ESAME 29.03.21

1. $F = F(2, t, e_{max}, e_{min})$

- determinare t, e_{max}, e_{min}

$$\begin{cases} realmin = 1/32 \\ realmax = 31 \\ N\mu = 10 \end{cases}$$

N: #el. di $F \neq 0$; μ: prec. macch.

$$\begin{cases} realmax = 2^{e_{max}}(1-2^{-t}) \\ real\ min = 2^{-e_{min}-1} \\ N = 2(2-1)\cdot 2^{t-1}(e_{max}+e_{min}+1) \\ \quad = 2^t(e_{max}+e_{min}+1) \\ \mu = \frac{2^{1-t}}{2} = 2^{-t} \end{cases}$$

REAL MIN $\Rightarrow \frac{1}{32} = 2^{-e_{min}-1} \Rightarrow 2^{-5} = 2^{-e_{min}-1}$

$\Rightarrow e_{min} = 4$

$N\cdot\mu = 10 \Rightarrow \overbrace{2^{-t}}^{\mu} \cdot \overbrace{2\cdot 2^{t-1}(e_{max}+e_{min}+1)}^{N} = 10$

$\Rightarrow e_{max} + e_{min} + 1 = 10$

$\Rightarrow e_{max} = 5$

$$31 = 2^5(1-2^{-t})$$

$$\Rightarrow 31 = 32(1-2^{-t})$$

$$\Rightarrow -1 = -2^{-t}\cdot 32 \Rightarrow 2^{-t} = \frac{1}{32} \Rightarrow 2^{-t} = 2^{-5} \Rightarrow t = 5$$

- dati $x = (1,\overline{101})_2$ $y = (10,\overline{101})_2$

trova $\tilde{x} = fl(x) \in \mathbb{F}$

$\tilde{y} = fl(y) \in \mathbb{F}$

$\tilde{z} = \tilde{y}\ fl(-)\ \tilde{x} \in \mathbb{F}$

$\tilde{x} = 0,1\overline{101}\cdot 2^1 \Rightarrow 0,11011|01\cdot 2^1 \Rightarrow 0,11011\cdot 2^1$

$\tilde{y} = 0,10\overline{101}|\overline{101}\cdot 2^2 \Rightarrow 0,10110\cdot 2^2$

$\tilde{z} = 0,10110\cdot 2^2 - 0,11011\cdot 2^1$

$= 0,10110\cdot 2^2 - 0,011011\cdot 2^2$

$= fl\left(2^2\cdot\begin{pmatrix} 0,101100\ - \\ 0,011011 \\ \hline 0,010001\ = \end{pmatrix}\right) = fl(2\cdot 0,10001) = (0,10001)_2\cdot 2$

- Determina e (esp.) tale che $\tilde{z} - 2^e realmin = 2\mu$ $\quad \mu = 2^{-5}$

$$\underbrace{(0,10001)_2\cdot 2}_{} - 2^e\cdot 2^{-5} = 2^{-4}$$

$$\left(1+2^{-4}\right)-2^{e-5}=2^{-4}$$

$$1+\cancel{2^{-4}}=\cancel{2^{-4}}+2^{e-5}$$

$$2^{e-5}=1 \quad \Rightarrow \quad e=5$$

2. $y=f(g(x))$

$$\frac{d}{dt}\left(\text{sen}(x^2)\right)=2x\cdot\cos(x^2)$$

- num cond: $\quad \text{cond}: \dfrac{|F'|\cdot|x|}{|F|}=\dfrac{|g'(x)\cdot f'(g(x))|\cdot|x|}{|f(g(x))|}$

- siano $f(x)=\sqrt{x}$ $\quad g(x)=x^4-1$ $\qquad \sqrt{x^4-1}$ ha DOM: $x\neq 0$

  studia il cond F

$$\cancel{\text{cond}:\ \frac{|x|\cdot\left|4x^3\cdot\frac{d}{dt}\left(\sqrt{x^4-1}\right)\right|}{\left|\sqrt{x^4-1}\right|}=\frac{|x|\cdot\left|4x^3\cdot 4x^3\cdot\frac{1}{2}(x^4-1)\right|}{\left|\sqrt{x^4-1}\right|}}$$

$$\text{cond } g(x):\ \frac{|4x^3|\cdot|x|}{|x^4-1|}=\frac{4x^4}{|x^4-1|} \qquad \text{cond } f(x):\ \frac{|x|\cdot\left|\frac{1}{2\sqrt{x}}\right|}{|\sqrt{x}|}=\frac{\frac{1}{2}\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}=\frac{1}{2}$$

