Department of Mathematics, University of Udine

Support Vector Machines and their use in IR

Lorenzo Zanolin.
lorenzo.zanolin@spes.uniud.it

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Outline

1. SVM over linearly separable data

2. SVM over non linearly separable data

3. SVM use in IR

Suppose you have to classify whether a document is *relevant* or not. We can think to use *terms* as features to divide properly the data. We will use a *t-dimensional* vector space to represent our documents.

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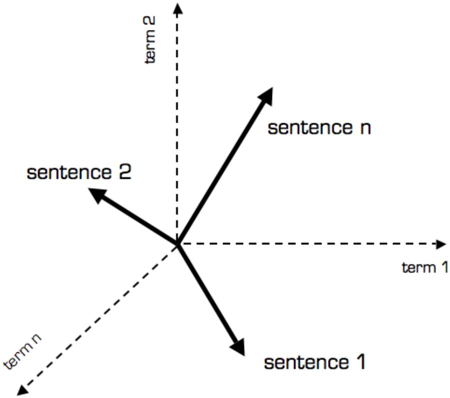
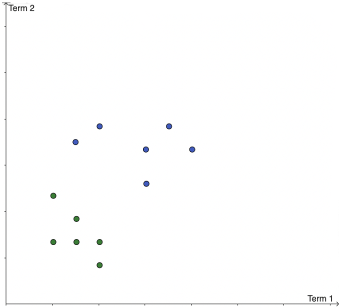


For simplicity, we can reason using a 2D Space.
Data can be separated using a *decision boundary*, which is an *hyperplane*.



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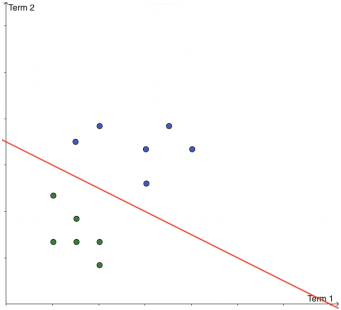

Support Vectors

In the previous example the *decision boundary* is a line, represented by the equation $a + bt_1 + ct_2 = 0$.
We can introduce two parallels hyperplanes (lines) to the decision boundary, called *support vectors* whose equations are

$$\begin{cases} a + bt_1 + ct_2 = 1 \\ a + bt_1 + ct_2 = -1 \end{cases} \tag{1}$$

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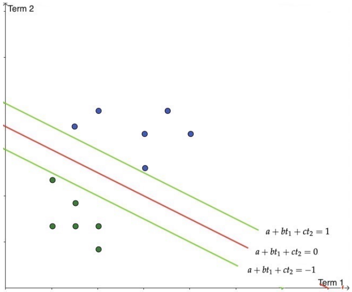

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We can consider two points x_1, x_2 that lie respectively on the two support vectors, their distance is $\lambda \|b\| = \frac{2}{\sqrt{b^T b}}$.

SVM [2, 1] are used to find the *maximum margin linear classifier*, thus we want to maximize the margin.

Remembering we want to classify documents, our goal is to find specific b s.t. given a document x belonging to class y the decision boundary behave the following:

$$\begin{cases} b^T x + a \geq 1 & \textit{if } y = 1 \\ b^T x + a \leq -1 & \textit{if } y = -1 \end{cases} \tag{3}$$

Cost Function

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We can define the *cost function* as a system of equations.

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} \\ \textit{subject to} \quad y_i(b^T x_i + a) \geq 1 \quad \forall x_i \end{cases} \tag{4}$$

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The new cost function will be

$$\begin{cases} \min_{b,a} \frac{\sqrt{b^T b}}{2} + C \sum_i \xi_i \\ \textit{subject to} \quad y_i(b^T x_i + a) \geq 1 - \xi_i \quad \textit{and } \xi_i > 0 \quad \forall x_i \end{cases} \tag{5}$$

Visualization

The larger is C the stricter the classification is, since a larger C will give more evidence to slack variables.

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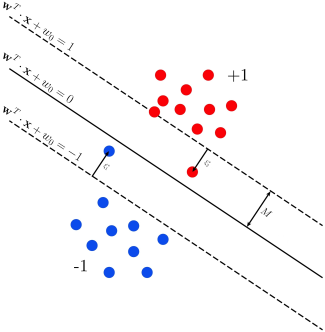

Non linearly separable data

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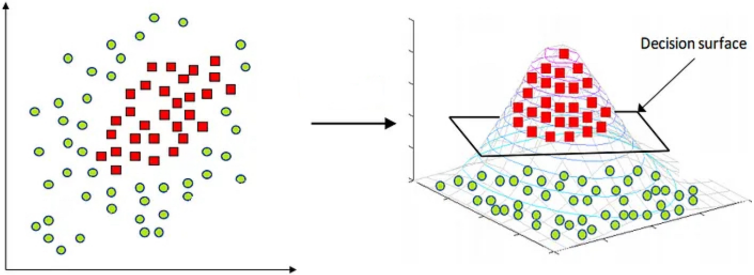

