ESAME 09.09.2020

4.
$$A = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 2 & 1 & -2 \\ \alpha+1 & -2 & 5 \end{pmatrix}$$

calcola la fatt. LU: ~~max tra $|\alpha+1|$ e 2 e $|2-\alpha| \Rightarrow |\alpha-2| = 2+\alpha$~~

~~pivot è $2-\alpha$ perciò $\alpha \neq 2$~~ NON SI USA IL PIVOT

$$\cancel{G = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{2+\alpha} & 1 & 0 \\ \frac{\alpha+1}{2+\alpha} & 0 & 1 \end{pmatrix}} \qquad \cancel{G\cdot A = \begin{pmatrix} 2-\alpha \\ 2 & \frac{-2}{2+\alpha} = \frac{4+2\alpha-2}{2-\alpha} \end{pmatrix}}$$

$$G = \begin{pmatrix} 1 & 0 & 0 \\ \frac{-2}{2-\alpha} & 1 & 0 \\ \frac{-\alpha+1}{2-\alpha} & 0 & 1 \end{pmatrix} \qquad G_1 A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ \frac{-2}{2-\alpha} & 1 & 0 \\ \frac{-\alpha+1}{2-\alpha} & 0 & 1 \end{pmatrix}}_{G_1} \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 2 & 1 & -2 \\ \alpha+1 & -2 & 5 \end{pmatrix} = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ -2+2=0 & \frac{-4}{2-\alpha}+1 & \frac{-2(\alpha+1)}{2-\alpha}-2 \\ -\alpha+1+\alpha+1=0 & \frac{-2(\alpha+1)}{2-\alpha}-2 & -\frac{(\alpha+1)^2}{2-\alpha}+5 \end{pmatrix}$$

$$\begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{-6}{2-\alpha} \\ 0 & \frac{-6}{2-\alpha} & \frac{10-(\alpha+1)^2-5\alpha}{2-\alpha} \end{pmatrix} \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-6}{\alpha+2} & 1 \end{pmatrix}$$

$$\frac{-6}{\cancel{2-\alpha}} \cdot \frac{\cancel{\alpha-2}}{\alpha+2} = \frac{6}{\alpha+2} \text{ e cioè } -$$

$$G_2\cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-6}{\alpha+2} & 1 \end{pmatrix}\begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{-6}{2-\alpha} \\ 0 & \frac{-6}{2-\alpha} & \frac{10-(\alpha+1)^2-5\alpha}{2-\alpha} \end{pmatrix} = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{-6}{2-\alpha} \\ 0 & 0 & \end{pmatrix}$$

$$\frac{10-(\alpha+1)^2-5\alpha}{2-\alpha} - \left(\frac{6}{2-\alpha}\right)^2$$

ora
$$U = G_2 \cdot G_1 \cdot A = \begin{pmatrix} 2-\alpha & 2 & \alpha+1 \\ 0 & \frac{\alpha+2}{\alpha-2} & \frac{-6}{2-\alpha} \\ 0 & 0 & \end{pmatrix}$$

$$\frac{10-(\alpha+1)^2-5\alpha}{2-\alpha} - \left(\frac{6}{2-\alpha}\right)^2$$

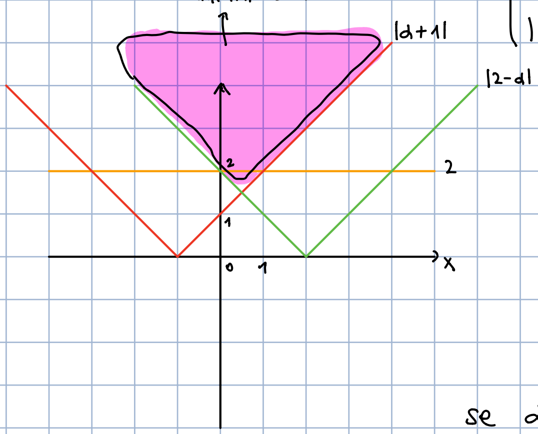
$$L = G_1^{-1} G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{2-\alpha} & 1 & 0 \\ \frac{\alpha-1}{2-\alpha} & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{6}{\alpha+2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{2-\alpha} & 1 & 0 \\ \frac{\alpha-1}{2-\alpha} & \frac{6}{\alpha+2} & 1 \end{pmatrix}$$