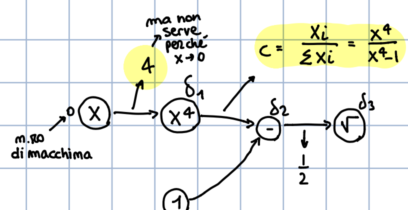
$$\text{cond } F:\ \frac{|g'(x)\cdot f'(g(x))|\cdot|x|}{|f(g(x))|}\cdot\frac{|g(x)|}{|g(x)|}=\underbrace{\frac{|g'(x)|\cdot|x|}{|g(x)|}}_{\text{cond}_{g(x)}}\cdot\underbrace{\frac{|f'(g(x))|\cdot|g(x)|}{|f(g(x))|}}_{\text{cond}_{f(g(x))}}$$

$$\text{cond } F=\underbrace{\text{cond } f(g(x))}_{\frac{1}{2}}\cdot\text{cond } g(x)=\frac{1}{\cancel{2}}\cdot\frac{\cancel{4}x^4}{|x^4-1|}=\frac{2x^4}{|x^4-1|}$$

- Supponi che $\sqrt{x}$ fornisca un'approssimaz. che ha $\varepsilon_{REL}<u$
  Studia la stabilità in avanti di F (con x numero di macchina)

F: $\sqrt{x^4-1}$

$$c=\frac{x_i}{\Sigma x_i}=\frac{x^4}{x^4-1}$$



$$\varepsilon_{ALGO}=\delta_3+\frac{1}{2}\left(\delta_2+\frac{x^4}{x^4-1}\cdot\delta_1\right)$$

$$\leq |u|+\left|\frac{1}{2}\left(u+x^4\cdot u\right)\right|$$

$$\leq |\mu| + \frac{1}{2}|\mu| + \left|\frac{x^4}{x^4-1}\right| |\mu| \cdot \frac{1}{2}$$

$$\leq |\mu| \left(\frac{3}{2} + \frac{1}{2}\left|\frac{x^4}{x^4-1}\right|\right) \Rightarrow \text{è INSTABILE se } x \approx \pm 1$$

ESAME 09.09.20

1. $F = F(2, t, e_{max}, e_{min})$ con ARROTONDAMENTO

– determina t, e_{max}, e_{min} in modo che
$$\begin{cases} realmin = 1/16 \\ e_{max} = t \\ \frac{realmax}{\mu} = 56 \end{cases}$$

so che $realmin = B^{-e_{min}-1}$

$\Rightarrow 2^{-4} = 2^{-e_{min}-1} \Rightarrow e_{min} = 3$

so che $realmax = B^{e_{max}}(1 - B^{-t})$

$\Rightarrow 56 = 2^t(1-2^{-t}) \cdot \frac{1}{\mu}$

$\Rightarrow 56 = (2^t - 1) \frac{1}{\mu}$ e $\mu = \frac{B^{1-t}}{2} = \frac{2^{1-t}}{2} = 2^{-t}$

$\Rightarrow 56 = (2^t - 1) \cdot 2^t$

$\Rightarrow 56 = 2^{2t} - 2^t$ $\quad y = 2^t \Rightarrow y^2 - y - 56 = 0$ $\quad y_{1,2} = \frac{1 \pm \sqrt{1+56\cdot 4}}{2} = 8, -7 \Rightarrow y = 8$

$\Rightarrow t = 3 \Rightarrow e_{max} = 3$

– $x = (1,\overline{101})_2$
$y = (10,\overline{101})_2$

$\tilde{x} = fl(x) = 0,1\overline{101} \cdot 2 \Rightarrow (0,111)_2 \cdot 2$

$\tilde{y} = fl(y) = 0,10\overline{101} \cdot 2^2 \Rightarrow (0,101)_2 \cdot 2^2$

$$\tilde{z} = \tilde{x}\ fl(+)\ \tilde{y} = fl\left(\left(\begin{array}{r} 0,0111 \ + \\ 0,1010 \\ \hline 1,0001 \ = \end{array}\right) \cdot 2^2\right) = (0,100)_2 \cdot 2^3$$

scrivi $\tilde{x}, \tilde{y}, \tilde{z}$ come frazioni in base 10: $\quad \tilde{x}: \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) \cdot 2 = 1 + \frac{1}{2} + \frac{1}{4} = \frac{7}{4}$

