Synthèse de contrôleur pour les réseaux de Petri temporels basée sur les classes d'états

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Introduction

- Controller synthesis for Time Petri Nets

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- Timed game for reachability

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- State classes

Example

The Tortoise and the Hare

For the race:

- the tortoise needs 5 to 6 minutes to arrive
- the hare needs only 3 to 4 minutes to arrive but,

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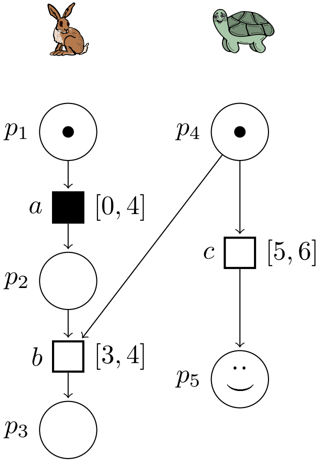
For the race:

- the tortoise needs 5 to 6 minutes to arrive
- the hare needs only 3 to 4 minutes to arrive but,
- the hare stays grazing between 0 and 4 minutes before starting the race

How long does the hare have to wait for the tortoise to win?

Time Petri Net (TPN)

Example



Time Petri Net (TPN)

Definition

A *time Petri net* (TPN) is a tuple $\mathcal{N} = (P, T, F, I_s)$ where:

- P is a finite non-empty set of *places*,
- T is a finite set of *transitions* such that $T \cap P = \emptyset$,
- $F : (P \times T) \cup (T \times P)$ is the *flow function*,
- $I_s : T \to \mathcal{I}(\mathbb{N})$ is the *static firing interval* function,

Semantics

States of the TPN: (m, θ) with

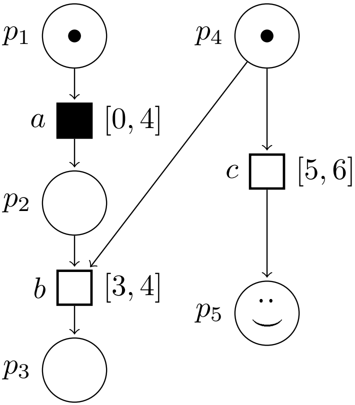
- $m \subseteq P$ a marking and
- θ the firing dates for every enabled transition in m

Firing dates are choosen when the transition become enable instead of at the firing time (moment of firing).

The transition relation $\rightarrow \subseteq S \times \Sigma \times S$:

- either $(m, \theta) \xrightarrow{t_f} (m', \theta')$ for the firing of a transition t_f
- or $(m, \theta) \xrightarrow{d} (m, \theta')$ for the time delay transition

Example



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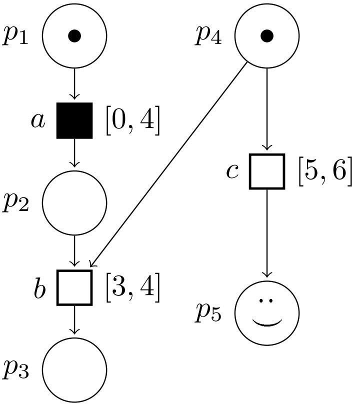
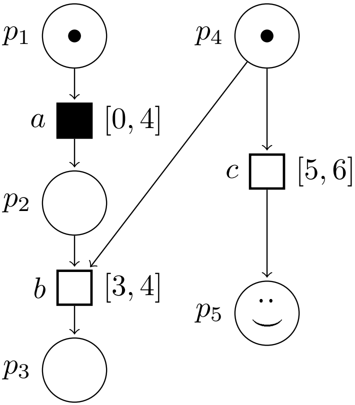
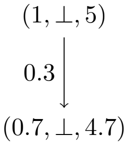


