



Department of Mathematics, University of Udine

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# Support Vector Machines and their use in IR

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May x, 2023



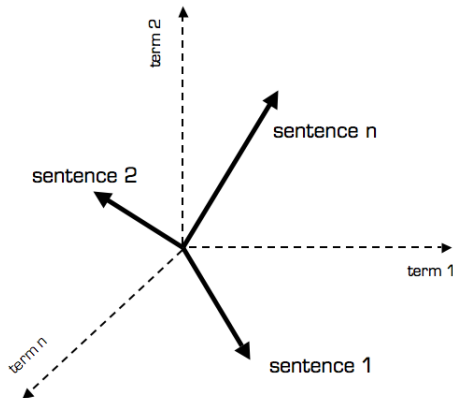
- 1 SVM over linearly separable data
- 2 SVM over non linearly separable data
- 3 SVM use in IR



# Vector Space

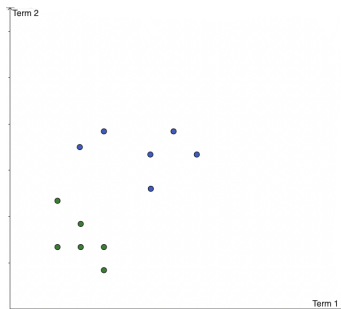
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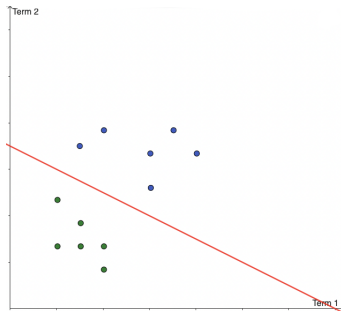


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# Support Vectors

In the previous example the *decision boundary* is a line, represented by the equation  $a + bt_1 + ct_2 = 0$ .

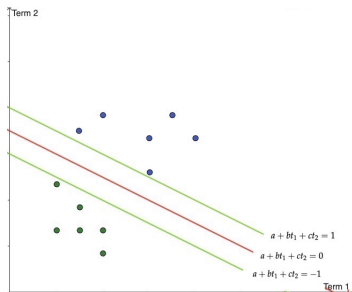
We can introduce two *parallels* hyperplanes (lines) to the decision boundary, called *support vectors* whose equations are

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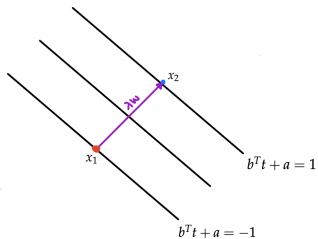
Intuitively, we can define the *margin* of the two support vectors as the distance between them.

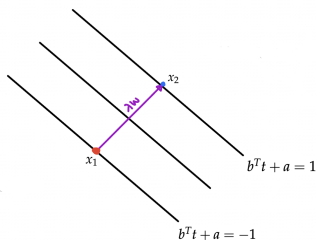
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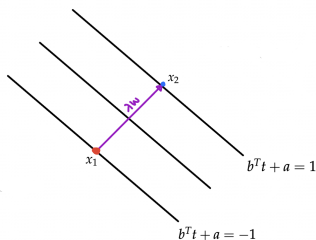
Consider an arbitrary point  $x_1$  that lies on support vector  $s_1$ . Since  $s_1, s_2$  are parallel, consider the closest point to  $x_1$  belonging to  $s_2$ , say  $x_2$ .





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SVM [2, 1] are used to find the *maximum margin linear classifier*, thus we want to maximize the margin.



Remembering that we want to classify documents, our goal is to find specific  $b$  s.t. given a document  $x$  belonging to class  $y$  the decision boundary behaves as follows:

$$\begin{cases} b^T x + a \geq 1 & \text{if } y = 1 \\ b^T x + a \leq -1 & \text{if } y = -1 \end{cases} \quad (3)$$

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We can define the *cost function* as a system of equations:

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} \\ \text{subject to } y_i(b^T x_i + a) \geq 1 \quad \forall x_i \end{cases} \quad (4)$$





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The new cost function will be

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} + C \sum_i \xi_i \\ \text{subject to } y_i(b^T x_i + a) \geq 1 - \xi_i \quad \text{and } \xi_i > 0 \quad \forall x_i \end{cases} \quad (5)$$

Intuitively,  $C$  represents how much the penalty counts.

