



Department of Mathematics, University of Udine

SVM and their use in IR

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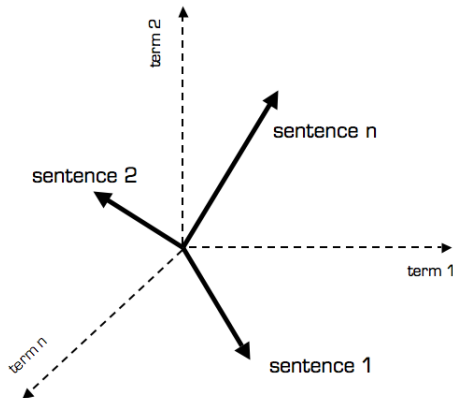
- ① SVM over linearly separable data
- ② SVM over non linearly separable data
- ③ SVM use in IR



Vector Space

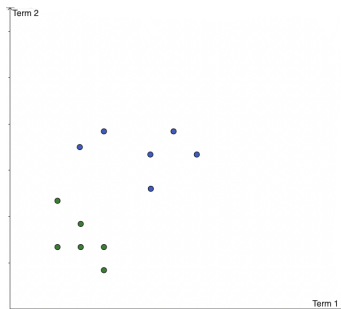
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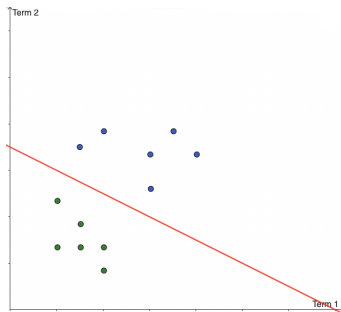


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Data can be separated using a *decision boundary*, which is an *hyperplane*.



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Support Vectors

In the previous example the *decision boundary* is a line, represented by the equation $a + bt_1 + ct_2 = 0$.

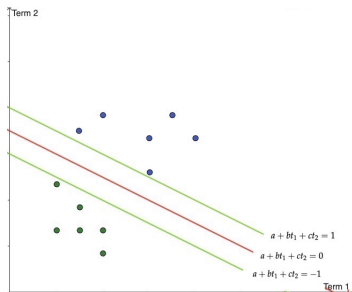
We can introduce two parallel hyperplanes (lines) to the decision boundary, called *support vectors* whose equations are

$$\begin{cases} a + bt_1 + ct_2 = 1 \\ a + bt_1 + ct_2 = -1 \end{cases} \quad (1)$$

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We can consider two points x_1, x_2 that lie respectively on the two support vectors, their distance is $\lambda \|b\| = \frac{2}{\sqrt{b^T b}}$.

SVM are used to find the *maximum margin linear classifier*, thus we want to maximize the margin.

Remembering we want to classify documents, our goal is to find specific b s.t. given a document x belonging to class y the decision boundary behave the following:

$$\begin{cases} b^T x + a \geq 1 & \text{if } y = 1 \\ b^T x + a \leq -1 & \text{if } y = -1 \end{cases} \quad (3)$$

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We can define the *cost function* as a system of equations.

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} \\ \text{subject to } y_i(b^T x_i + a) \geq 1 \quad \forall x_i \end{cases} \quad (4)$$



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So, we will use *slack variables* to introduce a penalty for each misclassified point.

The new cost function will be

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} + C \sum_i \xi_i \\ \text{subject to} & y_i(b^T x_i + a) \geq 1 - \xi_i \quad \text{and } \xi_i > 0 \quad \forall x_i \end{cases} \quad (5)$$

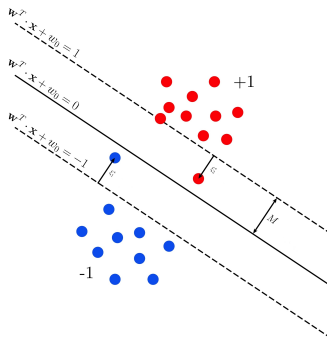


The larger is C the stricter the classification is, since a larger C will give more evidence to slack variables.