$\tilde{y}: \left(\frac{1}{2} + \frac{1}{8}\right) \cdot 2^2 = 2 + \frac{1}{2} = \frac{5}{2}$

$\tilde{z}: \ \frac{1}{2}\cdot 2^3 = 4$

- determina e tale che $\tilde{z}\cdot 2^e = \text{realmin}$

$$\Rightarrow \underset{\substack{\downarrow \\ 2^{-1}}}{(0{,}100)_2}\cdot 2^3 \cdot 2^e = 2^{-4}$$

$$\Rightarrow 2^2\cdot 2^e = 2^{-4} \Rightarrow 2^{2+e} = 2^{-4} \Rightarrow e = -6$$

- quanto fa realmax $- \tilde{z}$?

$$\Rightarrow \underbrace{2^3-1}_{\text{realmax}} - 2^2 = 3$$

2. $y = f(x) \quad f(x) = e^{g(x)}$

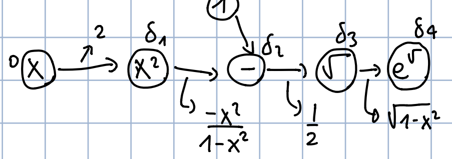
- $\text{cond}_y = \dfrac{|g'(x)|\cdot \cancel{|e^{g(x)}|}\cdot|x|}{\cancel{|e^{g(x)}|}}$ e se consideriamo $\text{cond}_{g(x)} = \dfrac{|g'(x)|\cdot|x|}{|g(x)|}$ allora $\text{cond}_f = \text{cond}_{g(x)}\cdot|g(x)|$

$g(x) = \sqrt{1-x^2} \qquad \text{dom}: -1 \leq x \leq 1$

allora $\text{cond}_{g(x)} = \left|\dfrac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\right| \dfrac{\cdot|x|}{|\sqrt{1-x^2}|} = \dfrac{x^2}{|1-x^2|}$

e $\text{cond}_f = \dfrac{x^2}{|1-x^2|}\cdot|\sqrt{1-x^2}| = \dfrac{x^2}{|\sqrt{1-x^2}|}$ è MALCONDIZIONATO se x vicino a 1 e -1

- studia la stabilità in AVANTI



$$\epsilon_{ALGO} = \delta_4 + \sqrt{1-x^2}\left(\delta_3 + \frac{1}{2}\left[\delta_2 - \frac{x^2}{1-x^2}\delta_1\right]\right)$$

$$= \delta_4 + \sqrt{1-x^2}\left(\delta_3 + \frac{1}{2}\delta_2 - \frac{1}{2}\cdot\frac{x^2}{1-x^2}\delta_1\right)$$

$$\leq |\delta_4| + |\sqrt{1-x^2}|\left[|\delta_3| + \frac{1}{2}|\delta_2| - \frac{1}{2}\left|\frac{x^2}{1-x^2}\right||\delta_1|\right]$$

$$\leq |\mu| \left(1 + |\sqrt{1-x^2}| + \frac{|\sqrt{1-x^2}|}{2} - \frac{1}{2}\left|\frac{x^2}{1-x^2}\right| \cdot |\sqrt{1-x^2}|\right)$$

ESAME 08.02.21

1 - $F = (2, t, e_{max}, e_{min})$ con ARROTONDAMENTO

$$\begin{cases} realmin = 1/64 \\ realmax = 15 \\ N \cdot \mu = 5 \end{cases}$$

$\hookrightarrow$ # el. >0

$realmin = 2^{-e_{min}-1} \Rightarrow 2^{-6} = 2^{-e_{min}-1} \Rightarrow e_{min} = 5$

