

Extended Version

Featherweight OCL

A Study for a Consistent Semantics of UML/OCL 2.3 in HOL

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At its origins, OCL was conceived as a strict semantics for undefinedness, with the exception of the logical connectives of type `Boolean` that constitute a three-valued propositional logic. Recent versions of the OCL standard added a second exception element, which, similar to the null references in programming languages, is given a non-strict semantics.

In this paper, we report on our results in formalizing the core of OCL in higher-order logic (HOL). This formalization revealed several inconsistencies and contradictions in the current version of the OCL standard. These inconsistencies and contradictions are reflected in the challenge to define and implement OCL tools in a uniform manner.

Further readings: This theory extends the paper “Featherweight OCL: A study for the consistent semantics of OCL 2.3 in HOL” [10] that is published as part of the proceedings of the OCL workshop 2012.

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1 Introduction

At its origins [17, 14], OCL was conceived as a strict semantics for undefinedness, with the exception of the logical connectives of type `Boolean` that constitute a three-valued propositional logic. Recent versions of the OCL standard [15, 16] added a second exception element, which is given a non-strict semantics. Unfortunately, this extension results in several inconsistencies and contradictions. These problems are reflected in difficulties to define interpreters, code-generators, specification animators or theorem provers for OCL in a uniform manner and resulting incompatibilities of various tools. For the OCL community, this results in the challenge to define a new formal semantics definition OCL that could replace the “Annex A” of the OCL standard [16].

In the paper “Extending OCL with Null-References” [4] we explored—based on mathematical arguments and paper and pencil proofs—a consistent formal semantics that comprises two exception elements: `invalid` (“bottom” in semantics terminology) and `null` (for “non-existing element”).

This short paper is based on a formalization of [4], called “Featherweight OCL,” in Isabelle/HOL [13]. This formalization is in its present form merely a semantical study and a proof of technology than a real tool. It focuses on the formalization of the key semantical constructions, i. e., the type `Boolean` and the logic, the type `Integer` and a standard strict operator library, and the collection type `Set(A)` with quantifiers, iterators and key operators.

2 Featherweight OCL

Featherweight OCL is a formalization of the core of OCL aiming at formally investigation the relationship between the different notions of “undefinedness,” i. e., `invalid` and `null`. As such, it does not attempt to define the complete OCL library. Instead, it concentrates on the core concepts of OCL as well as the types `Boolean`, `Integer`, and typed sets (`Set(T)`). Following the tradition of HOL-OCL [6, 5], Featherweight OCL is based on the following principles:

1. It is an embedding into a powerful semantic meta-language and environment, namely Isabelle/HOL [13].
2. It is a *shallow embedding* in HOL; types in OCL were injectively mapped to types in Featherweight OCL. Ill-typed OCL specifications cannot be represented in Featherweight OCL and a type in Featherweight OCL contains exactly the values that are possible in OCL. Thus, sets may contain `null` (`Set{null}` is a defined set) but not `invalid` (`Set{invalid}` is just `invalid`).
3. Any Featherweight OCL type contains at least `invalid` and `null` (the type `Void` contains only these instances). The logic is consequently four-valued, and there is a `null`-element in the type `Set(A)`.
4. It is a strongly typed language in the Hindley-Milner tradition. We assume that a pre-process eliminates all implicit conversions due to subtyping by introducing explicit casts (e. g., `oclAsType()`). The details of such a pre-processing are de-

scribed in [2]. Casts are semantic functions, typically injections, that may convert data between the different Featherweight OCL types.

5. All objects are represented in an object universe in the HOL-OCL tradition [7] the universe construction also gives semantics to type casts, dynamic type tests, as well as functions such as `oclAllInstances()`, or `isNewInState()`.
6. Featherweight OCL types may be arbitrarily nested: `Set{Set{1,2}} = Set{Set{2,1}}` is legal and true.
7. For demonstration purposes, the set-type in Featherweight OCL may be infinite, allowing infinite quantification and a constant that contains the set of all Integers. Arithmetic laws like commutativity may therefore expressed in OCL itself. The iterator is only defined on finite sets.
8. It supports equational reasoning and congruence reasoning, but this requires a differentiation of the different equalities like strict equality, strong equality, meta-equality (HOL). Strict equality and strong equality require a subcalculus, “cp” (a detailed discussion of the different equalities as well the subcalculus “cp”—for three-valued OCL 2.0—is given in [9]), which is nasty but can be hidden from the user inside tools.

```
theory
  OCL-core
imports
  Main
begin
```

2.1 Foundational Notations

2.1.1 Notations for the option type

First of all, we will use a more compact notation for the library option type which occur all over in our definitions and which will make the presentation more “textbook”-like:

```
notation Some (|(-)|)
notation None (⊥)
```

The following function (corresponding to *the* in the Isabelle/HOL library) is defined as the inverse of the injection *Some*.

```
fun drop :: 'α option ⇒ 'α (|(-)|)
where drop-lift[simp]: |v| = v
```

2.1.2 Minimal Notions of State and State Transitions

Next we will introduce the foundational concept of an object id (oid), which is just some infinite set.

```
type-synonym oid = ind
```

States are just a partial map from oid’s to elements of an object universe \mathcal{A} , and state transitions pairs of states...

type-synonym ($'\mathcal{A}$) $state = oid \rightarrow '\mathcal{A}$

type-synonym ($'\mathcal{A}$) $st = '\mathcal{A} \ state \times '\mathcal{A} \ state$

2.1.3 Prerequisite: An Abstract Interface for OCL Types

In order to have the possibility to nest collection types, such that we can give semantics to expressions like $Set\{Set\{\mathbf{2}\}, null\}$, it is necessary to introduce a uniform interface for types having the *invalid* (= bottom) element. The reason is that we impose a data-invariant on raw-collection **types_code** which assures that the *invalid* element is not allowed inside the collection; all raw-collections of this form were identified with the *invalid* element itself. The construction requires that the new collection type is uncomparable with the raw-types (consisting of nested option type constructions), such that the data-invariant must be expressed in terms of the interface. In a second step, our base-types will be shown to be instances of this interface.

This uniform interface consists in a type class requiring the existence of a bot and a null element. The construction proceeds by abstracting the null (which is defined by $\lfloor \perp \rfloor$ on $'a \ option \ option$ to a null - element, which may have an arbitrary semantic structure, and an undefinedness element \perp to an abstract undefinedness element *bot* (also written \perp whenever no confusion arises). As a consequence, it is necessary to redefine the notions of invalid, defined, valuation etc. on top of this interface.

This interface consists in two abstract type classes *bot* and *null* for the class of all types comprising a bot and a distinct null element.

instance *option* :: (*plus*) *plus* **by** *intro-classes*

instance *fun* :: (*type, plus*) *plus* **by** *intro-classes*

```
class bot =
  fixes bot :: 'a
  assumes nonEmpty :  $\exists x. x \neq bot$ 
```

```
class null = bot +
  fixes null :: 'a
  assumes null-is-valid :  $null \neq bot$ 
```

2.1.4 Accomodation of Basic Types to the Abstract Interface

In the following it is shown that the option-option type type is in fact in the *null* class and that function spaces over these classes again "live" in these classes. This motivates the default construction of the semantic domain for the basic types (Boolean, Integer, Reals, ...).

instantiation *option* :: (*type*)*bot*

begin

definition *bot-option-def*: $(bot::'a \ option) \equiv (None::'a \ option)$

```

instance proof show  $\exists x::'a \text{ option}. x \neq \text{bot}$ 
  by(rule-tac x=Some x in exI, simp add:bot-option-def)
qed
end

```

```

instantiation option :: (bot)null
begin
  definition null-option-def: (null::'a::bot option)  $\equiv$  [ bot ]
  instance proof show (null::'a::bot option)  $\neq$  bot
    by( simp add:null-option-def bot-option-def)
  qed
end

```

```

instantiation fun :: (type,bot) bot
begin
  definition bot-fun-def: bot  $\equiv$  ( $\lambda x. \text{bot}$ )

  instance proof show  $\exists (x::'a \Rightarrow 'b). x \neq \text{bot}$ 
    apply(rule-tac x= $\lambda -. (\text{SOME } y. y \neq \text{bot})$  in exI, auto)
    apply(drule-tac x=x in fun-cong, auto simp:bot-fun-def)
    apply(erule contrapos-pp, simp)
    apply(rule some-eq-ex[THEN iffD2])
    apply(simp add: nonEmpty)
    done
  qed
end

```

```

instantiation fun :: (type,null) null
begin
  definition null-fun-def: (null::'a  $\Rightarrow$  'b::null)  $\equiv$  ( $\lambda x. \text{null}$ )

  instance proof
    show (null::'a  $\Rightarrow$  'b::null)  $\neq$  bot
    apply(auto simp: null-fun-def bot-fun-def)
    apply(drule-tac x=x in fun-cong)
    apply(erule contrapos-pp, simp add: null-is-valid)
    done
  qed
end

```

A trivial consequence of this adaption of the interface is that abstract and concrete versions of null are the same on base types (as could be expected).

2.2 The Semantic Space of OCL Types: Valuations.

Valuations are now functions from a state pair (built upon data universe \mathcal{A}) to an arbitrary null-type (i.e. containing at least a distinguished *null* and *invalid* element).

type-synonym $(\mathcal{A}, 'a) \text{ val} = \mathcal{A} \text{ st} \Rightarrow 'a::\text{null}$

The definitions for the constants and operations based on valuations will be geared towards a format that Isabelle can check to be a "conservative" (i.e. logically safe) axiomatic definition. By introducing an explicit interpretation function (which happens to be defined just as the identity since we are using a shallow embedding of OCL into HOL), all these definitions can be rewritten into the conventional semantic "textbook" format as follows:

definition $\text{Sem} :: 'a \Rightarrow 'a \ (I[-])$
where $I[x] \equiv x$

As a consequence of semantic domain definition, any OCL type will have the two semantic constants *invalid* (for exceptional, aborted computation) and *null*; the latter, however is either defined

definition $\text{invalid} :: (\mathcal{A}, 'a::\text{bot}) \text{ val}$
where $\text{invalid} \equiv \lambda \tau. \text{bot}$

This conservative Isabelle definition of the polymorphic constant *invalid* is equivalent with the textbook definition:

lemma *invalid-def-textbook*: $I[\text{invalid}]\tau = \text{bot}$
by(*simp add: invalid-def Sem-def*)

Note that the definition :

definition $\text{null} :: ('\<AA>, '\<alpha>::\text{null}) \text{ val}$
where $\text{"null"} \quad \backslash\text{equiv} \quad \backslash\text{lambda} \quad \backslash\text{tau}. \text{null}"$

is not necessary since we defined the entire function space over null types again as null-types; the crucial definition is $\text{null} \equiv \lambda x. \text{null}$. Thus, the polymorphic constant *null* is simply the result of a general type class construction. Nevertheless, we can derive the semantic textbook definition for the OCL null constant based on the abstract null:

lemma *null-def-textbook*: $I[\text{null}::(\mathcal{A}, 'a::\text{null}) \text{ val}] \tau = (\text{null}::'a::\text{null})$
by(*simp add: null-fun-def Sem-def*)

2.3 Boolean Type and Logic

The semantic domain of the (basic) boolean type is now defined as standard: the space of valuation to *bool option option*:

type-synonym $(\mathcal{A})\text{Boolean} = (\mathcal{A}, \text{bool option option}) \text{ val}$

2.3.1 Basic Constants

lemma *bot-Boolean-def* : (*bot*::('A)Boolean) = ($\lambda \tau. \perp$)
by(*simp add: bot-fun-def bot-option-def*)

lemma *null-Boolean-def* : (*null*::('A)Boolean) = ($\lambda \tau. \lfloor \perp \rfloor$)
by(*simp add: null-fun-def null-option-def bot-option-def*)

definition *true* :: ('A)Boolean
where *true* $\equiv \lambda \tau. \lfloor \text{True} \rfloor$

definition *false* :: ('A)Boolean
where *false* $\equiv \lambda \tau. \lfloor \text{False} \rfloor$

lemma *bool-split*: $X \tau = \text{invalid } \tau \vee X \tau = \text{null } \tau \vee$
 $X \tau = \text{true } \tau \vee X \tau = \text{false } \tau$
apply(*simp add: invalid-def null-def true-def false-def*)
apply(*case-tac X \tau, simp-all add: null-fun-def null-option-def bot-option-def*)
apply(*case-tac a, simp*)
apply(*case-tac aa, simp*)
apply *auto*
done

lemma [*simp*]: *false* (*a*, *b*) = $\lfloor \text{False} \rfloor$
by(*simp add: false-def*)

lemma [*simp*]: *true* (*a*, *b*) = $\lfloor \text{True} \rfloor$
by(*simp add: true-def*)

lemma *true-def-textbook*: $I\llbracket \text{true} \rrbracket \tau = \lfloor \text{True} \rfloor$
by(*simp add: Sem-def true-def*)

lemma *false-def-textbook*: $I\llbracket \text{false} \rrbracket \tau = \lfloor \text{False} \rfloor$
by(*simp add: Sem-def false-def*)

Summary:

Name	Theorem
<i>invalid-def-textbook</i>	$I\llbracket \text{invalid} \rrbracket ?\tau = \text{OCL-core.bot-class.bot}$
<i>null-def-textbook</i>	$I\llbracket \text{null} \rrbracket ?\tau = \text{null}$
<i>true-def-textbook</i>	$I\llbracket \text{true} \rrbracket ?\tau = \lfloor \text{True} \rfloor$
<i>false-def-textbook</i>	$I\llbracket \text{false} \rrbracket ?\tau = \lfloor \text{False} \rfloor$

Table 1: Basic semantic constant definitions of the logic (except *null*)

2.3.2 Fundamental Predicates I: Validity and Definedness

However, this has also the consequence that core concepts like definedness, validness and even `cp` have to be redefined on this type class:

definition *valid* :: ($\mathfrak{A}, 'a::\text{null}$)*val* \Rightarrow (\mathfrak{A})*Boolean* (*v* - [100]100)
where $v\ X \equiv \lambda\ \tau . \text{if } X\ \tau = \text{bot } \tau \text{ then false } \tau \text{ else true } \tau$

lemma *valid1*[simp]: *v invalid* = *false*
by(rule *ext,simp* add: *valid-def bot-fun-def bot-option-def*
invalid-def true-def false-def)

lemma *valid2*[simp]: *v null* = *true*
by(rule *ext,simp* add: *valid-def bot-fun-def bot-option-def null-is-valid*
null-fun-def invalid-def true-def false-def)

lemma *valid3*[simp]: *v true* = *true*
by(rule *ext,simp* add: *valid-def bot-fun-def bot-option-def null-is-valid*
null-fun-def invalid-def true-def false-def)

lemma *valid4*[simp]: *v false* = *true*
by(rule *ext,simp* add: *valid-def bot-fun-def bot-option-def null-is-valid*
null-fun-def invalid-def true-def false-def)

lemma *cp-valid*: (*v X*) τ = (*v* ($\lambda\ .\ X\ \tau$)) τ
by(*simp* add: *valid-def*)

definition *defined* :: ($\mathfrak{A}, 'a::\text{null}$)*val* \Rightarrow (\mathfrak{A})*Boolean* (δ - [100]100)
where $\delta\ X \equiv \lambda\ \tau . \text{if } X\ \tau = \text{bot } \tau \vee X\ \tau = \text{null } \tau \text{ then false } \tau \text{ else true } \tau$

The generalized definitions of *invalid* and *definedness* have the same properties as the old ones :

lemma *defined1*[simp]: $\delta\ \text{invalid}$ = *false*
by(rule *ext,simp* add: *defined-def bot-fun-def bot-option-def*
null-def invalid-def true-def false-def)

lemma *defined2*[simp]: $\delta\ \text{null}$ = *false*
by(rule *ext,simp* add: *defined-def bot-fun-def bot-option-def*
null-def null-option-def null-fun-def invalid-def true-def false-def)

lemma *defined3*[simp]: $\delta\ \text{true}$ = *true*
by(rule *ext,simp* add: *defined-def bot-fun-def bot-option-def null-is-valid null-option-def*
null-fun-def invalid-def true-def false-def)

lemma *defined4*[simp]: $\delta\ \text{false}$ = *true*
by(rule *ext,simp* add: *defined-def bot-fun-def bot-option-def null-is-valid null-option-def*)

null-fun-def invalid-def true-def false-def)

lemma *defined5[simp]: $\delta \delta X = true$*
by(*rule ext, auto simp: defined-def true-def false-def*
bot-fun-def bot-option-def null-option-def null-fun-def)

lemma *defined6[simp]: $\delta v X = true$*
by(*rule ext,*
auto simp: valid-def defined-def true-def false-def
bot-fun-def bot-option-def null-option-def null-fun-def)

lemma *defined7[simp]: $\delta \delta X = true$*
by(*rule ext,*
auto simp: valid-def defined-def true-def false-def
bot-fun-def bot-option-def null-option-def null-fun-def)

lemma *valid6[simp]: $v \delta X = true$*
by(*rule ext,*
auto simp: valid-def defined-def true-def false-def
bot-fun-def bot-option-def null-option-def null-fun-def)

lemma *cp-defined: $(\delta X)\tau = (\delta (\lambda -. X \tau)) \tau$*
by(*simp add: defined-def*)

The definitions above for the constants *defined* and *valid* can be rewritten into the conventional semantic "textbook" format as follows:

lemma *defined-def-textbook: $I[\delta(X)] \tau = (if I[X] \tau = I[bot] \tau \vee I[X] \tau = I>null] \tau$*
then $I>false] \tau$
else $I>true] \tau$)
by(*simp add: Sem-def defined-def*)

lemma *valid-def-textbook: $I[v(X)] \tau = (if I[X] \tau = I[bot] \tau$*
then $I>false] \tau$
else $I>true] \tau$)
by(*simp add: Sem-def valid-def*)

Summary: These definitions lead quite directly to the algebraic laws on these predicates:

2.3.3 Fundamental Predicates II: Logical (Strong) Equality

Note that we define strong equality extremely generic, even for types that contain an *null* or \perp element:

Name	Theorem
<i>defined-def-textbook</i>	$I[\delta \text{ ?}X] \text{ ?}\tau = (\text{if } I[\text{?}X] \text{ ?}\tau = I[\text{OCL-core.bot-class.bot}] \text{ ?}\tau \vee I[\text{?}X] \text{ ?}\tau = I[\text{null}])$
<i>valid-def-textbook</i>	$I[v \text{ ?}X] \text{ ?}\tau = (\text{if } I[\text{?}X] \text{ ?}\tau = I[\text{OCL-core.bot-class.bot}] \text{ ?}\tau \text{ then } I[\text{false}] \text{ ?}\tau \text{ else } \dots)$

Table 2: Basic predicate definitions of the logic.)