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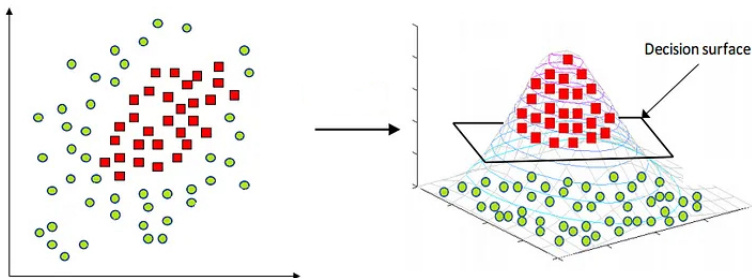


Non linearly separable data

What if our data is not linearly separable?

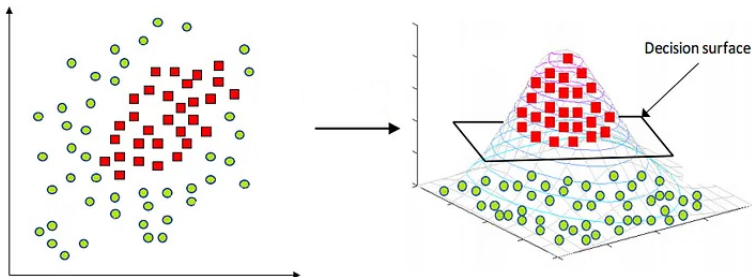
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But augmenting dimensions costs a lot...



Kernel Trick

Consider the function $\phi : \mathbb{R}^3 \mapsto \mathbb{R}^{10}$ used to map points in a new vector space. Calculating the *similarity* $\phi(x_i)^T \phi(x_j)$ between each point may be intractable.



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Instead of doing the complex computations in the 10-dimensional space, we reach the same result within the 3-dimensional space by calculating the dot product.



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The result will be a mapping for each point in a n -dimensional space. Finally, we can identify an hyperplane which can divide correctly the two classes of points.



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- **K-Means:** model for *unsupervised classification*;
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K-means algorithm is one of the most used clustering algorithms. It consists of partitioning unlabeled objects into k classes, where k is no predefined.

A brief explanation will follow.



K-means

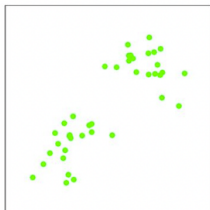
The aim is to create clusters that contain *similar documents*.

Given a training set $x^{(1)}, x^{(2)}, \dots, x^{(m)}$, the algorithm used is the following:

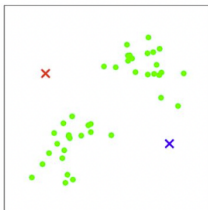
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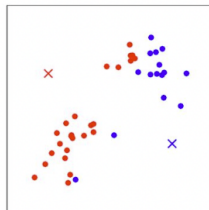
```
1:  $K \leftarrow \text{randomValue}$                                 ▷ multiple K will be tried
2: procedure K-MEANS( $K$ )
3:   initialize cluster centroids  $\mu_1, \dots, \mu_k$  randomly.
4:   repeat until convergence{
5:     for  $i \leftarrow 1$  to  $m$  do
6:        $c^{(i)} = \arg \min_j \|x^{(i)} - \mu_j\|$            ▷ Closest centroid to  $x^{(i)}$ 
7:     end for
8:     for  $k \leftarrow 1$  to  $K$  do
9:        $\mu_j = \frac{\sum_{i=1}^m 1\{c^{(i)}=j\}x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)}=j\}}$        ▷ New centroid of cluster  $k$ 
10:    end for
11:  }
12: end procedure
```



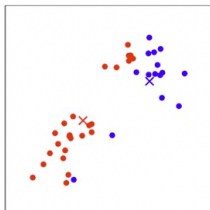
(a)



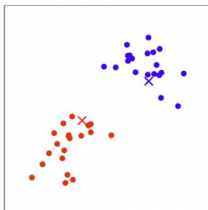
(b)



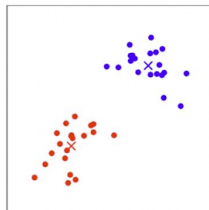
(c)



(d)



(e)

