The Logarithmic Wind Profile

H. TENNEKES [1]

Dept. of Aerospace Engineering, The Pennsylvania State University, University Park 16802

(Manuscript received 8 September 1972, in revised form 27 November 1972)

ABSTRACT

This paper explores the practical consequences of the asymptotic nature of the logarithmic wind profile in neutral, barotropic, planetary boundary layers. Recent developments in boundary-layer theory have shown that the von Kármán constant is a universal constant only in a very specific asymptotic sense; in typical atmospheric conditions its value is probably about 10% larger than the asymptotic one. Pending the development of a second-order theory, the value $\kappa=0.35\pm0.02$ is recommended for micrometeorological applications over smooth terrain. It is shown that K theory cannot be used in attempts to detect any trends or deviations from the logarithmic law.

1. Introduction

The logarithmic wind profile in adiabatic surface layers is one of the cornerstones of micrometeorology. Because the logarithmic law appears to be insensitive to the manner of its derivation (Lumley and Panofsky, 1964, p. 103), a great deal of folklore on its accuracy and applicability has become entrenched in the subject over the forty-odd years since it was first derived. It is the purpose of this paper to put that folklore in proper perspective. Recent developments in our understanding of boundary-layer theory allow us to make fairly specific statements on the logarithmic law, to establish guidelines for its use, and to provide warnings against abuses.

The issue is of immediate practical concern. Micrometeorology recently abolished its equivalent of the gold standard. The von Kármán constant, pegged at 0.4 since the Depression, is now allowed to float anywhere between 0.33 (Tennekes, 1968; Tennekes and Lumley, 1972) and 0.40, with a current value of 0.35 over smooth terrain as this goes to press (Businger *et al.*, 1971; Frenzen, 1972). The consequences of this uncertainty for the determination of stress from wind profiles are evident: in many experiments, the computed stress may have been some 20% above its true value, and the inferred surface friction velocity may have been off by 10% or so.

2. Review of boundary-layer theory

It is convenient (but not necessary) to restrict the derivation of the logarithmic wind profile according to current theory to planetary boundary layers that are stationary, horizontally homogeneous, adiabatic and barotropic. For simplicity, the surface stress ρu_*^2 (where u_* is the surface friction velocity) is taken parallel to the x direction; the geostrophic wind (equatorial boundary layers will be ignored) then has a positive x component and a negative y component if the boundary layer is located in the Northern Hemisphere. The magnitude of the geostrophic wind is called G; together with the Coriolis parameter f and the surface roughness length z_0 it determines the external non-dimensional parameter for this flow, the surface Rossby number $\mathrm{Ro}=G/(fz_0)$. As we start the analysis, the surface friction velocity u_* is unknown; indeed, one of the purposes of this work is to establish a functional relation between u_*/G and Ro.

The equations of motion for this boundary layer are (Blackadar and Tennekes, 1968; Tennekes and Lumley, 1972)

$$-f(V-V_g)=\frac{d}{dz}(-\overline{uw}), \tag{1}$$

$$+f(U-U_g)=\frac{d}{dz}(-\overline{vw}). \tag{2}$$

Here, U and V are the components of the wind in the boundary layer and U_g and V_g are the components of the geostrophic wind.

In the limit as $\mathrm{Ro}\rightarrow\infty$, these equations admit two kinds of self-similar solutions. For finite zf/u_* (i.e., finite relative height in the boundary layer), but with $z/z_0\rightarrow\infty$ and $\mathrm{Ro}\rightarrow\infty$, the wind profile is asymptotically, to first approximation, independent of Ro provided it is plotted as

$$\frac{U-U_g}{u_*}=F_x\left(\frac{zf}{u_*}\right),\quad \frac{V-V_g}{u_*}=F_y\left(\frac{zf}{u_*}\right). \tag{3}$$

It is understood here that F_x and F_y will remain finite, no matter how large Ro is, provided we keep zf/u_*

[1] On leave (1972–73) at the Royal Netherlands Meteorological Institute, De Bilt.

finite. The *velocity-defect law* (3) is quite similar to the ones used in laboratory flows (Hinze, 1959; Monin and Yaglom, 1971); in fact, the use of (3) was first proposed on the basis of that analogy (Csanady, 1967; Gill, 1967).