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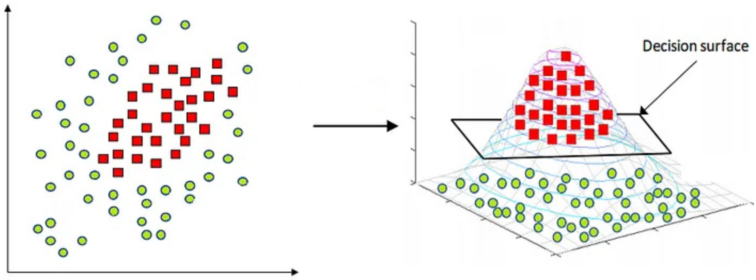

But augmenting dimensions costs a lot...

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It consists of a simple linear algebra reformulation,

$$\phi(x_i)^T \phi(x_j) = K(x_i, x_j) = (1 + x_i{}^T x_j)^2. \tag{7}$$

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Instead of doing the complex computations in the 10-dimensional space, we reach the same result within the 3-dimensional space by calculating the dot product.

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The result will be a mapping for each point in a n-dimensional space. Finally, we can identify an hyperplane which can divide correctly the two classes of points.

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- **K-Means**: model for *unsupervised classification*;
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Pratical application

An hybrid technique used in the IR field will be presented; it uses two components:

- **K-Means**: model for *unsupervised classification*;
- **SVM**: model for *supervised classification*.

K-means algorithm is one of the most used clustering algorithms. It consists of partitioning unlabeled objects into k classes, where k is no predefined.
A brief explanation will follow.

K-means

The aim is to create clusters that contain *similar documents*.
Given a training set $x^{(1)}, x^{(2)}, \dots, x^{(m)}$, the algorithm [5] used is the following:

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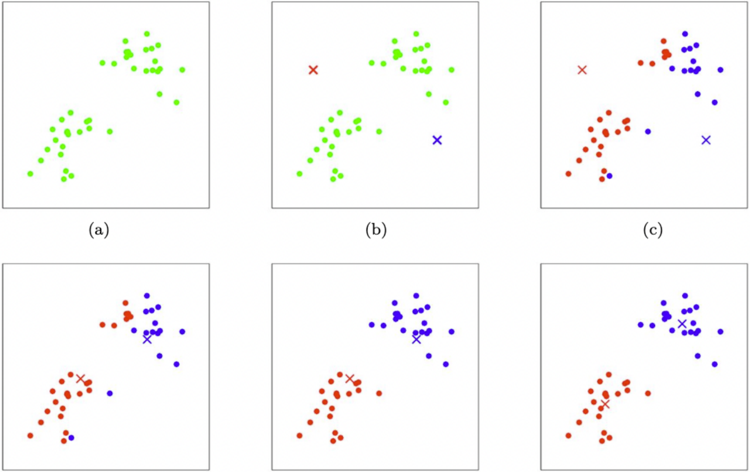
$K \leftarrow randomValue$ ▷ multiple K will be tried
procedure K-MEANS(K)
    `initialize cluster centroids` $\mu_1, \dots, \mu_k$ `randomly.`
    repeat until convergence{
        for $i \leftarrow 1$ to m **do**
            $c^{(i)} = arg\ min_j \|x^{(i)} - \mu_j\|$ ▷ Closest centroid to $x^{(i)}$
        end for
        for $k \leftarrow 1$ to K **do**
            $\mu_j = \frac{\sum_{i=1} m 1\{c^{(i)}=j\} x^{(i)}}{\sum_{i=1} m 1\{c^{(i)}=j\}}$ ▷ New centroid of cluster k
        end for
    }
end procedure



(a) (b) (c)

(d) (e) (f)

