726	1.631	1.161	1.127	
REMIND	LIMITSEnergyIndependence	0.708	0.700	0.743	0.703	0.728	0.995	0.977	0.936	0.981	0.966	1.097	1.035	0.931	0.989	0.966	
WITCH	LIMITS450	0.276	0.379	0.464	0.543	0.596	1.569	1.428	1.320	1.232	1.160	1.925	1.687	1.510	1.378	1.269	
WITCH	LIMITS500	0.292	0.309	0.429	0.494	0.550	1.652	1.571	1.339	1.253	1.191	2.525	2.074	1.696	1.547	1.456	
WITCH	LIMITSOilIndependence	0.906	0.897	0.887	0.885	0.885	0.922	0.893	0.899	0.891	0.877	1.048	0.945	0.952	0.912	0.855	
WITCH	LIMITSPledges	0.173	0.367	0.543	0.656	0.709	1.592	1.394	1.207	1.089	1.011	2.013	1.697	1.410	1.221	1.048	
WITCH	LIMITSEnergyIndependence	0.647	0.588	0.562	0.594	0.629	1.231	1.262	1.237	1.173	1.118	1.672	1.656	1.535	1.386	1.269	

Table 5: Total add-on for rating A issuers, expressed in percentage points

Model	Scenario	Green							Mix			Fossil					
		2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	
REMIND	LIMITS450	0.321	0.328	0.496	0.622	0.631	1.438	1.417	1.235	1.116	1.111	1.826	1.786	1.430	1.243	1.205	
REMIND	LIMITS500	0.281	0.323	0.397	0.424	0.552	1.504	1.466	1.366	1.315	1.192	1.694	1.636	1.476	1.370	1.235	
REMIND	LIMITSOilIndependence	0.908	0.844	0.871	0.855	0.846	0.899	0.966	0.951	0.962	0.976	0.962	1.047	1.045	1.046	1.059	
REMIND	LIMITSPledges	0.372	0.420	0.442	0.662	0.674	1.416	1.377	1.343	1.096	1.084	1.783	1.749	1.654	1.184	1.150	
REMIND	LIMITSEnergyIndependence	0.731	0.723	0.766	0.726	0.751	1.018	1.000	0.959	1.004	0.989	1.120	1.058	0.954	1.012	0.989	
WITCH	LIMITS450	0.299	0.402	0.487	0.566	0.619	1.592	1.451	1.343	1.255	1.183	1.948	1.710	1.533	1.401	1.292	
WITCH	LIMITS500	0.315	0.332	0.452	0.517	0.573	1.675	1.594	1.362	1.276	1.214	2.548	2.097	1.719	1.570	1.479	
WITCH	LIMITSOilIndependence	0.929	0.920	0.910	0.908	0.908	0.945	0.916	0.922	0.914	0.900	1.071	0.968	0.975	0.935	0.878	
WITCH	LIMITSPledges	0.196	0.390	0.566	0.679	0.732	1.615	1.417	1.230	1.112	1.034	2.036	1.720	1.433	1.244	1.071	
WITCH	LIMITSEnergyIndependence	0.670	0.611	0.585	0.617	0.652	1.254	1.285	1.260	1.196	1.141	1.695	1.679	1.558	1.409	1.292	