Again, in a limit process by which z/z_0 is kept finite, but $zf/u_* \rightarrow 0$ and $\mathrm{Ro} \rightarrow \infty$, the wind profile is asymptotically, to first approximation, independent of Ro provided it is plotted as

$$\frac{U}{u_*}=F_s\left(\frac{z}{z_0}\right), \quad \frac{V}{u_*}=0. \tag{4}$$

The second of these is a trivial identity because the x direction has been aligned with the surface stress (Blackadar and Tennekes, 1968). It should be noted that this surface-layer law is valid only for finite values of z/z_0 (at this point, the theory cannot make a statement on the largest permissible value of z/z_0). Also, it can be shown that (4) pertains asymptotically to a constant-stress layer, provided $\mathrm{Ro} \rightarrow \infty$ and z/z_0 remains finite (Blackadar and Tennekes, 1968). Finally, it should be emphasized that (4) does *not* make any commitment as to the shape of the wind profile in the surface layer.

Both (3) and (4) exhibit what is called *Rossby-number similarity:* in the particular nondimensional forms selected, they are independent of the surface Rossby number Ro. This universality of the first-order solutions cannot be achieved in any other way. It is worth noting that this asymptotic independence from the effects of Ro has been obtained at the expense of having to use the internal velocity scale u_* instead of the external velocity scale G. For a more detailed discussion of the mathematical issues involved, see Blackadar and Tennekes (1968) or Tennekes and Lumley (1972).

Though (3) is valid only well outside the surface layer ($z/z_0 \rightarrow \infty$), while (4) is valid only inside the surface layer (z/z_0 finite), they do have to have a common region of validity if Ro is large enough. That region is called the region of overlap (Clauser, 1956) or the matched layer, or the *inertial sublayer* (Blackadar and Tennekes, 1968). In the inertial sublayer ($z/z_0 \rightarrow \infty$, $zf/u_* \rightarrow 0$), Eqs. (3) and (4), including all their derivatives, have to agree with each other; the only way in which that can be done is through a logarithmic wind profile:

$$\frac{U-U_g}{u_*}=\frac{1}{\kappa}\ln\left(\frac{zf}{u_*}\right)+\frac{B}{\kappa}, \tag{5}$$

$$\frac{U}{u_*}=\frac{1}{\kappa}\ln\frac{z}{z_0}. \tag{6}$$

These expressions are valid only as first approximations in the limit as $\mathrm{Ro} \rightarrow \infty$, providing we stay within a range of heights such that $z/z_0 \rightarrow \infty$ and $zf/u_* \rightarrow 0$. In particular, (6) is not valid in the surface layer proper, where z/z_0 is not very large (Monin and Yaglom, 1971, p. 288). Also, the von Kármán constant occurring in (5) and (6) is not necessarily the same as that measured in any experiment at finite Ro—only in an asymptotic sense, in the limit as $\mathrm{Ro} \rightarrow \infty$, can one consider the von Kármán constant to be universal. More about that later.

Because (5) and (6) are valid simultaneously within the inertial sublayer, they may be subtracted from each other. This yields

$$\frac{U_g}{u_*}=\frac{1}{\kappa}\ln\left(\frac{u_*}{fz_0}\right)-\frac{B}{\kappa}. \tag{7}$$

The y component of the wind, described by the second members of (3) and (4), also has to be matched inside the inertial sublayer. That exercise yields (Blackadar and Tennekes, 1968)

$$\frac{V_g}{u_*}=-\frac{A}{\kappa}. \tag{8}$$

From (7) and (8) one can compute the geostrophic drag coefficient u_*/G and the angle α between the geostrophic wind and the surface stress if Ro is known (Csanady, 1967; Kazanski and Monin, 1961; Monin and Yaglom, 1971). The parameters A and B in (7) and (8) conform to Russian terminology; in Blackadar and Tennekes (1968) the definitions of A and B were reversed.

It needs to be emphasized that the logarithmic drag law based on (7) and (8) arises because the logarithmic wind profile exhibits asymptotic Rossby-number similarity in *two* frames of reference; both (5) and (6) are asymptotically independent of Ro. In other words, we could not have obtained a relation between u_*/G and Ro if the logarithmic wind profile had been a feature of the surface layer only.

The logarithmic wind profile thus plays a crucial role in the determination of the surface stress and of the surface friction velocity u_* from the external parameters of the problem (G, f, z_0). The importance of this feature cannot be underestimated: after all, u_* is an internal parameter!

