



Department of Mathematics, University of Udine

Support Vector Machines and their use in IR

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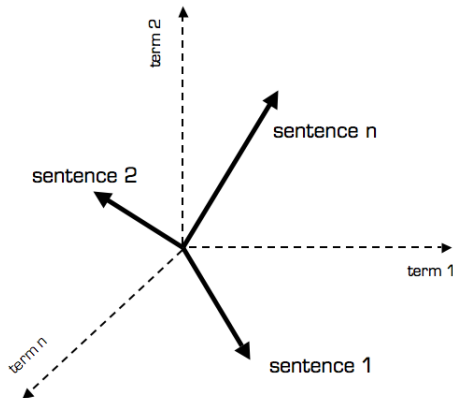
- 1 SVM over linearly separable data
- 2 SVM over non linearly separable data
- 3 SVM use in IR



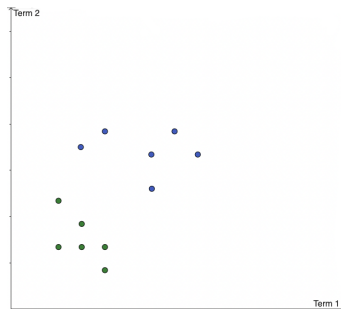
Vector Space

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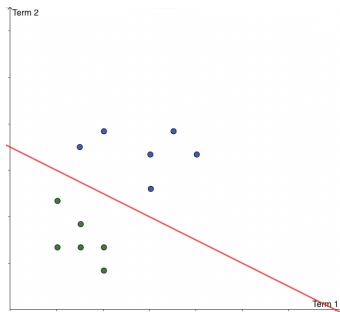
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Support Vectors

In the previous example the *decision boundary* is a line, represented by the equation $a + bt_1 + ct_2 = 0$.

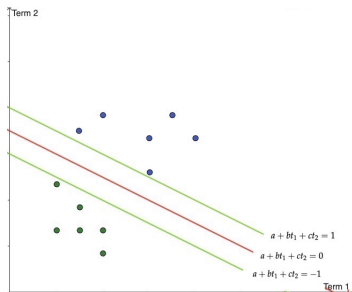
We can introduce two *parallels* hyperplanes (lines) to the decision boundary, called *support vectors* whose equations are

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Intuitively, we can define the *margin* of the two support vectors as the distance between them.

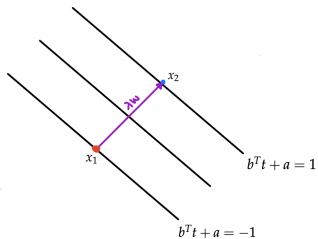


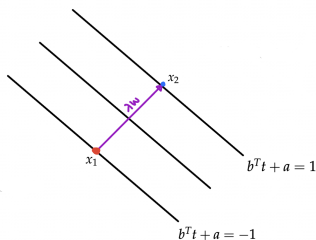
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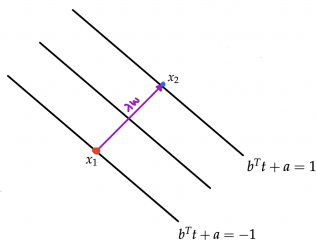
Consider an arbitrary point x_1 that lies on support vector s_1 . Since s_1, s_2 are parallel, consider the closest point to x_1 belonging to s_2 , say x_2 .





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SVM [2, 1] are used to find the *maximum margin linear classifier*, thus we want to maximize the margin.



Remembering that we want to classify documents, our goal is to find specific b s.t. given a document x belonging to class y the decision boundary behaves as follows:

$$\begin{cases} b^T x + a \geq 1 & \text{if } y = 1 \\ b^T x + a \leq -1 & \text{if } y = -1 \end{cases} \quad (3)$$

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We can define the *cost function* as a system of equations:

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} \\ \text{subject to } y_i(b^T x_i + a) \geq 1 \quad \forall x_i \end{cases} \quad (4)$$



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The new cost function will be

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} + C \sum_i \xi_i \\ \text{subject to } y_i(b^T x_i + a) \geq 1 - \xi_i \quad \text{and } \xi_i > 0 \quad \forall x_i \end{cases} \quad (5)$$

Intuitively, C represents how much the penalty counts.

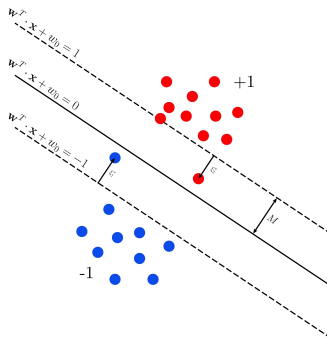


The larger is C the stricter the classification is, since a larger C will give more evidence to slack variables.