Name	Theorem
<i>defined1</i>	$\delta \text{ invalid} = \text{false}$
<i>defined2</i>	$\delta \text{ null} = \text{false}$
<i>defined3</i>	$\delta \text{ true} = \text{true}$
<i>defined4</i>	$\delta \text{ false} = \text{true}$
<i>defined5</i>	$\delta \delta \text{ ?}X = \text{true}$
<i>defined6</i>	$\delta v \text{ ?}X = \text{true}$
<i>defined7</i>	$\delta \delta \text{ ?}X = \text{true}$

Table 3: Laws of the basic predicates of the logic.)

definition *StrongEq*:: $[\text{'}\mathfrak{A} \text{ st} \Rightarrow \text{'}\alpha, \text{'}\mathfrak{A} \text{ st} \Rightarrow \text{'}\alpha] \Rightarrow (\text{'}\mathfrak{A})\text{Boolean}$ (**infixl** $\triangleq 30$)
where $X \triangleq Y \equiv \lambda \tau. \llbracket X \tau = Y \tau \rrbracket$

Equality reasoning in OCL is not humpty dumpty. While strong equality is clearly an equivalence:

lemma *StrongEq-refl* [*simp*]: $(X \triangleq X) = \text{true}$
by(*rule ext*, *simp add: null-def invalid-def true-def false-def StrongEq-def*)

lemma *StrongEq-sym*: $(X \triangleq Y) = (Y \triangleq X)$
by(*rule ext*, *simp add: eq-sym-conv invalid-def true-def false-def StrongEq-def*)

lemma *StrongEq-trans-strong* [*simp*]:
assumes $A: (X \triangleq Y) = \text{true}$
and $B: (Y \triangleq Z) = \text{true}$
shows $(X \triangleq Z) = \text{true}$
apply(*insert A B*) **apply**(*rule ext*)
apply(*simp add: null-def invalid-def true-def false-def StrongEq-def*)
apply(*drule-tac x=x in fun-cong*)
by *auto*

... it is only in a limited sense a congruence, at least from the point of view of this semantic theory. The point is that it is only a congruence on OCL- expressions, not arbitrary HOL expressions (with which we can mix Essential OCL expressions. A semantic — not syntactic — characterization of OCL-expressions is that they are *context-passing* or *context-invariant*, i.e. the context of an entire OCL expression, i.e. the pre-and post-state it refers to, is passed constantly and unmodified to the sub-expressions, i.e. all sub-expressions inside an OCL expression refer to the same context. Expressed formally,

this boils down to:

```

lemma StrongEq-subst :
  assumes cp:  $\bigwedge X. P(X)\tau = P(\lambda \cdot. X \tau)\tau$ 
  and eq:  $(X \triangleq Y)\tau = \text{true } \tau$ 
  shows  $(P X \triangleq P Y)\tau = \text{true } \tau$ 
  apply(insert cp eq)
  apply(simp add: null-def invalid-def true-def false-def StrongEq-def)
  apply(subst cp[of X])
  apply(subst cp[of Y])
  by simp

```

2.3.4 Fundamental Predicates III

And, last but not least,

```

lemma defined8[simp]:  $\delta (X \triangleq Y) = \text{true}$ 
  by(rule ext,
    auto simp: valid-def defined-def true-def false-def StrongEq-def
    bot-fun-def bot-option-def null-option-def null-fun-def)

```

```

lemma valid5[simp]:  $v (X \triangleq Y) = \text{true}$ 
  by(rule ext,
    auto simp: valid-def true-def false-def StrongEq-def
    bot-fun-def bot-option-def null-option-def null-fun-def)

```

```

lemma cp-StrongEq:  $(X \triangleq Y) \tau = ((\lambda \cdot. X \tau) \triangleq (\lambda \cdot. Y \tau)) \tau$ 
by(simp add: StrongEq-def)

```

The semantics of strict equality of OCL is constructed by overloading: for each base type, there is an equality.

2.3.5 Logical Connectives and their Universal Properties

It is a design goal to give OCL a semantics that is as closely as possible to a "logical system" in a known sense; a specification logic where the logical connectives can not be understood other than having the truth-table aside when reading fails its purpose in our view.

Practically, this means that we want to give a definition to the core operations to be as close as possible to the lattice laws; this makes also powerful symbolic normalizations of OCL specifications possible as a pre-requisite for automated theorem provers. For example, it is still possible to compute without any definedness- and validity reasoning the DNF of an OCL specification; be it for test-case generations or for a smooth transition to a two-valued representation of the specification amenable to fast standard SMT-solvers, for example.

Thus, our representation of the OCL is merely a 4-valued Kleene-Logics with *invalid* as least, *null* as middle and *true* resp. *false* as unrelated top-elements.

definition *not* :: (\mathcal{A})Boolean \Rightarrow (\mathcal{A})Boolean

where $\text{not } X \equiv \lambda \tau . \text{case } X \tau \text{ of}$

$$\begin{aligned} & \perp \Rightarrow \perp \\ & | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ & | \lfloor \lfloor x \rfloor \rfloor \Rightarrow \lfloor \lfloor \neg x \rfloor \rfloor \end{aligned}$$

lemma *cp-not*: $(\text{not } X)\tau = (\text{not } (\lambda \neg . X \tau)) \tau$

by(*simp add: not-def*)

lemma *not1[simp]*: *not invalid = invalid*

by(*rule ext, simp add: not-def null-def invalid-def true-def false-def bot-option-def*)

lemma *not2[simp]*: *not null = null*

by(*rule ext, simp add: not-def null-def invalid-def true-def false-def
bot-option-def null-fun-def null-option-def*)

lemma *not3[simp]*: *not true = false*

by(*rule ext, simp add: not-def null-def invalid-def true-def false-def*)

lemma *not4[simp]*: *not false = true*

by(*rule ext, simp add: not-def null-def invalid-def true-def false-def*)

lemma *not-not[simp]*: *not (not X) = X*

apply(*rule ext, simp add: not-def null-def invalid-def true-def false-def*)

apply(*case-tac X x, simp-all*)

apply(*case-tac a, simp-all*)

done

definition *ocl-and* :: [(\mathcal{A}) Boolean, (\mathcal{A}) Boolean] \Rightarrow (\mathcal{A})Boolean (**infixl** and 30)

where $X \text{ and } Y \equiv (\lambda \tau . \text{case } X \tau \text{ of}$

$$\begin{aligned} & \perp \Rightarrow (\text{case } Y \tau \text{ of} \\ & \quad \perp \Rightarrow \perp \\ & \quad | \lfloor \perp \rfloor \Rightarrow \perp \\ & \quad | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow \perp \\ & \quad | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor) \\ & | \lfloor \perp \rfloor \Rightarrow (\text{case } Y \tau \text{ of} \\ & \quad \perp \Rightarrow \perp \\ & \quad | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ & \quad | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow \lfloor \perp \rfloor \\ & \quad | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor) \\ & | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow (\text{case } Y \tau \text{ of} \\ & \quad \perp \Rightarrow \perp \\ & \quad | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ & \quad | \lfloor \lfloor y \rfloor \rfloor \Rightarrow \lfloor \lfloor y \rfloor \rfloor) \\ & | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor) \end{aligned}$$

Note that *not* is *not* defined as a strict function; proximity to lattice laws implies that

we need a definition of *not* that satisfies $\text{not}(\text{not}(x))=x$.

In textbook notation, the logical core constructs *not* and *op and* were represented as follows:

lemma *textbook-not*:

$$I[\text{not}(X)] \tau = (\text{case } I[X] \tau \text{ of } \perp \Rightarrow \perp \\ | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ | \lfloor \lfloor x \rfloor \rfloor \Rightarrow \lfloor \lfloor \neg x \rfloor \rfloor)$$

by(*simp add: Sem-def not-def*)

lemma *textbook-and*:

$$I[X \text{ and } Y] \tau = (\text{case } I[X] \tau \text{ of } \perp \Rightarrow (\text{case } I[Y] \tau \text{ of } \perp \Rightarrow \perp \\ | \lfloor \perp \rfloor \Rightarrow \perp \\ | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow \perp \\ | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor) \\ | \lfloor \perp \rfloor \Rightarrow (\text{case } I[Y] \tau \text{ of } \perp \Rightarrow \perp \\ | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow \lfloor \perp \rfloor \\ | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor) \\ | \lfloor \lfloor \text{True} \rfloor \rfloor \Rightarrow (\text{case } I[Y] \tau \text{ of } \perp \Rightarrow \perp \\ | \lfloor \perp \rfloor \Rightarrow \lfloor \perp \rfloor \\ | \lfloor \lfloor y \rfloor \rfloor \Rightarrow \lfloor \lfloor y \rfloor \rfloor) \\ | \lfloor \lfloor \text{False} \rfloor \rfloor \Rightarrow \lfloor \lfloor \text{False} \rfloor \rfloor)$$

by(*simp add: Sem-def ocl-and-def split: option.split*)

definition *ocl-or* :: $[(\mathfrak{A})\text{Boolean}, (\mathfrak{A})\text{Boolean}] \Rightarrow (\mathfrak{A})\text{Boolean}$
(**infixl** *or* 25)

where $X \text{ or } Y \equiv \text{not}(\text{not } X \text{ and } \text{not } Y)$

definition *ocl-implies* :: $[(\mathfrak{A})\text{Boolean}, (\mathfrak{A})\text{Boolean}] \Rightarrow (\mathfrak{A})\text{Boolean}$
(**infixl** *implies* 25)

where $X \text{ implies } Y \equiv \text{not } X \text{ or } Y$

lemma *cp-ocl-and*: $(X \text{ and } Y) \tau = ((\lambda -. X \tau) \text{ and } (\lambda -. Y \tau)) \tau$
by(*simp add: ocl-and-def*)

lemma *cp-ocl-or*: $((X::(\mathfrak{A})\text{Boolean}) \text{ or } Y) \tau = ((\lambda -. X \tau) \text{ or } (\lambda -. Y \tau)) \tau$
apply(*simp add: ocl-or-def*)
apply(*subst cp-not[of not ($\lambda -. X \tau$) and not ($\lambda -. Y \tau$)]*)
apply(*subst cp-ocl-and[of not ($\lambda -. X \tau$) not ($\lambda -. Y \tau$)]*)
by(*simp add: cp-not[symmetric] cp-ocl-and[symmetric]*)

lemma *cp-ocl-implies*: $(X \text{ implies } Y) \tau = ((\lambda \cdot. X \tau) \text{ implies } (\lambda \cdot. Y \tau)) \tau$
apply(*simp add: ocl-implies-def*)
apply(*subst cp-ocl-or[of not ($\lambda \cdot. X \tau$) ($\lambda \cdot. Y \tau$)]*)
by(*simp add: cp-not[symmetric] cp-ocl-or[symmetric]*)

lemma *ocl-and1*[*simp*]: $(\text{invalid and true}) = \text{invalid}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def*)
lemma *ocl-and2*[*simp*]: $(\text{invalid and false}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def*)
lemma *ocl-and3*[*simp*]: $(\text{invalid and null}) = \text{invalid}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)
lemma *ocl-and4*[*simp*]: $(\text{invalid and invalid}) = \text{invalid}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def*)

lemma *ocl-and5*[*simp*]: $(\text{null and true}) = \text{null}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)
lemma *ocl-and6*[*simp*]: $(\text{null and false}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)
lemma *ocl-and7*[*simp*]: $(\text{null and null}) = \text{null}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)
lemma *ocl-and8*[*simp*]: $(\text{null and invalid}) = \text{invalid}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)

lemma *ocl-and9*[*simp*]: $(\text{false and true}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)
lemma *ocl-and10*[*simp*]: $(\text{false and false}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)
lemma *ocl-and11*[*simp*]: $(\text{false and null}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)
lemma *ocl-and12*[*simp*]: $(\text{false and invalid}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)

lemma *ocl-and13*[*simp*]: $(\text{true and true}) = \text{true}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)
lemma *ocl-and14*[*simp*]: $(\text{true and false}) = \text{false}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def*)
lemma *ocl-and15*[*simp*]: $(\text{true and null}) = \text{null}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)
lemma *ocl-and16*[*simp*]: $(\text{true and invalid}) = \text{invalid}$
by(*rule ext, simp add: ocl-and-def null-def invalid-def true-def false-def bot-option-def null-fun-def null-option-def*)

lemma *ocl-and-idem*[*simp*]: $(X \text{ and } X) = X$

```

apply(rule ext,simp add: ocl-and-def null-def invalid-def true-def false-def)
apply(case-tac X x, simp-all)
apply(case-tac a, simp-all)
apply(case-tac aa, simp-all)
done

lemma ocl-and-commute:  $(X \text{ and } Y) = (Y \text{ and } X)$ 
  by(rule ext,auto simp:true-def false-def ocl-and-def invalid-def
      split: option.split option.split-asm
      bool.split bool.split-asm)

lemma ocl-and-false1[simp]:  $(\text{false and } X) = \text{false}$ 
  apply(rule ext, simp add: ocl-and-def)
  apply(auto simp:true-def false-def invalid-def
      split: option.split option.split-asm)
  done

lemma ocl-and-false2[simp]:  $(X \text{ and false}) = \text{false}$ 
  by(simp add: ocl-and-commute)

lemma ocl-and-true1[simp]:  $(\text{true and } X) = X$ 
  apply(rule ext, simp add: ocl-and-def)
  apply(auto simp:true-def false-def invalid-def
      split: option.split option.split-asm)
  done

lemma ocl-and-true2[simp]:  $(X \text{ and true}) = X$ 
  by(simp add: ocl-and-commute)

lemma ocl-and-assoc:  $(X \text{ and } (Y \text{ and } Z)) = (X \text{ and } Y \text{ and } Z)$ 
  apply(rule ext, simp add: ocl-and-def)
  apply(auto simp:true-def false-def null-def invalid-def
      split: option.split option.split-asm
      bool.split bool.split-asm)
  done

lemma ocl-or-idem[simp]:  $(X \text{ or } X) = X$ 
  by(simp add: ocl-or-def)

lemma ocl-or-commute:  $(X \text{ or } Y) = (Y \text{ or } X)$ 
  by(simp add: ocl-or-def ocl-and-commute)

lemma ocl-or-false1[simp]:  $(\text{false or } Y) = Y$ 
  by(simp add: ocl-or-def)

lemma ocl-or-false2[simp]:  $(Y \text{ or false}) = Y$ 

```


by(simp add: ocl-or-def)

lemma ocl-or-true1[simp]: (true or Y) = true
by(simp add: ocl-or-def)

lemma ocl-or-true2: (Y or true) = true
by(simp add: ocl-or-def)

lemma ocl-or-assoc: (X or (Y or Z)) = (X or Y or Z)
by(simp add: ocl-or-def ocl-and-assoc)

lemma deMorgan1: not(X and Y) = ((not X) or (not Y))
by(simp add: ocl-or-def)

lemma deMorgan2: not(X or Y) = ((not X) and (not Y))
by(simp add: ocl-or-def)

2.4 A Standard Logical Calculus for OCL

Besides the need for algebraic laws for OCL in order to normalize

definition OclValid :: [(\mathcal{A})st, (\mathcal{A})Boolean] \Rightarrow bool ((I(-)/ \models (-)) 50)
where $\tau \models P \equiv ((P \ \tau) = \text{true} \ \tau)$

2.4.1 Global vs. Local Judgements

lemma transform1: P = true $\implies \tau \models P$
by(simp add: OclValid-def)

lemma transform1-rev: $\forall \tau. \tau \models P \implies P = \text{true}$
by(rule ext, auto simp: OclValid-def true-def)

lemma transform2: (P = Q) $\implies ((\tau \models P) = (\tau \models Q))$
by(auto simp: OclValid-def)

lemma transform2-rev: $\forall \tau. (\tau \models \delta \ P) \wedge (\tau \models \delta \ Q) \wedge (\tau \models P) = (\tau \models Q) \implies P = Q$
apply(rule ext, auto simp: OclValid-def true-def defined-def)
apply(erule-tac x=a in allE)
apply(erule-tac x=b in allE)
apply(auto simp: false-def true-def defined-def bot-Boolean-def null-Boolean-def
split: option.split-asm HOL.split-if-asm)
done

However, certain properties (like transitivity) can not be *transformed* from the global level to the local one, they have to be re-proven on the local level.

lemma transform3:
assumes H : P = true $\implies Q = \text{true}$
shows $\tau \models P \implies \tau \models Q$
apply(simp add: OclValid-def)

```

apply(rule  $H[THEN\ fun-cong]$ )
apply(rule ext)
oops

```

2.4.2 Local Validity and Meta-logic

```

lemma foundation1[simp]:  $\tau \models true$ 
by(auto simp: OclValid-def)

```

```

lemma foundation2[simp]:  $\neg(\tau \models false)$ 
by(auto simp: OclValid-def true-def false-def)

```

```

lemma foundation3[simp]:  $\neg(\tau \models invalid)$ 
by(auto simp: OclValid-def true-def false-def invalid-def bot-option-def)

```

```

lemma foundation4[simp]:  $\neg(\tau \models null)$ 
by(auto simp: OclValid-def true-def false-def null-def null-fun-def null-option-def bot-option-def)

```

```

lemma bool-split-local[simp]:
( $\tau \models (x \triangleq invalid)$ )  $\vee$  ( $\tau \models (x \triangleq null)$ )  $\vee$  ( $\tau \models (x \triangleq true)$ )  $\vee$  ( $\tau \models (x \triangleq false)$ )
apply(insert bool-split[of  $x\ \tau$ ], auto)
apply(simp-all add: OclValid-def StrongEq-def true-def null-def invalid-def)
done

```

```

lemma def-split-local:
( $\tau \models \delta\ x$ ) = ( $((\neg(\tau \models (x \triangleq invalid))) \wedge (\neg(\tau \models (x \triangleq null))))$ )
by(simp add: defined-def true-def false-def invalid-def null-def
      StrongEq-def OclValid-def bot-fun-def null-fun-def)

```

```

lemma foundation5:
 $\tau \models (P\ and\ Q) \implies (\tau \models P) \wedge (\tau \models Q)$ 
by(simp add: ocl-and-def OclValid-def true-def false-def defined-def
      split: option.split option.split-asm bool.split bool.split-asm)

```

```

lemma foundation6:
 $\tau \models P \implies \tau \models \delta\ P$ 
by(simp add: not-def OclValid-def true-def false-def defined-def
      null-option-def null-fun-def bot-option-def bot-fun-def
      split: option.split option.split-asm)

```

```

lemma foundation7[simp]:
( $\tau \models not\ (\delta\ x)$ ) = ( $\neg(\tau \models \delta\ x)$ )
by(simp add: not-def OclValid-def true-def false-def defined-def
      split: option.split option.split-asm)

```

```

lemma foundation7'[simp]:
( $\tau \models not\ (v\ x)$ ) = ( $\neg(\tau \models v\ x)$ )
by(simp add: not-def OclValid-def true-def false-def valid-def)

```

split: option.split option.split-asm)

Key theorem for the Delta-closure: either an expression is defined, or it can be replaced (substituted via **StrongEq_L_subst2**; see below) by invalid or null. Strictness-reduction rules will usually reduce these substituted terms drastically.

lemma *foundation8*:

$(\tau \models \delta x) \vee (\tau \models (x \triangleq \text{invalid})) \vee (\tau \models (x \triangleq \text{null}))$

proof –

have 1 : $(\tau \models \delta x) \vee (\neg(\tau \models \delta x))$ **by** *auto*

have 2 : $(\neg(\tau \models \delta x)) = ((\tau \models (x \triangleq \text{invalid})) \vee (\tau \models (x \triangleq \text{null})))$

by (*simp only: def-split-local, simp*)

show *?thesis* **by** (*insert 1, simp add:2*)

qed

lemma *foundation9*:

$\tau \models \delta x \implies (\tau \models \text{not } x) = (\neg (\tau \models x))$

apply (*simp add: def-split-local*)

by (*auto simp: not-def null-fun-def null-option-def bot-option-def*
OclValid-def invalid-def true-def null-def StrongEq-def)

lemma *foundation10*:

$\tau \models \delta x \implies \tau \models \delta y \implies (\tau \models (x \text{ and } y)) = ((\tau \models x) \wedge (\tau \models y))$

apply (*simp add: def-split-local*)

by (*auto simp: ocl-and-def OclValid-def invalid-def*
true-def null-def StrongEq-def null-fun-def null-option-def bot-option-def
split:bool.split-asm)

lemma *foundation11*:

$\tau \models \delta x \implies \tau \models \delta y \implies (\tau \models (x \text{ or } y)) = ((\tau \models x) \vee (\tau \models y))$

apply (*simp add: def-split-local*)

by (*auto simp: not-def ocl-or-def ocl-and-def OclValid-def invalid-def*
true-def null-def StrongEq-def null-fun-def null-option-def bot-option-def
split:bool.split-asm bool.split)

lemma *foundation12*:

$\tau \models \delta x \implies \tau \models \delta y \implies (\tau \models (x \text{ implies } y)) = ((\tau \models x) \longrightarrow (\tau \models y))$

apply (*simp add: def-split-local*)

by (*auto simp: not-def ocl-or-def ocl-and-def ocl-implies-def bot-option-def*
OclValid-def invalid-def true-def null-def StrongEq-def null-fun-def null-option-def
split:bool.split-asm bool.split)

lemma *foundation13*: $(\tau \models A \triangleq \text{true}) = (\tau \models A)$

by (*auto simp: not-def OclValid-def invalid-def true-def null-def StrongEq-def*
split:bool.split-asm bool.split)

lemma *foundation14*: $(\tau \models A \triangleq \text{false}) = (\tau \models \text{not } A)$
by(*auto simp: not-def OclValid-def invalid-def false-def true-def null-def StrongEq-def*
split: bool.split-asm bool.split option.split)

lemma *foundation15*: $(\tau \models A \triangleq \text{invalid}) = (\tau \models \text{not}(v A))$
by(*auto simp: not-def OclValid-def valid-def invalid-def false-def true-def null-def*
StrongEq-def bot-option-def null-fun-def null-option-def bot-option-def bot-fun-def
split: bool.split-asm bool.split option.split)

lemma *foundation16*: $\tau \models (\delta X) = (X \tau \neq \text{bot} \wedge X \tau \neq \text{null})$
by(*auto simp: OclValid-def defined-def false-def true-def bot-fun-def null-fun-def*
split: split-if-asm)

lemmas *foundation17* = *foundation16*[*THEN iffD1, standard*]

lemma *foundation18*: $\tau \models (v X) = (X \tau \neq \text{invalid } \tau)$
by(*auto simp: OclValid-def valid-def false-def true-def bot-fun-def invalid-def*
split: split-if-asm)

lemma *foundation18'*: $\tau \models (v X) = (X \tau \neq \text{bot})$
by(*auto simp: OclValid-def valid-def false-def true-def bot-fun-def*
split: split-if-asm)

lemmas *foundation19* = *foundation18*[*THEN iffD1, standard*]

lemma *foundation20* : $\tau \models (\delta X) \implies \tau \models v X$
by(*simp add: foundation18 foundation16 invalid-def*)

lemma *foundation21*: $(\text{not } A \triangleq \text{not } B) = (A \triangleq B)$
by(*rule ext, auto simp: not-def StrongEq-def*
split: bool.split-asm HOL.split-if-asm option.split)

lemma *foundation22*: $(\tau \models (X \triangleq Y)) = (X \tau = Y \tau)$
by(*auto simp: StrongEq-def OclValid-def true-def*)

lemma *foundation23*: $(\tau \models P) = (\tau \models (\lambda \cdot . P \tau))$
by(*auto simp: OclValid-def true-def*)

lemmas *cp-validity*=*foundation23*

lemma *defined-not-I* : $\tau \models \delta(x) \implies \tau \models \delta(\text{not } x)$
by(*auto simp: not-def null-def invalid-def defined-def valid-def OclValid-def*
true-def false-def bot-option-def null-option-def null-fun-def bot-fun-def
split: option.split-asm HOL.split-if-asm)

lemma *valid-not-I* : $\tau \models v(x) \implies \tau \models v(\text{not } x)$
by(*auto simp*: *not-def null-def invalid-def defined-def valid-def OclValid-def*
true-def false-def bot-option-def null-option-def null-fun-def bot-fun-def
split: *option.split-asm option.split HOL.split-if-asm*)

lemma *defined-and-I* : $\tau \models \delta(x) \implies \tau \models \delta(y) \implies \tau \models \delta(x \text{ and } y)$
apply(*simp add*: *ocl-and-def null-def invalid-def defined-def valid-def OclValid-def*
true-def false-def bot-option-def null-option-def null-fun-def bot-fun-def
split: *option.split-asm HOL.split-if-asm*)
apply(*auto simp*: *null-option-def split*: *bool.split*)
by(*case-tac ya, simp-all*)

lemma *valid-and-I* : $\tau \models v(x) \implies \tau \models v(y) \implies \tau \models v(x \text{ and } y)$
apply(*simp add*: *ocl-and-def null-def invalid-def defined-def valid-def OclValid-def*
true-def false-def bot-option-def null-option-def null-fun-def bot-fun-def
split: *option.split-asm HOL.split-if-asm*)
by(*auto simp*: *null-option-def split*: *option.split bool.split*)

2.4.3 Local Judgements and Strong Equality

lemma *StrongEq-L-refl*: $\tau \models (x \triangleq x)$
by(*simp add*: *OclValid-def StrongEq-def*)

lemma *StrongEq-L-sym*: $\tau \models (x \triangleq y) \implies \tau \models (y \triangleq x)$
by(*simp add*: *StrongEq-sym*)

lemma *StrongEq-L-trans*: $\tau \models (x \triangleq y) \implies \tau \models (y \triangleq z) \implies \tau \models (x \triangleq z)$
by(*simp add*: *OclValid-def StrongEq-def true-def*)

In order to establish substitutivity (which does not hold in general HOL-formulas we introduce the following predicate that allows for a calculus of the necessary side-conditions.

definition *cp* :: $((\mathfrak{A}, \alpha) \text{ val} \implies (\mathfrak{A}, \beta) \text{ val}) \implies \text{bool}$
where $cp\ P \equiv (\exists f. \forall X\ \tau. P\ X\ \tau = f\ (X\ \tau)\ \tau)$

The rule of substitutivity in HOL-OCL holds only for context-passing expressions - i.e. those, that pass the context τ without changing it. Fortunately, all operators of the OCL language satisfy this property (but not all HOL operators).

lemma *StrongEq-L-subst1*: $\bigwedge \tau. cp\ P \implies \tau \models (x \triangleq y) \implies \tau \models (P\ x \triangleq P\ y)$
by(*auto simp*: *OclValid-def StrongEq-def true-def cp-def*)

lemma *StrongEq-L-subst2*:
 $\bigwedge \tau. cp\ P \implies \tau \models (x \triangleq y) \implies \tau \models (P\ x) \implies \tau \models (P\ y)$
by(*auto simp*: *OclValid-def StrongEq-def true-def cp-def*)

lemma *cpI1*:
 $(\forall X\ \tau. f\ X\ \tau = f(\lambda\cdot. X\ \tau)\ \tau) \implies cp\ P \implies cp(\lambda X. f\ (P\ X))$
apply(*auto simp*: *true-def cp-def*)
apply(*rule exI, (rule allI)+*)

by(*erule-tac* $x=P\ X$ **in** *allE*, *auto*)

lemma *cpI2*:

$(\forall\ X\ Y\ \tau. f\ X\ Y\ \tau = f(\lambda\cdot. X\ \tau)(\lambda\cdot. Y\ \tau)\ \tau) \implies$

$cp\ P \implies cp\ Q \implies cp(\lambda X. f\ (P\ X)\ (Q\ X))$

apply(*auto simp: true-def cp-def*)

apply(*rule exI, (rule allI)+*)

by(*erule-tac* $x=P\ X$ **in** *allE*, *auto*)

lemma *cp-const* : $cp(\lambda\cdot. c)$

by (*simp add: cp-def, fast*)

lemma *cp-id* : $cp(\lambda X. X)$

by (*simp add: cp-def, fast*)

lemmas *cp-intro*[*simp,intro!*] =

cp-const

cp-id

cp-defined[*THEN allI*[*THEN allI*[*THEN cpI1*], *of defined*]]

cp-valid[*THEN allI*[*THEN allI*[*THEN cpI1*], *of valid*]]

cp-not[*THEN allI*[*THEN allI*[*THEN cpI1*], *of not*]]

cp-ocl-and[*THEN allI*[*THEN allI*[*THEN allI*[*THEN cpI2*]], *of op and*]]

cp-ocl-or[*THEN allI*[*THEN allI*[*THEN allI*[*THEN cpI2*]], *of op or*]]

cp-ocl-implies[*THEN allI*[*THEN allI*[*THEN allI*[*THEN cpI2*]], *of op implies*]]

cp-StrongEq[*THEN allI*[*THEN allI*[*THEN allI*[*THEN cpI2*]],
of StrongEq]]

2.4.4 Laws to Establish Definedness (Delta-Closure)

For the logical connectives, we have — beyond $?\tau \models ?P \implies ?\tau \models \delta\ ?P$ — the following facts:

lemma *ocl-not-defargs*:

$\tau \models (not\ P) \implies \tau \models \delta\ P$

by(*auto simp: not-def OclValid-def true-def invalid-def defined-def false-def*

bot-fun-def bot-option-def null-fun-def null-option-def

split: bool.split-asm HOL.split-if-asm option.split option.split-asm)

So far, we have only one strict Boolean predicate (-family): The strict equality.

2.5 Miscellaneous: OCL's if then else endif

definition *if-ocl* :: $[(\mathfrak{A})\text{Boolean}, (\mathfrak{A}, \alpha::\text{null})\ \text{val}, (\mathfrak{A}, \alpha)\ \text{val}] \Rightarrow (\mathfrak{A}, \alpha)\ \text{val}$

$(if\ (-)\ then\ (-)\ else\ (-)\ endif\ [10,10,10]50)$

where $(if\ C\ then\ B_1\ else\ B_2\ endif) = (\lambda\ \tau. if\ (\delta\ C)\ \tau = true\ \tau$

$then\ (if\ (C\ \tau) = true\ \tau$

$then\ B_1\ \tau$

$else\ B_2\ \tau)$

$else\ invalid\ \tau)$

lemma *cp-if-ocl*:((if *C* then *B*₁ else *B*₂ endif) τ =
 (if ($\lambda \cdot$. *C* τ) then ($\lambda \cdot$. *B*₁ τ) else ($\lambda \cdot$. *B*₂ τ) endif) τ)
by(*simp only: if-ocl-def, subst cp-defined, rule refl*)

lemma *if-ocl-invalid* [*simp*]: (if *invalid* then *B*₁ else *B*₂ endif) = *invalid*
by(*rule ext, auto simp: if-ocl-def*)

lemma *if-ocl-null* [*simp*]: (if *null* then *B*₁ else *B*₂ endif) = *invalid*
by(*rule ext, auto simp: if-ocl-def*)

lemma *if-ocl-true* [*simp*]: (if *true* then *B*₁ else *B*₂ endif) = *B*₁
by(*rule ext, auto simp: if-ocl-def*)

lemma *if-ocl-true'* [*simp*]: $\tau \models P \implies (if\ P\ then\ B_1\ else\ B_2\ endif)\tau = B_1\ \tau$
apply(*subst cp-if-ocl, auto simp: OclValid-def*)
by(*simp add: cp-if-ocl[symmetric]*)

lemma *if-ocl-false* [*simp*]: (if *false* then *B*₁ else *B*₂ endif) = *B*₂
by(*rule ext, auto simp: if-ocl-def*)

lemma *if-ocl-false'* [*simp*]: $\tau \models not\ P \implies (if\ P\ then\ B_1\ else\ B_2\ endif)\tau = B_2\ \tau$
apply(*subst cp-if-ocl*)
apply(*auto simp: foundation14[symmetric] foundation22*)
by(*auto simp: cp-if-ocl[symmetric]*)

lemma *if-ocl-idem1* [*simp*]:(if $\delta\ X$ then *A* else *A* endif) = *A*
by(*rule ext, auto simp: if-ocl-def*)

lemma *if-ocl-idem2* [*simp*]:(if $v\ X$ then *A* else *A* endif) = *A*
by(*rule ext, auto simp: if-ocl-def*)

end

theory *OCL-lib*
imports *OCL-core*
begin

2.6 Basic Types like Void, Boolean and Integer

Since Integer is again a basic type, we define its semantic domain as the valuations over
int option option

type-synonym (\mathfrak{A})*Integer* = ($\mathfrak{A}, int\ option\ option$) *val*

by(rule ext, simp add: StrictRefEq-bool true-def false-def)

lemma strictEqBool-vs-strongEq:
 $\tau \models (v \ x) \implies \tau \models (v \ y) \implies (\tau \models (((x::('A) Boolean) \doteq y) \triangleq (x \triangleq y)))$
apply(simp add: StrictRefEq-bool OclValid-def)
apply(subst cp-StrongEq)**back**
by simp

lemma strictEqInt-vs-strongEq:
 $\tau \models (v \ x) \implies \tau \models (v \ y) \implies (\tau \models (((x::('A) Integer) \doteq y) \triangleq (x \triangleq y)))$
apply(simp add: StrictRefEq-int OclValid-def)
apply(subst cp-StrongEq)**back**
by simp

lemma strictEqBool-defargs:
 $\tau \models ((x::('A) Boolean) \doteq y) \implies (\tau \models (v \ x)) \wedge (\tau \models (v \ y))$
by(simp add: StrictRefEq-bool OclValid-def true-def invalid-def
bot-option-def
split: bool.split-asm HOL.split-if-asm)

lemma strictEqInt-defargs:
 $\tau \models ((x::('A) Integer) \doteq y) \implies (\tau \models (v \ x)) \wedge (\tau \models (v \ y))$
by(simp add: StrictRefEq-int OclValid-def true-def invalid-def valid-def bot-option-def
split: bool.split-asm HOL.split-if-asm)

lemma strictEqBool-valid-args-valid:
 $(\tau \models \delta((x::('A) Boolean) \doteq y)) = ((\tau \models (v \ x)) \wedge (\tau \models (v \ y)))$
by(auto simp: StrictRefEq-bool OclValid-def true-def valid-def false-def StrongEq-def
defined-def invalid-def null-fun-def bot-fun-def null-option-def bot-option-def
split: bool.split-asm HOL.split-if-asm option.split)

lemma strictEqInt-valid-args-valid:
 $(\tau \models \delta((x::('A) Integer) \doteq y)) = ((\tau \models (v \ x)) \wedge (\tau \models (v \ y)))$
by(auto simp: StrictRefEq-int OclValid-def true-def valid-def false-def StrongEq-def
defined-def invalid-def null-fun-def bot-fun-def null-option-def bot-option-def
split: bool.split-asm HOL.split-if-asm option.split)

lemma StrictRefEq-int-strict :
assumes A: $v \ (x::('A) Integer) = true$
and B: $v \ y = true$
shows $v \ (x \doteq y) = true$
apply(insert A B)
apply(rule ext, simp add: StrongEq-def StrictRefEq-int true-def valid-def defined-def
bot-fun-def bot-option-def)

done

lemma *StrictRefEq-int-strict'* :
 assumes $A: v ((x::('A)Integer)) \doteq y) = true$
 shows $v\ x = true \wedge v\ y = true$
 apply(insert A, rule conjI)
 apply(rule ext, drule-tac $x=xa$ in fun-cong)
 prefer 2
 apply(rule ext, drule-tac $x=xa$ in fun-cong)
 apply(simp-all add: StrongEq-def StrictRefEq-int
 false-def true-def valid-def defined-def)
 apply(case-tac y xa, auto)
 apply(simp-all add: true-def invalid-def bot-fun-def)
 done

lemma *StrictRefEq-int-strict''* : $\delta ((x::('A)Integer)) \doteq y) = (v(x) \text{ and } v(y))$
 by(auto intro!: transform2-rev defined-and-I simp:foundation10 strictEqInt-valid-args-valid)

lemma *StrictRefEq-bool-strict''* : $\delta ((x::('A)Boolean)) \doteq y) = (v(x) \text{ and } v(y))$
 by(auto intro!: transform2-rev defined-and-I simp:foundation10 strictEqBool-valid-args-valid)

lemma *cp-StrictRefEq-bool*:
 $((X::('A)Boolean) \doteq Y) \tau = ((\lambda -. X \tau) \doteq (\lambda -. Y \tau)) \tau$
 by(auto simp: StrictRefEq-bool StrongEq-def defined-def valid-def cp-defined[symmetric])

lemma *cp-StrictRefEq-int*:
 $((X::('A)Integer) \doteq Y) \tau = ((\lambda -. X \tau) \doteq (\lambda -. Y \tau)) \tau$
 by(auto simp: StrictRefEq-int StrongEq-def valid-def cp-defined[symmetric])

lemmas *cp-intro*[simp,intro!] =
 cp-intro
 cp-StrictRefEq-bool[THEN allI[THEN allI[THEN allI[THEN cpI2]], of StrictRefEq]]
 cp-StrictRefEq-int[THEN allI[THEN allI[THEN allI[THEN cpI2]], of StrictRefEq]]

definition *ocl-zero* :: ('A)Integer (**0**)
 where **0** = ($\lambda -. \lfloor 0::int \rfloor$)

definition *ocl-one* :: ('A)Integer (**1**)
 where **1** = ($\lambda -. \lfloor 1::int \rfloor$)

definition *ocl-two* :: ('A)Integer (**2**)
 where **2** = ($\lambda -. \lfloor 2::int \rfloor$)

definition *ocl-three* :: (' \mathcal{A})Integer (3)
where **3** = (λ - . $\llbracket 3::int \rrbracket$)

definition *ocl-four* :: (' \mathcal{A})Integer (4)
where **4** = (λ - . $\llbracket 4::int \rrbracket$)

definition *ocl-five* :: (' \mathcal{A})Integer (5)
where **5** = (λ - . $\llbracket 5::int \rrbracket$)

definition *ocl-six* :: (' \mathcal{A})Integer (6)
where **6** = (λ - . $\llbracket 6::int \rrbracket$)

definition *ocl-seven* :: (' \mathcal{A})Integer (7)
where **7** = (λ - . $\llbracket 7::int \rrbracket$)

definition *ocl-eight* :: (' \mathcal{A})Integer (8)
where **8** = (λ - . $\llbracket 8::int \rrbracket$)

definition *ocl-nine* :: (' \mathcal{A})Integer (9)
where **9** = (λ - . $\llbracket 9::int \rrbracket$)

definition *ten-nine* :: (' \mathcal{A})Integer (10)
where **10** = (λ - . $\llbracket 10::int \rrbracket$)

Here is a way to cast in standard operators via the type class system of Isabelle.

Here follows a list of code-examples, that explain the meanings of the above definitions by compilation to code and execution to "True".

2.6.3 Test Statements on Basic Types.

Elementary computations on Booleans

value $\tau_0 \models v(true)$
value $\tau_0 \models \delta(false)$
value $\neg(\tau_0 \models \delta(null))$
value $\neg(\tau_0 \models \delta(invalid))$
value $\tau_0 \models v((null::('A)Boolean))$
value $\neg(\tau_0 \models v(invalid))$
value $\tau_0 \models (true \text{ and } true)$
value $\tau_0 \models (true \text{ and } true \triangleq true)$
value $\tau_0 \models ((null \text{ or } null) \triangleq null)$
value $\tau_0 \models ((null \text{ or } null) \doteq null)$
value $\tau_0 \models ((true \triangleq false) \triangleq false)$
value $\tau_0 \models ((invalid \triangleq false) \triangleq false)$
value $\tau_0 \models ((invalid \doteq false) \triangleq invalid)$

Elementary computations on Integer

value $\tau_0 \models v(4)$

```

value  $\tau_0 \models \delta(4)$ 
value  $\tau_0 \models v((null::('A)Integer))$ 
value  $\tau_0 \models (invalid \triangleq invalid)$ 
value  $\tau_0 \models (null \triangleq null)$ 
value  $\tau_0 \models (4 \triangleq 4)$ 
value  $\neg(\tau_0 \models (9 \triangleq 10))$ 
value  $\neg(\tau_0 \models (invalid \triangleq 10))$ 
value  $\neg(\tau_0 \models (null \triangleq 10))$ 
value  $\neg(\tau_0 \models (invalid \doteq (invalid::('A)Integer)))$ 
value  $\tau_0 \models (null \doteq (null::('A)Integer))$ 
value  $\tau_0 \models (null \doteq (null::('A)Integer))$ 
value  $\tau_0 \models (4 \doteq 4)$ 
value  $\neg(\tau_0 \models (4 \doteq 10))$ 

```

lemma $\delta(null::('A)Integer) = false$ **by** *simp*

lemma $v(null::('A)Integer) = true$ **by** *simp*

2.6.4 More algebraic and logical layer on basic types

lemma $[simp, code-unfold]: v\ 0 = true$
by (*simp add: ocl-zero-def valid-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: \delta\ 1 = true$
by (*simp add: ocl-one-def defined-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: v\ 1 = true$
by (*simp add: ocl-one-def valid-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: \delta\ 2 = true$
by (*simp add: ocl-two-def defined-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: v\ 2 = true$
by (*simp add: ocl-two-def valid-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: v\ 6 = true$
by (*simp add: ocl-six-def valid-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma $[simp, code-unfold]: v\ 8 = true$
by (*simp add: ocl-eight-def valid-def true-def*
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma [simp,code-unfold]: $v \mathbf{9} = \text{true}$
by(simp add:ocl-nine-def valid-def true-def
bot-fun-def bot-option-def null-fun-def null-option-def)

lemma zero-non-null [simp]: $(\mathbf{0} \doteq \text{null}) = \text{false}$
by(rule ext,auto simp:ocl-zero-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)
lemma null-non-zero [simp]: $(\text{null} \doteq \mathbf{0}) = \text{false}$
by(rule ext,auto simp:ocl-zero-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)

lemma one-non-null [simp]: $(\mathbf{1} \doteq \text{null}) = \text{false}$
by(rule ext,auto simp:ocl-one-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)
lemma null-non-one [simp]: $(\text{null} \doteq \mathbf{1}) = \text{false}$
by(rule ext,auto simp:ocl-one-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)

lemma two-non-null [simp]: $(\mathbf{2} \doteq \text{null}) = \text{false}$
by(rule ext,auto simp:ocl-two-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)
lemma null-non-two [simp]: $(\text{null} \doteq \mathbf{2}) = \text{false}$
by(rule ext,auto simp:ocl-two-def null-def StrictRefEq-int valid-def invalid-def
bot-fun-def bot-option-def null-fun-def null-option-def StrongEq-def)

Here is a common case of a built-in operation on built-in types. Note that the arguments must be both defined (non-null, non-bot).