It is also worth noting that (7) and (8) can be used backwards; i.e., if one measures u_* and z_0 from a logarithmic wind profile, one can compute the geostrophic wind if the boundary layer in question is stationary, homogeneous, adiabatic, barotropic, and non-equatorial. Obviously, we are discussing a matter of principle, not one of practical applicability.

3. The nature of the logarithmic law

As we have seen, the very existence of the logarithmic wind profile is based on the asymptotic links that couple the surface layer (finite z/z_0, even in the limit as $\mathrm{Ro} \rightarrow \infty$) to the outer part of the planetary boundary layer (finite zf/u_*, no matter how large Ro becomes). From this, we have to conclude that the practice of treating the surface layer as an independent part of the planetary boundary layer is not justified from the point of view of current boundary-layer theory. If, as Eqs. (5) and (6) demonstrate, even the logarithmic wind profile is a feature that is shared between the upper part of the surface layer ($z/z_0 \gg 1$) and the lower part of the Ekman layer ($zf/u_* \ll 1$), we should be prepared to see evidence of sharing in all other properties of the upper part of the surface layer. Let me give an example: from the point of view developed here it is not surprising that the low-frequency ends of u and v spectra measured in the surface layer do not seem to satisfy simple surface-layer scaling (Busch, 1972; Lumley and Panofsky, 1964). In summary, no longer can we pretend to do honest surface-layer meteorology without accounting for some of the properties of the Ekman layer as a whole.

At this point, it is necessary to dwell on the relations between this approach and the classical one. In the most frequently encountered derivation of the logarithmic wind profile, dimensional analysis is employed to state that, in a layer where $z/z_0 \gg 1$ and $z/h \ll 1$ (where h is the height of the boundary layer), dU/dz can be a function of u_* and z only. This statement leads to $dU/dz = u_*/(\kappa z)$ and thence to (6). We now realize that this is an extremely compact way to state one (not all!) of the results (not assumptions!) of the asymptotic theory; indeed, the rather ad hoc assumptions made in the classical approach now receive a firm foundation within the entire framework of current boundary-layer theory. This does not imply that the classical approach continues to deserve the standing it had for many years. On the contrary, though it intuitively anticipated one of the results of the asymptotic theory, it failed to discover the simultaneous validity of (5) and (6) and thus did not see the close link between the logarithmic profile and the corresponding drag law. Also, the classical approach (now some 40 years old) had to postulate that the von Kármán constant is "universal," it was not equipped to deal with the possibility that the value of κ might depend on an external parameter, and could not cope with the issues regarding the inherent limitations to the accuracy of the logarithmic law. It is perhaps unfortunate that the asymptotic theory requires a fairly sophisticated and roundabout approach, but the perspective gained this way is so much wider that the classical approach has to yield. After all, the latter disqualified itself by permitting the use of an internal parameter (u_*) without giving a prescription for the way in which that parameter depends on the external ones.

Now let us turn to the von Kármán constant. The theory states unequivocally that κ is a universal constant only in the asymptotic sense, in the limit as $\mathrm{Ro} \to \infty$. Because no experiments at infinite values of Ro can be performed, there is no experimental way of determining the asymptotic value of κ; instead, we have to rely on theory. What kind of theory is needed to do that job? The analysis given in Section 2 is a first-order, asymptotic one, in which the "constants" κ, A, B are allowed to be weak functions of the surface Rossby number, as long as those functions tend to universal constants in the limit as $\mathrm{Ro} \to \infty$. An honest statement on the variation of κ with Ro can be made only if a theory that predicts such second-order asymptotic effects can be developed. This is a formidable task, unlikely to be completed in the next few years. It so happens that a relatively assumption-free second-order theory was developed for turbulent pipe flow a few years ago (Tennekes, 1968); that analysis predicted that the asymptotic value of κ is about 0.33 (with an estimated rms error of about 3%). Now, if we are willing to assume that the asymptotic value of κ in the limit as the Reynolds number approaches infinity is the same as the asymptotic value of κ in the limit as $\mathrm{Ro} \to \infty$, we may conclude that $\kappa = 0.33 \pm 0.01$ is the lower bound for the range of values of κ that will be encountered in the atmosphere. Indeed, in the experiments performed by Businger *et al.* (1971), the calculated value of κ is ~ 0.35, a far cry from the value 0.4 that has been in use for the last 30 years. Because the experimentally determined value of κ is a function, however weakly, of Ro, the best advice that we can give at this point is that $\kappa = 0.35$ is probably a good choice for surface Rossby numbers corresponding to smooth terrain. Over rough terrain, the value 0.40 may be more appropriate (see also Section 6).