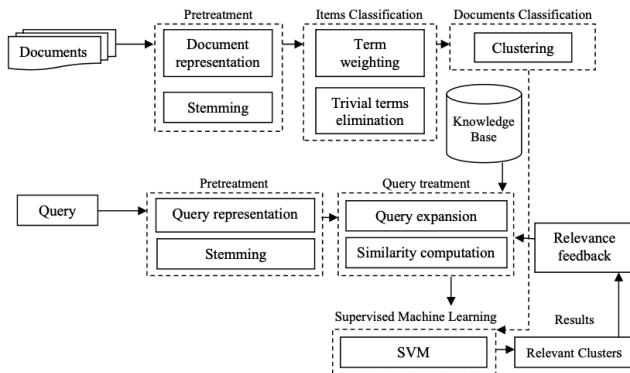


(f)

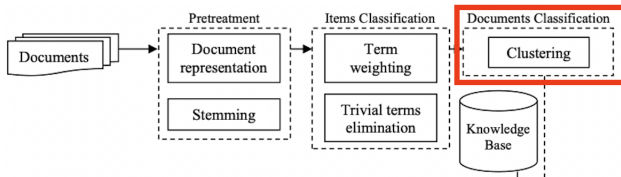


The proposed system can be summarized by the following figure:

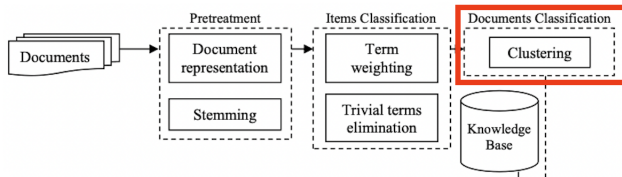
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Documents clustering

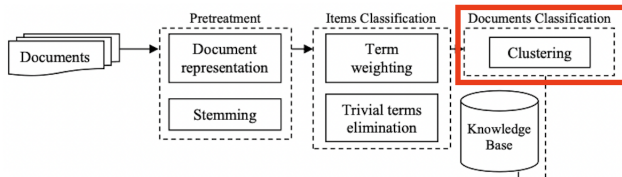


Documents clustering



First part consist of text pre-processing, followed by a vector representation of each document.

Next, there is the clustering phase.



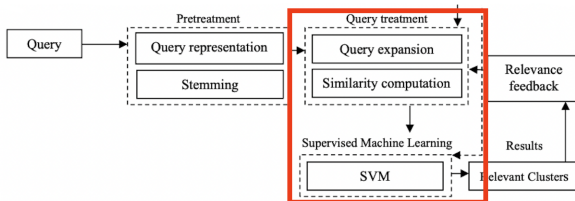
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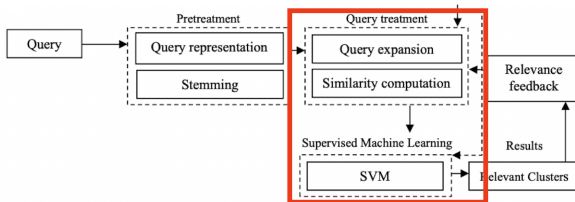
Using *k-means*, documents are classified into classes of similar vectors according to the retained terms and the similarity used.

Finally, terms are classified as *Trivial*, *Decisive* or *Standard*. Only the last two are preserved and used as indicator of the class.

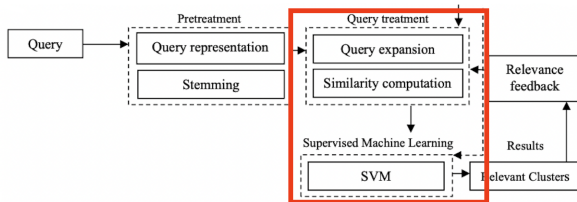
Queries classification



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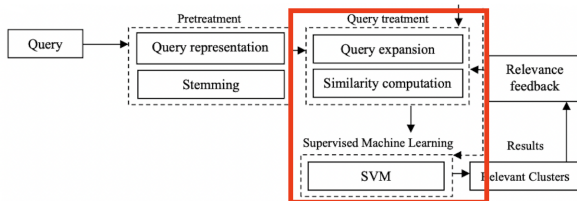


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So, this process allows the selection of documents from the returned class to return first. [1]



- [1] Galen Erso. “Construction plans of the Death Star 1 Orbital Battle Station”. In: *Journal of Horrible Plot Flaws 2* (0BBY), pp. 1977–2015.