Table 6: Total add-on for rating BBB issuers, expressed in percentage points

Model	Scenario			Green					Mix			Fossil					
		2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	
REMIND	LIMITS450	0.383	0.390	0.558	0.684	0.693	1.500	1.479	1.297	1.178	1.173	1.888	1.848	1.492	1.305	1.267	
REMIND	LIMITS500	0.343	0.385	0.459	0.486	0.614	1.566	1.528	1.428	1.377	1.254	1.756	1.698	1.538	1.432	1.297	
REMIND	LIMITSOilIndependence	0.970	0.906	0.933	0.917	0.908	0.961	1.028	1.013	1.024	1.038	1.024	1.109	1.107	1.108	1.121	
REMIND	LIMITSPledges	0.434	0.482	0.504	0.724	0.736	1.478	1.439	1.405	1.158	1.146	1.845	1.811	1.716	1.246	1.212	
REMIND	LIMITSEnergyIndependence	0.793	0.785	0.828	0.788	0.813	1.080	1.062	1.021	1.066	1.051	1.182	1.120	1.016	1.074	1.051	
WITCH	LIMITS450	0.361	0.464	0.549	0.628	0.681	1.654	1.513	1.405	1.317	1.245	2.010	1.772	1.595	1.463	1.354	
WITCH	LIMITS500	0.377	0.394	0.514	0.579	0.635	1.737	1.656	1.424	1.338	1.276	2.610	2.159	1.781	1.632	1.541	
WITCH	LIMITSOilIndependence	0.991	0.982	0.972	0.970	0.970	1.007	0.978	0.984	0.976	0.962	1.133	1.030	1.037	0.997	0.940	
WITCH	LIMITSPledges	0.258	0.452	0.628	0.741	0.794	1.677	1.479	1.292	1.174	1.096	2.098	1.782	1.495	1.306	1.133	
WITCH	LIMITSEnergyIndependence	0.732	0.673	0.647	0.679	0.714	1.316	1.347	1.322	1.258	1.203	1.757	1.741	1.620	1.471	1.354	

Table 7: Total add-on for rating BB issuers, expressed in percentage points

Model	Scenario	Green							Mix			Fossil					
		2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	
REMIND	LIMITS450	0.514	0.521	0.689	0.815	0.824	1.631	1.610	1.428	1.309	1.304	2.019	1.979	1.623	1.436	1.398	
REMIND	LIMITS500	0.474	0.516	0.590	0.617	0.745	1.697	1.659	1.559	1.508	1.385	1.887	1.829	1.669	1.563	1.428	
REMIND	LIMITSOilIndependence	1.101	1.037	1.064	1.048	1.039	1.092	1.159	1.144	1.155	1.169	1.155	1.240	1.238	1.239	1.252	
REMIND	LIMITSPledges	0.565	0.613	0.635	0.855	0.867	1.609	1.570	1.536	1.289	1.277	1.976	1.942	1.847	1.377	1.343	
REMIND	LIMITSEnergyIndependence	0.924	0.916	0.959	0.919	0.944	1.211	1.193	1.152	1.197	1.182	1.313	1.251	1.147	1.205	1.182	
WITCH	LIMITS450	0.492	0.595	0.680	0.759	0.812	1.785	1.644	1.536	1.448	1.376	2.141	1.903	1.726	1.594	1.485	
WITCH	LIMITS500	0.508	0.525	0.645	0.710	0.766	1.868	1.787	1.555	1.469	1.407	2.741	2.290	1.912	1.763	1.672	
WITCH	LIMITSOilIndependence	1.122	1.113	1.103	1.101	1.101	1.138	1.109	1.115	1.107	1.093	1.264	1.161	1.168	1.128	1.071	
WITCH	LIMITSPledges	0.389	0.583	0.759	0.872	0.925	1.808	1.610	1.423	1.305	1.227	2.229	1.913	1.626	1.437	1.264	
WITCH	LIMITSEnergyIndependence	0.863	0.804	0.778	0.810	0.845	1.447	1.478	1.453	1.389	1.334	1.888	1.872	1.751	1.602	1.485	