In principle, it is possible that κ ranges anywhere between 0.33 and 0.40 [even larger values are sometimes used in laboratory flows, e.g. Hinze (1959)]. A cautious micrometeorologist, therefore, should not assume that he can determine u_* within 1% from a wind profile; systematic errors of 5% or more are quite likely. Not altogether facetiously, I propose to refer to this issue as the uncertainty problem of micrometeorology. Until someone solves the second-order problem, there will be errors that cannot be made arbitrarily small by careful measurements.

4. The constant-stress layer: Fact and fiction

The derivation of the logarithmic wind profile presented in Section 2 does not require that the stress be independent of height in the inertial sublayer. If we assume that the logarithmic law (6) is a good approximation to the state of affairs at large values of Ro, we can determine the stress profile in the upper part of the surface layer ($z/z_0 \gg 1$, $zf/u_* \ll 1$) by substituting the second member of (4), as well as (6), (7) and (8), into the equations of motion (1) and (2). Upon integration, there results (with some approximations based on $z/z_0 \gg 1$)

$$-\frac{\overline{uw}}{u_*^2} = 1 - \frac{A}{\kappa}\frac{fz}{u_*}, \tag{9}$$

$$-\frac{\overline{vw}}{u_*^2} = \frac{fz}{\kappa u_*}\left[\ln\left(\frac{fz}{u_*}\right) + B - 1\right]. \tag{10}$$

Strictly speaking, these relations are valid only if $zf/u_* \to 0$ as a function of the surface Rossby number, no matter how slowly that limit is approached. For practical purposes, however, it is fair to keep zf/u_* fixed at some very small value, even though $\mathrm{Ro} \to \infty$ [for a discussion of this issue, see Tennekes and Lumley (1972, chapter 5)]. Now, the height h of the neutral boundary layer is about $0.3\ u_*/f$ (Blackadar and Tennekes, 1968). Therefore, keeping zf/u_* fixed at some small value amounts to keeping z/h fixed at some level near the base of the planetary boundary layer. Laboratory experiments have shown that the logarithmic profile extends to about 10% of the boundary-layer thickness, with surprising accuracy (Hinze, 1959; Clauser, 1956); on these grounds, we tentatively put the upper limit of validity of the logarithmic law at $z/h=0.1$, i.e., $zf/u_*=0.03$ [corresponding to $\sim$100 m under typical conditions (Panofsky, 1972)]. It should be kept in mind that this is a *practical* upper limit: from a theoretical point of view, the inertial sublayer is a constant-stress layer in the asymptotic sense, provided $zf/u_* \to 0$ as $\mathrm{Ro} \to \infty$. This is clear from the behavior of (9) and (10) in the limit as $zf/u_* \to 0$.

With $zf/u_*=0.03$, $A=5$, $B=1$ (averages of the values reported in the literature), and with $\kappa=0.4$, the stresses at that level are computed to be

$$\frac{\overline{uw}}{u_*^2}=0.62, \quad \frac{\overline{vw}}{u_*^2}=-0.26. \tag{11}$$

This implies that the modulus of the stress "vector" has decreased by some 30% from its surface value, and that the stress has turned by some 20°! These deviations are large, but they most emphatically do not imply that the neutral wind profile cannot be logarithmic up to $zf/u_*=0.03$. In turbulent pipe flow, for example, the stress has decreased to some 75% of its surface value before any deviations from the logarithmic profile are discernible (Hinze, 1959, p. 517). Also, the stress stays within 1% of its surface value only if we are willing to stay below $zf/u_*=10^{-3}$, say. In typical conditions, this amounts to $z=3$ m; clearly, the logarithmic law is useful and accurate well above that height if the boundary layer is an adiabatic one (this paper does not deal with diabatic corrections to the theory). We conclude that the accuracy of the logarithmic law is not at all comparable to the accuracy of the constant-stress assumption.