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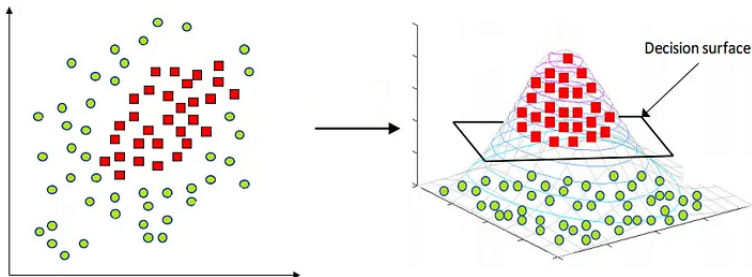


Non linearly separable data

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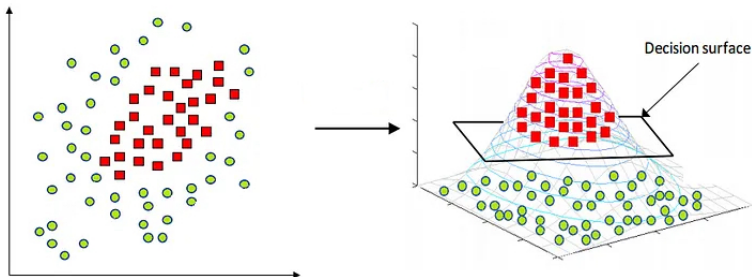
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But augmenting dimensions costs a lot...



Kernel Trick

Consider the function $\phi : \mathbb{R}^3 \mapsto \mathbb{R}^{10}$ used to map points in a new vector space. Calculating the *similarity* $\phi(x_i)^T \phi(x_j)$ between each point may be intractable.



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K is a *kernel function* that operates on the lower dimension vectors x_i and x_j to produce a value equivalent to the dot-product of the higher-dimensional vectors.

Instead of doing the complex computations in the 10-dimensional space, we reach the same result within the 3-dimensional space by calculating the dot product.



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The result will be a mapping for each point in a n -dimensional space. Finally, we can identify an hyperplane which can divide correctly the two classes of points.



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- **K-Means:** model for *unsupervised classification*;
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K-means algorithm is one of the most used clustering algorithms. It consists of partitioning unlabeled objects into k classes, where k is no predefined.

A brief explanation will follow.



K-means

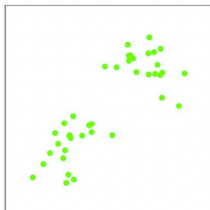
The aim is to create clusters that contain *similar documents*.

Given a training set $x^{(1)}, x^{(2)}, \dots, x^{(m)}$, the algorithm [5] used is the following:

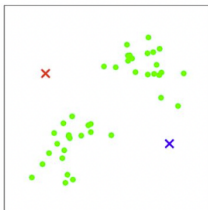
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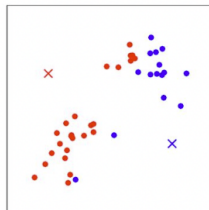
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1:  $K \leftarrow \text{randomValue}$                                 ▷ multiple K will be tried
2: procedure K-MEANS( $K$ )
3:   initialize cluster centroids  $\mu_1, \dots, \mu_k$  randomly.
4:   repeat until convergence{
5:     for  $i \leftarrow 1$  to  $m$  do
6:        $c^{(i)} = \arg \min_j \|x^{(i)} - \mu_j\|$            ▷ Closest centroid to  $x^{(i)}$ 
7:     end for
8:     for  $k \leftarrow 1$  to  $K$  do
9:        $\mu_j = \frac{\sum_{i=1}^m 1\{c^{(i)}=j\} x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)}=j\}}$        ▷ New centroid of cluster  $k$ 
10:    end for
11:  }
12: end procedure
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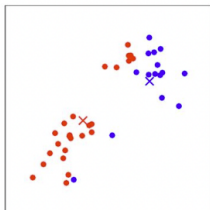
(a)



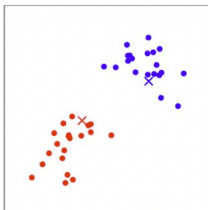
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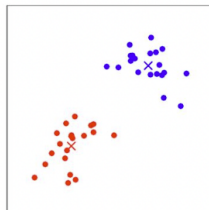
(c)



(d)



(e)

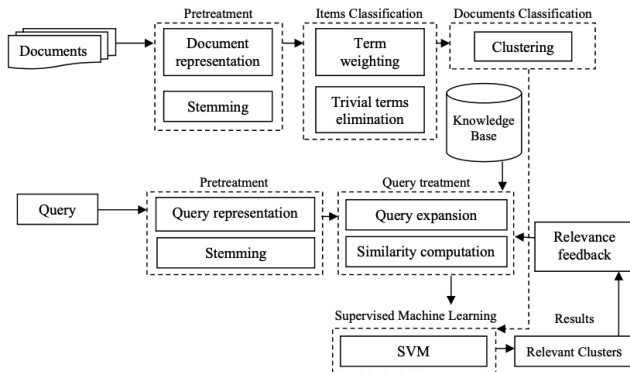


(f)