$realmax = 2^{e_{max}}(1-2^{-t})$

$N = (B-1) \cdot B^{t-1}(e_{max} + e_{min} + 1)$

$\overset{!}{=} 2^{t-1}(e_{max} + e_{min} + 1)$

$\mu = \frac{2^{1-t}}{2} = 2^{-t}$

$2^{t-1}(e_{max} + e_{min} + 1) \cdot 2^{-t} = 5$

$\frac{1}{2} \cdot (e_{max} + 5 + 1) = 5$

$\frac{e_{max}}{2} + 3 = 5 \Rightarrow \frac{e_{max}}{2} = 2 \Rightarrow e_{max} = 4$

$15 = 2^4(1-2^{-t}) \Rightarrow 15 = 2^4 - 2^{-t+4} \Rightarrow 2^{-t+4} = 1 \Rightarrow -t+4 = 0 \Rightarrow t = 4$

- $x = (10,\overline{101})_2$

$y = (11,\overline{101})_2$

$\tilde{x} \Rightarrow 0,10\overline{101} \cdot 2^2 \Rightarrow (0,1011)_2 \cdot 2^2$

$\tilde{y} \Rightarrow 0,11\overline{101} \cdot 2^2 \Rightarrow (0,1111)_2 \cdot 2^2$

$\tilde{z} = 2\tilde{x}\ fl(-)\ \tilde{y} \Rightarrow 2\tilde{x} \Rightarrow$

$$\begin{array}{r} 0,1011 \ + \\ 0,1011 \ = \\ \hline 1,0110 \end{array} \Rightarrow \begin{array}{r} 1,0110 \ - \\ 0,1111 \ = \\ \hline 0,0111 \end{array}$$

$\Rightarrow 0,0111 \Rightarrow (0,1110)_2 \cdot 2^{-1}$

- scrivi $x, y, \tilde{x}, \tilde{y}$ come frazioni in base 10

$\tilde{x} \Rightarrow (0,1011)_2 \cdot 2^2 \Rightarrow 4 \cdot \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{16}\right) = 2 + \frac{1}{2} + \frac{1}{4} = \frac{11}{4}$

$\tilde{y} \Rightarrow (0,1111)_2 \cdot 2^2 \Rightarrow 4 \cdot \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) = 2 + 1 + \frac{1}{2} + \frac{1}{4} = \frac{15}{4}$

$x \Rightarrow (10,\overline{101})_2 \Rightarrow 2 +$

- trova e (minimo) tale che $\tilde{z} \cdot 2^e \in F$

$(0,1110)_2 \cdot 2^{-1}$ $e = -4$? così arriva al max a 2^{-5} ↳ emin

$f(x)^m = m \cdot f(x)^{m-1}$ $(f(x))^m$

$(3x)^4$ $4(3x)^3 \cdot 3$

2. $f(x)$ funz.
$m > 1$ intero

- se $F = f(x)^m$ scrivi il num. condiz. $\Rightarrow$ $cond_F = \dfrac{|x| \cdot |m \cdot \cancel{f(x)^{m-1}}| \cdot |f'(x)|}{|\cancel{f(x)^m}|} = \dfrac{|m| \cdot |x| \cdot |f'(x)|}{|f(x)|}$

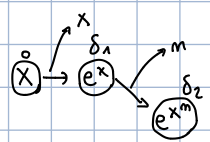
- se $G = f(x^m)$ " " $\Rightarrow$ $cond_G = \dfrac{|x| \cdot |m \cdot x^{m-1}| \cdot |f'(x^m)|}{|f(x^m)|}$

- considera $f(x) = e^x$. Per quali x risulta $cond_F(x) > cond_G(x)$

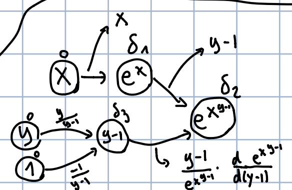
$cond_F = \dfrac{|m| \cdot |x| \cdot \cancel{|e^x|}}{\cancel{|e^x|}}$ $cond_G = \dfrac{|x| \cdot |m \cdot x^{m-1}| \cdot \cancel{|e^{x^m}|}}{\cancel{|e^{x^m}|}} = |m| \cdot |x^m|$ $\Rightarrow$ $\cancel{m} \cdot x > \cancel{m} \cdot x^m$