Table 8: Total add-on for rating B issuers, expressed in percentage points

Model	Scenario	Green							Mix			Fossil					
		2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	2030	2035	2040	2045	2050	
REMIND	LIMITS450	0.703	0.710	0.878	1.004	1.013	1.820	1.799	1.617	1.498	1.493	2.208	2.168	1.812	1.625	1.587	
REMIND	LIMITS500	0.663	0.705	0.779	0.806	0.934	1.886	1.848	1.748	1.697	1.574	2.076	2.018	1.858	1.752	1.617	
REMIND	LIMITSOilIndependence	1.290	1.226	1.253	1.237	1.228	1.281	1.348	1.333	1.344	1.358	1.344	1.429	1.427	1.428	1.441	
REMIND	LIMITSPledges	0.754	0.802	0.824	1.044	1.056	1.798	1.759	1.725	1.478	1.466	2.165	2.131	2.036	1.566	1.532	
REMIND	LIMITSEnergyIndependence	1.113	1.105	1.148	1.108	1.133	1.400	1.382	1.341	1.386	1.371	1.502	1.440	1.336	1.394	1.371	
WITCH	LIMITS450	0.681	0.784	0.869	0.948	1.001	1.974	1.833	1.725	1.637	1.565	2.330	2.092	1.915	1.783	1.674	
WITCH	LIMITS500	0.697	0.714	0.834	0.899	0.955	2.057	1.976	1.744	1.658	1.596	2.930	2.479	2.101	1.952	1.861	
WITCH	LIMITSOilIndependence	1.311	1.302	1.292	1.290	1.290	1.327	1.298	1.304	1.296	1.282	1.453	1.350	1.357	1.317	1.260	
WITCH	LIMITSPledges	0.578	0.772	0.948	1.061	1.114	1.997	1.799	1.612	1.494	1.416	2.418	2.102	1.815	1.626	1.453	
WITCH	LIMITSEnergyIndependence	1.052	0.993	0.967	0.999	1.034	1.636	1.667	1.642	1.578	1.523	2.077	2.061	1.940	1.791	1.674	

Table 9: Total add-on for rating CCC issuers, expressed in percentage points

C Economic Classification, CPRS and IAMs

Holdings in financial instituions' balance sheets are typically classified under regional system, such as NACE or NAICS (Roncoroni et al., 2021) to categorize related economic activities. The NACE classification, used across all European countries, employs a hierarchical structure of categories and subcategories of economic activities, culminating in a four-digits code classification (European Parliament and the Council, 2006).

According to Pörtner et al. (2022), de-carbonization policies are expected to affect economic sectors differently, depending on the specific mitigation scenarios and the technology used for energy and electricity production. Information available to policymakers and investors related to the development of economic sectors following the implementation of a climate policy comes from IAMs (Integrated Assessment Models) (Battiston and Monasterolo, 2020), which consist of a set of forward-looking trajectories designed to model future changes in the energy system and GHG emissions. We have considered two IAMs in our elaboration: REMIND and WITCH.

IAM variables do not directly correspond to NACE or NAICS sectors, that lack a classification of their exposures to climate transition risk (Battiston et al., 2022). This poses a challenge for the climate risk analysis of financial portfolios, which requires translating IAM trajectories into climate transition risk exposure and their effect on economic activities. To address this gap, (Battiston et al., 2017) introduced Climate Policy Relevant Sectors (CPRS) to define a correspondence between these two databases and execute climate stress tests. This methodology, introduced by Battiston et al. (2017), overcomes the shortcomings of previous climate data, which relied on carbon footprint and GHG emissions. These metrics suffered from a lack of transparency and standardized reporting, which could lead to misleading assessment of investors' climate risk exposure and climate alignment Battiston et al. (2022). Furthermore, such information was primarily disclosed by listed firms, excluding the reporting of Small and Medium companies, which constitute the core of many regional economies.

CPRS has been developed over the last years to provide a more specific categorization of relevant data, both in terms of granularity and alignment with climate scenarios. CPRS has progressively evolved from a broad sectoral classification (CPRS 1) to a highly detailed, scenario-aligned model (CPRS Granular), enabling a quantitative and targeted approach to understanding and mitigating exposure to transition risks in financial portfolios. The mapping process, as described in Roncoroni et al. (2021), involves the following steps:

1. Mapping NACE/ NAICS codes of the securities within a portfolio to corresponding

CPRS.

2. Associating each CPRS with corresponding LIMITS variables.

This can be executed by the table available in Battiston (2022).