Note that we can not follow the lexis of standard OCL for Isabelle- technical reasons; these operators are heavily overloaded in the library that a further overloading would lead to heavy technical buzz in this document...

definition ocl-add-int :: $(\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Integer}$ (**infix** \oplus 40)
where $x \oplus y \equiv \lambda \tau. \text{if } (\delta x) \tau = \text{true} \tau \wedge (\delta y) \tau = \text{true} \tau$
then $[[[x \tau]] + [[y \tau]]]$
else *invalid* τ

definition ocl-less-int :: $(\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Boolean}$ (**infix** \prec 40)
where $x \prec y \equiv \lambda \tau. \text{if } (\delta x) \tau = \text{true} \tau \wedge (\delta y) \tau = \text{true} \tau$
then $[[[x \tau]] < [[y \tau]]]$
else *invalid* τ

definition ocl-le-int :: $(\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Integer} \Rightarrow (\mathfrak{A})\text{Boolean}$ (**infix** \preceq 40)
where $x \preceq y \equiv \lambda \tau. \text{if } (\delta x) \tau = \text{true} \tau \wedge (\delta y) \tau = \text{true} \tau$
then $[[[x \tau]] \leq [[y \tau]]]$
else *invalid* τ

Here follows a list of code-examples, that explain the meanings of the above definitions

by compilation to code and execution to "True".

```

value  $\tau_0 \models (9 \preceq 10)$ 
value  $\tau_0 \models ((4 \oplus 4) \preceq 10)$ 
value  $\neg(\tau_0 \models ((4 \oplus (4 \oplus 4)) \prec 10))$ 

```

2.7 Example for Complex Types: The Set-Collection Type

```

no-notation None ( $\perp$ )
notation bot ( $\perp$ )

```

2.7.1 The construction of the Set-Collection Type

For the semantic construction of the collection types, we have two goals:

1. we want the types to be *fully abstract*, i.e. the type should not contain junk-elements that are not representable by OCL expressions.
2. We want a possibility to nest collection types (so, we want the potential to talking about $Set(Set(Sequences(Pairs(X, Y))))$), and

The former principle rules out the option to define $'\alpha$ *Set* just by $(\mathfrak{A}, (' \alpha \text{ option option set}) \text{ val})$. This would allow sets to contain junk elements such as $\{\perp\}$ which we need to identify with undefinedness itself. Abandoning fully abstractness of rules would later on produce all sorts of problems when quantifying over the elements of a type. However, if we build an own type, then it must conform to our abstract interface in order to have nested types: arguments of type-constructors must conform to our abstract interface, and the result type too.

The core of an own type construction is done via a type definition which provides the raw-type $'\alpha$ *Set-0*. it is shown that this type "fits" indeed into the abstract type interface discussed in the previous section.

```

typedef  $'\alpha$  Set-0 = { $X :: (' \alpha :: \text{null}) \text{ set option option}.$ 
 $X = \text{bot} \vee X = \text{null} \vee (\forall x \in [X]. x \neq \text{bot})$ }
by (rule-tac  $x = \text{bot}$  in  $exI$ , simp)

```

```

instantiation Set-0 :: (null)bot
begin

```

```

definition bot-Set-0-def: ( $\text{bot} :: (' a :: \text{null}) \text{ Set-0}$ )  $\equiv$  Abs-Set-0 None

```

```

instance proof show  $\exists x :: 'a \text{ Set-0}. x \neq \text{bot}$ 
  apply(rule-tac  $x = \text{Abs-Set-0 } [None]$  in  $exI$ )
  apply(simp add: bot-Set-0-def)
  apply(subst Abs-Set-0-inject)
  apply(simp-all add: Set-0-def bot-Set-0-def
    null-option-def bot-option-def)
  done
qed

```

```

end

```

instantiation *Set-0* :: (*null*)*null*
begin

definition *null-Set-0-def*: (*null*::('a::*null*) *Set-0*) \equiv *Abs-Set-0* [*None*]

instance proof show (*null*::('a::*null*) *Set-0*) \neq *bot*
apply(*simp add:null-Set-0-def bot-Set-0-def*)
apply(*subst Abs-Set-0-inject*)
apply(*simp-all add: Set-0-def bot-Set-0-def*
null-option-def bot-option-def)
done
qed

end

... and lifting this type to the format of a valuation gives us:

type-synonym ($\mathfrak{A}, 'a$) *Set* = ($\mathfrak{A}, 'a$ *Set-0*) *val*

lemma *Set-inv-lemma*: $\tau \models (\delta \ X) \implies (X \ \tau = \text{Abs-Set-0} \ [bot])$
 $\vee (\forall x \in \llbracket \text{Rep-Set-0} \ (X \ \tau) \rrbracket. x \neq bot)$
apply(*insert OCL-lib.Set-0.Rep-Set-0 [of X τ], simp add:Set-0-def*)
apply(*auto simp: OclValid-def defined-def false-def true-def cp-def*
bot-fun-def bot-Set-0-def null-Set-0-def null-fun-def
split:split-if-asm)
apply(*erule contrapos-pp [of Rep-Set-0 (X τ) = bot]*)
apply(*subst Abs-Set-0-inject[symmetric], simp add:Rep-Set-0*)
apply(*simp add: Set-0-def*)
apply(*simp add: Rep-Set-0-inverse bot-Set-0-def bot-option-def*)
apply(*erule contrapos-pp [of Rep-Set-0 (X τ) = null]*)
apply(*subst Abs-Set-0-inject[symmetric], simp add:Rep-Set-0*)
apply(*simp add: Set-0-def*)
apply(*simp add: Rep-Set-0-inverse null-option-def*)
done

lemma *invalid-set-not-defined* [*simp,code-unfold*]: $\delta(\text{invalid}::(\mathfrak{A}, 'a::\text{null}) \ \text{Set}) = \text{false}$ **by** *simp*

lemma *null-set-not-defined* [*simp,code-unfold*]: $\delta(\text{null}::(\mathfrak{A}, 'a::\text{null}) \ \text{Set}) = \text{false}$

by(*simp add: defined-def null-fun-def*)

lemma *invalid-set-valid* [*simp,code-unfold*]: $v(\text{invalid}::(\mathfrak{A}, 'a::\text{null}) \ \text{Set}) = \text{false}$

by *simp*

lemma *null-set-valid* [*simp,code-unfold*]: $v(\text{null}::(\mathfrak{A}, 'a::\text{null}) \ \text{Set}) = \text{true}$

apply(*simp add: valid-def null-fun-def bot-fun-def bot-Set-0-def null-Set-0-def*)

apply(*subst Abs-Set-0-inject,simp-all add: Set-0-def null-option-def bot-option-def*)

done

... which means that we can have a type ($\mathfrak{A}, (\mathfrak{A}, (\mathfrak{A}) \ \text{Integer}) \ \text{Set}) \ \text{Set}$ corresponding exactly to $\text{Set}(\text{Set}(\text{Integer}))$ in OCL notation. Note that the parameter \mathfrak{A} still refers to the object universe; making the OCL semantics entirely parametric in the object universe makes it possible to study (and prove) its properties independently from a concrete class

diagram.

2.7.2 Constants on Sets

definition $mtSet::('A, 'α::null) Set (Set\{\})$
where $Set\{\} \equiv (\lambda \tau. Abs-Set-0 \llbracket \{\}::'α \text{ set} \rrbracket)$

lemma $mtSet-defined[simp, code-unfold]: \delta(Set\{\}) = true$
apply(rule ext, auto simp: mtSet-def defined-def null-Set-0-def
 bot-Set-0-def bot-fun-def null-fun-def)
apply(simp-all add: Abs-Set-0-inject Set-0-def bot-option-def null-Set-0-def null-option-def)
done

lemma $mtSet-valid[simp, code-unfold]: v(Set\{\}) = true$
apply(rule ext, auto simp: mtSet-def valid-def null-Set-0-def
 bot-Set-0-def bot-fun-def null-fun-def)
apply(simp-all add: Abs-Set-0-inject Set-0-def bot-option-def null-Set-0-def null-option-def)
done

Note that the collection types in OCL allow for null to be included; however, there is the null-collection into which inclusion yields invalid.

2.7.3 Strict Equality on Sets

This section of foundational operations on sets is closed with a paragraph on equality. Strong Equality is inherited from the OCL core, but we have to consider the case of the strict equality. We decide to overload strict equality in the same way we do for other value's in OCL:

defs $StrictRefEq-set :$
 $(x::('A, 'α::null)Set) \doteq y \equiv \lambda \tau. \text{if } (v\ x)\ \tau = true\ \tau \wedge (v\ y)\ \tau = true\ \tau$
 $\quad \text{then } (x \triangleq y)\tau$
 $\quad \text{else invalid } \tau$

lemma $RefEq-set-refl[simp, code-unfold]:$
 $((x::('A, 'α::null)Set) \doteq x) = (\text{if } (v\ x) \text{ then true else invalid endif})$
by(rule ext, simp add: StrictRefEq-set if-ocl-def)

lemma $StrictRefEq-set-strict1: ((x::('A, 'α::null)Set) \doteq invalid) = invalid$
by(simp add: StrictRefEq-set false-def true-def)

lemma $StrictRefEq-set-strict2: (invalid \doteq (y::('A, 'α::null)Set)) = invalid$
by(simp add: StrictRefEq-set false-def true-def)

lemma $StrictRefEq-set-strictEq-valid-args-valid:$
 $(\tau \models \delta((x::('A, 'α::null)Set) \doteq y)) = ((\tau \models (v\ x)) \wedge (\tau \models v\ y))$
proof –


```

have A:  $\tau \models \delta (x \doteq y) \implies \tau \models v\ x \wedge \tau \models v\ y$ 
  apply(simp add: StrictRefEq-set valid-def OclValid-def defined-def)
  apply(simp add: invalid-def bot-fun-def split: split-if-asm)
  done
have B:  $(\tau \models v\ x) \wedge (\tau \models v\ y) \implies \tau \models \delta (x \doteq y)$ 
  apply(simp add: StrictRefEq-set, elim conjE)
  apply(drule foundation13[THEN iffD2], drule foundation13[THEN iffD2])
  apply(rule cp-validity[THEN iffD2])
  apply(subst cp-defined, simp add: foundation22)
  apply(simp add: cp-defined[symmetric] cp-validity[symmetric])
  done
show ?thesis by(auto intro!: A B)
qed

lemma cp-StrictRefEq-set:  $((X::('A, 'a::null)Set) \doteq Y) \tau = ((\lambda-. X\ \tau) \doteq (\lambda-. Y\ \tau)) \tau$ 
by(simp add: StrictRefEq-set cp-StrongEq[symmetric] cp-valid[symmetric])

lemma strictRefEq-set-vs-strongEq:
 $\tau \models v\ x \implies \tau \models v\ y \implies (\tau \models (((x::('A, 'a::null)Set) \doteq y) \triangleq (x \triangleq y)))$ 
apply(drule foundation13[THEN iffD2], drule foundation13[THEN iffD2])
by(simp add: StrictRefEq-set foundation22)

```

2.7.4 Algebraic Properties on Strict Equality on Sets

One might object here that for the case of objects, this is an empty definition. The answer is no, we will restrain later on states and objects such that any object has its id stored inside the object (so the ref, under which an object can be referenced in the store will be represented in the object itself). For such well-formed stores that satisfy this invariant (the WFF - invariant), the referential equality and the strong equality — and therefore the strict equality on sets in the sense above) coincides.

To become operational, we derive:

```

lemma StrictRefEq-set-refl :
 $((x::('A, 'a::null)Set) \doteq x) = (if\ (v\ x)\ then\ true\ else\ invalid\ endif)$ 
by(rule ext, simp add: StrictRefEq-set if-ocl-def)

```

The key for an operational definition is OclForall given below.

The case of the size definition is somewhat special, we admit explicitly in Essential OCL the possibility of infinite sets. For the size definition, this requires an extra condition that assures that the cardinality of the set is actually a defined integer.

2.7.5 Library Operations on Sets

```

definition OclSize ::  $('A, 'a::null)Set \Rightarrow 'A\ Integer$ 
where OclSize x =  $(\lambda\ \tau.\ if\ (\delta\ x)\ \tau = true\ \tau \wedge finite([Rep-Set-0\ (x\ \tau)])$ 
   $then\ [\ int(card\ [Rep-Set-0\ (x\ \tau)]) ] ]$ 

```

$$else \perp)$$

definition $OclIncluding :: ([(\mathfrak{A}, \alpha :: null) Set, (\mathfrak{A}, \alpha) val] \Rightarrow (\mathfrak{A}, \alpha) Set$
where $OclIncluding\ x\ y = (\lambda\ \tau.\ if\ (\delta\ x)\ \tau = true\ \tau \wedge (v\ y)\ \tau = true\ \tau$
 $then\ Abs\text{-}Set\text{-}0\ \llbracket \llbracket Rep\text{-}Set\text{-}0\ (x\ \tau) \rrbracket \rrbracket \cup \{y\ \tau\} \rrbracket$
 $else\ \perp)$

definition $OclIncludes$ $:: [(\mathfrak{A}, \alpha :: null) \text{ Set}, (\mathfrak{A}, \alpha) \text{ val}] \Rightarrow \mathfrak{A} \text{ Boolean}$
where $OclIncludes \ x \ y = (\lambda \ \tau. \text{ if } (\delta \ x) \ \tau = true \ \tau \wedge (v \ y) \ \tau = true \ \tau$
 $\text{ then } \llbracket (y \ \tau) \in \llbracket Rep\text{-}Set\text{-}0 \ (x \ \tau) \rrbracket \rrbracket$
 $\text{ else } \perp)$

definition $OclExcluding :: [(\mathfrak{A}, \alpha :: null) Set, (\mathfrak{A}, \alpha) val] \Rightarrow (\mathfrak{A}, \alpha) Set$
where $OclExcluding\ x\ y = (\lambda\ \tau.\ \text{if } (\delta\ x)\ \tau = true\ \tau \wedge (v\ y)\ \tau = true\ \tau$
 $\text{then } Abs\text{-}Set\text{-}0\ \llbracket \llbracket Rep\text{-}Set\text{-}0\ (x\ \tau) \rrbracket - \{y\ \tau\} \rrbracket$
 $\text{else } \perp)$

definition $OclExcludes :: [('A, 'a :: null) Set, ('A, 'a) val] \Rightarrow 'A Boolean$
where $OclExcludes\ x\ y = (not(OclIncludes\ x\ y))$

The following definition follows the requirement of the standard to treat null as neutral element of sets. It is a well-documented exception from the general strictness rule and the rule that the distinguished argument self should be non-null.

definition $OclIsEmpty :: ('A, 'a::null) Set \Rightarrow 'A \text{ Boolean}$
where $OclIsEmpty\ x = ((x \doteq null) \text{ or } ((OclSize\ x) \doteq 0))$

definition *OclNotEmpty* :: ($\mathcal{A}, 'a :: \text{null}$) *Set* \Rightarrow $'a$ *Boolean*
where *OclNotEmpty* $x = \text{not}(\text{OclIsEmpty } x)$

$$\begin{array}{ll}
\textbf{definition} & \textit{OclForall} \quad :: [(\mathfrak{A}, ' \alpha : null) Set, (' \mathfrak{A}, ' \alpha) val \Rightarrow (' \mathfrak{A}) Boolean] \Rightarrow ' \mathfrak{A} Boolean \\
\textbf{where} & \textit{OclForall} \ S \ P = (\lambda \ \tau. \text{ if } (\delta \ S) \ \tau = \text{true } \tau \\
& \quad \text{then if } (\forall x \in [\textit{Rep-Set-0} \ (S \ \tau)] . P \ (\lambda \ -. \ x) \ \tau = \text{true } \tau) \\
& \quad \quad \text{then true } \tau \\
& \quad \quad \text{else if } (\forall x \in [\textit{Rep-Set-0} \ (S \ \tau)] . P(\lambda \ -. \ x) \ \tau = \text{true } \tau \vee \\
& \quad \quad \quad P(\lambda \ -. \ x) \ \tau = \text{false } \tau) \\
& \quad \quad \text{then false } \tau \\
& \quad \quad \text{else } \perp \\
& \text{else } \perp)
\end{array}$$

definition $OclExists \quad :: [\langle \mathfrak{A}, ' \alpha :: null \rangle \text{ Set}, \langle \mathfrak{A}, ' \alpha \rangle \text{ val} \Rightarrow \langle \mathfrak{A} \rangle \text{ Boolean}] \Rightarrow \mathfrak{A} \text{ Boolean}$
where $OclExists \ S \ P = not(OclForall \ S \ (\lambda \ X. not \ (P \ X)))$

$$\begin{array}{l} \text{syntax} \\ \text{-OclForall} :: [(\mathfrak{A}, \alpha :: \text{null}) \text{ Set}, id, (\mathfrak{A}) \text{ Boolean}] \Rightarrow \mathfrak{A} \text{ Boolean} \quad ((-) \rightarrow \text{forall}'(-|')) \\ \text{translations} \end{array}$$

$X \rightarrow \text{forall}(x \mid P) == \text{CONST OclForall } X \ (\%x. P)$

syntax

$\text{-OclExist} :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, \text{id}, (\mathfrak{A}) \text{ Boolean}] \Rightarrow \mathfrak{A} \text{ Boolean} \quad ((-) \rightarrow \text{exists}'(-|'))$

translations

$X \rightarrow \text{exists}(x \mid P) == \text{CONST OclExists } X \ (\%x. P)$

consts

$\text{OclUnion} \quad :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, (\mathfrak{A}, ' \alpha) \text{ Set}] \Rightarrow (\mathfrak{A}, ' \alpha) \text{ Set}$
 $\text{OclIntersection} :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, (\mathfrak{A}, ' \alpha) \text{ Set}] \Rightarrow (\mathfrak{A}, ' \alpha) \text{ Set}$
 $\text{OclIncludesAll} \quad :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, (\mathfrak{A}, ' \alpha) \text{ Set}] \Rightarrow \mathfrak{A} \text{ Boolean}$
 $\text{OclExcludesAll} \quad :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, (\mathfrak{A}, ' \alpha) \text{ Set}] \Rightarrow \mathfrak{A} \text{ Boolean}$
 $\text{OclComplement} \quad :: (\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set} \Rightarrow (\mathfrak{A}, ' \alpha) \text{ Set}$
 $\text{OclSum} \quad \quad :: (\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set} \Rightarrow \mathfrak{A} \text{ Integer}$
 $\text{OclCount} \quad \quad :: [(\mathfrak{A}, ' \alpha :: \text{null}) \text{ Set}, (\mathfrak{A}, ' \alpha) \text{ Set}] \Rightarrow \mathfrak{A} \text{ Integer}$

notation

$\text{OclSize} \quad \quad \quad (\rightarrow \text{size}'(') [66])$
and
 $\text{OclCount} \quad \quad (\rightarrow \text{count}'(-) [66, 65] 65)$
and
 $\text{OclIncludes} \quad (\rightarrow \text{includes}'(-) [66, 65] 65)$
and
 $\text{OclExcludes} \quad (\rightarrow \text{excludes}'(-) [66, 65] 65)$
and
 $\text{OclSum} \quad \quad \quad (\rightarrow \text{sum}'(') [66])$
and
 $\text{OclIncludesAll} \quad (\rightarrow \text{includesAll}'(-) [66, 65] 65)$
and
 $\text{OclExcludesAll} \quad (\rightarrow \text{excludesAll}'(-) [66, 65] 65)$
and
 $\text{OclIsEmpty} \quad \quad (\rightarrow \text{isEmpty}'(') [66])$
and
 $\text{OclNotEmpty} \quad (\rightarrow \text{notEmpty}'(') [66])$
and
 $\text{OclIncluding} \quad (\rightarrow \text{including}'(-))$
and
 $\text{OclExcluding} \quad (\rightarrow \text{excluding}'(-))$
and
 $\text{OclComplement} \quad (\rightarrow \text{complement}'('))$

and
 $OclUnion \quad (\rightarrow union'(-') \quad [66,65]65)$
and
 $OclIntersection(\rightarrow intersection'(-') \quad [71,70]70)$

lemma *cp-OclIncluding*:
 $(X \rightarrow including(x)) \tau = ((\lambda -. X \tau) \rightarrow including(\lambda -. x \tau)) \tau$
by(*auto simp: OclIncluding-def StrongEq-def invalid-def*
cp-defined[symmetric] cp-valid[symmetric])

lemma *cp-OclExcluding*:
 $(X \rightarrow excluding(x)) \tau = ((\lambda -. X \tau) \rightarrow excluding(\lambda -. x \tau)) \tau$
by(*auto simp: OclExcluding-def StrongEq-def invalid-def*
cp-defined[symmetric] cp-valid[symmetric])

lemma *cp-OclIncludes*:
 $(X \rightarrow includes(x)) \tau = (OclIncludes (\lambda -. X \tau) (\lambda -. x \tau) \tau)$
by(*auto simp: OclIncludes-def StrongEq-def invalid-def*
cp-defined[symmetric] cp-valid[symmetric])

