ESAME 9 luglio 2019

1. $F = (2, t, e_{min}, e_{max})$

- $\begin{cases} e_{max} = e_{min} + 2 \\ realmin = 1/32 \\ realmax = 62 \end{cases} \Rightarrow \quad 2^{-e_{min}-1} = 2^{-5} \Rightarrow e_{min} = 4 \quad e_{max} = 6$

$$2^{e_{max}}(1 - 2^{-t}) = 62$$

$$2^6 - 2^{6-t} = 62 \Rightarrow 2^{6-t} = 2^1 \Rightarrow t = 5$$

- $|F| = ?$ $\quad 1 + 2(1) \cdot 2^{t-1}(e_{max} + e_{min} + 1)$

$$1 + 2^5(11) = 353$$

- numeri denormalizzati : $2 \cdot (B^{t-1} - \overset{\text{o gli } 0,000\ldots}{1}) \Rightarrow 2(2^{t-1} - 1) = 2^t - 2 = 30$

- $u = \frac{B^{1-t}}{2}$ e quella di F è $u = \frac{2^{-4}}{2} = 2^{-5} = 1/32$

- $x = (1,\overline{0111})_2$

$y = (11,\overline{0111})_2$

$$\tilde{x} = (0,10111) \cdot 2 = (0,010111) \cdot 2^2$$

$$\tilde{y} = (0,11100) \cdot 2^2$$

$$\tilde{z} = \tilde{x}\, fl(+)\, \tilde{y} = \left(\begin{array}{l} 0,0\,1\,0\,1\,1\,1 \\ 0,1\,1\,1\,0\,0\,0 \\ \hline 1,0\,0\,1\,1\,1\,1 \end{array} \right) \cdot 2^2 = 0,1001111 \cdot 2^3$$

$$= (0,10100) \cdot 2^3$$

$11\cancel{0}\cancel{1}\cancel{1}1$

- $\tilde{x} = 2\left(\frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}\right) = 1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{16+4+2+1}{16} = \frac{23}{16}$

$$\tilde{y} = 4\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = 2 + 1 + \frac{1}{2} = \frac{4+2+1}{2} = \frac{7}{2}$$

$$x = \frac{(10111)_2 - (1)_2}{(1111)_2} = \frac{(10110)_2}{(1111)_2} = \frac{2+4+16}{1+2+4+8} = \frac{22}{15}$$

$$y = \frac{(110111)_2 - (11)_2}{(1111)_2} = \frac{(110100)_2}{15} = 4 + 16 + 32 = \frac{52}{15}$$

2. $y = f(x)$ definisci l'errore in.

$$|\epsilon_{IN}| = \frac{|f(x) - f(\tilde{x})|}{|f(\tilde{x})|} \qquad cond_f = \frac{|x|\,|f'(x)|}{|f(x)|}$$

- $f(x) = \sqrt{x^2 - 1}$ $\quad D: x^2 - 1 \geq 0 \quad x \leq -1 \lor x \geq 1$

$$f'(x) = \cancel{2}x \cdot \frac{1}{\cancel{2}\sqrt{x^2-1}} = \frac{x}{\sqrt{x^2-1}} \qquad cond_f = \frac{|x|\,|x|}{|\sqrt{x^2-1}|} \cdot \frac{1}{|\sqrt{x^2-1}|} = \frac{x^2}{|x^2-1|}$$

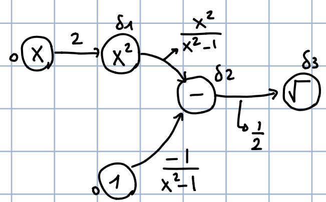
mal cond se $x \approx \pm 1$

algo in arit. esatta

algo in arit. macchina

- $$|\epsilon_{ALGO}| = \frac{|g(\tilde{x}) - \tilde{g}(\tilde{x})|}{|g(\tilde{x})|}$$

