ES. 5. 19.9.16

Data $A = \begin{pmatrix} -2 & 6 & 0 \\ a & 3 & 1+a \\ 0 & -3 & -a \end{pmatrix}$.

- calcola la fattorizzaz. LU:

$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{a}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad G_1 \cdot A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 3(a+1) & 1+a \\ 0 & 0 & 1 \end{pmatrix}$

$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{a+1} & 1 \end{pmatrix}$ $\quad U = G_2 G_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 3(a+1) & 1+a \\ 0 & 0 & 1-a \end{pmatrix}$ che è TRIANG. SUP

$a+1 \neq 0 \Rightarrow a \neq -1$

$L = G_1^{-1} \cdot G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{a}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-1}{a+1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{a}{2} & 1 & 0 \\ 0 & \frac{-1}{a+1} & 1 \end{pmatrix}$

La fattorizz. si può calcolare per tutti gli $a \neq -1$

- calcola il det(A). Sappiamo che $\det(A) = \det(L) \cdot \det(U) \Rightarrow$ $\det(L) = 1 \cdot 1 \cdot 1$

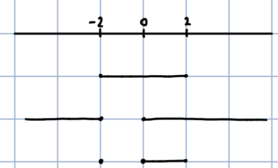
$\det(U) = -2 \cdot 3(a+1) \cdot (1-a)$

A è singolare se $\det(A) = 0 \iff a = -1 \vee a = 1$

- Per quali valori di a si ottiene $PA = LU$ con pivot parz e $P = I$?

1. $|a| \leq 2$ così non si scambiano le righe nella 1ª colonna
2. $|3(a+1)| \geq 3$ così non si scambiano le righe nella 2ª colonna

$\begin{cases} |a| \leq 2 \\ |3(a+1)| \geq 3 \end{cases} \rightarrow \begin{cases} -2 \leq a \leq 2 \\ 3|a+1| \geq 3 \end{cases} \rightarrow \begin{cases} -2 \leq a \leq 2 \\ |a+1| \geq 1 \end{cases} \rightarrow \begin{cases} -2 \leq a \leq 2 \\ a+1 \geq 1 \vee a+1 \leq -1 \end{cases} \Rightarrow -2 \leq a \leq 2 \wedge (a \leq -2 \vee a \geq 0)$ $\Rightarrow a = -2 \vee a \in [0,2]$



- se $a = -\frac{1}{3}$ $\quad A = \begin{pmatrix} -2 & 6 & 0 \\ -\frac{1}{3} & 3 & \frac{2}{3} \\ 0 & -3 & \frac{1}{3} \end{pmatrix}$ $\quad P_1 = I$ e $P_1 A = A$

non devo scambiare niente

$G_1 = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{6} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad G_1 P_1 A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & 2 & \frac{2}{3} \\ 0 & -3 & \frac{1}{3} \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

$P_2 =$ in G_1P_1A va scambiato -3 con 2 $= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$$P_2G_1P_1A = \begin{pmatrix} -2 & 6 & 0 \\ 0 & -3 & 2/3 \\ 0 & 2 & 1/3 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{pmatrix} \qquad U = \underbrace{G_2P_2G_1P_1A}_{L^{-1}\cdot P} = \begin{pmatrix} -2 & 6 & 0 \\ 0 & -3 & 1/3 \\ 0 & 0 & \frac{8}{9} \end{pmatrix} \qquad P = P_2 \cdot \overset{I}{P_1} = P_2$$

e sappiamo $PA = LU \Rightarrow U = L^{-1}PA$

$L^{-1}\cdot P = G_2P_2G_1P_1$

$L^{-1} = G_2P_2G_1P_1P^{-1}$

$L = PP_1^{-1}G_1^{-1}P_2^{-1}G_2^{-1} = P_2 \cdot \cancel{P_1 \cdot P_1^{-1}} \cdot G_1^{-1} \cdot P_2^{-1} \cdot G_2^{-1} = P_2G_1^{-1}P_2G_2^{-1}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1/6 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{2}{3} & 1 \end{pmatrix}$$