- la scelta di tale parametri esiste se $\alpha \neq \pm 2$

- Studia al variare di α il comportamento del metodo di Gauss con Pivot PARZ. al primo passo

va scelto il max $\{|2-\alpha|, 2, |\alpha+1|\}$ = $\begin{cases} |2-\alpha| \\ 2 \end{cases}$
mi interessa

$|\alpha+1|$.



e da qui scelgo $\Rightarrow$ $|2-\alpha|$ se $\alpha \leq 0$
2 se $0 < \alpha < 1$
$|\alpha+1|$ se $\alpha \geq 1$

se $\alpha \leq 0$ non si fanno scambi $\rightarrow$ $|2-\alpha|$ è già PIVOT
se $0<\alpha<1$ si scambia prima con seconda riga
se $\alpha \geq 1$ si scambia prima e terza riga

- se $\alpha = -6$. Calcola la fattorizz. $PA = LU$ con PIVOT PARZIALE

$$A = \begin{pmatrix} 8 & 2 & -5 \\ 2 & 1 & -2 \\ -5 & -2 & 5 \end{pmatrix} \quad \text{nel 1° il pivot è giusto} \quad P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 \\ \frac{5}{8} & 0 & 1 \end{pmatrix} \begin{pmatrix} 8 & 2 & -5 \\ 2 & 1 & -2 \\ -5 & -2 & 5 \end{pmatrix} \quad e \quad G_1 \cdot P_1 \cdot A = \begin{pmatrix} 8 & 2 & -5 \\ 0 & \frac{1}{2} & -\frac{3}{4} \\ 0 & -\frac{3}{4} & \frac{15}{8} \end{pmatrix}$$

$$\text{ora il pivot è } -\frac{3}{4} \Rightarrow P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad e \quad P_2 \cdot (G_1 P_1 A) = \begin{pmatrix} 8 & 2 & -5 \\ 0 & -\frac{3}{4} & \frac{15}{8} \\ 0 & \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{2}{3} & 1 \end{pmatrix} \cdot \quad P_2 G_1 P_1 A = \begin{pmatrix} 8 & 2 & -5 \\ 0 & -\frac{3}{4} & \frac{15}{8} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \Rightarrow U = \begin{pmatrix} 8 & 2 & -5 \\ 0 & -\frac{3}{4} & \frac{15}{8} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad e \quad U = L^{-1} \cdot P$$

$P = P_2 \cdot P_1$ e sappiamo $PA = LU \Rightarrow U = L^{-1}PA$

$$L = P \cdot P_1^{-1} G_1^{-1} P_2^{-1} G_2^{-1}$$
$$= P_2 \cdot \cancel{P_1} \cdot \cancel{P_1^{-1}} \cdot G_1^{-1} \cdot P_2^{-1} \cdot G_2^{-1}$$
$$= P_2 \cdot G_1^{-1} \cdot P_2^{-1} \cdot G_2^{-1}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ -\frac{5}{8} & 0 & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -\frac{5}{8} & 0 & 1 \\ \frac{1}{4} & 1 & 0 \end{pmatrix}}_{P_2 \cdot G_1^{-1}}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -\frac{5}{8} & 0 & 1 \\ \frac{1}{4} & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 \\ 0 & 1 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -\frac{5}{8} & 1 & 0 \\ \frac{1}{4} & -\frac{2}{3} & 1 \end{pmatrix}}_{L}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 \\ \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -\frac{2}{3} & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$$

$P_2^{-1} \cdot G_2^{-1}$

ESAME 29.04.21

- $A = \begin{pmatrix} \alpha-3 & 2 & 6-\alpha \\ 2 & 1 & -2 \\ 6-\alpha & -2 & -12 \end{pmatrix}$ $\quad G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{\alpha-3} & 1 & 0 \\ -\frac{6-\alpha}{\alpha-3} & 0 & 1 \end{pmatrix}$ $\quad G_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{\alpha-3} & 1 & 0 \\ -\frac{6-\alpha}{\alpha-3} & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha-3 & 2 & 6-\alpha \\ 2 & 1 & -2 \\ 6-\alpha & -2 & -12 \end{pmatrix} = \begin{pmatrix} \alpha-3 & 2 & 6-\alpha \\ 0 & \frac{\alpha-7}{\alpha-3} & \frac{-6}{\alpha-3} \\ 0 & \frac{-6}{\alpha-3} & \frac{-\alpha^2}{\alpha-3} \end{pmatrix}$