Lacking a second-order theory that could explain these strange phenomena, we have to accept as a fact of life that the neutral wind profile is very nearly logarithmic up to relatively great height, even though the stress both decreases and turns appreciably. Also, from the point of view developed in Section 2, there is no use in attempting to "explain" deviations in the logarithmic law through such artifacts as the introduction of a friction velocity derived from the local stress at any height. It bears repeating that the u_* used in this paper is the *surface* friction velocity; it is a true parameter, and most definitely not a function of height. In this context, it is worth noting that, completely consistent with the asymptotic theory, the wind profile appears to follow the logarithmic law to greater heights if the surface friction velocity is used instead of local friction velocities (Panofsky, 1972).

5. Relations with K theory

The asymptotic theory states that the logarithmic law is a first approximation, valid only if $z/z_0 \to \infty$, $zf/u_* \to 0$ and $\mathrm{Ro} \to \infty$. Now, Eqs. (9) and (19) show that the stress becomes constant in the limit as $zf/u_* \to 0$. Therefore, it is formally correct to *derive* an exchange coefficient K_m and a mixing length l from (6), as long as all of the associated limit processes are kept in mind. This yields

$$K_m \equiv \frac{u_*^2}{dU/dz}=\kappa u_* z, \tag{12}$$

$$l \equiv \frac{u_*}{dU/dz}=\kappa z. \tag{13}$$

These are *not* assumptions or postulates; they are immediate consequences of the asymptotic theory. Indeed, one of the major points in favor of the theory in Section 2 is that the derivation of the logarithmic law did not require any ad hoc assumption about the relation between stress and wind shear. We now find that the models used in K theory are consistent with modern boundary-layer theory, provided those models, such as (12) and (13), are understood as asymptotic approximations. At any finite value of Ro, Eqs. (12) and (13) are subject to small, and as yet indeterminable, systematic errors; those errors are related to the corrections of the logarithmic law that are required at finite values of Ro (recall that even the von Kármán constant is a function of the surface Rossby number).

In K theory, of course, the logarithmic law is derived with the aid of (12) or (13), instead of the other way around. That practice, though formally incorrect, is harmless for most practical purposes. However, (12) and (13) are not harmless any more if they (or their relatives) are used to hunt for small deviations from exact Rossby-number similarity in an inertial sublayer at finite values of Ro. Why is this so? If one wants to detect small deviations in the logarithmic profile, they have to be computed from the full equations of motion, assuming that such expressions as (12) and (13) themselves are not subject to weak trends associated with finite Rossby numbers. As we saw above, that crucial assumption is not justified. In other words, the logarithmic law should not be turned against itself: its first-order consequences should not be used to compute second-order effects. The temptation to try this kind of approach is strong, but it should be resisted by all possible means.

6. Speculations on a second-order theory

Some thoughts on the prospects of a second-order theory are in order. In the only flow for which second-order corrections have been calculated (Tennekes, 1968), the major effect is a weak dependence of κ on the Reynolds number. This implies that a significant part of the velocity profile remains logarithmic at finite Reynolds numbers, even though the logarithmic slope is not independent of the Reynolds number any more. This feature is related to the fact that the second-order correction also appears to have a logarithmic profile in the inertial sublayer [for a full discussion, see Tennekes and Lumley (1972, chapter 5)].

Perhaps we may expect the second-order corrections of the flow in planetary boundary layers to be somewhat similar to those needed in pipe flow. If that is a fair assumption, we anticipate that the departures from exact Rossby number similarity show up not as deviations from the *shape* of the wind profile, but mainly as corrections to its logarithmic *slope*. How large are these corrections likely to be? Again taking our clues from turbulent pipe flow, we expect that the corrections are associated with the small, but finite, effects caused by the horizontal pressure gradient in a surface layer at finite surface Rossby number. In the flow described by (1) and (2), the pressure gradient is equal to ρfG (where ρ is the density). If we form a characteristic velocity (velocity scale) with the pressure gradient and the surface roughness length z_0, we obtain

$$u_p=(z_0fG)^{\frac{1}{2}}. \tag{14}$$

The relation between u_p and u_* is

$$\frac{u_p}{u_*}=\left(\frac{z_0fG}{u_*{}^2}\right)^{\frac{1}{2}}=\mathrm{Ro}^{-\frac{1}{2}}\frac{G}{u_*}. \tag{15}$$

For $\mathrm{Ro}=10^5$, we have $G/u_*\approx 22$ (Monin and Yaglom, 1972), so that $u_p/u_*=0.07$ in that case. If our experience with pipe flow is any indication, we may expect that the logarithmic slope behaves as

$$\frac{z}{u_*}\frac{dU}{dz}=\frac{1}{\kappa}\left(1-\gamma\frac{u_p}{u_*}\right), \tag{16}$$

where γ is an undetermined constant. Very roughly speaking, the actual value of κ at $\mathrm{Ro}=10^5$ is probably near 0.4; if the asymptotic value of κ is indeed 0.33, γ has to be approximately equal to 2.5. It need not be mentioned that this is an extremely crude result; it is offered only as preliminary guidance for the experimentalist who is interested in looking for a trend such as expressed in (16).