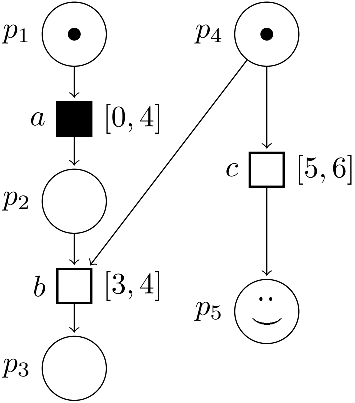
$(1, \perp, 5)$

Example



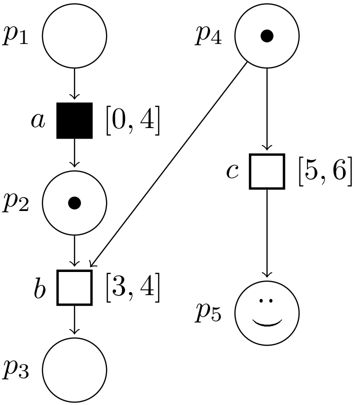


Example



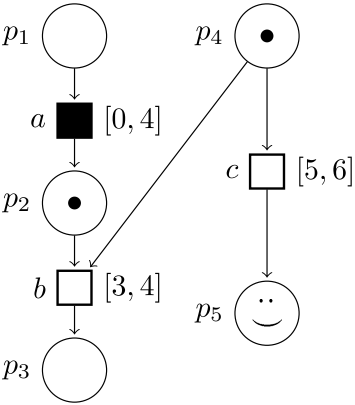
$$\begin{tikzcd}
(1, \perp, 5) \arrow[d, "0.3"'] \\
(0.7, \perp, 4.7) \arrow[d, "0.7"'] \\
(0, \perp, 4)
\end{tikzcd}$$

Example



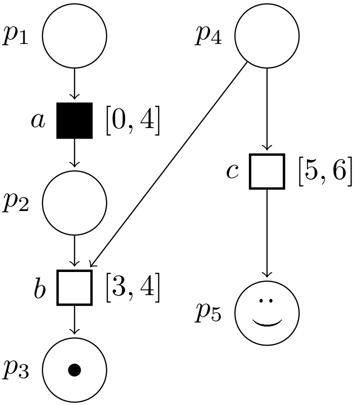
$$
\begin{tikzcd}
(1, \perp, 5) \arrow[d, "0.3"'] \\
(0.7, \perp, 4.7) \arrow[d, "0.7"'] \\
(0, \perp, 4) \arrow[d, "a"'] \\
(\perp, 3, 4)
\end{tikzcd}
$$

Example



$$
\begin{array}{c}
(1, \perp, 5) \\
\Big\downarrow{\scriptstyle 0.3} \\
(0.7, \perp, 4.7) \\
\Big\downarrow{\scriptstyle 0.7} \\
(0, \perp, 4) \\
\Big\downarrow{\scriptstyle a} \\
(\perp, 3, 4) \\
\Big\downarrow{\scriptstyle 3} \\
(\perp, 0, 1)
\end{array}
$$

Example



$$
\begin{array}{c}
(1,\bot,5) \\
\big\downarrow{\scriptstyle 0.3} \\
(0.7,\bot,4.7) \\
\big\downarrow{\scriptstyle 0.7} \\
(0,\bot,4) \\
\big\downarrow{\scriptstyle a} \\
(\bot,3,4) \\
\big\downarrow{\scriptstyle 3} \\
(\bot,0,1) \\
\big\downarrow{\scriptstyle b} \\
(\bot,\bot,\bot)
\end{array}
$$

Reachability game

Definition

$\mathcal{R} = (\mathcal{A}, \mathsf{Goal})$ with:

- an arena $\mathcal{A} = (S, \to, Pl, (Mov_i)_{i \in Pl}, Trans)$
  - Pl_c : controllable transitions
  - Pl_u : uncontrollable transitions
- a set of target states $\mathsf{Goal} \in S$ that Pl_c wants to reach and Pl_u wants to avoid.

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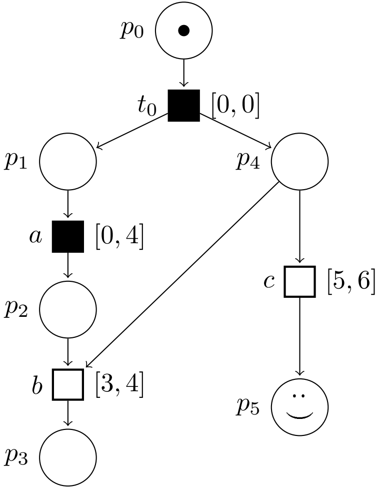
Turn :

1. Pl_c chooses $t_c \in T_c$
2. Pl_u chooses $t_u \in T_u \cup \{t_c\}$
3. Both player chooses firing times for their newly enabled transitions, controllable or uncontrollable.