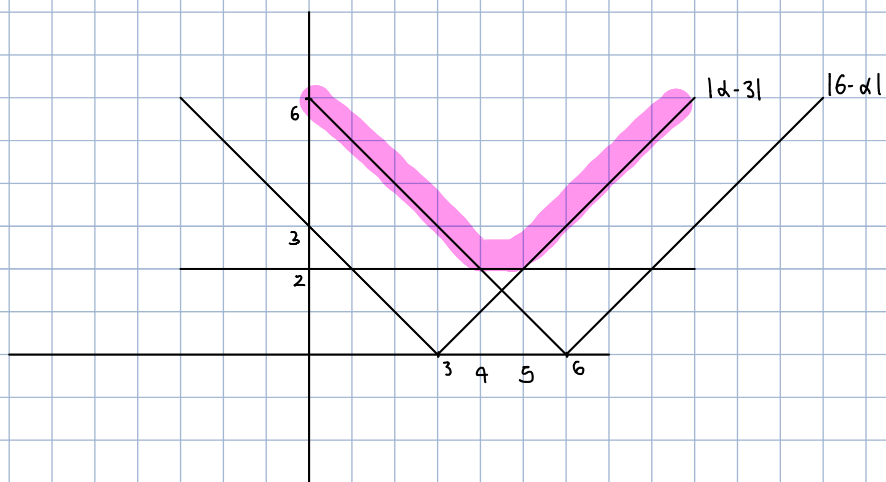
$\alpha \neq 3$

$\alpha \neq 7 \quad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{6}{\alpha-7} & 1 \end{pmatrix}$ $\quad G_2 \cdot G_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{6}{\alpha-7} & 1 \end{pmatrix} \cdot \begin{pmatrix} \alpha-3 & 2 & 6-\alpha \\ 0 & \frac{\alpha-7}{\alpha-3} & \frac{-6}{\alpha-3} \\ 0 & \frac{-6}{\alpha-3} & \frac{-\alpha^2}{\alpha-3} \end{pmatrix} = \begin{pmatrix} \alpha-3 & 2 & 6-\alpha \\ 0 & \frac{\alpha-7}{\alpha-3} & \frac{-6}{\alpha-3} \\ 0 & 0 & \frac{-36-\alpha^2(\alpha-7)}{(\alpha-7)(\alpha-3)} \end{pmatrix} = U$

$\frac{-36}{\alpha-7} - \frac{\alpha^2}{\alpha-3}$

$L = G_1^{-1} \cdot G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{\alpha-3} & 1 & 0 \\ \frac{6-\alpha}{\alpha-3} & \frac{6}{7-\alpha} & 1 \end{pmatrix}$

- al variare di α, il pivot parziale al primo passo: $\max\{|\alpha-3|, 2, |6-\alpha|\}$

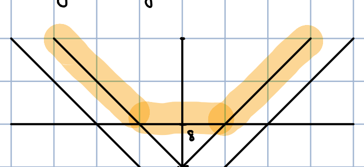


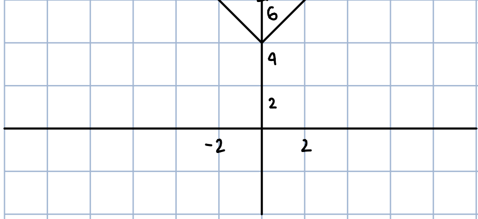
$2 \quad$ se $\quad 4 < \alpha < 5$
$|6-\alpha| \quad$ se $\quad \alpha \leq 3$
$|\alpha-3| \quad$ se $\quad \alpha \geq 5$