In this context it is worthwhile repeating that the logarithmic law—even as amended by (16)—is valid over a restricted range of heights. If we put the upper limit of validity at $zf/u_*=0.03$ (as in Section 4), and the lower limit at $z/z_0=100$ (of course, that is relatively arbitrary, because firm estimates of the accuracy cannot yet be made), then the vertical range of the logarithmic law vanishes when $u_*/fz_0=3.3\times10^3$, i.e., when $\mathrm{Ro}\approx6\times10^4$. This is in accord with experience: over very rough terrain it is extremely difficult (if not impossible) to find a logarithmic section in an observed wind profile. On the other hand, the vertical range increases with increasing values of Ro; for $\mathrm{Ro}=10^8$, $G/u_*\approx40$ (Monin and Yaglom, 1971), so that $u_*/fz_0=2.5\times10^6$, putting almost three decades of z between the upper and lower bounds.

It is impossible to give guidance on the possible effects of other complicating parameters at this stage. Thermal wind, inhomogeneous or nonstationary boundary conditions, and the presence of a vertical heat flux all may, or may not, affect the logarithmic slope and other features of the wind profile in the inertial sublayer; however, nothing even remotely approaching a careful specification of those problems is available at the present time.

Acknowledgments. This research was supported in part by the Atmospheric Sciences Section of the National Science Foundation, under Grants GA-18109 and GA-35422X. While writing this paper, the author was on sabbatical leave at the Royal Netherlands Meteorological Institute in De Bilt; the cooperation of the Director-in-Chief of the Institute and his staff is gratefully acknowledged.

REFERENCES

Blackadar, A. K., and H. Tennekes, 1968: Asymptotic similarity in neutral, barotropic, atmospheric boundary layers. *J. Atmos. Sci.*, **25**, 1015–1020.

Busch, N. E., 1972: Spectra of turbulence in the atmospheric surface layer. *Boundary Layer Meteor.* (in press).

Businger, J. A., J. C. Wyngaard, Y. Izumi and E. F. Bradley, 1971: Flux-profile relations in the atmospheric surface layer. *J. Atmos. Sci.*, **28**, 181–189.

Clauser, F. H., 1956: The turbulent boundary layer. *Advances in Applied Mechanics*, Vol. 4, New York, Academic Press, 1–51.

Csanady, G. T., 1967: On the resistance law of a turbulent Ekman layer. *J. Atmos. Sci.*, **24**, 467–471.

Frenzen, P., 1972: The observed relation between the Kolmogorov and von Kármán constants in the surface boundary layer. *Boundary Layer Meteor.* (in press).

Gill, A. E., 1967: The turbulent Ekman layer. Unpublished manuscript.

Hinze, J. O., 1959: *Turbulence.* New York, McGraw-Hill, 586 pp.

Kazanski, A. B., and A. S. Monin, 1961: On the dynamical interaction between the atmosphere and the earth's surface. *Bull. Acad. Sci. USSR, Ser. Geophys.*, No. 5, 786–788.

Lumley, J. L., and H. A. Panofsky, 1964: *The Structure of Atmospheric Turbulence.* New York, Wiley/Interscience, 239 pp.

Monin, A. S., and A. M. Yaglom, 1971: *Statistical Fluid Mechanics*, Vol. 1. Cambridge, Mass., The MIT Press, 769 pp.

Panofsky, H. A., 1972: Tower micrometeorology. *Boundary Layer Meteor.* (in press).

Tennekes, H. 1968: Outline of a second-order theory of turbulent pipe flow. *AIAA J.*, **6**, 1735–1740.

——, and J. L. Lumley, 1972: *A First Course in Turbulence.* Cambridge, Mass., The MIT Press, 300 pp.