When a company operates across multiples business lines, corresponding to different NACE codes, CPRS are allocated to the issuer (j) proportionally to the respective revenue shares w_j , s(B). If more information on the segregation of the firms' revenues by energy technology is available, this classification can be refined from CPRS Main to CPRS2, reflecting the contribution of each energy technology to the firm's revenue. Furthermore, CPRS granular adds another layer of detail by incorporating information about the energy technology used in the firm's various plants, considering both their respective business lines and their contribution to overall revenue. This level of granularity is crucial because many companies, such as those in transport or utilities sectors, may have plants with different technology mixes (e.g. combustion cars vs EV). Consequently, different business lines within the same firm can be affected in different ways by climate transition Battiston et al. (2022). CPRS are used to associate to each holding (j) a climate policy shocks $u_j(BP)$, derived from corresponding IAM variables, proportionally to the weight of the transition affected business lines. This assignment is essential to execute the financial valuation described in the equation (18).

In our article, we begin by CPRS main and consider the following categories for which a transition connotation can be defined:

- 1. 1-fossil-fuel.
- 2. 2-utility.
- 3. 3-energy-intensive.

Using the mapping table, we define the corresponding IAM energy variables and select the twelve most relevant energy sources. These are then categorized into three main categories:

- Primary energy—fossil
- Secondary energy electricity— fossil
- Secondary energy electricity renewable

For each of the IAM variables involved, we calculated the shock using Equation (4), with LIMITSBaseline considered as the BAU scenario. Assuming thaty assuming the shock on IAM outputs being proportional to the shock on issuer j's GVA (a realistic assumption for concentrated sectors like utilities and energy, as discussed in (Battiston and Monasterolo, 2020)) we derive the climate shock on issuer j as a weighted sum of the climate shocks related to the different energy sources.

Primary Energy Fossil	Secondary Energy Electricity Fossil	Secondary Energy Electricity Renewable
Primary Energy Coal	Secondary Energy Electricity Coal	Secondary Energy Electricity Biomass
Primary Energy Oil	Secondary Energy Electricity Gas	Secondary Energy Electricity Geothermal
Primary Energy Gas	Secondary Energy Electricity Oil	Secondary Energy Electricity Hydro
		Secondary Energy Electricity Nuclear
		Secondary Energy Electricity Solar CSP
		Secondary Energy Electricity Solar PV
		Secondary Energy Electricity Wind

Table 10: Classification of energy variables by categories.

For each analysis, we defined a possible technology mix, consisting of a maximum of three simulated different energy sources for each of the three categories involved represented in Table 10. Green portfolios (see table 2) are constructed by assigning climate shocks based on 3 energy sources from Secondary energy — electricity — renewable. Fossil portfolios (see table 3) are constructed using six energy sources (three from "Primary energy—fossil" and three from "Secondary energy — electricity—fossil"). Mixed portfolios (see Table 4) are constructed using a total of nine energy sources (three of each category), with the sum of the weights equal to 0.50 for renewable energy sources and 0.25 for the other two categories.

We can now identify the NACE3 codes relevant to our analysis by filtering the mapping table for the chosen IAM variables. These codes can be interpreted as the economic sectors from which we should choose the issuers to produce results comparable to the ones presented in section 4.

- 05.1-Mining of hard coal.
- 05.2-Mining of lignite.
- 06.1-Extraction of crude petroleum.
- 06.2-Extraction of natural gas.
- 08.9-Mining and quarrying n.e.c.
- 19.1-Manufacture of coke oven products.
- 19.2-Manufacture of refined petroleum products.
- 20.1-Manufacture of basic chemicals, fertilisers and nitrogen compounds, plastics and synthetic rubber in primary forms.
- 35.1-Electric power generation, transmission and distribution.

Finally, we note that LIMITS WP4 database, used for calculating output shocks, divides the global economy into six main regions (r):

- 1. Europe.
- 2. Asia.
- 3. North America.
- 4. Latin America
- 5. Africa
- 6. Rest of the World.

These are used to assign climate shocks to issuers according to their operating regions, since IAM trajectories are defined for each region.

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