$0 > x^m - x \Rightarrow x(x^{m-1} - 1) < 0$ ↳ $x_1 = 0$, $x_2 = \sqrt[m-1]{1} = 1 \; 0 \pm 1$

- stabilità:



es. a parte



$\delta_2 + (y-1)(\delta_1 + x \cdot 0) + u(\delta_3 + y \ldots + -1 \ldots)$

$\varepsilon_{ALGO} = \delta_2 + m(\delta_1) + x \cdot 0$
$\leq |\underset{u}{\delta_2}| + |m| \cdot |\underset{u}{\delta_1}|$ $\Rightarrow |\varepsilon_{ALGO}| \leq |u|(1 + |m|)$ instabile se $|m| \gg 1$

se $m = 50$ quante cifre sig. potresti avere in meno rispetto a quelle garantite da u?

ESAME 12.02.18

1. $F = (2, t, e_{max}, e_{min})$ con ARROTONDAMENTO

$\begin{cases} t = e_{min} + 1 \\ realmin = 1/32 \\ realmax = 62 \end{cases}$ $\Rightarrow$ $2^{-e_{min}-1} = 2^{-5} \Rightarrow -e_{min} - 1 = -5 \Rightarrow e_{min} = 4 \Rightarrow t = 5$

$2^{e_{max}}(1 - 2^{-t}) = 62 \Rightarrow 2^{e_{max}} = \dfrac{62}{1 - 2^{-5}} \Rightarrow 2^{e_{max}} = \dfrac{\cancel{62}^{\,2}}{\cancel{31}_{\,1}} \cdot 32 \Rightarrow 2^{e_{max}} = 2^6 \Rightarrow e_{max} = 6$

↳ $1 - \dfrac{1}{32} = \dfrac{31}{32}$

- $|F| = 1 + \overset{\text{POSITIVO/NEG}}{2}(2-1) + 2^{t-1}(e_{max} + e_{min} + 1)$
$= 1 + 2 \cdot 2^4 (4 + 6 + 1) = 1 + 2^5 (11) = 1 \cdot 32 \cdot 11 = 353$

- i numeri denormalizzati sono quelli con 0,0... con esponente = e_{min} $\Rightarrow \pm(0,0 d_2 d_3 \dots)_B \cdot B^{-e_{min}}$

$|den| = \cancel{2^{t-1}(e_{min}) = 2^4 \cdot 4 = 2^6 = 64}$

$= 2 \cdot (\underset{\substack{\downarrow \\ \text{quando} \\ \text{sono tutti} = 0}}{2^{t-1} - 1}) \Rightarrow 2(2^4 - 1) = 30$

- gli elementi positivi di F sono: $1 + (2-1) + 2^{t-1}(e_{max} + e_{min} + 1)$

$= 1 + 1 + 2^4(11) = 2 + 176 = 178$

- $u = \frac{B^{1-t}}{2}$ perché c'è arrotondamento. $u = \frac{2^{1-5}}{2} = 2^{-5} = \frac{1}{32}$

- $x = (1,\overline{01})_2$ $\qquad \tilde{x} = (1,\overline{01})_2 = (0,1\overline{01})_2 \cdot 2 = (0,10101)_2 \cdot 2$

$y = (10,\overline{01})_2$ $\qquad \tilde{y} = (10,\overline{01})_2 = (0,10011) \cdot 2^2$

- $x, y, \tilde{x}, \tilde{y}$ come interi in base 10 e gli errori relativi di numeri interi in base 10

$\tilde{x} = 2 \cdot \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{32}\right) = 1 + \frac{1}{4} + \frac{1}{16} = \frac{16+4+1}{16} = \frac{21}{16}$

$\tilde{y} = 2^2\left(\frac{1}{2} + \frac{1}{16} + \frac{1}{32}\right) = 2 + \frac{1}{4} + \frac{1}{8} = \frac{16+2+1}{8} = \frac{19}{8}$

$y = \frac{(1001)_2 - (10)_2}{(11)_2} = \frac{(111)_2}{(11)_2} = \frac{7}{3}$

$x =$ metodo della fraz. generatrice

num. senza punti e periodi

$= \frac{(101)_2 - (1)_2}{(11)_2}$ → le cifra prima del periodo

↳ metto tanti uni quanto è lungo il PERIODO

$= \frac{(100)_2}{(11)_2} = \frac{4}{3}$

$E_{Rel\,x} = \frac{|x - \tilde{x}|}{|x|} = \frac{\left|\frac{4}{3} - \frac{21}{16}\right|}{\left|\frac{4}{3}\right|}$