ESAME 18 luglio 2017

4. $A = \begin{pmatrix} -4 & 2 & \alpha \\ \alpha & 2 & -2 \\ 6 & 0 & -2 \end{pmatrix}$ $\begin{matrix} \to 4+2+|\alpha| = |\alpha|+6 \\ \to 4+|\alpha| \\ \to 8 \end{matrix}$ max tra essi

- disegna il grafico $\alpha \to \|A(\alpha)\|_\infty$





- fattorizz. LU:

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\alpha}{4} & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -4 & 2 & \alpha \\ \alpha & 2 & -2 \\ 6 & 0 & -2 \end{pmatrix} = \begin{pmatrix} -4 & 2 & \alpha \\ 0 & \frac{\alpha+4}{2} & \frac{\alpha^2-8}{4} \\ 0 & 3 & \frac{3\alpha-4}{2} \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-6}{\alpha+4} & 1 \end{pmatrix} \begin{pmatrix} -4 & 2 & \alpha \\ 0 & \frac{\alpha+4}{2} & \frac{\alpha^2-8}{4} \\ 0 & 3 & \frac{3\alpha-4}{2} \end{pmatrix} = \begin{pmatrix} -4 & 2 & \alpha \\ 0 & \frac{\alpha+4}{2} & \frac{\alpha^2-8}{4} \\ 0 & 0 & \frac{-6(\alpha^2-8)}{4(\alpha+4)} + \frac{3\alpha-4}{2} \end{pmatrix} = U$$

$$L = G_1^{-1} \cdot G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{\alpha}{4} & 1 & 0 \\ -\frac{3}{2} & \frac{6}{\alpha+4} & 1 \end{pmatrix} \qquad \alpha \neq -4$$

- per quale α $\det(A) = \det(L) \cdot \det(U) = 0 \Rightarrow$ $\det(U) = -4 \cdot \frac{\alpha+4}{2} \cdot \left(\frac{-\cancel{6}^{3}(\alpha^2-8)}{\cancel{4}_{2}(\alpha+4)} + \frac{3\alpha-4}{2} \right)$

$\alpha = -1$

$$= \frac{-3(\alpha^2-8) + 3\alpha(\alpha+4) - 4(\alpha+4)}{2(\alpha+4)}$$

$$= -\cancel{3\alpha^2} + 24 + \cancel{3\alpha^2} + 12\alpha - 4\alpha - 16 = \frac{8\alpha + 8}{2(\alpha+4)}$$

$8\alpha + 8 = 0$ se $\alpha = -1$

- per quali valori di α pivot parziale scambia al primo passo

$\max\{|-4|, |\alpha|, |6|\} =$ 6 se $\alpha \leq 6$ → scambia 1ª e 3ª
$|\alpha|$ se $\alpha > 6$ → scambia 1ª e 2ª

- sia $\alpha = -5$ calcola $PA = LU$

$$A = \begin{pmatrix} -4 & 2 & -5 \\ -5 & 2 & -2 \\ 6 & 0 & -2 \end{pmatrix} \qquad P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -4 & 2 & -5 \\ -5 & 2 & -2 \\ 6 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 0 & -2 \\ -5 & 2 & -2 \\ -4 & 2 & -2 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{5}{6} & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 0 & -2 \\ -5 & 2 & -2 \\ -4 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 0 & -2 \\ 0 & 2 & -\frac{11}{3} \\ 0 & 2 & -\frac{19}{3} \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 6 & 0 & -2 \\ 0 & 2 & -\frac{11}{3} \\ 0 & 2 & -\frac{19}{3} \end{pmatrix} = \begin{pmatrix} 6 & 0 & -2 \\ 0 & 2 & -\frac{11}{3} \\ 0 & 0 & -\frac{8}{3} \end{pmatrix} = U$$

$L = P_2 G_1^{-1} P_2 G_2^{-1}$

$= \quad P_2 \cdot G_1^{-1} = G_1^{-1} \quad \cdot \quad P_2 G_2^{-1} = G_2^{-1}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -\frac{5}{6} & 1 & 0 \\ -\frac{2}{3} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{5}{6} & 1 & 0 \\ -\frac{2}{3} & 1 & 1 \end{pmatrix}$$