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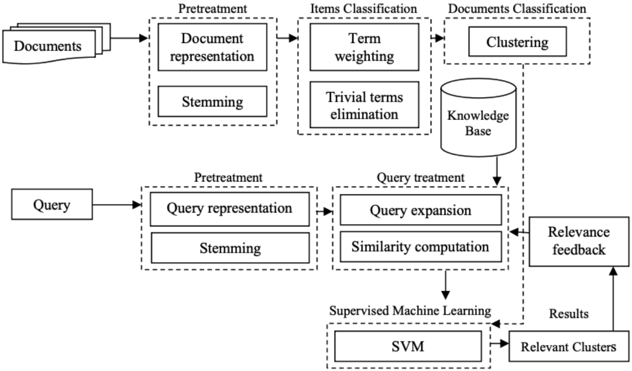
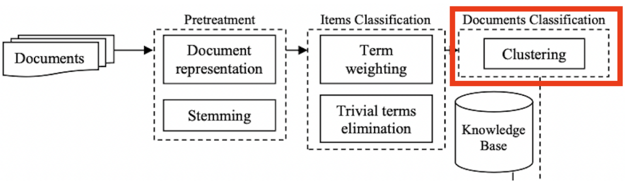
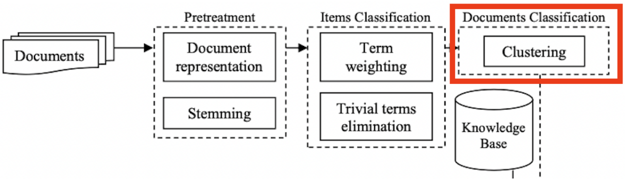
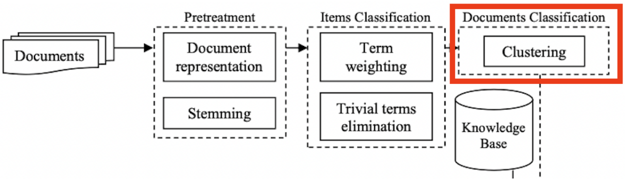


Documents clustering





First part consist of text pre-processing, followed by a vector representation of each document.
Next, there is the clustering phase.



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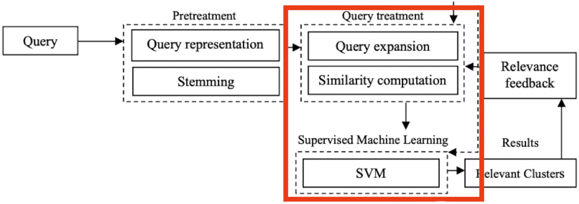
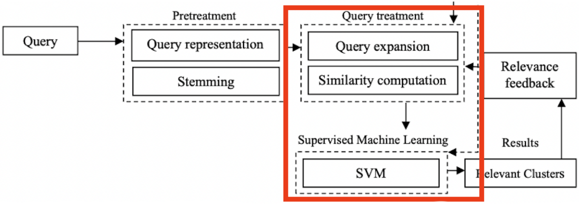
Next, there is the clustering phase.

Using *k-means*, documents are classified into classes of similar vectors according to the retained terms and the similarity used.

Finally, terms are classified as *Trivial*, *Decisive* or *Standard*. Only the last two are preserved and used as indicator of the class.

Queries classification





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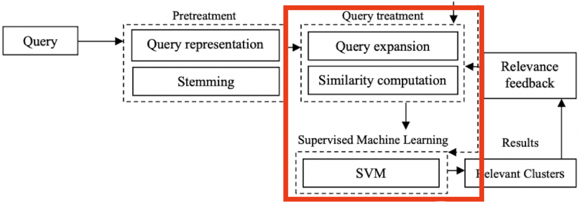

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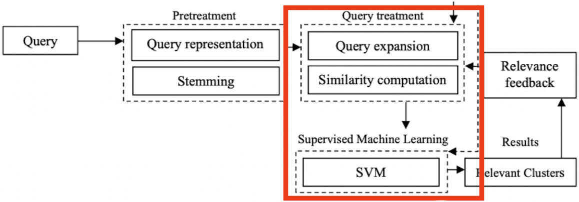

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So, this process allows the selection of documents from the returned class to return first.

[1] Nello Cristianini and John Shawe-Taylor. “Support Vector Machines”. In: *An Introduction to Support Vector Machines and Other Kernel-based Learning Methods*. Cambridge University Press, 2000, pp. 93–124. DOI: 10.1017/CBO9780511801389.008.

[2] Vikramaditya Jakkula. “Tutorial on support vector machine (svm)”. In: *School of EECS, Washington State University* 37.2.5 (2006), p. 3.

[3] Hamid Khalifi, Abderrahim Elqadi, and Youssef Ghanou. “Support vector machines for a new hybrid information retrieval system”. In: *Procedia Computer Science* 127 (2018), pp. 139–145.

[4] B. Scholkopf et al. “Input space versus feature space in kernel-based methods”. In: *IEEE Transactions on Neural Networks* (1999). DOI: 10.1109/72.788641.

[5] Kristina P. Sinaga and Miin-Shen Yang. "Unsupervised K-Means Clustering Algorithm". In: *IEEE Access* 8 (2020), pp. 80716–80727. DOI: 10.1109/ACCESS.2020.2988796.