ESAME 8.02.21

1. $F = (2, t, e_{max}, e_{min})$ con ARROT.

- $\begin{cases} realmin = 1/64 \\ realmax = 15 \\ N \cdot u = 5 \end{cases}$

  ↳ elementi >0

$realmin = \beta^{-p_{min}-1} = 2^{-p_{min}-1} = 2^{-6} \Rightarrow e_{min} = 5$

$realmax = \beta^{e_{max}}(1-\beta^{-t}) \Rightarrow 2^{e_{max}}(1-2^{-t}) = 15$

$(\beta-1)\beta^{t-1}(e_{max}+e_{min}+1) \cdot \frac{\beta^{1-t}}{2} = 5$

$2^{t-1}(e_{max}+6) \cdot 2^{-t} = 5$

$\frac{1}{2}(e_{max}+6) = 5 \Rightarrow e_{max} = 4$

$2^4(1-2^{-t}) = 15 \Rightarrow 16 - 2^{-t+4} = 15 \Rightarrow -t+4 = 0 \Rightarrow t = 4$

- $x = (10,\overline{101})_2$

  $y = (11,\overline{101})_2$

$\tilde{x} = (0,10\overline{101})_2 \cdot 2^2 = (0,1011)_2 \cdot 2^2$

$\tilde{y} = (0,11\overline{101})_2 \cdot 2^2 = (0,1111)_2 \cdot 2^2$

$\tilde{z} = 2\tilde{x}\ fl(-)\ \tilde{y}$

$$2\tilde{x} = \begin{array}{r} 0,1011 \\ +\ 0,1011 \\ \hline 1,0110 \end{array} \qquad 2\tilde{x}\ fl(-)\ \tilde{y} = \left(\begin{array}{r} 1,0110 \\ -\ 0,1111 \\ \hline 0,0111 \end{array}\right) \Rightarrow (0,0111) \cdot 2^2 = (0,1110)_2 \cdot 2$$

- In base 10: $\tilde{x} = 4\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{16}\right) = 2+\frac{1}{2}+\frac{1}{4} = \frac{11}{4}$

$\tilde{y} = 4\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right) = 2+1+\frac{1}{2}+\frac{1}{4} = \frac{15}{4}$

$x = \frac{(10101)_2 - (10)_2}{(111)_2} = \frac{21-2}{7} = \frac{19}{7}$

- esponente min e tale che $\tilde{z} \cdot 2^e \in F \Rightarrow e = -6$ così $2^{1-6} = 2^{-5}$

2. $F(x) = [F(x)]^m$

$G(x) = f(x^m)$

- scrivi cond$_F$ e cond$_G$: $\quad \text{cond}_F = \dfrac{|x|\cdot|F'(x)|}{|F(x)|} = \dfrac{|x|\cdot|n\cdot \cancel{f(x)^{n-1}}|\cdot|f'(x)|}{\cancel{|f(x)^{n}|}} = \dfrac{|x|\cdot|n|\cdot|f'(x)|}{|f(x)|}$

$$\text{cond}_G = \frac{|x|\cdot|f'(x^n)|\cdot|n\cdot x^{n-1}|}{|f(x^n)|} = \frac{|f'(x^n)|\cdot|n\cdot x^n|}{|f(x^n)|}$$

- $f(x) = e^x$. Per quali x risulta $\text{cond}_F(x) > \text{cond}_G(x)$

$F = e^{x^n}$
$G = e^{x^n}$

$$\text{cond}_F = \frac{|x|\cdot|n|\cdot\cancel{|e^x|}}{\cancel{|e^x|}} = |x|\cdot|n|$$

$\Rightarrow \quad |x|\cdot\cancel{|n|} > \cancel{|n|}\cdot|x^n|$

$$\text{cond}_G = \frac{\cancel{|e^{x^n}|}\cdot|n\cdot x^n|}{\cancel{|e^{x^n}|}} = |n|\cdot|x^n|$$