2.7.6 Logic and Algebraic Layer on Set Operations

lemma *including-strict1*[*simp,code-unfold*]:(*invalid* $\rightarrow including(x)$) = *invalid*
by(*simp add: bot-fun-def OclIncluding-def invalid-def defined-def valid-def false-def true-def*)

lemma *including-strict2*[*simp,code-unfold*]:(*X* $\rightarrow including(invalid)$) = *invalid*
by(*simp add: OclIncluding-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *including-strict3*[*simp,code-unfold*]:(*null* $\rightarrow including(x)$) = *invalid*
by(*simp add: OclIncluding-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *excluding-strict1*[*simp,code-unfold*]:(*invalid* $\rightarrow excluding(x)$) = *invalid*
by(*simp add: bot-fun-def OclExcluding-def invalid-def defined-def valid-def false-def true-def*)

lemma *excluding-strict2*[*simp,code-unfold*]:(*X* $\rightarrow excluding(invalid)$) = *invalid*
by(*simp add: OclExcluding-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *excluding-strict3*[*simp,code-unfold*]:(*null* $\rightarrow excluding(x)$) = *invalid*
by(*simp add: OclExcluding-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *includes-strict1*[*simp,code-unfold*]:(*invalid* $\rightarrow includes(x)$) = *invalid*
by(*simp add: bot-fun-def OclIncludes-def invalid-def defined-def valid-def false-def true-def*)

lemma *includes-strict2*[*simp,code-unfold*]:(*X* $\rightarrow includes(invalid)$) = *invalid*
by(*simp add: OclIncludes-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *includes-strict3*[simp,code-unfold]:(*null*->*includes*(*x*)) = *invalid*
by(simp add: *OclIncludes-def invalid-def bot-fun-def defined-def valid-def false-def true-def*)

lemma *including-defined-args-valid*:
 $(\tau \models \delta(X \rightarrow \text{including}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$
proof –
have *A* : $\perp \in \text{Set-0}$ **by**(simp add: *Set-0-def bot-option-def*)
have *B* : $\lfloor \perp \rfloor \in \text{Set-0}$ **by**(simp add: *Set-0-def null-option-def bot-option-def*)
have *C* : $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies \lfloor \lfloor \text{insert } (x \ \tau) \llbracket \text{Rep-Set-0 } (X \ \tau) \rrbracket \rfloor \rfloor \in \text{Set-0}$
apply(frule *Set-inv-lemma*)
apply(simp add: *Set-0-def bot-option-def null-Set-0-def null-fun-def foundation18 foundation16 invalid-def*)
done
have *D*: $(\tau \models \delta(X \rightarrow \text{including}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$
by(auto simp: *OclIncluding-def OclValid-def true-def valid-def false-def StrongEq-def defined-def invalid-def bot-fun-def null-fun-def split: bool.split-asm HOL.split-if-asm option.split*)
have *E*: $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models \delta(X \rightarrow \text{including}(x)))$
apply(frule *C*, simp)
apply(auto simp: *OclIncluding-def OclValid-def true-def false-def StrongEq-def defined-def invalid-def valid-def bot-fun-def null-fun-def split: bool.split-asm HOL.split-if-asm option.split*)
apply(simp-all add: *null-Set-0-def bot-Set-0-def bot-option-def*)
apply(simp-all add: *Abs-Set-0-inject A B bot-option-def[symmetric]*,
simp-all add: bot-option-def)
done
show ?thesis **by**(auto dest:*D intro:E*)
qed

lemma *including-valid-args-valid*:
 $(\tau \models v(X \rightarrow \text{including}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$
proof –
have *A* : $\perp \in \text{Set-0}$ **by**(simp add: *Set-0-def bot-option-def*)
have *B* : $\lfloor \perp \rfloor \in \text{Set-0}$ **by**(simp add: *Set-0-def null-option-def bot-option-def*)
have *C* : $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies \lfloor \lfloor \text{insert } (x \ \tau) \llbracket \text{Rep-Set-0 } (X \ \tau) \rrbracket \rfloor \rfloor \in \text{Set-0}$
apply(frule *Set-inv-lemma*)
apply(simp add: *Set-0-def bot-option-def null-Set-0-def null-fun-def foundation18 foundation16 invalid-def*)
done
have *D*: $(\tau \models v(X \rightarrow \text{including}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$
by(auto simp: *OclIncluding-def OclValid-def true-def valid-def false-def StrongEq-def defined-def invalid-def bot-fun-def null-fun-def split: bool.split-asm HOL.split-if-asm option.split*)
have *E*: $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models v(X \rightarrow \text{including}(x)))$
apply(frule *C*, simp)

```

apply(auto simp: OclIncluding-def OclValid-def true-def false-def StrongEq-def
      defined-def invalid-def valid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
apply(simp-all add: null-Set-0-def bot-Set-0-def bot-option-def)
apply(simp-all add: Abs-Set-0-inject A B bot-option-def[symmetric],
      simp-all add: bot-option-def)
done
show ?thesis by(auto dest:D intro:E)
qed

```

```

lemma including-defined-args-valid'[simp,code-unfold]:
 $\delta(X \rightarrow \text{including}(x)) = ((\delta X) \text{ and } (v x))$ 
by(auto intro!: transform2-rev simp:including-defined-args-valid foundation10 defined-and-I)

```

```

lemma including-valid-args-valid'[simp,code-unfold]:
 $v(X \rightarrow \text{including}(x)) = ((\delta X) \text{ and } (v x))$ 
by(auto intro!: transform2-rev simp:including-valid-args-valid foundation10 defined-and-I)

```

```

lemma excluding-defined-args-valid:
 $(\tau \models \delta(X \rightarrow \text{excluding}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 
proof -
  have  $A : \perp \in \text{Set-0}$  by(simp add: Set-0-def bot-option-def)
  have  $B : \lfloor \perp \rfloor \in \text{Set-0}$  by(simp add: Set-0-def null-option-def bot-option-def)
  have  $C : (\tau \models (\delta X)) \implies (\tau \models (v x)) \implies \lfloor \lfloor \text{Rep-Set-0 } (X \ \tau) \rfloor \rfloor - \{x \ \tau\} \rfloor \in \text{Set-0}$ 
    apply(frule Set-inv-lemma)
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def
      foundation18 foundation16 invalid-def)
  done
  have  $D : (\tau \models \delta(X \rightarrow \text{excluding}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 
    by(auto simp: OclExcluding-def OclValid-def true-def valid-def false-def StrongEq-def
      defined-def invalid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
  have  $E : (\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models \delta(X \rightarrow \text{excluding}(x)))$ 
    apply(frule C, simp)
    apply(auto simp: OclExcluding-def OclValid-def true-def false-def StrongEq-def
      defined-def invalid-def valid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
    apply(simp-all add: null-Set-0-def bot-Set-0-def bot-option-def)
    apply(simp-all add: Abs-Set-0-inject A B bot-option-def[symmetric],
      simp-all add: bot-option-def)
  done
show ?thesis by(auto dest:D intro:E)
qed

```

```

lemma excluding-valid-args-valid:
 $(\tau \models v(X \rightarrow \text{excluding}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 

```

```

proof –
  have A :  $\perp \in \text{Set-0}$  by(simp add: Set-0-def bot-option-def)
  have B :  $\lfloor \perp \rfloor \in \text{Set-0}$  by(simp add: Set-0-def null-option-def bot-option-def)
  have C :  $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies \lfloor \lfloor \text{Rep-Set-0 } (X \ \tau) \rfloor \rfloor - \{x \ \tau\} \rfloor \in \text{Set-0}$ 
    apply(frule Set-inv-lemma)
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def
      foundation18 foundation16 invalid-def)
    done
  have D:  $(\tau \models v(X \rightarrow \text{excluding}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 
    by(auto simp: OclExcluding-def OclValid-def true-def valid-def false-def StrongEq-def
      defined-def invalid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
  have E:  $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models v(X \rightarrow \text{excluding}(x)))$ 
    apply(frule C, simp)
    apply(auto simp: OclExcluding-def OclValid-def true-def false-def StrongEq-def
      defined-def invalid-def valid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
    apply(simp-all add: null-Set-0-def bot-Set-0-def bot-option-def)
    apply(simp-all add: Abs-Set-0-inject A B bot-option-def[symmetric],
      simp-all add: bot-option-def)
    done
show ?thesis by(auto dest:D intro:E)
qed

```

```

lemma excluding-valid-args-valid'[simp,code-unfold]:
 $\delta(X \rightarrow \text{excluding}(x)) = ((\delta X) \text{ and } (v x))$ 
by(auto intro!: transform2-rev simp:excluding-defined-args-valid foundation10 defined-and-I)

```

```

lemma excluding-valid-args-valid''[simp,code-unfold]:
 $v(X \rightarrow \text{excluding}(x)) = ((\delta X) \text{ and } (v x))$ 
by(auto intro!: transform2-rev simp:excluding-valid-args-valid foundation10 defined-and-I)

```

```

lemma includes-defined-args-valid:
 $(\tau \models \delta(X \rightarrow \text{includes}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 
proof –
  have A:  $(\tau \models \delta(X \rightarrow \text{includes}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$ 
    by(auto simp: OclIncludes-def OclValid-def true-def valid-def false-def StrongEq-def
      defined-def invalid-def bot-fun-def null-fun-def
      split: bool.split-asm HOL.split-if-asm option.split)
  have B:  $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models \delta(X \rightarrow \text{includes}(x)))$ 
    by(auto simp: OclIncludes-def OclValid-def true-def false-def StrongEq-def
      defined-def invalid-def valid-def bot-fun-def null-fun-def
      bot-option-def null-option-def
      split: bool.split-asm HOL.split-if-asm option.split)
show ?thesis by(auto dest:A intro:B)

```

qed

lemma *includes-valid-args-valid*:

$(\tau \models v(X \rightarrow \text{includes}(x))) = ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$

proof –

have *A*: $(\tau \models v(X \rightarrow \text{includes}(x))) \implies ((\tau \models (\delta X)) \wedge (\tau \models (v x)))$

by (*auto simp: OclIncludes-def OclValid-def true-def valid-def false-def StrongEq-def
defined-def invalid-def bot-fun-def null-fun-def
split: bool.split-asm HOL.split-if-asm option.split*)

have *B*: $(\tau \models (\delta X)) \implies (\tau \models (v x)) \implies (\tau \models v(X \rightarrow \text{includes}(x)))$

by (*auto simp: OclIncludes-def OclValid-def true-def false-def StrongEq-def
defined-def invalid-def valid-def bot-fun-def null-fun-def
bot-option-def null-option-def
split: bool.split-asm HOL.split-if-asm option.split*)

show *?thesis* **by** (*auto dest:A intro:B*)

qed

lemma *includes-valid-args-valid'[simp,code-unfold]*:

$\delta(X \rightarrow \text{includes}(x)) = ((\delta X) \text{ and } (v x))$

by (*auto intro!: transform2-rev simp:includes-defined-args-valid foundation10 defined-and-I*)

lemma *includes-valid-args-valid''[simp,code-unfold]*:

$v(X \rightarrow \text{includes}(x)) = ((\delta X) \text{ and } (v x))$

by (*auto intro!: transform2-rev simp:includes-valid-args-valid foundation10 defined-and-I*)

Some computational laws: **lemma** *including-chn0[simp]*:

assumes *val-x*: $\tau \models (v x)$

shows $\tau \models \text{not}(\text{Set}\{\} \rightarrow \text{includes}(x))$

using *val-x*

apply (*auto simp: OclValid-def OclIncludes-def not-def false-def true-def*)

apply (*auto simp: mtSet-def OCL-lib.Set-0.Abs-Set-0-inverse Set-0-def*)

done

lemma *including-chn0'[simp,code-unfold]*:

$\text{Set}\{\} \rightarrow \text{includes}(x) = (\text{if } v x \text{ then false else invalid endif})$

proof –

have *A*: $\bigwedge \tau. (\text{Set}\{\} \rightarrow \text{includes}(\text{invalid})) \tau = (\text{if } (v \text{ invalid}) \text{ then false else invalid endif}) \tau$
by *simp*

have *B*: $\bigwedge \tau x. \tau \models (v x) \implies (\text{Set}\{\} \rightarrow \text{includes}(x)) \tau = (\text{if } v x \text{ then false else invalid endif})$

τ

apply (*frule including-chn0, simp add: OclValid-def*)

apply (*rule foundation21[THEN fun-cong, simplified StrongEq-def,simplified,
THEN iffD1, of - - false]*)

by *simp*

show *?thesis*

apply (*rule ext, rename-tac τ*)

apply (*case-tac τ ⊨ (v x)*)

apply (*simp-all add: B foundation18*)


```

    apply(subst cp-OclIncludes, simp add: cp-OclIncludes[symmetric] A)
  done
qed

lemma including-cha1:
assumes def-X:  $\tau \models (\delta X)$ 
assumes val-x:  $\tau \models (v x)$ 
shows  $\tau \models (X \rightarrow \text{including}(x) \rightarrow \text{includes}(x))$ 
proof -
  have A :  $\perp \in \text{Set-0}$  by(simp add: Set-0-def bot-option-def)
  have B :  $\lfloor \perp \rfloor \in \text{Set-0}$  by(simp add: Set-0-def null-option-def bot-option-def)
  have C :  $\lfloor \lfloor \text{insert } (x \ \tau) \rfloor \rfloor \in \text{Set-0}$ 
    apply(insert def-X[THEN foundation17] val-x[THEN foundation19] Set-inv-lemma[OF
def-X])
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def invalid-def)
  done
show ?thesis
  apply(insert def-X[THEN foundation17] val-x[THEN foundation19])
  apply(auto simp: OclValid-def bot-fun-def OclIncluding-def OclIncludes-def false-def true-def
    invalid-def defined-def valid-def
    bot-Set-0-def null-fun-def null-Set-0-def bot-option-def)
  apply(simp-all add: Abs-Set-0-inject A B C bot-option-def[symmetric],
    simp-all add: bot-option-def Abs-Set-0-inverse C)
done
qed

```

```

lemma including-cha2:
assumes def-X:  $\tau \models (\delta X)$ 
and val-x:  $\tau \models (v x)$ 
and val-y:  $\tau \models (v y)$ 
and neq :  $\tau \models \text{not}(x \triangleq y)$ 
shows  $\tau \models (X \rightarrow \text{including}(x) \rightarrow \text{includes}(y)) \triangleq (X \rightarrow \text{includes}(y))$ 
proof -
  have A :  $\perp \in \text{Set-0}$  by(simp add: Set-0-def bot-option-def)
  have B :  $\lfloor \perp \rfloor \in \text{Set-0}$  by(simp add: Set-0-def null-option-def bot-option-def)
  have C :  $\lfloor \lfloor \text{insert } (x \ \tau) \rfloor \rfloor \in \text{Set-0}$ 
    apply(insert def-X[THEN foundation17] val-x[THEN foundation19] Set-inv-lemma[OF
def-X])
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def invalid-def)
  done
  have D :  $y \ \tau \neq x \ \tau$ 
    apply(insert neq)
    by(auto simp: OclValid-def bot-fun-def OclIncluding-def OclIncludes-def
      false-def true-def defined-def valid-def bot-Set-0-def
      null-fun-def null-Set-0-def StrongEq-def not-def)
show ?thesis

```

```

apply(insert def-X[THEN foundation17] val-x[THEN foundation19])
apply(auto simp: OclValid-def bot-fun-def OclIncluding-def OclIncludes-def false-def true-def
      invalid-def defined-def valid-def bot-Set-0-def null-fun-def null-Set-0-def
      StrongEq-def)
apply(simp-all add: Abs-Set-0-inject Abs-Set-0-inverse A B C D)
apply(simp-all add: Abs-Set-0-inject A B C bot-option-def[symmetric],
      simp-all add: bot-option-def Abs-Set-0-inverse C)
done
qed

```

One would like a generic theorem of the form:

```

lemma includes_execute[code_unfold]:
"(X->including(x)->includes(y)) = (if \<delta> X then if x \<doteq> y
                                     then true
                                     else X->includes(y)
                                     endif
                                     else invalid endif)"

```

Unfortunately, this does not hold in general, since referential equality is an overloaded concept and has to be defined for each type individually. Consequently, it is only valid for concrete type instances for Boolean, Integer, and Sets thereof..

The computational law **includes_execute** becomes generic since it uses strict equality which in itself is generic. It is possible to prove the following generic theorem and instantiate it if a number of properties that link the polymorphic logical, Strong Equality with the concrete instance of strict quality.

```

lemma includes_execute-generic:
assumes strict1: (x ≐ invalid) = invalid
and strict2: (invalid ≐ y) = invalid
and strictEq-valid-args-valid:  $\bigwedge (x::('A,'a::null)val) y \tau.$ 
      ( $\tau \models \delta (x \doteq y) = ((\tau \models (v x)) \wedge (\tau \models v y))$ )
and cp-StrictRefEq:  $\bigwedge (X::('A,'a::null)val) Y \tau. (X \doteq Y) \tau = ((\lambda-. X \tau) \doteq (\lambda-. Y \tau)) \tau$ 
and strictEq-vs-strongEq:  $\bigwedge (x::('A,'a::null)val) y \tau.$ 
       $\tau \models v x \implies \tau \models v y \implies (\tau \models ((x \doteq y) \triangleq (x \triangleq y)))$ 
shows
  (X->including(x::('A,'a::null)val)->includes(y)) =
  (if  $\delta X$  then if  $x \doteq y$  then true else X->includes(y) endif else invalid endif)
proof -
  have A:  $\bigwedge \tau. \tau \models (X \triangleq invalid) \implies$ 
    (X->including(x)->includes(y))  $\tau$  = invalid  $\tau$ 
    apply(subst cp-OclIncludes, subst cp-OclIncluding)
    apply(drule foundation22[THEN iffD1], simp)
    apply(simp only: cp-OclIncluding[symmetric] cp-OclIncludes[symmetric])
    by simp
  have B:  $\bigwedge \tau. \tau \models (X \triangleq null) \implies$ 
    (X->including(x)->includes(y))  $\tau$  = invalid  $\tau$ 