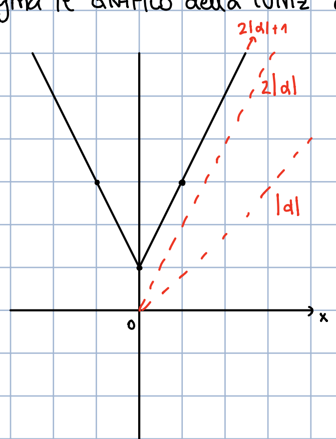
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{1}{6} & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{6} & -\frac{2}{3} & 1 \end{pmatrix}$$

- se $a = 3$ $\quad A = \begin{pmatrix} -2 & 6 & 0 \\ 3 & 3 & 4 \\ 0 & -3 & -3 \end{pmatrix} \quad P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad P_1 \cdot A = \begin{pmatrix} 3 & 3 & 4 \\ -2 & 6 & 0 \\ 0 & -3 & -3 \end{pmatrix}$ ... stessa cosa di prima

ES. 4 $\qquad$ 24.9.2018

$$A = \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & 1 & \alpha \\ 0 & \alpha & 1 \end{pmatrix} \begin{array}{l} \to 1+|\alpha| \\ \to 1+2|\alpha| \\ \to 1+|\alpha| \end{array}$$

- disegna il GRAFICO della funz $\alpha \to \|\alpha\|_\infty = \max\limits_{\substack{i\in\{1,2,3\} \\ \uparrow \\ \text{RIGHE}}} \sum_{j=1}^{3} |a_{ij}| = \max\{1+|\alpha|, 1+2|\alpha|, 1+|\alpha|\} = 1+2|\alpha|$



- per quali valori di α non esiste la fattorizz. di A

$$G_1=\begin{pmatrix}1&0&0\\-\alpha&1&0\\0&0&1\end{pmatrix}\quad G_1A=\begin{pmatrix}1&\alpha&0\\0&1-\alpha^2&\alpha\\0&0&1\end{pmatrix}\quad G_2=\begin{pmatrix}1&0&0\\0&1&0\\0&\frac{-\alpha}{1-\alpha^2}&1\end{pmatrix}$$

$1-\alpha^2\neq 0 \Rightarrow \alpha^2\neq 1 \Rightarrow \alpha\neq\pm 1$

$$U=G_2G_1A=\begin{pmatrix}1&\alpha&0\\0&1-\alpha^2&\alpha\\0&0&\frac{1-2\alpha^2}{1-\alpha^2}\end{pmatrix}\quad L=G_1^{-1}G_2^{-1}=\begin{pmatrix}1&0&0\\\alpha&1&0\\0&\frac{\alpha}{1-\alpha^2}&1\end{pmatrix}$$

$A=LU$ per $\alpha\neq\pm1$

$\frac{-\alpha^2}{1-\alpha^2}+1$

$-\ \alpha>0 \ \wedge\ \|A\|_\infty=2 \Rightarrow \begin{cases}\alpha>0\\ 1+2|\alpha|=2\end{cases} \Rightarrow \alpha=\frac{1}{2}$

$$A=\begin{pmatrix}1&\frac12&0\\\frac12&1&\frac12\\0&\frac12&1\end{pmatrix}\quad U=\begin{pmatrix}1&\frac12&0\\0&\frac34&\frac12\\0&0&\frac23\end{pmatrix}\quad L=\begin{pmatrix}1&0&0\\\frac12&1&0\\0&\frac23&1\end{pmatrix}$$

- calcolando $PA=LU$ con pivot p. si scambiano righe al primo passo se $|\alpha|>1$ ovvero $\alpha<-1 \vee \alpha>1$

$-\ \begin{cases}\alpha<0\\ \|\alpha\|_\infty=5\end{cases} \Rightarrow \begin{array}{l}1+2|\alpha|=5\\2|\alpha|=4\end{array} \Rightarrow \begin{array}{l}\alpha<0\\|\alpha|=2\end{array} \Rightarrow \alpha=-2 \Rightarrow A=\begin{pmatrix}1&-2&0\\-2&1&-2\\0&-2&1\end{pmatrix}$ e poi faccio gli scambi

esercizio APPROSSIMAZIONE

es. 5 12.02.18

$f(x)=2\log_2(x)$

$P_0=\left(\frac12, f\left(\frac12\right)\right)=\left(\frac12,-2\right)$
$P_1=(1,f(1))=(1,0)$
$P_2=(2,f(2))=(2,2)$

polinomio p interpolante P_0,P_1,P_2 nella forma di Newton

$\frac12$	-2		
		$\frac{0+2}{1-\frac12}=4$	
1	0		$\frac{2-4}{2-\frac12}=-\frac43$
		$\frac{2-0}{2-1}=2$	
2	2		