$|x^n| - |x| < 0 \qquad x(x^{n-1}-1) < 0$

$x^{n-1} < 1 \qquad \pm\sqrt[n-1]{1} = \pm 1$

$\Rightarrow -1 < x < 1$

- $0\; \underset{\delta_0}{x} \xrightarrow{|x|} \underset{\delta_1}{e^x} \xrightarrow{|n|} \underset{\delta_2}{e^{x^n}}$

$|\epsilon_{ALGO}| = |\delta_2| + |n|\cdot(|\delta_1| + |x|\cdot 0)$

$\leq |u| + |n|\cdot(|u| + 0)$

$\leq |u|\,(|n|+1) \qquad$ se $n \gg 1$ è instabile

- e^{x^n} e calcola stabilità

$0\; \underset{\delta_0}{x} \xrightarrow{|n|} \underset{\delta_1}{x^n} \xrightarrow{|x^n|} \underset{\delta_2}{e^{x^n}}$

$|\epsilon_{ALGO}| = |\delta_2| + |x^n|\,(|\delta_1| + |n|\cdot 0)$

$= |\delta_2| + |x^n|\cdot|\delta_1|$

$\leq |u|\,(1 + |x^n|) \qquad$ se $x > 1$ è instabile

3. $f(x) = \frac{1}{2}(x^3 - x^2) - 4x + 6 = \frac{x^3}{2} - \frac{x^2}{2} - 4x + 6$

$\lim_{x\to+\infty} f(x) = +\infty \qquad \lim_{x\to-\infty} f(x) = -\infty$

$f'(x) = \frac{3x^2}{2} - x - 4 = 3x^2 - 2x - 8 \qquad x_{1,2} = \frac{2 \pm \sqrt{4+96}}{6} = \frac{2 \pm 10}{6} \begin{cases} 2 \\ -\frac{4}{3} \end{cases}$ RAD. MULTIPLA

$f'(x) > 0$ se $x < -\frac{4}{3} \lor x > 2$
$= 0$ se $x = -4/3 \lor x = 2$
< 0 se $-4/3 < x < 2$

$f(0) = 6$

$x = 2 \; d \circ \beta$

$f(2) = 0$

$-\,32 \cdot 1 + 8 - 16 + 6$

$x = 2$ PT. MIN

$f(-4/3) = \quad \frac{\;}{3^3} \quad \frac{\;}{2} \quad \frac{\;}{3^2} \quad \frac{\;}{3}$

$= \frac{-32}{27} - \frac{8}{9} + \frac{16}{3} + 6 = \frac{-32-24+144+122}{27} = \frac{250}{27} \approx 10$

$x = -4/3$ pt. MAX

$f''(x) = 6x - 2$

$f''(x) > 0$ se $x < 1/3$
$= 0$ se $x = 1/3$ $\quad x = 1/3$ FLESSO $\quad f(1/3) =$
< 0 se $x > 1/3$

$= \frac{1}{54} - \frac{1}{18} - \frac{4}{3} + 6 = \frac{1-3-72+324}{54} > 0$

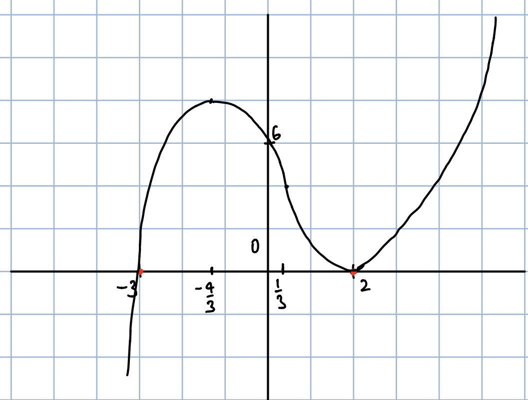
	$\frac{1}{2}$	$-\frac{1}{2}$	-4	6
2		$+1$	$+1$	-6
	$\frac{1}{2}$	$\frac{1}{2}$	-3	0

$\Rightarrow (x-2)\left(\frac{1}{2}x^2 + \frac{1}{2}x - 3\right) = 0$

$\downarrow$

$x^2 + x - 6 = 0 \quad , x_2 = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \begin{cases} 2 \\ -3 \end{cases}$

$(x-2)^2(x+3) \qquad \alpha = -3 \quad \beta = 2$



- convergenza ad α

- $x_0 \in]-\infty, -3[$ $\quad f \cdot f'' > 0 \Rightarrow$ converge superlineare mon.