```

```

    apply(subst cp-OclIncludes, subst cp-OclIncluding)
    apply(drule foundation22[THEN iffD1], simp)
    apply(simp only: cp-OclIncluding[symmetric] cp-OclIncludes[symmetric])
    by simp
have C:  $\bigwedge \tau. \tau \models (x \triangleq \text{invalid}) \implies$ 
  ( $X \rightarrow \text{including}(x) \rightarrow \text{includes}(y)$ )  $\tau =$ 
  ( $\text{if } x \doteq y \text{ then true else } X \rightarrow \text{includes}(y) \text{ endif}$ )  $\tau$ 
  apply(subst cp-if-ocl, subst cp-StrictRefEq)
  apply(subst cp-OclIncludes, subst cp-OclIncluding)
  apply(drule foundation22[THEN iffD1], simp)
  apply(simp only: cp-if-ocl[symmetric] cp-OclIncluding[symmetric]
    cp-StrictRefEq[symmetric] cp-OclIncludes[symmetric] )
  by (simp add: strict2)
have D:  $\bigwedge \tau. \tau \models (y \triangleq \text{invalid}) \implies$ 
  ( $X \rightarrow \text{including}(x) \rightarrow \text{includes}(y)$ )  $\tau =$ 
  ( $\text{if } x \doteq y \text{ then true else } X \rightarrow \text{includes}(y) \text{ endif}$ )  $\tau$ 
  apply(subst cp-if-ocl, subst cp-StrictRefEq)
  apply(subst cp-OclIncludes, subst cp-OclIncluding)
  apply(drule foundation22[THEN iffD1], simp)
  apply(simp only: cp-if-ocl[symmetric] cp-OclIncluding[symmetric]
    cp-StrictRefEq[symmetric] cp-OclIncludes[symmetric])
  by (simp add: strict1)
have E:  $\bigwedge \tau. \tau \models v \ x \implies \tau \models v \ y \implies$ 
  ( $\text{if } x \doteq y \text{ then true else } X \rightarrow \text{includes}(y) \text{ endif}$ )  $\tau =$ 
  ( $\text{if } x \triangleq y \text{ then true else } X \rightarrow \text{includes}(y) \text{ endif}$ )  $\tau$ 
  apply(subst cp-if-ocl)
  apply(subst strictEq-vs-strongEq[THEN foundation22[THEN iffD1]])
  by(simp-all add: cp-if-ocl[symmetric])
have F:  $\bigwedge \tau. \tau \models (x \triangleq y) \implies$ 
  ( $X \rightarrow \text{including}(x) \rightarrow \text{includes}(y)$ )  $\tau = (X \rightarrow \text{including}(x) \rightarrow \text{includes}(x)) \ \tau$ 
  apply(subst cp-OclIncludes)
  apply(drule foundation22[THEN iffD1], drule sym, simp)
  by(simp add: cp-OclIncludes[symmetric])
show ?thesis
  apply(rule ext, rename-tac  $\tau$ )
  apply(case-tac  $\neg (\tau \models (\delta \ X))$ , simp add: def-split-local, elim disjE A B)
  apply(case-tac  $\neg (\tau \models (v \ x))$ ,
    simp add: foundation18 foundation22[symmetric],
    drule StrongEq-L-sym)
  apply(simp add: foundation22 C)
  apply(case-tac  $\neg (\tau \models (v \ y))$ ,
    simp add: foundation18 foundation22[symmetric],
    drule StrongEq-L-sym, simp add: foundation22 D, simp)
  apply(subst E, simp-all)
  apply(case-tac  $\tau \models \text{not}(x \triangleq y)$ )
  apply(simp add: including-charn2[simplified foundation22] E)
  apply(simp add: foundation9 F
    including-charn1[THEN foundation13[THEN iffD2],
      THEN foundation22[THEN iffD1]])

```

done
qed

schematic-lemma *includes-execute-int*[code-unfold]: ?X
by(rule *includes-execute-generic*[OF *StrictRefEq-int-strict1 StrictRefEq-int-strict2*
strictEqInt-valid-args-valid cp-StrictRefEq-int
strictEqInt-vs-strongEq], *simp-all*)

schematic-lemma *includes-execute-bool*[code-unfold]: ?X
by(rule *includes-execute-generic*[OF *StrictRefEq-bool-strict1 StrictRefEq-bool-strict2*
strictEqBool-valid-args-valid cp-StrictRefEq-bool
strictEqBool-vs-strongEq], *simp-all*)

schematic-lemma *includes-execute-set*[code-unfold]: ?X
by(rule *includes-execute-generic*[OF *StrictRefEq-set-strict1 StrictRefEq-set-strict2*
StrictRefEq-set-strictEq-valid-args-valid cp-StrictRefEq-set
strictRefEq-set-vs-strongEq], *simp-all*)

lemma *excluding-cha0*[simp]:
assumes *val-x*: $\tau \models (v\ x)$
shows $\tau \models ((Set\{\} \rightarrow excluding(x)) \triangleq Set\{\})$
proof –
have *A* : $[None] \in Set-0$ **by**(simp add: *Set-0-def null-option-def bot-option-def*)
have *B* : $[[\{\}]] \in Set-0$ **by**(simp add: *Set-0-def bot-option-def*)
show ?thesis **using** *val-x*
apply(auto simp: *OclValid-def OclIncludes-def not-def false-def true-def StrongEq-def*
OclExcluding-def mtSet-def defined-def bot-fun-def null-fun-def null-Set-0-def)
apply(auto simp: *mtSet-def Set-0-def OCL-lib.Set-0.Abs-Set-0-inverse*
OCL-lib.Set-0.Abs-Set-0-inject[OF *B*, OF *A*])
done
qed

lemma *excluding-cha0-exec*[code-unfold]:
 $(Set\{\} \rightarrow excluding(x)) = (if\ (v\ x)\ then\ Set\{\}\ else\ invalid\ endif)$
proof –
have *A*: $\bigwedge \tau. (Set\{\} \rightarrow excluding(invalid))\ \tau = (if\ (v\ invalid)\ then\ Set\{\}\ else\ invalid\ endif)$
 τ
by *simp*
have *B*: $\bigwedge \tau\ x. \tau \models (v\ x) \implies (Set\{\} \rightarrow excluding(x))\ \tau = (if\ (v\ x)\ then\ Set\{\}\ else\ invalid\ endif)\ \tau$
by(simp add: *excluding-cha0*[THEN *foundation22*[THEN *iffD1*]])
show ?thesis
apply(rule *ext*, *rename-tac* τ)

```

apply(case-tac  $\tau \models (v\ x)$ )
  apply(simp add: B)
  apply(simp add: foundation18)
  apply(subst cp-OclExcluding, simp)
  apply(simp add: cp-if-ocl[symmetric] cp-OclExcluding[symmetric] cp-valid[symmetric] A)
done
qed

lemma excluding-charn1:
assumes def-X: $\tau \models (\delta\ X)$ 
and    val-x: $\tau \models (v\ x)$ 
and    val-y: $\tau \models (v\ y)$ 
and    neq : $\tau \models \text{not}(x \triangleq y)$ 
shows    $\tau \models ((X \rightarrow \text{including}(x)) \rightarrow \text{excluding}(y)) \triangleq ((X \rightarrow \text{excluding}(y)) \rightarrow \text{including}(x))$ 
proof -
  have A :  $\perp \in \text{Set-0}$  by(simp add: Set-0-def bot-option-def)
  have B :  $\lfloor \perp \rfloor \in \text{Set-0}$  by(simp add: Set-0-def null-option-def bot-option-def)
  have C :  $\lfloor \lfloor \text{insert } (x\ \tau) \lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor \rfloor \rfloor \in \text{Set-0}$ 
    apply(insert def-X[THEN foundation17] val-x[THEN foundation19] Set-inv-lemma[OF
def-X])
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def invalid-def)
    done
  have D :  $\lfloor \lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor - \{y\ \tau\} \rfloor \rfloor \in \text{Set-0}$ 
    apply(insert def-X[THEN foundation17] val-x[THEN foundation19] Set-inv-lemma[OF
def-X])
    apply(simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def invalid-def)
    done
  have E :  $x\ \tau \neq y\ \tau$ 
    apply(insert neq)
    by(auto simp: OclValid-def bot-fun-def OclIncluding-def OclIncludes-def
      false-def true-def defined-def valid-def bot-Set-0-def
      null-fun-def null-Set-0-def StrongEq-def not-def)
  have G :  $(\delta\ (\lambda\_. \text{Abs-Set-0 } \lfloor \lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor \rfloor))\ \tau = \text{true}\ \tau$ 
    apply(auto simp: OclValid-def false-def true-def defined-def
      bot-fun-def bot-Set-0-def null-fun-def null-Set-0-def )
    by(simp-all add: Abs-Set-0-inject A B C bot-option-def[symmetric],
      simp-all add: bot-option-def)
  have H :  $(\delta\ (\lambda\_. \text{Abs-Set-0 } \lfloor \lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor - \{y\ \tau\} \rfloor))\ \tau = \text{true}\ \tau$ 
    apply(auto simp: OclValid-def false-def true-def defined-def
      bot-fun-def bot-Set-0-def null-fun-def null-Set-0-def )
    by(simp-all add: Abs-Set-0-inject A B D bot-option-def[symmetric],
      simp-all add: bot-option-def)
  have Z:insert  $(x\ \tau) \lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor - \{y\ \tau\} = \text{insert } (x\ \tau) (\lfloor \lfloor \text{Rep-Set-0 } (X\ \tau) \rfloor \rfloor - \{y\ \tau\})$ 
    by(auto simp: E)
  show ?thesis
    apply(insert def-X[THEN foundation13[THEN iffD2]] val-x[THEN foundation13[THEN
iffD2]])
      val-y[THEN foundation13[THEN iffD2]])

```

```

apply(simp add: foundation22 OclIncluding-def OclExcluding-def def-X [THEN foundation17])
apply(subst cp-defined, simp) apply(subst cp-defined, simp)
apply(subst cp-defined, simp) apply(subst cp-defined, simp)
apply(subst cp-defined, simp)
apply(simp add: G H Abs-Set-0-inverse[OF C] Abs-Set-0-inverse[OF D] Z)
done
qed

```

```

lemma excluding-cha2:
assumes def-X:  $\tau \models (\delta X)$ 
and val-x:  $\tau \models (v x)$ 
shows  $\tau \models (((X \rightarrow \text{including}(x)) \rightarrow \text{excluding}(x)) \triangleq (X \rightarrow \text{excluding}(x)))$ 
proof -
  have A :  $\perp \in \text{Set-0}$  by (simp add: Set-0-def bot-option-def)
  have B :  $\lfloor \perp \rfloor \in \text{Set-0}$  by (simp add: Set-0-def null-option-def bot-option-def)
  have C :  $\lfloor \lfloor \text{insert } (x \ \tau) \lfloor \lfloor \text{Rep-Set-0 } (X \ \tau) \rfloor \rfloor \rfloor \in \text{Set-0}$ 
    apply (insert def-X [THEN foundation17] val-x [THEN foundation19] Set-inv-lemma[OF
def-X])
    apply (simp add: Set-0-def bot-option-def null-Set-0-def null-fun-def invalid-def)
    done
show ?thesis
  apply (insert def-X [THEN foundation17] val-x [THEN foundation19])
  apply (auto simp: OclValid-def bot-fun-def OclIncluding-def OclIncludes-def false-def true-def
invalid-def defined-def valid-def bot-Set-0-def null-fun-def null-Set-0-def
StrongEq-def)
  apply (subst cp-OclExcluding) back
  apply (auto simp: OclExcluding-def)
  apply (simp add: Abs-Set-0-inverse[OF C])
  apply (simp-all add: false-def true-def defined-def valid-def
null-fun-def bot-fun-def null-Set-0-def bot-Set-0-def
split: bool.split-asm HOL.split-if-asm option.split)
  apply (simp-all add: Abs-Set-0-inject A B C bot-option-def[symmetric],
simp-all add: bot-option-def Abs-Set-0-inverse C)
done
qed

```

```

lemma excluding-cha-exec[code-unfold]:
 $(X \rightarrow \text{including}(x) \rightarrow \text{excluding}(y)) = (\text{if } \delta X \text{ then if } x \doteq y$ 
   $\text{then } X \rightarrow \text{excluding}(y)$ 
   $\text{else } X \rightarrow \text{excluding}(y) \rightarrow \text{including}(x)$ 
   $\text{endif}$ 
   $\text{else invalid endif})$ 
sorry

```

```

syntax
  -OclFinset :: args => ('A, 'a::null) Set (Set{(-)})
translations
  Set{x, xs} == CONST OclIncluding (Set{xs}) x

```

$Set\{x\} == CONST\ OclIncluding\ (Set\{\})\ x$

lemma *syntax-test*: $Set\{\mathbf{2}, \mathbf{1}\} = (Set\{\}->including(\mathbf{1})->including(\mathbf{2}))$
by (*rule refl*)

lemma *set-test1*: $\tau \models (Set\{\mathbf{2}, null\}->includes(null))$
by(*simp add: includes-execute-int*)

lemma *set-test2*: $\neg(\tau \models (Set\{\mathbf{2}, \mathbf{1}\}->includes(null)))$
by(*simp add: includes-execute-int*)

Here is an example of a nested collection. Note that we have to use the abstract null (since we did not (yet) define a concrete constant *null* for the non-existing Sets) :

lemma *semantic-test2*:
assumes $H:(Set\{\mathbf{2}\} \doteq null) = (false::('A)Boolean)$
shows $(\tau::('A)st) \models (Set\{Set\{\mathbf{2}\}, null\}->includes(null))$
by(*simp add: includes-execute-set H*)

lemma *semantic-test3*: $\tau \models (Set\{null, \mathbf{2}\}->includes(null))$
by(*simp-all add: including-cha1 including-defined-args-valid*)

lemma *StrictRefEq-set-exec*[*simp, code-unfold*] :
 $((x::('A, 'a::null)Set) \doteq y) =$
 (if $\delta\ x$ then (if $\delta\ y$
 then $((x->forall(z\mid y->includes(z))\ and\ (y->forall(z\mid x->includes(z))))$
 else if $v\ y$
 then $false\ (*\ x'->includes = null\ *)$
 else *invalid*
 endif)
 else if $v\ x\ (*\ null = ???\ *)$
 then if $v\ y$ then $not(\delta\ y)$ else *invalid* endif
 else *invalid*
 endif)
 endif)
sorry

lemma *forall-set-null-exec*[*simp, code-unfold*] :
 $(null->forall(z\mid P(z))) = invalid$

sorry

lemma *forall-set-mt-exec*[simp,code-unfold] :
 $((Set\{\}) \rightarrow \text{forall}(z \mid P(z))) = \text{true}$
sorry

lemma *exists-set-null-exec*[simp,code-unfold] :
 $(\text{null} \rightarrow \text{exists}(z \mid P(z))) = \text{invalid}$
sorry

lemma *exists-set-mt-exec*[simp,code-unfold] :
 $((Set\{\}) \rightarrow \text{exists}(z \mid P(z))) = \text{false}$
sorry

lemma *forall-set-including-exec*[simp,code-unfold] :
 $((S \rightarrow \text{including}(x)) \rightarrow \text{forall}(z \mid P(z))) = (\text{if } (\delta \ S) \text{ and } (v \ x) \\ \text{then } P(x) \text{ and } S \rightarrow \text{forall}(z \mid P(z)) \\ \text{else invalid} \\ \text{endif})$
sorry

lemma *not-if*[simp]:
 $\text{not}(\text{if } P \text{ then } C \text{ else } E \text{ endif}) = (\text{if } P \text{ then not } C \text{ else not } E \text{ endif})$
sorry

lemma *exists-set-including-exec*[simp,code-unfold] :
 $((S \rightarrow \text{including}(x)) \rightarrow \text{exists}(z \mid P(z))) = (\text{if } (\delta \ S) \text{ and } (v \ x) \\ \text{then } P(x) \text{ or } S \rightarrow \text{exists}(z \mid P(z)) \\ \text{else invalid} \\ \text{endif})$
by(simp add: OclExists-def ocl-or-def)

lemma *set-test4* : $\tau \models (Set\{\mathbf{2}, \text{null}, \mathbf{2}\} \doteq Set\{\text{null}, \mathbf{2}\})$
by(simp add: includes-execute-int)

definition *OclIterate*_{Set} :: $[('A, 'a :: \text{null}) \ Set, ('A, 'b :: \text{null}) \ val, \\ ('A, 'a) \ val \Rightarrow ('A, 'b) \ val \Rightarrow ('A, 'b) \ val] \Rightarrow ('A, 'b) \ val$
where *OclIterate*_{Set} *S A F* = $(\lambda \tau. \text{if } (\delta \ S) \ \tau = \text{true} \ \tau \wedge (v \ A) \ \tau = \text{true} \ \tau \wedge \text{finite}[\llbracket \text{Rep-Set-0} \\ (S \ \tau) \rrbracket] \\ \text{then } (Finite-Set.fold \ (F) \ (A) \ ((\lambda a \ \tau. a) \ ' \llbracket \text{Rep-Set-0} \ (S \ \tau) \rrbracket)) \tau \\ \text{else } \perp)$

syntax

-*OclIterate* :: $[('A, 'a :: \text{null}) \ Set, \text{idt}, \text{idt}, 'a, 'b] \Rightarrow ('A, 'a) \ val \\ (- \rightarrow \text{iterate} \ '(-; -) \mid -) \ [71, 100, 70] 50)$

translations

$X \rightarrow \text{iterate}(a; x = A \mid P) == \text{CONST } \text{OclIterate}_{\text{Set}} X A (\%a. (\% x. P))$

lemma $\text{OclIterate}_{\text{Set-strict1}}[\text{simp}]: \text{invalid} \rightarrow \text{iterate}(a; x = A \mid P a x) = \text{invalid}$
by ($\text{simp add: bot-fun-def invalid-def OclIterate}_{\text{Set-def}} \text{defined-def valid-def false-def true-def}$)

lemma $\text{OclIterate}_{\text{Set-null1}}[\text{simp}]: \text{null} \rightarrow \text{iterate}(a; x = A \mid P a x) = \text{invalid}$
by ($\text{simp add: bot-fun-def invalid-def OclIterate}_{\text{Set-def}} \text{defined-def valid-def false-def true-def}$)

lemma $\text{OclIterate}_{\text{Set-strict2}}[\text{simp}]: S \rightarrow \text{iterate}(a; x = \text{invalid} \mid P a x) = \text{invalid}$
by ($\text{simp add: bot-fun-def invalid-def OclIterate}_{\text{Set-def}} \text{defined-def valid-def false-def true-def}$)

An open question is this ...

lemma $\text{OclIterate}_{\text{Set-null2}}[\text{simp}]: S \rightarrow \text{iterate}(a; x = \text{null} \mid P a x) = \text{invalid}$
oops

In the definition above, this does not hold in general. And I believe, this is how it should be ...

lemma $\text{OclIterate}_{\text{Set-infinite}}$:
assumes $\text{non-finite}: \tau \models \text{not}(\delta(S \rightarrow \text{size}()))$
shows $(\text{OclIterate}_{\text{Set}} S A F) \tau = \text{invalid } \tau$
sorry

lemma $\text{OclIterate}_{\text{Set-empty}}[\text{simp}]: ((\text{Set}\{\}) \rightarrow \text{iterate}(a; x = A \mid P a x)) = A$
sorry

In particular, this does hold for $A = \text{null}$.

lemma $\text{OclIterate}_{\text{Set-including}}$:
assumes $S\text{-finite}: \tau \models \delta(S \rightarrow \text{size}())$
shows $((S \rightarrow \text{including}(a)) \rightarrow \text{iterate}(a; x = A \mid F a x)) \tau =$
 $((S \rightarrow \text{excluding}(a)) \rightarrow \text{iterate}(a; x = F a A \mid F a x)) \tau$
sorry

lemma $[\text{simp}]: \delta (\text{Set}\{\} \rightarrow \text{size}()) = \text{true}$
sorry

lemma $[\text{simp}]: \delta ((X \rightarrow \text{including}(x)) \rightarrow \text{size}()) = (\delta(X) \text{ and } v(x))$
sorry

2.7.7 Test Statements

lemma $\text{short-cut}'[\text{simp}]: (8 \doteq 6) = \text{false}$
sorry

lemma *GogollasChallenge-on-sets:*

$$(Set\{ \mathbf{6}, \mathbf{8} \} \rightarrow iterate(i; r1 = Set\{ \mathbf{9} \} | \\ r1 \rightarrow iterate(j; r2 = r1 | \\ r2 \rightarrow including(\mathbf{0}) \rightarrow including(i) \rightarrow including(j))) = Set\{ \mathbf{0}, \mathbf{6}, \mathbf{9} \})$$

apply(*rule ext*,

simp add: excluding-chaon-exec OclIterate_{Set}-including excluding-chaon0-exec)

sorry

Elementary computations on Sets.

```

value  $\neg (\tau_0 \models v(invalid::('A, 'a::null) Set))$ 
value  $\tau_0 \models v(null::('A, 'a::null) Set)$ 
value  $\neg (\tau_0 \models \delta(null::('A, 'a::null) Set))$ 
value  $\tau_0 \models v(Set\{\})$ 
value  $\tau_0 \models v(Set\{Set\{\mathbf{2}\}, null\})$ 
value  $\tau_0 \models \delta(Set\{Set\{\mathbf{2}\}, null\})$ 
value  $\tau_0 \models (Set\{\mathbf{2}, \mathbf{1}\} \rightarrow includes(\mathbf{1}))$ 
value  $\neg (\tau_0 \models (Set\{\mathbf{2}\} \rightarrow includes(\mathbf{1})))$ 
value  $\neg (\tau_0 \models (Set\{\mathbf{2}, \mathbf{1}\} \rightarrow includes(null)))$ 
value  $\tau_0 \models (Set\{\mathbf{2}, null\} \rightarrow includes(null))$ 
value  $\tau \models ((Set\{\mathbf{2}, \mathbf{1}\}) \rightarrow forall(z \mid \mathbf{0} \prec z))$ 
value  $\neg (\tau \models ((Set\{\mathbf{2}, \mathbf{1}\}) \rightarrow exists(z \mid z \prec \mathbf{0})))$ 

value  $\neg (\tau \models ((Set\{\mathbf{2}, null\}) \rightarrow forall(z \mid \mathbf{0} \prec z)))$ 
value  $\tau \models ((Set\{\mathbf{2}, null\}) \rightarrow exists(z \mid \mathbf{0} \prec z))$ 

value  $\tau \models (Set\{\mathbf{2}, null, \mathbf{2}\} \doteq Set\{null, \mathbf{2}\})$ 
value  $\tau \models (Set\{\mathbf{1}, null, \mathbf{2}\} <> Set\{null, \mathbf{2}\})$ 

value  $\tau \models (Set\{Set\{\mathbf{2}, null\}\} \doteq Set\{Set\{null, \mathbf{2}\}\})$ 
value  $\tau \models (Set\{Set\{\mathbf{2}, null\}\} <> Set\{Set\{null, \mathbf{2}\}, null\})$ 