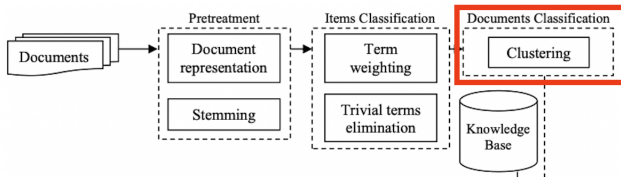


The proposed system [3] can be summarized by the following figure:

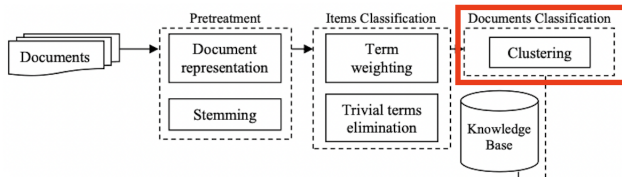
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Documents clustering

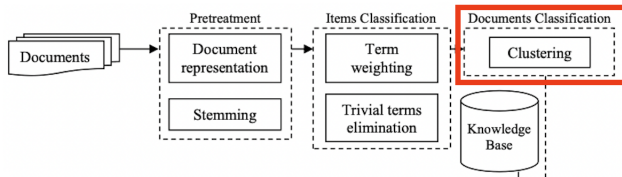


Documents clustering



First part consist of text pre-processing, followed by a vector representation of each document.

Next, there is the clustering phase.



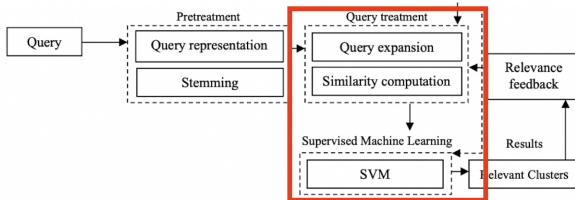
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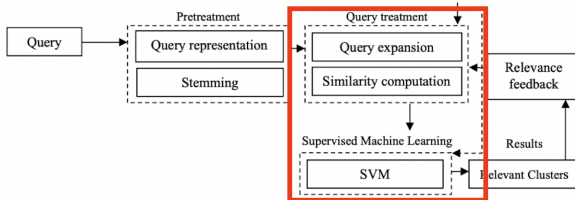
Using *k-means*, documents are classified into classes of similar vectors according to the retained terms and the similarity used.

Finally, terms are classified as *Trivial*, *Decisive* or *Standard*. Only the last two are preserved and used as indicator of the class.

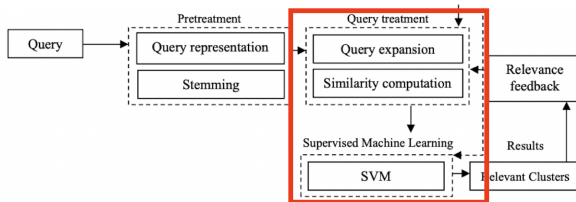
Queries classification



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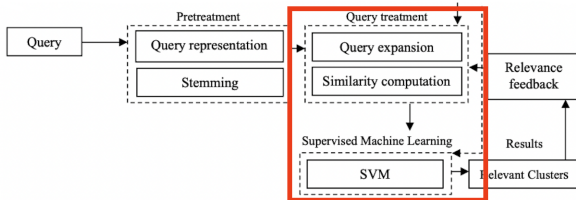


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So, this process allows the selection of documents from the returned class to return first.



- [1] Nello Cristianini and John Shawe-Taylor. "Support Vector Machines". In: *An Introduction to Support Vector Machines and Other Kernel-based Learning Methods*. Cambridge University Press, 2000, pp. 93–124. DOI: 10.1017/CBO9780511801389.008.
- [2] Vikramaditya Jakkula. "Tutorial on support vector machine (svm)". In: *School of EECS, Washington State University* 37.2.5 (2006), p. 3.
- [3] Hamid Khalifi, Abderrahim Elqadi, and Youssef Ghanou. "Support vector machines for a new hybrid information retrieval system". In: *Procedia Computer Science* 127 (2018), pp. 139–145.
- [4] B. Scholkopf et al. "Input space versus feature space in kernel-based methods". In: *IEEE Transactions on Neural Networks* (1999). DOI: 10.1109/72.788641.



- [5] Kristina P. Sinaga and Miin-Shen Yang. “Unsupervised K-Means Clustering Algorithm”. In: *IEEE Access* 8 (2020), pp. 80716–80727. DOI: 10.1109/ACCESS.2020.2988796.