- $x_0 \in]-3, -\frac{4}{3}[$ converge dal passo x_1 superlin. mon.

- $x_0 = -3$ OK

- $x_0 = -4/3$ NON CONVERGE

- convergenza a β

$x_0 \in]-4/3, 1/3[$ $\quad f \cdot f'' < 0$ ma da x_1 converge in maniera monotona lineare $\ell = 1/2$

$x_0 \in]1/3, 2[$ $\quad f \cdot f'' > 0$ conv. monotona lineare $\ell = 1/2$

$x_0 = 2$ ok

$x_0 = 1/3$ NO

$x_0 \in]2, +\infty[$ $\quad f \cdot f'' > 0$ conv. monotona lineare $\ell = 1/2$

a) si, α, slineare da x_1 $\qquad$ d) si, β, lineare

b) si, α, superl. $\qquad$ e) si, β, lin.

c) no

f) si, β, lin da x_1

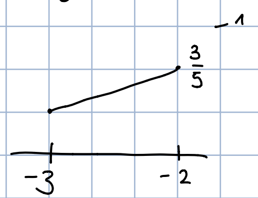
$g(x) = x - \frac{f(x)}{m}$ $\quad \alpha = -3 \quad \beta = 2 \qquad g(\alpha) = \alpha \quad g(\beta) = \beta$

- trova m tale che converge ad α con $\ell = 1/6$ $\quad f'(\alpha) \cdot \frac{3x^2}{2} - x - 4 \Rightarrow \frac{f'(\alpha)}{2} = 3x^2 - x - 8$

$g'(x) = 1 - \frac{f'(\alpha)}{m} \Rightarrow \frac{1}{6} = 1 - \frac{3x^2 - 2x - 8}{2 \cdot m} \Rightarrow \frac{1}{6} = 1 - \frac{25}{2m} \Rightarrow -\frac{5}{6} = -\frac{25}{2m} \Rightarrow 2m = -\cancel{25}^{5} \cdot -\frac{6}{\cancel{5}} = 30 \Rightarrow m = 15$

con $x_0 = -2$ è convergente? $\quad g'(-2) = 1 - \frac{f'(-2)}{15} = 1 - \frac{4}{15} = \frac{3}{5} < 1$

$g'(x) \in [-2, \alpha]$ $\quad g'(\alpha) = 1 - \frac{f'(\alpha)}{15} = 1 - \frac{25}{30} = \frac{1}{6} < 1$



$g''(x) = \frac{-f''(x)}{m}$ quando $-f''(x) > 0$ quando $f''(x) < 0 \Rightarrow x < 1/3$

g'' lo studi dove è negativo $\Rightarrow$ è convergente ad α

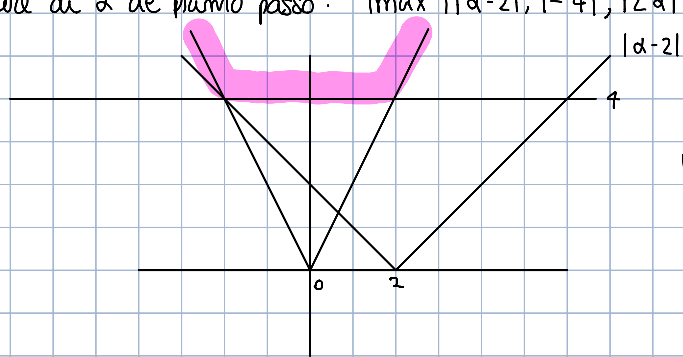
4. $A = \begin{pmatrix} \alpha-2 & 3 & 4 \\ -4 & -9 & 4 \\ 2\alpha & 13 & 8 \end{pmatrix}$