```

end

theory *OCL-state*

imports *OCL-lib*

begin

2.7.8 Recall: The generic structure of States

Next we will introduce the foundational concept of an object id (oid), which is just some infinite set.

type-synonym *oid* = *ind*

States are just a partial map from oid's to elements of an object universe $'A$, and state transitions pairs of states...

type-synonym $('A)state = oid \rightarrow 'A$

type-synonym ($'\mathcal{A}$)*st* = $'\mathcal{A}$ *state* \times $'\mathcal{A}$ *state*

Now we refine our state-interface. In certain contexts, we will require that the elements of the object universe have a particular structure; more precisely, we will require that there is a function that reconstructs the oid of an object in the state (we will settle the question how to define this function later).

class *object* = **fixes** *oid-of* :: $'a \Rightarrow \text{oid}$

Thus, if needed, we can constrain the object universe to objects by adding the following type class constraint:

typ $'\mathcal{A}$:: *object*

2.7.9 Referential Object Equality in States

Generic referential equality - to be used for instantiations with concrete object types ...

definition *gen-ref-eq* :: $('\mathcal{A}, 'a :: \{\text{object}, \text{null}\}) \text{val} \Rightarrow ('\mathcal{A}, 'a) \text{val} \Rightarrow ('\mathcal{A}) \text{Boolean}$

where *gen-ref-eq* *x y*
 $\equiv \lambda \tau. \text{if } (\delta \ x) \ \tau = \text{true} \ \tau \wedge (\delta \ y) \ \tau = \text{true} \ \tau$
 $\text{then if } x \ \tau = \text{null} \vee y \ \tau = \text{null}$
 $\text{then } \llbracket x \ \tau = \text{null} \wedge y \ \tau = \text{null} \rrbracket$
 $\text{else } \llbracket (\text{oid-of } (x \ \tau)) = (\text{oid-of } (y \ \tau)) \rrbracket$
 $\text{else invalid } \tau$

lemma *gen-ref-eq-object-strict1* [*simp*] :
 $(\text{gen-ref-eq } x \ \text{invalid}) = \text{invalid}$
by (*rule ext*, *simp add: gen-ref-eq-def true-def false-def*)

lemma *gen-ref-eq-object-strict2* [*simp*] :
 $(\text{gen-ref-eq } \text{invalid } x) = \text{invalid}$
by (*rule ext*, *simp add: gen-ref-eq-def true-def false-def*)

lemma *gen-ref-eq-object-strict3* [*simp*] :
 $(\text{gen-ref-eq } x \ \text{null}) = \text{invalid}$
by (*rule ext*, *simp add: gen-ref-eq-def true-def false-def*)

lemma *gen-ref-eq-object-strict4* [*simp*] :
 $(\text{gen-ref-eq } \text{null } x) = \text{invalid}$
by (*rule ext*, *simp add: gen-ref-eq-def true-def false-def*)

lemma *cp-gen-ref-eq-object*:
 $(\text{gen-ref-eq } x \ y \ \tau) = (\text{gen-ref-eq } (\lambda \cdot. x \ \tau) (\lambda \cdot. y \ \tau)) \ \tau$
by (*auto simp: gen-ref-eq-def StrongEq-def invalid-def cp-defined symmetric*)

lemmas *cp-intro* [*simp, intro!*] =
 OCL-core.cp-intro
 $\text{cp-gen-ref-eq-object} [\text{THEN allI} [\text{THEN allI} [\text{THEN allI} [\text{THEN cpI2}]]],$

of *gen-ref-eq*]]

Finally, we derive the usual laws on definedness for (generic) object equality:

lemma *gen-ref-eq-defargs*:

$\tau \models (\text{gen-ref-eq } x \ (y :: (\mathcal{A}, 'a :: \{\text{null}, \text{object}\}) \text{val})) \implies (\tau \models (\delta \ x)) \wedge (\tau \models (\delta \ y))$

by (*simp add: gen-ref-eq-def OclValid-def true-def invalid-def
defined-def invalid-def bot-fun-def bot-option-def
split: bool.split-asm HOL.split-if-asm*)

2.7.10 Further requirements on States

A key-concept for linking strict referential equality to logical equality: in well-formed states (i.e. those states where the self (oid-of) field contains the pointer to which the object is associated to in the state), referential equality coincides with logical equality.

definition *WFF* :: $(\mathcal{A} :: \text{object}) \text{st} \Rightarrow \text{bool}$

where *WFF* $\tau = ((\forall x \in \text{ran}(\text{fst } \tau). \lceil \text{fst } \tau \ (\text{oid-of } x) \rceil = x) \wedge$
 $(\forall x \in \text{ran}(\text{snd } \tau). \lceil \text{snd } \tau \ (\text{oid-of } x) \rceil = x))$

This is a generic definition of referential equality: Equality on objects in a state is reduced to equality on the references to these objects. As in HOL-OCL, we will store the reference of an object inside the object in a (ghost) field. By establishing certain invariants ("consistent state"), it can be assured that there is a "one-to-one-correspondance" of objects to their references — and therefore the definition below behaves as we expect.

Generic Referential Equality enjoys the usual properties: (quasi) reflexivity, symmetry, transitivity, substitutivity for defined values. For type-technical reasons, for each concrete object type, the equality \doteq is defined by generic referential equality.

theorem *strictEqGen-vs-strongEq*:

WFF $\tau \implies \tau \models (\delta \ x) \implies \tau \models (\delta \ y) \implies$
 $(x \ \tau \in \text{ran} \ (\text{fst } \tau) \wedge y \ \tau \in \text{ran} \ (\text{fst } \tau)) \wedge$
 $(x \ \tau \in \text{ran} \ (\text{snd } \tau) \wedge y \ \tau \in \text{ran} \ (\text{snd } \tau)) \implies (* \ x \text{ and } y \text{ must be object representations}$
 $\text{that exist in either the pre or post state } *)$
 $(\tau \models (\text{gen-ref-eq } x \ y)) = (\tau \models (x \ \hat{=} \ y))$

apply(*auto simp: gen-ref-eq-def OclValid-def WFF-def StrongEq-def true-def Ball-def*)

apply(*erule-tac x=x \tau in allE', simp-all*)

done

So, if two object descriptions live in the same state (both pre or post), the referential equality on objects implies in a WFF state the logical equality. Uffz.

2.8 Miscillaneous: Initial States (for Testing and Code Generation)

definition $\tau_0 :: (\mathcal{A}) \text{st}$

where $\tau_0 \equiv (\text{Map.empty}, \text{Map.empty})$

2.8.1 Generic Operations on States

In order to denote OCL-types occuring in OCL expressions syntactically — as, for example, as "argument" of `allInstances` — we use the inverses of the injection functions into the object universes; we show that this is sufficient "characterization".

definition $allinstances :: ('\mathfrak{A} \Rightarrow '\alpha) \Rightarrow ('\mathfrak{A}::object, '\alpha\ option\ option)\ Set$
 $(- .oclAllInstances'())$
where $((H).oclAllInstances()) \tau =$
 $Abs-Set-0 \ \llbracket (Some\ o\ Some\ o\ H) \ ' \ (ran(snd\ \tau) \cap \{x. \exists\ y. y=H\ x\}) \ \rrbracket$

definition $allinstancesATpre :: ('\mathfrak{A} \Rightarrow '\alpha) \Rightarrow ('\mathfrak{A}::object, '\alpha\ option\ option)\ Set$
 $(- .oclAllInstances@pre'())$
where $((H).oclAllInstances@pre()) \tau =$
 $Abs-Set-0 \ \llbracket (Some\ o\ Some\ o\ H) \ ' \ (ran(fst\ \tau) \cap \{x. \exists\ y. y=H\ x\}) \ \rrbracket$

lemma $\tau_0 \models H .oclAllInstances() \triangleq Set\{\}$
sorry

lemma $\tau_0 \models H .oclAllInstances@pre() \triangleq Set\{\}$
sorry

theorem *state-update-vs-allInstances:*
assumes $oid \notin dom\ \sigma'$
and $cp\ P$
shows $((\sigma, \sigma'(oid \mapsto Object)) \models (P(Type .oclAllInstances()))) =$
 $((\sigma, \sigma') \models (P((Type .oclAllInstances()) \rightarrow including(\lambda -. Some(Some((the-inv\ Type)$
 $Object))))))$
sorry

theorem *state-update-vs-allInstancesATpre:*
assumes $oid \notin dom\ \sigma$
and $cp\ P$
shows $((\sigma(oid \mapsto Object), \sigma') \models (P(Type .oclAllInstances@pre()))) =$
 $((\sigma, \sigma') \models (P((Type .oclAllInstances@pre()) \rightarrow including(\lambda -. Some(Some((the-inv\ Type)$
 $Object))))))$
sorry

definition $oclisnew :: ('\mathfrak{A}, '\alpha::\{null, object\})val \Rightarrow ('\mathfrak{A})Boolean \ \ ((-).oclIsNew'())$
where $X .oclIsNew() \equiv (\lambda\tau . \text{if } (\delta\ X) \ \tau = true \ \tau$
 $\text{then } \llbracket oid-of\ (X\ \tau) \notin dom(fst\ \tau) \wedge oid-of\ (X\ \tau) \in dom(snd\ \tau) \rrbracket$
 $\text{else } invalid\ \tau)$

The following predicate — which is not part of the OCL standard descriptions — provides a simple, but powerful means to describe framing conditions. For any formal approach, be it animation of OCL contracts, test-case generation or die-hard theorem proving, the specification of the part of a system transistion that DOES NOT CHANGE is of premordial importance. The following operator establishes the equality between old and

new objects in the state (provided that they exist in both states), with the exception of those objects

definition *oclismodified* :: ($\mathcal{A}::\text{object}, 'a::\{\text{null}, \text{object}\}\text{Set} \Rightarrow 'a \text{ Boolean}$
 $(- \rightarrow \text{oclisModifiedOnly}')()$)
where $X \rightarrow \text{oclisModifiedOnly}() \equiv (\lambda(\sigma, \sigma'). \text{ let } X' = (\text{oid-of } ' \llbracket \text{Rep-Set-0}(X(\sigma, \sigma')) \rrbracket \rrbracket);$
 $S = ((\text{dom } \sigma \cap \text{dom } \sigma') - X')$
 $\text{in if } (\delta X) (\sigma, \sigma') = \text{true } (\sigma, \sigma')$
 $\text{then } \llbracket \forall x \in S. \sigma x = \sigma' x \rrbracket$
 $\text{else invalid } (\sigma, \sigma')$)

definition *atSelf* :: ($\mathcal{A}::\text{object}, 'a::\{\text{null}, \text{object}\}\text{val} \Rightarrow$
 $(\mathcal{A} \Rightarrow 'a) \Rightarrow$
 $(\mathcal{A}::\text{object}, 'a::\{\text{null}, \text{object}\}\text{val } ((-)\text{@pre}(-))$)
where $x \text{@pre } H = (\lambda\tau . \text{ if } (\delta x) \tau = \text{true } \tau$
 $\text{then if oid-of } (x \tau) \in \text{dom}(\text{fst } \tau) \wedge \text{oid-of } (x \tau) \in \text{dom}(\text{snd } \tau)$
 $\text{then } H \llbracket (\text{fst } \tau)(\text{oid-of } (x \tau)) \rrbracket$
 $\text{else invalid } \tau$
 $\text{else invalid } \tau)$

theorem *framing*:

assumes *modifiesclause*: $\tau \models (X \rightarrow \text{excluding}(x)) \rightarrow \text{oclisModifiedOnly}()$
and *represented-x*: $\tau \models \delta(x \text{@pre } H)$
and *H-is-typeprepr*: $\text{inj } H$
shows $\tau \models (x \triangleq (x \text{@pre } H))$

sorry

end

theory *OCL-tools*
imports *OCL-core*
begin

end

theory *OCL-main*
imports *OCL-lib OCL-state OCL-tools*
begin

end

```

theory
  OCL-linked-list
imports
  ../OCL-main
begin

```

2.8.2 Introduction

For certain concepts like Classes and Class-types, only a generic definition for its resulting semantics can be given. Generic means, there is a function outside HOL that "compiles" a concrete, closed-world class diagram into a "theory" of this data model, consisting of a bunch of definitions for classes, accessors, method, casts, and tests for actual types, as well as proofs for the fundamental properties of these operations in this concrete data model.

Such generic function or "compiler" can be implemented in Isabelle on the ML level. This has been done, for a semantics following the open-world assumption, for UML 2.0 in [7]. In this paper, we follow another approach for UML 2.4: we define the concepts of the compilation informally, and present a concrete example which is verified in Isabelle/HOL.

2.8.3 Outlining the Example

2.8.4 Example Data-Universe and its Infrastructure

Should be generated entirely from a class-diagram.

Our data universe consists in the concrete class diagram just of node's, and implicitly of the class object. Each class implies the existence of a class type defined for the corresponding object representations as follows:

```

datatype node = mknode oid
                  int option
                  oid option

```

```

datatype object = mkobject oid
                  (int option × oid option) option

```

Now, we construct a concrete "universe of object types" by injection into a sum type containing the class types. This type of objects will be used as instance for all resp. type-variables ...

```

datatype  $\mathfrak{A}$  = innode node | inobject object

```

Recall that in order to denote OCL-types occurring in OCL expressions syntactically — as, for example, as "argument" of allInstances — we use the inverses of the injection functions into the object universes; we show that this is sufficient "characterization".

definition *Node* :: $\mathcal{A} \Rightarrow node$
where *Node* $\equiv (the_inv\ in_node)$

definition *Object* :: $\mathcal{A} \Rightarrow object$
where *Object* $\equiv (the_inv\ in_object)$

Having fixed the object universe, we can introduce type synonyms that exactly correspond to OCL types. Again, we exploit that our representation of OCL is a "shallow embedding" with a one-to-one correspondance of OCL-types to types of the meta-language HOL.

type-synonym *Boolean* = (\mathcal{A})*Boolean*
type-synonym *Integer* = (\mathcal{A})*Integer*
type-synonym *Void* = (\mathcal{A})*Void*
type-synonym *Object* = ($\mathcal{A}, object\ option\ option$) *val*
type-synonym *Node* = ($\mathcal{A}, node\ option\ option$) *val*
type-synonym *Set-Integer* = ($\mathcal{A}, int\ option\ option$) *Set*
type-synonym *Set-Node* = ($\mathcal{A}, node\ option\ option$) *Set*

Just a little check:

typ *Boolean*

In order to reuse key-elements of the library like referential equality, we have to show that the object universe belongs to the type class "object", i.e. each class type has to provide a function *oid-of* yielding the object id (oid) of the object.

instantiation *node* :: *object*
begin
definition *oid-of-node-def*: *oid-of* *x* = (case *x* of *mk_{node}* *oid* - \Rightarrow *oid*)
instance ..
end

instantiation *object* :: *object*
begin
definition *oid-of-object-def*: *oid-of* *x* = (case *x* of *mk_{object}* *oid* - \Rightarrow *oid*)
instance ..
end

instantiation \mathcal{A} :: *object*
begin
definition *oid-of-A-def*: *oid-of* *x* = (case *x* of
 $\quad in_node\ node \Rightarrow oid_of\ node$
 $\quad | in_object\ obj \Rightarrow oid_of\ obj$)
instance ..
end

instantiation *option* :: (*object*)*object*
begin
definition *oid-of-option-def*: *oid-of* *x* = *oid-of* (*the* *x*)
instance ..
end

2.9 Instantiation of the generic strict equality. We instantiate the referential equality on *Node* and *Object*

```

defs(overloaded) StrictRefEqnode : (x::Node)  $\doteq$  y  $\equiv$  gen-ref-eq x y
defs(overloaded) StrictRefEqobject : (x::Object)  $\doteq$  y  $\equiv$  gen-ref-eq x y

```

```

lemmas strict-eq-node =
  cp-gen-ref-eq-object[of x::Node y::Node  $\tau$ ,
    simplified StrictRefEqnode[symmetric]]
  cp-intro(9) [ of P::Node  $\Rightarrow$  NodeQ::Node  $\Rightarrow$  Node,
    simplified StrictRefEqnode[symmetric] ]
  gen-ref-eq-def [ of x::Node y::Node,
    simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-defargs [ of - x::Node y::Node,
    simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-object-strict1
    [ of x::Node,
      simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-object-strict2
    [ of x::Node,
      simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-object-strict3
    [ of x::Node,
      simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-object-strict3
    [ of x::Node,
      simplified StrictRefEqnode[symmetric]]
  gen-ref-eq-object-strict4
    [ of x::Node,
      simplified StrictRefEqnode[symmetric]]

```

```

thm strict-eq-node

```

2.9.1 AllInstances

```

lemma (Node .oclAllInstances()) =
  ( $\lambda\tau$ . Abs-Set-0 [  $\ll$  (Some  $\circ$  Some  $\circ$  (the-inv innode))'(ran(snd  $\tau$ ))  $\ll$  ] )
by(rule ext, simp add:allinstances-def Node-def)

```

```

lemma (Object .oclAllInstances@pre()) =
  ( $\lambda\tau$ . Abs-Set-0 [  $\ll$  (Some  $\circ$  Some  $\circ$  (the-inv inobject))'(ran(fst  $\tau$ ))  $\ll$  ] )
by(rule ext, simp add:allinstancesATpre-def Object-def)

```

For each Class *C*, we will have an casting operation *.oclAsType*(*C*), a test on the actual type *.oclIsTypeOf*(*C*) as well as its relaxed form *.oclIsKindOf*(*C*) (corresponding exactly to Java's *instanceof*-operator).

Thus, since we have two class-types in our concrete class hierarchy, we have two operations to declare and and to provide two overloading definitions for the two static types.