Reachability game

Example



Reachability game

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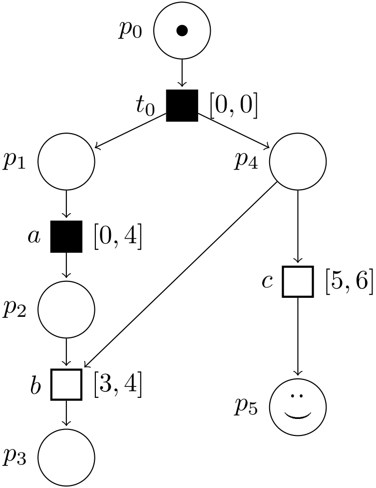
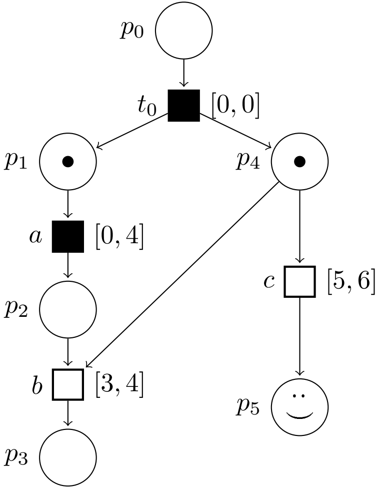