calcola fatt. LU $\rightarrow$ $\quad G_1 = \begin{pmatrix} 1 & 0 & 0 \\ \frac{4}{\alpha-2} & 1 & 0 \\ \frac{2\alpha}{2-\alpha} & 0 & 1 \end{pmatrix} \quad G_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ \frac{4}{\alpha-2} & 1 & 0 \\ \frac{2\alpha}{2-\alpha} & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha-2 & 3 & 4 \\ -4 & -9 & 4 \\ 2\alpha & 13 & 8 \end{pmatrix} = \begin{pmatrix} \alpha-2 & 3 & 4 \\ 0 & \frac{30-9\alpha}{\alpha-2} & \frac{4\alpha+8}{\alpha-2} \\ 0 & \frac{26-7\alpha}{2-\alpha} & \frac{16}{2-\alpha} \end{pmatrix}$

$\alpha \neq \pm 2, \frac{1}{9}$

$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{26-7\alpha}{30-9\alpha} & 1 \end{pmatrix} \quad G_2 \cdot G_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{26-7\alpha}{30-9\alpha} & 1 \end{pmatrix} \begin{pmatrix} \alpha-2 & 3 & 4 \\ 0 & \frac{30-9\alpha}{\alpha-2} & \frac{4\alpha+8}{\alpha-2} \\ 0 & \frac{26-7\alpha}{2-\alpha} & \frac{16}{2-\alpha} \end{pmatrix} = \begin{pmatrix} \alpha-2 & 3 & 4 \\ 0 & \frac{30-9\alpha}{\alpha-2} & \frac{4\alpha+8}{\alpha-2} \\ 0 & 0 & \frac{4\alpha+8}{\alpha-2} \cdot \frac{26-7\alpha}{30-9\alpha} \end{pmatrix} = U$

- variare di α al primo passo: $\max \{|\alpha-2|, |-4|, |2\alpha|\}$ $\qquad L = G_1^{-1} \cdot G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{4}{2-\alpha} & 1 & 0 \\ \frac{2\alpha}{\alpha-2} & \frac{26-7\alpha}{9\alpha-30} & 1 \end{pmatrix}$



$|2\alpha|$ se $\alpha < -2 \vee \alpha > 2$

$|-4|$ se $-2 \leq \alpha \leq 2$

- se $\alpha = 4$ calcola $PA = LU$

$$A = \begin{pmatrix} 2 & 3 & 4 \\ -4 & -9 & 4 \\ 8 & 13 & 8 \end{pmatrix} \quad P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad P_1 \cdot A = \begin{pmatrix} 8 & 13 & 8 \\ -4 & -9 & 4 \\ 2 & 3 & 4 \end{pmatrix} \quad G_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/4 & 0 & 1 \end{pmatrix}$$

$$G_1 \cdot P_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/4 & 0 & 1 \end{pmatrix} \begin{pmatrix} 8 & 13 & 8 \\ -4 & -9 & 4 \\ 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 8 & 13 & 8 \\ 0 & -5/2 & 8 \\ 0 & -1/4 & 2 \end{pmatrix}$$

$$P_2 = I \quad G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{-1}{10} & 1 \end{pmatrix} \quad G_2 \cdot P_2 \cdot G_1 \cdot P_1 \cdot A = \begin{pmatrix} 8 & 13 & 8 \\ 0 & -5/2 & 8 \\ 0 & 0 & 6/5 \end{pmatrix} = U$$

$$P = P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$L = P_2^{-1} G_1^{-1} \cdot P_2^{-1} \cdot G_2^{-1} = \cdot \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 1/4 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{10} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 1/4 & \frac{7}{20} & 5/4 \end{pmatrix}$$