$E_{Rel\,y} = \frac{|y - \tilde{y}|}{|y|} = \frac{\left|\frac{7}{3} - \frac{19}{8}\right|}{\left|\frac{7}{3}\right|}$

- Determina $\tilde{z} = \tilde{x}\, fl(+)\, \tilde{y} \in F$ $\qquad \tilde{z} = fl\begin{pmatrix} 0,010101 \cdot 2^2 \to \tilde{x} \\ 0,100110 + \cdot 2^2 \to \tilde{y} \\ \hline 0,111011 \end{pmatrix} \Rightarrow (0,11110)_2 \cdot 2^2 \Rightarrow 2^2 \cdot \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right)$

$= 2 + 1 + \frac{1}{2} + \frac{1}{4} = 3 + \frac{3}{4} = \frac{15}{4}$

calcola e.t.c. $\tilde{z} <$ realmin $< \tilde{z} \Rightarrow \frac{15}{4} < \frac{1}{4} < \frac{15}{4}$

$\frac{\quad}{2^{e+1}} \quad \frac{\quad}{2^{e}} \quad \frac{\quad}{2^{e+1}} \quad \frac{\quad}{32} \quad \frac{\quad}{2^{e}}$

$\Rightarrow \frac{15}{2^{e+3}} < \frac{1}{32} < \frac{15}{2^{e+2}}$ moltiplico per $2^{e+3} \Rightarrow 15 < 2^{e+3-5} < 15 \cdot 2$

$\Rightarrow 15 < 2^{e-2} < 30$ e l'unico compreso è

$2^{e-2} = 16 \Rightarrow e-2 = 4 \Rightarrow e = 6$

2. $y = f(x)$

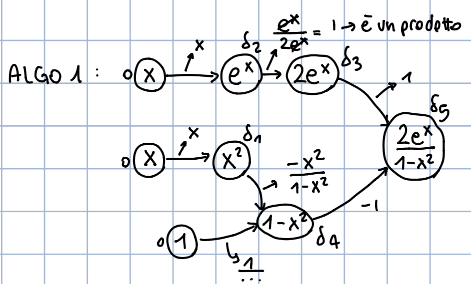
- $\epsilon_{IN} = \frac{|f(\tilde{x}) - f(x)|}{|f(\tilde{x})|}$ $\quad cond = \frac{|f'(x)| \cdot |x|}{|f(x)|}$

- $f(x) = \frac{2e^x}{1-x^2}$ studia il condizionamento: $f'(x) = \frac{2e^x(1-x^2) - 2e^x(-2x)}{(1-x^2)^2} = \frac{2e^x(1-x^2+2x)}{(1-x^2)^2}$

$$cond_{f(x)} = \frac{\cancel{2e^x}(1-x^2+2x)}{(1-x^2)^2} \cdot \frac{x}{\cancel{2e^x}} \Rightarrow \text{è MAL CONDIZ. se } x = \pm 1$$

- $\epsilon_{ALGO} = \frac{|f(\tilde{x}) - \tilde{f}(\tilde{x})|}{|f(\tilde{x})|}$ e un algo è stabile se

- studia la stabilità di $\overset{\text{ALGO 1}}{\frac{2e^x}{1-x^2}} = \overset{\text{ALGO 2}}{e^x\left(\frac{1}{1-x^2} + \frac{1}{1+x}\right)}$

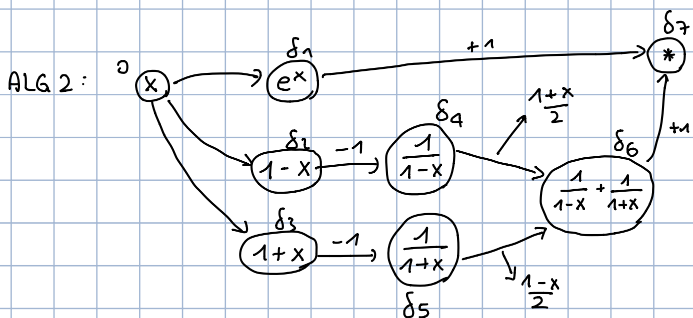


$\epsilon_{ALGO} = \cancel{\delta_5\left(1 + \delta_3\left(1 + \delta_2\right) - \delta_4\left(\frac{-x^2}{1-x^2} + \delta_1\right)\right)}$