- grafo di $\sqrt{x^2-1}$ $\Rightarrow$



$$|\epsilon_{ALGO}| = \delta_3 + \frac{1}{2}\left(\delta_2 + \frac{x^2}{|x^2-1|}\delta_1\right)$$

$$\leq |u| + \frac{1}{2}|u| + \frac{x^2}{2|x^2-1|}|u| = |u|\left(\frac{3}{2} + \frac{x^2}{2|x^2-1|}\right)$$

$\rightarrow$ non stabile in avanti se $x \approx \pm 1$

3. $f(x) = -\frac{x^4}{4} + x^3 - 4x + 4$

$\lim_{x \to +\infty} f(x) = -\infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = -x^3 + 3x^2 - 4 \qquad x = -1$

	-1	$+3$	0	-4
-1		$+1$	-4	$+4$
	-1	4	-4	0

va messo perchè col ² lo perdi il segno

$(x+1)(-x^2+4x-4) = -(x+1)(x-2)^2$

↳ sempre positivo, si annulla in 2

studio solo in $x+1>0$

$x_{1,2} = \frac{-4 \pm \sqrt{16-16}}{-2} = 2$

$f(-1) = \frac{1}{4} - 1 + 4 + 4 = \frac{1}{4} + 7 = \frac{29}{4}$

$f(2) = -4 + 8 - 8 + 4 = 0 \qquad x = 2$ radice multipla di $\mu = 3$

$f'(x) > 0$ se $x > -1$
< 0 se $x < -1$
$= 0$ se $x = -1 \lor x = 2$

$\Rightarrow$ però c'è il $-$ $\Rightarrow$ $f'(x) > 0$ se $x < -1$
< 0 se $x > -1$
$= 0$ se $x = -1 \lor x = 2$

$x = -1$ pt. MAX $\quad x = 2$ PT. MIN

$f''(x) = -3x^2 + 6x \qquad x_{1,2} = -3x(x-2) \qquad x_1 = 2 \quad x_2 = 0$

$x = 0$ e $x = 2$ PT. FLESSO

$f''(x) > 0$ se $x < 0 \lor x > 2$
< 0 se $0 < x < 2$

$= 0$ se $x = 0$ e $x = 2$

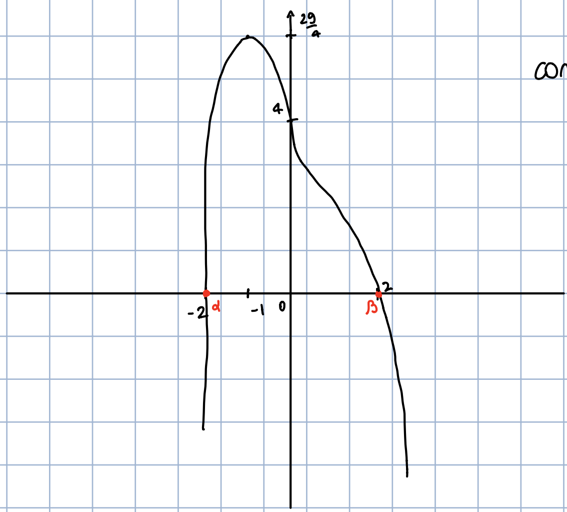
$f(0) = 4$
$f(2) = 0$
$f(-1) = 29/4$

Determina $\beta \Rightarrow$

	$-\frac{1}{4}$	1	0	-4	4
2		$-\frac{1}{2}$	$+1$	$+2$	-4
	$-\frac{1}{4}$	$\frac{1}{2}$	1	-2	0

$\Rightarrow (x-2)\left(-\frac{1}{4}x^3 + \frac{1}{2}x^2 + x - 2\right) \rightarrow \alpha = -2 \quad \beta = -2$

$\mu =$



convergenza ad α: $x_0 \in]-\infty, -2[$ $f \cdot f'' > 0$ allora converge superlin. (α)

$x_0 \in]-2, -1[$ $f \cdot f'' < 0$ converge con x_1 (α)

$x_0 \in]-1, 0[$ $f \cdot f'' < 0$ converge con x_1 (β)

$x_0 \in]0, 2[$ $f \cdot f'' > 0$ converge linearmente (β)

