Exercise 10

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Exercise 10. To be delivered before 4-XII-2021 (`Ex10-YourSurname.pdf`)

Prove that the number of faces of dimension p of a n-dimensional simplex is equal to

$$\begin{pmatrix} n+1 \\ p+1 \end{pmatrix} = \frac{(n+1)!}{(p+1)!(n-p)!}$$

A n-dimensional simplex S is a n-dimensional convex polyhedra with $n+1$ vertices. A p-dimensional face of S is a p-dimensional simplex formed by $p+1$ vertices of said simplex, with $0 \leq p \leq n$.

There are as many p-dimensional faces in a simplex as combinations of $p+1$ vertices we can form from the $n+1$ vertices of the simplex. Therefore, we can compute the number of faces through combinatorial analysis.

In order to compute how many subsets of $p+1$ items out of a set of $n+1$ items, with no repetition, we should compute:

$$\binom{n+1}{p+1} = \frac{(n+1)!}{(p+1)!((n+1)-(p+1))!} = \frac{(n+1)!}{(p+1)!(n-p)!} \tag{1}$$