2.10 Selector Definition

Should be generated entirely from a class-diagram.

```

typ Node  $\Rightarrow$  Node
fun dot-next:: Node  $\Rightarrow$  Node ((1(-).next) 50)
  where (X).next = ( $\lambda$   $\tau$ . case X  $\tau$  of
     $\perp \Rightarrow$  invalid  $\tau$  (* undefined pointer *)
    |  $\lfloor \perp \rfloor \Rightarrow$  invalid  $\tau$  (* dereferencing null pointer *)
    |  $\lfloor \lfloor mk_{node} oid i \perp \rfloor \rfloor \Rightarrow$  null  $\tau$  (* object contains null pointer *)
    |  $\lfloor \lfloor mk_{node} oid i [next] \rfloor \rfloor \Rightarrow$  (* We assume here that oid is indeed 'the' oid of the
Node,
                                     ie. we assume that  $\tau$  is well-formed. *)
    case (snd  $\tau$ ) next of
       $\perp \Rightarrow$  invalid  $\tau$ 
      |  $\lfloor in_{node} (mk_{node} a b c) \rfloor \Rightarrow \lfloor \lfloor mk_{node} a b c \rfloor \rfloor$ 
      |  $\lfloor - \rfloor \Rightarrow$  invalid  $\tau$ )

fun dot-i:: Node  $\Rightarrow$  Integer ((1(-).i) 50)
  where (X).i = ( $\lambda$   $\tau$ . case X  $\tau$  of
     $\perp \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \perp \rfloor \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \lfloor mk_{node} oid \perp - \rfloor \rfloor \Rightarrow$  null  $\tau$ 
    |  $\lfloor \lfloor mk_{node} oid [i] - \rfloor \rfloor \Rightarrow \lfloor \lfloor i \rfloor \rfloor$ )

fun dot-next-at-pre:: Node  $\Rightarrow$  Node ((1(-).next@pre) 50)
  where (X).next@pre = ( $\lambda$   $\tau$ . case X  $\tau$  of
     $\perp \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \perp \rfloor \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \lfloor mk_{node} oid i \perp \rfloor \rfloor \Rightarrow$  null  $\tau$  (* object contains null pointer. REALLY ?
                                     And if this pointer was defined in the pre-state ?*)
    |  $\lfloor \lfloor mk_{node} oid i [next] \rfloor \rfloor \Rightarrow$  (* We assume here that oid is indeed 'the' oid of the
Node,
                                     ie. we assume that  $\tau$  is well-formed. *)
    (case (fst  $\tau$ ) next of
       $\perp \Rightarrow$  invalid  $\tau$ 
      |  $\lfloor in_{node} (mk_{node} a b c) \rfloor \Rightarrow \lfloor \lfloor mk_{node} a b c \rfloor \rfloor$ 
      |  $\lfloor - \rfloor \Rightarrow$  invalid  $\tau$ ))

fun dot-i-at-pre:: Node  $\Rightarrow$  Integer ((1(-).i@pre) 50)
where (X).i@pre = ( $\lambda$   $\tau$ . case X  $\tau$  of
     $\perp \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \perp \rfloor \Rightarrow$  invalid  $\tau$ 
    |  $\lfloor \lfloor mk_{node} oid - - \rfloor \rfloor \Rightarrow$ 
      if oid  $\in$  dom (fst  $\tau$ )
      then (case (fst  $\tau$ ) oid of
         $\perp \Rightarrow$  invalid  $\tau$ 
        |  $\lfloor in_{node} (mk_{node} oid \perp next) \rfloor \Rightarrow$  null  $\tau$ 
        |  $\lfloor in_{node} (mk_{node} oid [i] next) \rfloor \Rightarrow \lfloor \lfloor i \rfloor \rfloor$ 
        |  $\lfloor - \rfloor \Rightarrow$  invalid  $\tau$ )

```

else invalid τ)

lemma *cp-dot-next*: $((X).next) \tau = ((\lambda\cdot. X \tau).next) \tau$ **by**(*simp*)

lemma *cp-dot-i*: $((X).i) \tau = ((\lambda\cdot. X \tau).i) \tau$ **by**(*simp*)

lemma *cp-dot-next-at-pre*: $((X).next@pre) \tau = ((\lambda\cdot. X \tau).next@pre) \tau$ **by**(*simp*)

lemma *cp-dot-i-pre*: $((X).i@pre) \tau = ((\lambda\cdot. X \tau).i@pre) \tau$ **by**(*simp*)

lemmas *cp-dot-nextI* [*simp*, *intro!*]=
cp-dot-next[*THEN allI*[*THEN allI*], *of* $\lambda X \cdot. X \lambda \cdot \tau. \tau$, *THEN cpI1*]

lemmas *cp-dot-nextI-at-pre* [*simp*, *intro!*]=
cp-dot-next-at-pre[*THEN allI*[*THEN allI*],
of $\lambda X \cdot. X \lambda \cdot \tau. \tau$, *THEN cpI1*]

lemma *dot-next-nullstrict* [*simp*]: $(null).next = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

lemma *dot-next-at-pre-nullstrict* [*simp*] : $(null).next@pre = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

lemma *dot-next-strict*[*simp*] : $(invalid).next = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

lemma *dot-next-strict'*[*simp*] : $(null).next = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

lemma *dot-nextATpre-strict*[*simp*] : $(invalid).next@pre = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

lemma *dot-nextATpre-strict'*[*simp*] : $(null).next@pre = invalid$
by(*rule ext*, *simp add*: *null-fun-def null-option-def bot-option-def null-def invalid-def*)

2.10.1 Casts

consts *oclastype_{object}* :: $'\alpha \Rightarrow Object \ ((-).oclAsType'(Object'))$

consts *oclastype_{node}* :: $'\alpha \Rightarrow Node \ ((-).oclAsType'(Node'))$

defs (**overloaded**) *oclastype_{object}-Object*:
 $(X::Object).oclAsType(Object) \equiv$
 $(\lambda\tau. \text{case } X \text{ of}$
 $\quad \perp \Rightarrow invalid \ \tau$
 $\quad | \lfloor \perp \rfloor \Rightarrow invalid \ \tau \quad (* \text{ to avoid: } null.oclAsType(Object) = null ? *)$
 $\quad | \lfloor mk_{object} \text{ oid } a \rfloor \Rightarrow \lfloor \lfloor mk_{object} \text{ oid } a \rfloor \rfloor)$

defs (**overloaded**) *oclastype_{object}-Node*:

```

(X::Node) .oclAsType(Object) ≡
  (λτ. case X τ of
    ⊥ ⇒ invalid τ
    | ⊥ ⇒ invalid τ (* OTHER POSSIBILITY : null ??? Really excluded
                     by standard *)
    | [[mknode oid a b]] ⇒ [[ (mkobject oid [(a,b)]) ]])

```

defs (overloaded) oclastype_{node}-Object:

```

(X::Object) .oclAsType(Node) ≡
  (λτ. case X τ of
    ⊥ ⇒ invalid τ
    | ⊥ ⇒ invalid τ
    | [[mkobject oid ⊥]] ⇒ invalid τ (* down-cast exception *)
    | [[mkobject oid [(a,b)]]] ⇒ [[mknode oid a b]])

```

defs (overloaded) oclastype_{node}-Node:

```

(X::Node) .oclAsType(Node) ≡
  (λτ. case X τ of
    ⊥ ⇒ invalid τ
    | ⊥ ⇒ invalid τ (* to avoid: null .oclAsType(Object) = null ? *)
    | [[mknode oid a b]] ⇒ [[mknode oid a b]])

```

lemma oclastype_{object}-Object-strict[simp] : (invalid::Object) .oclAsType(Object) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
 oclastype_{object}-Object)

lemma oclastype_{object}-Object-nullstrict[simp] : (null::Object) .oclAsType(Object) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
 oclastype_{object}-Object)

lemma oclastype_{node}-Object-strict[simp] : (invalid::Node) .oclAsType(Object) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
 oclastype_{object}-Node bot-Boolean-def)

lemma oclastype_{node}-Object-nullstrict[simp] : (null::Node) .oclAsType(Object) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
 oclastype_{object}-Node)

2.11 Tests for Actual Types

```

consts oclistypeofobject :: 'α ⇒ Boolean ((-).oclIsTypeOf'(Object'))
consts oclistypeofnode :: 'α ⇒ Boolean ((-).oclIsTypeOf'(Node'))

```

defs (overloaded) oclistypeof_{object}-Object:

```

(X::Object) .oclIsTypeOf(Object) ≡
  (λτ. case X τ of
    ⊥ ⇒ invalid τ
    | ⊥ ⇒ invalid τ
    | [[mkobject oid ⊥]] ⇒ true τ

```

$| \llbracket mk_{object} \text{ oid } [-] \rrbracket \Rightarrow false \ \tau$

defs (overloaded) *oclistypeof_{object}-Node*:

(*X::Node*) .*oclIsTypeOf*(*Object*) \equiv
 $(\lambda\tau. \text{ case } X \ \tau \text{ of}$
 $\quad \perp \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket \perp \rrbracket \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket - \rrbracket \Rightarrow false \ \tau)$

defs (overloaded) *oclistypeof_{node}-Object*:

(*X::Object*) .*oclIsTypeOf*(*Node*) \equiv
 $(\lambda\tau. \text{ case } X \ \tau \text{ of}$
 $\quad \perp \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket \perp \rrbracket \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket mk_{object} \text{ oid } \perp \rrbracket \Rightarrow false \ \tau$
 $\quad | \llbracket mk_{object} \text{ oid } [-] \rrbracket \Rightarrow true \ \tau)$

defs (overloaded) *oclistypeof_{node}-Node*:

(*X::Node*) .*oclIsTypeOf*(*Node*) \equiv
 $(\lambda\tau. \text{ case } X \ \tau \text{ of}$
 $\quad \perp \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket \perp \rrbracket \Rightarrow \text{invalid } \tau$
 $\quad | \llbracket - \rrbracket \Rightarrow true \ \tau)$

lemma *oclistypeof_{object}-Object-strict1*[simp]:

(*invalid::Object*) .*oclIsTypeOf*(*Object*) = *invalid*

by(rule *ext*, simp add: *null-fun-def null-option-def bot-option-def null-def invalid-def*
oclistypeof_{object}-Object)

lemma *oclistypeof_{object}-Object-strict2*[simp]:

(*null::Object*) .*oclIsTypeOf*(*Object*) = *invalid*

by(rule *ext*, simp add: *null-fun-def null-option-def bot-option-def null-def invalid-def*
oclistypeof_{object}-Object)

lemma *oclistypeof_{object}-Node-strict1*[simp]:

(*invalid::Node*) .*oclIsTypeOf*(*Object*) = *invalid*

by(rule *ext*, simp add: *null-fun-def null-option-def bot-option-def null-def invalid-def*
oclistypeof_{object}-Node)

lemma *oclistypeof_{object}-Node-strict2*[simp]:

(*null::Node*) .*oclIsTypeOf*(*Object*) = *invalid*

by(rule *ext*, simp add: *null-fun-def null-option-def bot-option-def null-def invalid-def*
oclistypeof_{object}-Node)

lemma *oclistypeof_{node}-Object-strict1*[simp]:

(*invalid::Object*) .*oclIsTypeOf*(*Node*) = *invalid*

by(rule *ext*, simp add: *null-fun-def null-option-def bot-option-def null-def invalid-def*
oclistypeof_{node}-Object)

lemma *oclistypeof_{node}-Object-strict2*[simp]:

(*null::Object*) .*oclIsTypeOf*(*Node*) = *invalid*

```

by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
    oclistypeofnode-Object)
lemma oclistypeofnode-Node-strict1[simp]:
  (invalid::Node) .oclIsTypeOf(Node) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
    oclistypeofnode-Node)
lemma oclistypeofnode-Node-strict2[simp]:
  (null::Node) .oclIsTypeOf(Node) = invalid
by(rule ext, simp add: null-fun-def null-option-def bot-option-def null-def invalid-def
    oclistypeofnode-Node)

lemma actualType-larger-staticType:
assumes isdef:  $\tau \models (\delta \ X)$ 
shows  $\tau \models (X::Node) .oclIsTypeOf(Object) \triangleq \text{false}$ 
using isdef
by(auto simp : bot-fun-def null-fun-def null-option-def bot-option-def null-def invalid-def
    oclistypeofobject-Node foundation22 foundation16
    split: option.split object.split node.split)

lemma down-cast:
assumes isObject:  $\tau \models (X::Object) .oclIsTypeOf(Object)$ 
shows  $\tau \models (X .oclAsType(Node)) \triangleq \text{invalid}$ 
using isObject
apply(auto simp : bot-fun-def null-fun-def null-option-def bot-option-def null-def invalid-def
    oclastypeobject-Node oclastypenode-Object foundation22 foundation16
    split: option.split object.split node.split)
by(simp add: oclistypeofobject-Object OclValid-def false-def true-def)

lemma up-down-cast :
assumes isdef:  $\tau \models (\delta \ X)$ 
shows  $\tau \models ((X::Node) .oclAsType(Object) .oclAsType(Node)) \triangleq X$ 
using isdef
by(auto simp : null-fun-def null-option-def bot-option-def null-def invalid-def
    oclastypeobject-Node oclastypenode-Object foundation22 foundation16
    split: option.split node.split)

```

2.12 Standard State Infrastructure

These definitions should be generated — again — from the class diagram.

2.13 Invariant

These recursive predicates can be defined conservatively by greatest fix-point constructions - automatically. See HOL-OCL Book for details. For the purpose of this example, we state them as axioms here.

axiomatization *inv-Node* :: *Node* \Rightarrow *Boolean*

where $A : (\tau \models (\delta \text{ self})) \longrightarrow$
 $(\tau \models \text{inv-Node}(\text{self})) =$
 $((\tau \models (\text{self}.\text{next} \doteq \text{null})) \vee$
 $(\tau \models (\text{self}.\text{next} <> \text{null}) \wedge (\tau \models (\text{self}.\text{next}.i \prec \text{self}.i)) \wedge$
 $(\tau \models (\text{inv-Node}(\text{self}.\text{next}))))$

axiomatization $\text{inv-Node-at-pre} :: \text{Node} \Rightarrow \text{Boolean}$

where $B : (\tau \models (\delta \text{ self})) \longrightarrow$
 $(\tau \models \text{inv-Node-at-pre}(\text{self})) =$
 $((\tau \models (\text{self}.\text{next@pre} \doteq \text{null})) \vee$
 $(\tau \models (\text{self}.\text{next@pre} <> \text{null}) \wedge (\tau \models (\text{self}.\text{next@pre}.i@pre \prec \text{self}.i@pre)))$
 \wedge
 $(\tau \models (\text{inv-Node-at-pre}(\text{self}.\text{next@pre}))))$

A very first attempt to characterize the axiomatization by an inductive definition - this can not be the last word since too weak (should be equality!)

coinductive $\text{inv} :: \text{Node} \Rightarrow (\mathbb{A})\text{st} \Rightarrow \text{bool}$ **where**

$(\tau \models (\delta \text{ self})) \implies ((\tau \models (\text{self}.\text{next} \doteq \text{null})) \vee$
 $(\tau \models (\text{self}.\text{next} <> \text{null}) \wedge (\tau \models (\text{self}.\text{next}.i \prec \text{self}.i)) \wedge$
 $(\text{inv}(\text{self}.\text{next})\tau)))$
 $\implies (\text{inv self } \tau)$

2.14 The contract of a recursive query :

The original specification of a recursive query :

```
context Node::contents():Set(Integer)
post:  result = if self.next = null
          then Set{i}
          else self.next.contents()->including(i)
        endif
```

consts $\text{dot-contents} :: \text{Node} \Rightarrow \text{Set-Integer } ((1(-).\text{contents}'()) \ 50)$

axiomatization dot-contents-def **where**

$(\tau \models ((\text{self}).\text{contents}() \triangleq \text{result})) =$
 $(\text{if } (\delta \text{ self}) \ \tau = \text{true} \ \tau$
 $\text{then } ((\tau \models \text{true}) \wedge$
 $(\tau \models (\text{result} \triangleq \text{if } (\text{self}.\text{next} \doteq \text{null})$
 $\text{then } (\text{Set}\{\text{self}.i\})$
 $\text{else } (\text{self}.\text{next}.\text{contents}()->\text{including}(\text{self}.i))$
 $\text{endif})))$
 $\text{else } \tau \models \text{result} \triangleq \text{invalid})$

consts $\text{dot-contents-AT-pre} :: \text{Node} \Rightarrow \text{Set-Integer } ((1(-).\text{contents@pre}'()) \ 50)$

axiomatization where *dot-contents-AT-pre-def*:

```
( $\tau \models (self).contents@pre() \triangleq result$ ) =
  (if ( $\delta self$ )  $\tau = true$   $\tau$ 
    then  $\tau \models true \wedge$ 
       $\tau \models (result \triangleq if (self).next@pre \doteq null (* post *)$ 
        then  $Set\{(self).i@pre\}$ 
        else  $(self).next@pre.contents@pre()->including(self.i@pre)$ 
        endif)
    else  $\tau \models result \triangleq invalid$ )
```

Note that these @pre variants on methods are only available on queries, i.e. operations without side-effect.

2.15 The contract of a method.

The specification in high-level OCL input syntax reads as follows:

```
context Node::insert(x:Integer)
post: contents():Set(Integer)
contents() = contents@pre()->including(x)
```

```
consts dot-insert :: Node  $\Rightarrow$  Integer  $\Rightarrow$  Void ((1(-).insert'(-)) 50)
```

axiomatization where *dot-insert-def*:

```
( $\tau \models ((self).insert(x) \triangleq result)$ ) =
  (if ( $\delta self$ )  $\tau = true$   $\tau \wedge (v x) \tau = true$   $\tau$ 
    then  $\tau \models true \wedge$ 
       $\tau \models ((self).contents() \triangleq (self).contents@pre()->including(x))$ 
    else  $\tau \models ((self).insert(x) \triangleq invalid)$ )
```

end

3 Lessons Learned

While our paper and pencil arguments, given in [4], turned out to be essentially correct, there had also been a lesson to be learned: If the logic is not defined as a Kleene-Logic, having a structure similar to a complete partial order (CPO), reasoning becomes complicated: several important algebraic laws break down which makes reasoning in OCL inherent messy and a semantically clean compilation of OCL formulae to a two-valued presentation, that is amenable to animators like KodKod [18] or SMT-solvers like Z3 [11] completely impractical. Concretely, if the expression `not(null)` is defined `invalid` (as is the case in the present standard [16]), than standard involution does not hold, i.e.,

$\text{not}(\text{not}(A)) = A$ does not hold universally. Similarly, if `null` and `null` is `invalid`, then not even idempotence `X and X = X` holds. We strongly argue in favor of a lattice-like organization, where `null` represents “more information” than `invalid` and the logical operators are monotone with respect to this semantical “information ordering.”

Featherweight OCL makes these two deviations from the standard, builds all logical operators on Kleene-**not** and Kleene-**and**, and shows that the entire construction of our paper “Extending OCL with Null-References” [4] is then correct, and the DNF-normaliation as well as δ -closure laws (necessary for a transition into a two-valued presentation of OCL specifications ready for interpretation in SMT solvers (see [3] for details) are valid in Featherweight OCL.

4 Conclusion and Future Work

Featherweight OCL concentrates on formalizing the semantics of a core subset of OCL in general and in particular on formalizing the consequences of a four-valued logic (i. e., OCL versions that support, besides the truth values `true` and `false` also the two exception values `invalid` and `null`).

In the following, we outline the necessary steps for turning Featherweight OCL into a fully fledged tool for OCL, e. g., similar to HOL-OCL as well as for supporting test case generation similar to HOL-TestGen [8]. There are essentially five extensions necessary:

- extension of the library to support all OCL data types, e. g., `Sequence(T)`, `OrderedSet(T)`. This formalization of the OCL standard library can be used for checking the consistency of the formal semantics (known as “Annex A”) with the informal and semi-formal requirements in the normative part of the OCL standard.
- development of a compiler that compiles a textual or CASE tool representation (e. g., using XMI or the textual syntax of the USE tool [17]) of class models. Such compiler could also generate the necessary casts when converting standard OCL to Featherweight OCL as well as providing “normalizations” such as converting multiplicities of class attributes to into OCL class invariants.
- a setup for translating Featherweight OCL into a two-valued representation as described in [3]. As, in real-world scenarios, large parts of UML/OCL specifications are defined (e. g., from the default multiplicity 1 of an attributes `x`, we can directly infer that for all valid states `x` is neither `invalid` nor `null`), such a translation enables an efficient test case generation approach.
- a setup in Featherweight OCL of the Nitpick animator [1]. It remains to be shown that the standard, Kodkod [18] based animator in Isabelle can give a similar quality of animation as the OCLexec Tool [12]
- a code-generator setup for Featherweight OCL for Isabelle’s code generator. For example, the Isabelle code generator supports the generation of F#, which would allow to use OCL specifications for testing arbitrary .net-based applications.

The first two extensions are sufficient to provide a formal proof environment for OCL 2.3 similar to HOL-OCL while the remaining extensions are geared towards increasing

the degree of proof automation and usability as well as providing a tool-supported test methodology for UML/OCL.

Our work shows that developing a machine-checked formal semantics of recent OCL standards still reveals significant inconsistencies—even though this type of research is not new. In fact, we started our work already with the 1.x series of OCL. The reasons for this ongoing consistency problems of OCL standard are manifold. For example, the consequences of adding an additional exception value to OCL 2.2 are widespread across the whole language and many of them are also quite subtle. Here, a machine-checked formal semantics is of great value, as one is forced to formalize all details and subtleties. Moreover, the standardization process of the OMG, in which standards (e.g., the UML infrastructure and the OCL standard) that need to be aligned closely are developed quite independently, are prone to ad-hoc changes that attempt to align these standards. And, even worse, updating a standard document by voting on the acceptance (or rejection) of isolated text changes does not help either. Here, a tool for the editor of the standard that helps to check the consistency of the whole standard after each and every modifications can be of great value as well.

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