$x_0 \in]2, +\infty[$ $f \cdot f'' < 0$ converge con x_1 (β)

se $x_0 = -1{,}5$ converge ad α
se $x_0 = 1$ converge a β

$g(x) = x - \frac{f(x)}{m}$. verifica che α e β sono di p. fisso

$g'(x) = 1 - \frac{f'(x)}{m}$ $\qquad g(\alpha) = \alpha \qquad g(\beta) = \beta$

$- m = 32$ $\qquad$ conv. ad α $\qquad g'(-2) = 1 - \frac{f'(-2)}{32} = \left|1 - \frac{10}{32}\right| < 1$ ok

$f'(-2) = 10$

se $x_0 = -1{,}5$ converge ad α? $\quad [-2, -1{,}5] \quad g(-1{,}5) = 1 - \frac{f'(-1{,}5)}{32} = 1 - \frac{49}{8} \cdot \frac{1}{32} < 1$ OK

$f'(-1{,}5) = \frac{49}{8}$

$g''(x) = \frac{-f''(x)}{32} = \frac{3x^2 - 6x}{32}$ $\qquad g''(x) > 0$ se $0 < x < 2$
< 0 se $x < 0 \vee x > 2$
$= 0$ se $x = 0 \vee x = 2$

$\Rightarrow$ converge perché $|g'(x)| < 1$ in maniera lineare

4. $A = \begin{pmatrix} -3 & \alpha & -2 \\ \alpha & 1 & -1 \\ -2 & -1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} -3 & \alpha & -2 \end{pmatrix} \begin{pmatrix} -3 & \alpha & -2 \end{pmatrix}$

- calcola LU fatt: $G_1 = \begin{pmatrix} \frac{\alpha}{3} & 1 & 0 \\ -\frac{2}{3} & 0 & 1 \end{pmatrix}$ $\quad G_1 \cdot A = \begin{pmatrix} \frac{\alpha}{3} & 1 & 0 \\ -\frac{2}{3} & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha & 1 & -1 \\ -2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{\alpha^2+3}{3} & \frac{-2\alpha-3}{3} \\ 0 & \frac{-2\alpha-3}{3} & \frac{4}{3} \end{pmatrix}$

$$G_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{2\alpha+3}{\alpha^2+3} & 1 \end{pmatrix} \qquad G_2 \cdot G_1 \cdot A = U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{2\alpha+3}{\alpha^2+3} & 1 \end{pmatrix} \begin{pmatrix} -3 & \alpha & -2 \\ 0 & \frac{\alpha^2+3}{3} & \frac{-2\alpha-3}{3} \\ 0 & \frac{-2\alpha-3}{3} & \frac{4}{3} \end{pmatrix} = \begin{pmatrix} -3 & \alpha & -2 \\ 0 & \frac{\alpha^2+3}{3} & \frac{-2\alpha-3}{3} \\ 0 & 0 & \frac{4}{3} - \frac{(2\alpha+3)^2}{3(\alpha^2+3)} \end{pmatrix}$$

$$L = G_1^{-1} \cdot G_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{\alpha}{3} & 1 & 0 \\ \frac{2}{3} & \frac{-2\alpha-3}{\alpha^2+3} & 1 \end{pmatrix}$$

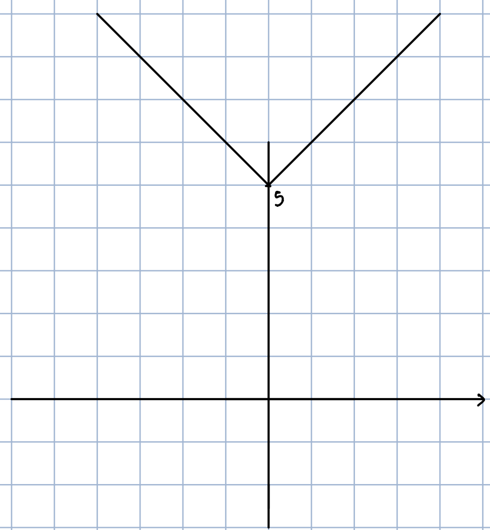
$\alpha \neq \pm\sqrt{3}$

- grafico di $\alpha \to \|A\|_1$ $\Rightarrow$ sommo per colonne $\max\{\overset{\text{ottengo}}{|\alpha|+5}, |\alpha|+2, 3\}$

$$\begin{pmatrix} -3 & \alpha & -2 \\ \alpha & 1 & -1 \\ -2 & -1 & 0 \end{pmatrix}$$