$= \delta_5 + \delta_3 + \delta_2 - \delta_4 - \frac{x^2}{1-x^2}\delta_1$

$|\epsilon_{ALGO}| = |\delta_5| + |\delta_3| + |\delta_2| + |\delta_4| + \left|\frac{x^2}{1-x^2}\right||\delta_1|$

$= |u|\left(4 + \left|\frac{x^2}{1-x^2}\right|\right) \to$ mal cond se $x = \pm 1$



$\frac{\frac{1}{1-x}}{\frac{1}{1-x} + \frac{1}{1+x}} = \frac{1+x}{2}$

$\epsilon_{ALG2} = \delta_7 + \delta_1 + \delta_6 + \frac{1+x}{2}\left(\delta_4 - \delta_2\right) + \frac{1-x}{2}\left(\delta_5 - \delta_3\right)$ $\quad |\delta_i| \leq u$

$|\epsilon_{ALG2}| \leq |\delta_7| + |\delta_1| + |\delta_6| + |1+x|\left(|\delta_4| + |\delta_2|\right) + |1-x|\left(|\delta_5| + |\delta_3|\right)$

$$\leq 3u + \frac{|1+x|}{\cancel{2}}\cancel{2}u + \frac{|1-x|}{\cancel{2}}\cancel{2}u = u\left(|1+x|+|1-x|+3\right)$$

ALG 2 è INSTABILE se $|x|$ è molto grande

ESAME 9.4.20

$- g(x) = \sqrt{z - \sqrt{z^2+x}} \qquad z>0 \qquad D: \; z - \sqrt{z^2+x} \geq 0 \quad z \geq \sqrt{z^2+x} \Rightarrow z^2+x \geq 0 \quad x \geq -z^2$

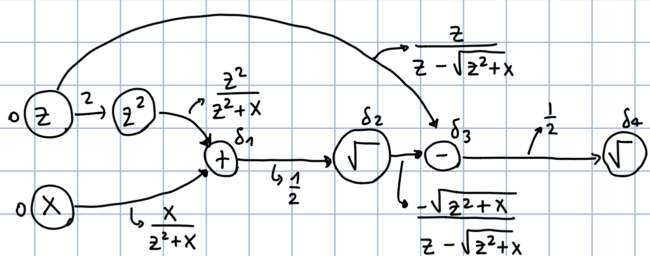
$\hookrightarrow > 0$

studia $\text{cond}_{e^{g(x)}} = \dfrac{|x| \cdot |f'(x)|}{|f(x)|} = \dfrac{|x| \cdot |g'(x)| \cancel{|e^{g(x)}|}}{\cancel{|e^{g(x)}|}} = |x| \cdot |g'(x)| = \dfrac{|x|}{2\left|\sqrt{z-\sqrt{z^2+x}}\right| \cdot 2\left|\sqrt{z^2+x}\right|}$

$$g'(x) = \frac{1}{2\sqrt{z-\sqrt{z^2+x}}} \cdot \frac{-1}{2\sqrt{z^2+x}} \cdot 1$$

$\rightarrow$ mal condiz. se $x=0$ o se $x = -z^2$

calcolo $g(x)$ con grafo:



$$|\epsilon_{ALGO}| = |\delta_4| + \frac{1}{2}\left(|\delta_3| - \frac{\sqrt{z^2+x}}{z-\sqrt{z^2+x}}\left(|\delta_2| + \frac{|\delta_1|}{2}\right)\right)$$

$$\leq |u| + \frac{1}{2}|u| - \frac{\sqrt{z^2+x}}{z-\sqrt{z^2+x}}|u| - \frac{\sqrt{z^2+x}}{2(z-\sqrt{z^2+x})} \cdot |u|$$

$$\leq |u|\left(\frac{3}{2} - \frac{3\sqrt{z^2+x}}{2(z-\sqrt{z^2+x})}\right)$$