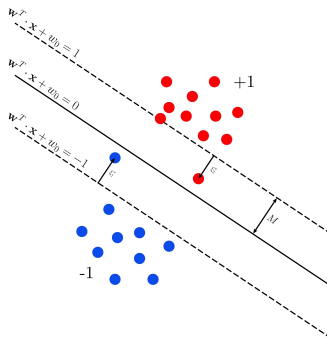


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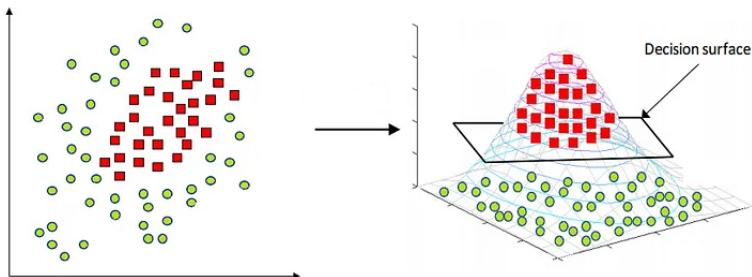


# Non linearly separable data

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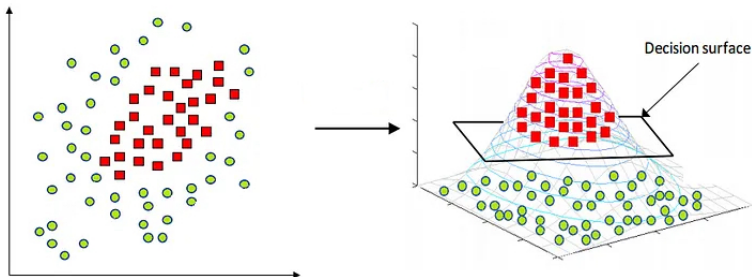
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But augmenting dimensions costs a lot...





# Kernel Trick

Consider the function  $\phi : \mathbb{R}^3 \mapsto \mathbb{R}^{10}$  used to map points in a new vector space. Calculating the *similarity*  $\phi(x_i)^T \phi(x_j)$  between each point may be intractable.



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$K$  is a *kernel function* that operates on the lower dimension vectors  $x_i$  and  $x_j$  to produce a value equivalent to the dot-product of the higher-dimensional vectors.

Instead of doing the complex computations in the 10-dimensional space, we reach the same result within the 3-dimensional space by calculating the dot product.



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The result will be a mapping for each point in a  $n$ -dimensional space. Finally, we can identify an hyperplane which can divide correctly the two classes of points.





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- **K-Means:** model for *unsupervised classification*;
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K-means algorithm is one of the most used clustering algorithms. It consists of partitioning unlabeled objects into  $k$  classes, where  $k$  is not predefined.

A brief explanation will follow.



# K-means

The aim is to create clusters that contain *similar documents*.

Given a training set  $x^{(1)}, x^{(2)}, \dots, x^{(m)}$ , the algorithm [5] used is the following:

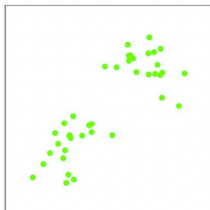
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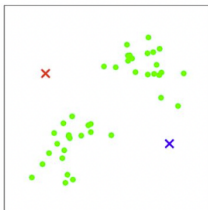
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```
1:  $K \leftarrow \text{randomValue}$                                 ▷ multiple K will be tried
2: procedure K-MEANS( $K$ )
3:   initialize cluster centroids  $\mu_1, \dots, \mu_k$  randomly.
4:   repeat until convergence{
5:     for  $i \leftarrow 1$  to  $m$  do
6:        $c^{(i)} = \arg \min_j \|x^{(i)} - \mu_j\|$            ▷ Closest centroid to  $x^{(i)}$ 
7:     end for
8:     for  $k \leftarrow 1$  to  $K$  do
9:        $\mu_j = \frac{\sum_{i=1}^m 1\{c^{(i)}=j\} x^{(i)}}{\sum_{i=1}^m 1\{c^{(i)}=j\}}$        ▷ New centroid of cluster  $k$ 
10:    end for
11:  }
12: end procedure
```

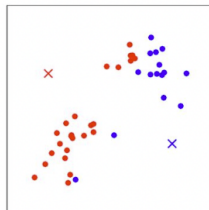
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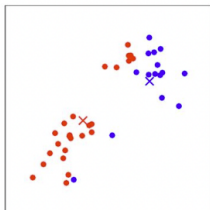
(a)



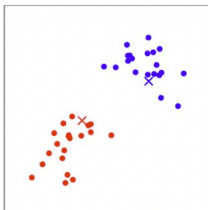
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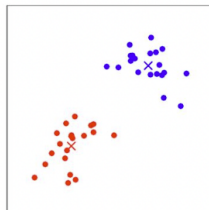
(c)



(d)



(e)

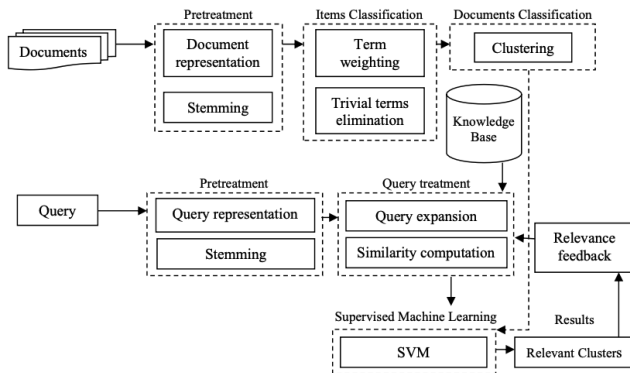


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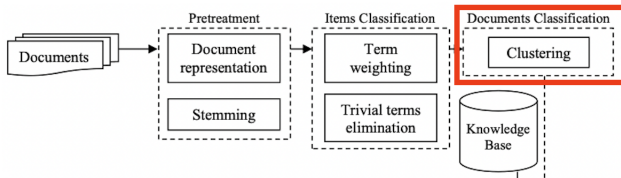


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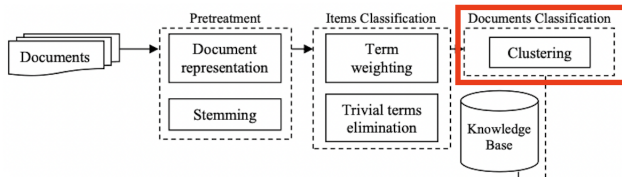


# Documents clustering

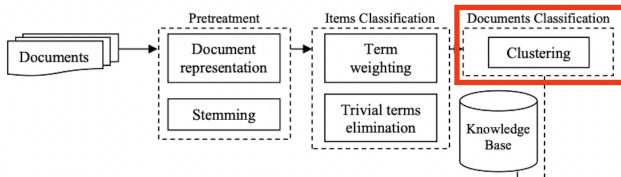




# Documents clustering



First part consists of text pre-processing, followed by a vector representation of each document using stemmed terms. Next, there is the clustering phase.

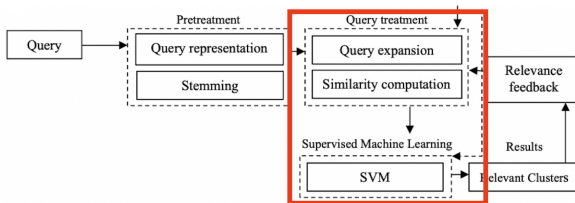


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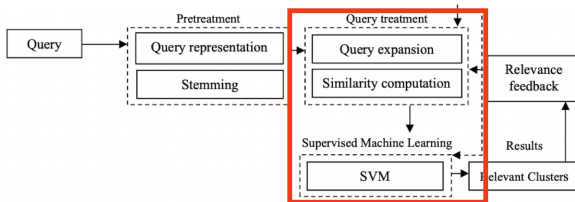
Using *k-means*, documents are classified into classes of similar vectors according to the selected terms (dimensions).

Finally, terms are classified as *Trivial*, *Decisive* or *Standard*. Only the last two are preserved and used as indicator of the class.

# Queries classification

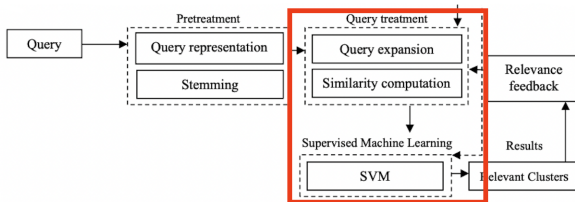


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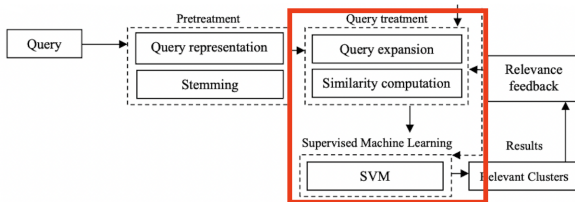
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So, this process allows the selection of documents from the correct cluster (the one with the highest  $p_i$ ).



We have seen the combination of use of SVMs and Clustering in the Information Retrieval field, with great results [3].

To conclude, SVMs are a great tool utilized for classification and regression.

Since they are less expensive than Neural Networks, usually researchers start experimenting using them before NN.



- [1] Nello Cristianini and John Shawe-Taylor. “Support Vector Machines”. In: *An Introduction to Support Vector Machines and Other Kernel-based Learning Methods*. Cambridge University Press, 2000, pp. 93–124. DOI: 10.1017/CBO9780511801389.008.
- [2] Vikramaditya Jakkula. “Tutorial on support vector machine (svm)”. In: *School of EECS, Washington State University* 37.2.5 (2006), p. 3.
- [3] Hamid Khalifi, Abderrahim Elqadi, and Youssef Ghanou. “Support vector machines for a new hybrid information retrieval system”. In: *Procedia Computer Science* 127 (2018), pp. 139–145.
- [4] B. Scholkopf et al. “Input space versus feature space in kernel-based methods”. In: *IEEE Transactions on Neural Networks* (1999). DOI: 10.1109/72.788641.





- [5] Kristina P. Sinaga and Miin-Shen Yang. “Unsupervised K-Means Clustering Algorithm”. In: *IEEE Access* 8 (2020), pp. 80716–80727. DOI: 10.1109/ACCESS.2020.2988796.