$\downarrow$

$|-3|+|\alpha|+|-2|$
$= |\alpha|+5$



- per quale scelta di α $Ax=b$ ha unica soluz.

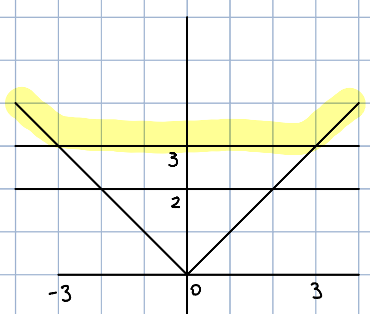
$\alpha = \pm\sqrt{3}$

$$\det(LU) \neq 0 \quad \underset{\downarrow\ 1}{\det(L)} \cdot \det(U) = \det(U) = -\cancel{3} \cdot \frac{\alpha^2+3}{\cancel{3}} \cdot \left(\frac{4}{3} - \frac{(2\alpha+3)^2}{3(\alpha^2+3)}\right)$$

$\Rightarrow$ quando $\frac{4}{3} - \frac{(2\alpha+3)^2}{3(\alpha^2+3)} = 0$

- per quali valori di α P. PARZIALE al primo passo scambia con la seconda?

$\max\{3, |\alpha|, 2\}$



3 se $-3 < x < 3$
$|\alpha|$ se $x \leq -3 \vee x \geq 3$

5. $- f(x) = 3\log_2(x^2)$

$P_0 = \left(\frac{1}{4}, f\left(\frac{1}{4}\right)\right) = \left(\frac{1}{4}, -12\right)$

$P_1 = \left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, -6\right)$

$P_2 = \left(1, f(1)\right) = \left(1, 0\right)$

x	f(x)		
$\frac{1}{4}$	-12	$\frac{-6+12}{\frac{1}{2}-\frac{1}{4}} = \frac{6}{\frac{1}{4}} = 24$	$\frac{12-24}{1-\frac{1}{4}} = \frac{-12}{\frac{3}{4}} = -16$
$\frac{1}{2}$	-6	$\frac{6}{\frac{1}{2}} = 12$	
1	0		

$$p(x) = -12 + 24\left(x - \tfrac{1}{4}\right) - 16\left(x - \tfrac{1}{4}\right)\left(x - \tfrac{1}{2}\right)$$

- Dato l'ulteriore punto $P_3 = (2, f(2))$ determina $\tilde{p}(x)$

$P_3 = (2, 6)$

x	f(x)			
$\frac{1}{4}$	-12	$\frac{-6+12}{\frac{1}{2}-\frac{1}{4}} = \frac{6}{\frac{1}{4}} = 24$	$\frac{12-24}{1-\frac{1}{4}} = \frac{-12}{\frac{3}{4}} = -16$	$\frac{-4+16}{2-\frac{1}{4}} = \frac{12}{\frac{7}{4}} = \frac{48}{7}$
$\frac{1}{2}$	-6	$\frac{6}{\frac{1}{2}} = 12$	$\frac{6-12}{2-\frac{1}{2}} = \frac{-6}{\frac{3}{2}} = -4$	
1	0	$\frac{6}{2-1} = 6$		
2	6			

$$\tilde{p}(x) = p(x) + \frac{48}{7}\underbrace{\left(x - \tfrac{1}{4}\right)\left(x - \tfrac{1}{2}\right)(x - 1)}_{\#\text{ volte che fai unione}}$$

- formula dell'errore $f(x) - p(x)$