$p(x)=-2+4\left(x-\frac12\right)-\frac43\left(x-\frac12\right)(x-1)$

$p(2)=2$ ok

ERRORE in $\left[\frac12,2\right]$: $f(x)-p(x)=\frac{f^{(3)}(\xi)}{3!}\left(x-\frac12\right)(x-1)(x-2)$ $\quad \xi\in\left[\frac12,2\right]$

maggiorazione: $\max\limits_{x\in[\frac12,2]}|f(x)-p(x)|\leq\frac{\max\limits_{x\in[\frac12,2]}|f^{(3)}(x)|}{3!}\cdot\left(2-\frac12\right)^3$

$f(x)=2\log_2(x)\quad f'(x)=2\cdot\frac{1}{\ }\cdot\frac{1}{\ }\quad D(\ln x)=\frac1x$

$x \quad \ell n 2$

$$D(\log_a x) = \frac{1}{\ell n\, a} \cdot \frac{1}{x}$$

$$f'''(x) = \frac{4}{\ell n 2} \cdot \frac{1}{x^3}$$

$\vdots$

- aggiungo $P_3 = (4, f(4)) = (4, 4)$

e con forma di Newton aggiungo una riga in tabella

$$\begin{array}{cccccc}
\frac{1}{2} & -2 & & & & \\
 & & \frac{0+2}{1-\frac{1}{2}} = 4 & & & \\
1 & 0 & & \frac{2-4}{2-\frac{1}{2}} = -\frac{4}{3} & & \\
 & & \frac{2-0}{2-1} = 2 & & \frac{2}{7} & \\
2 & 2 & & \frac{1-2}{4-1} = -\frac{1}{3} & & \\
 & & \frac{4-2}{4-2} = 1 & & & \\
4 & 4 & & & &
\end{array}$$

$$\tilde{p}(x) = -2 + 4\left(x - \frac{1}{2}\right) - \frac{4}{3}\left(x - \frac{1}{2}\right)(x-1) + \frac{2}{7}\left(x - \frac{1}{2}\right)(x-1)(x-2)$$ $\rightarrow$ verifico $\tilde{p}(4) = 4$ ok

- calcola 2 polinomi di miglior approssimaz.

- $q \rightarrow P_0, P_1, P_2$ $\quad q(x) = q_0 + q_1 x$

$$\begin{pmatrix} 1 & 1 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \frac{1}{2} & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 7/2 \\ 7/2 & 21/4 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \Rightarrow \begin{cases} 3q_0 + \frac{7}{2} q_1 = 0 \\ \frac{7}{2} q_0 + \frac{21}{4} q_1 = 3 \end{cases} \Rightarrow \begin{cases} q_0 = -\frac{7}{6} q_1 \\ \frac{7}{2}\left(-\frac{7}{6} q_1\right) + \frac{21}{4} q_1 = 3 \end{cases}$$

$$\Rightarrow q_1\left(-\frac{49}{12} + \frac{21}{4}\right) = 3 = q_1 \cdot \frac{14}{12} = 3$$

$$\Rightarrow q_1 = \frac{18}{7} \qquad q_0 = -3$$

$$q(x) = -3 + \frac{18}{7} x$$

- r di P_1, P_2, P_3

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{pmatrix} \cdot \begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & 7 \\ 7 & 21 \end{pmatrix} \begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \end{pmatrix} \Rightarrow \begin{cases} 3r_0 + 7r_1 = 6 \\ 7r_0 + 21 r_1 = 20 \end{cases} \Rightarrow \begin{array}{l} r_0 = -1 \\ r_1 = 9/7 \end{array}$$

$$r(x) = -1 + \frac{9}{7} x$$

ES. 5 $\quad$ 8.2.21

$P_0 = (-1, 18)$

$P_1 = (0, 12)$

$P_2 = (2, 0)$

$\frac{12-18}{0+1} = -6$

$\frac{0-12}{2-0} = -6$

0

$p(x) = 18 - 6(x+1)$