$$s_0 = (\{p_0\}, \theta(t_0) = 0)$$

Reachability game

Example

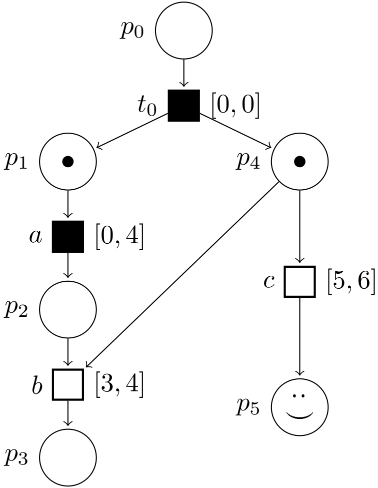


$$s_0 = (\{p_0\}, \theta(t_0) = 0)$$

$$t_c = t_0, t_u = t_c, \theta(a) = 2, \theta(c) = 6$$

Reachability game

Example

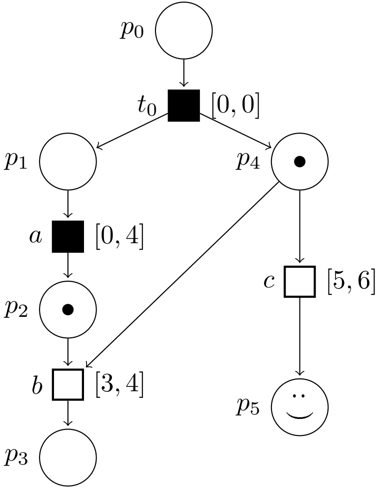


$$s_0 = (\{p_0\}, \theta(t_0) = 0)$$

$$s_1 = (\{p_1, p_4\}, \theta(a) = 2, \theta(c) = 6)$$

Reachability game

Example



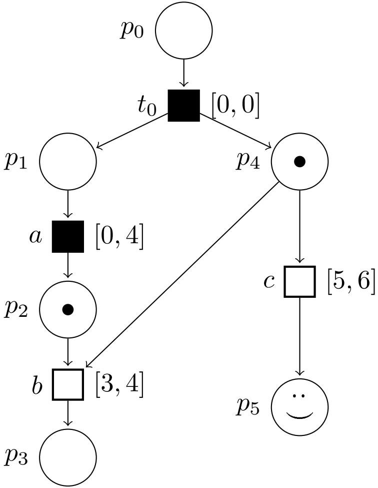
$$s_0 = (\{p_0\}, \theta(t_0) = 0)$$

$$s_1 = (\{p_1, p_4\}, \theta(a) = 2, \theta(c) = 6)$$

$$t_c = a, t_u = t_c, \theta(b) = 4$$

Reachability game

Example



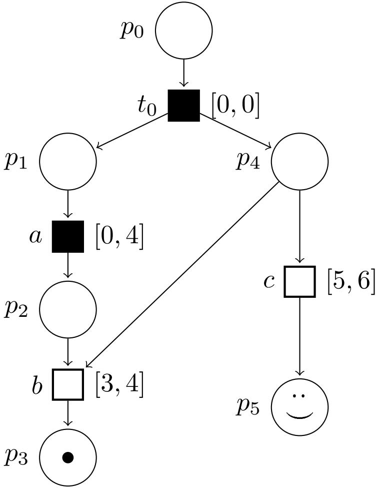
$$s_0 = (\{p_0\}, \theta(t_0) = 0)$$

$$s_1 = (\{p_1, p_4\}, \theta(a) = 2, \theta(c) = 6)$$

$$s_2 = (\{p_2, p_4\}, \theta(b) = 4, \theta(c) = 4)$$

Reachability game

Example



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$$t_u = b$$

Reachability game

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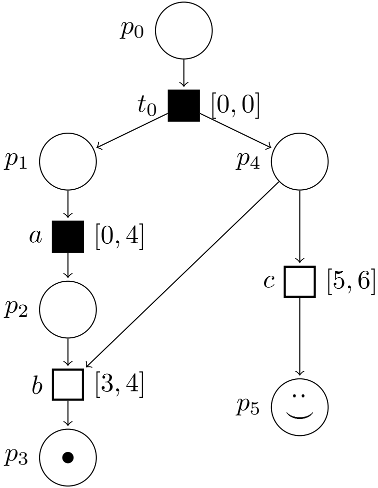

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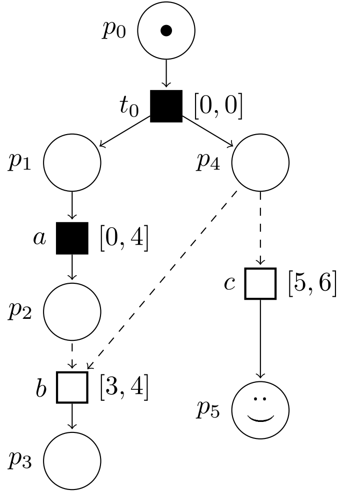
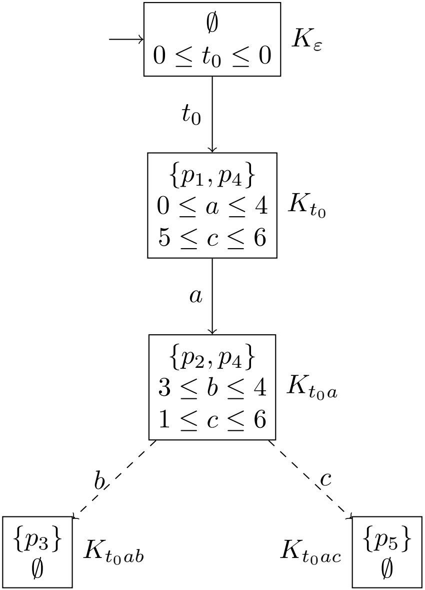
$$t_u = b$$

$$s_3 = (\{p_3\}, \perp)$$

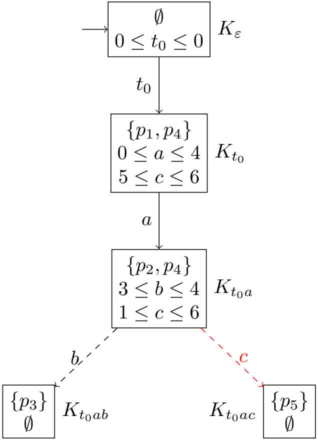
State-Class Graph (SCG)[1]





[1] Berthomieu and Menasche, "An Enumerative Approach For Analyzing Time Petri Nets".

Subset of winning states



$$C = \begin{pmatrix} \{p_2, p_4\} \\ 3 \leq b \leq 4 \\ 1 \leq c \leq 6 \\ c < b \end{pmatrix}$$

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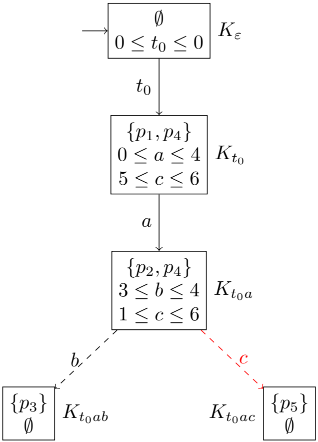
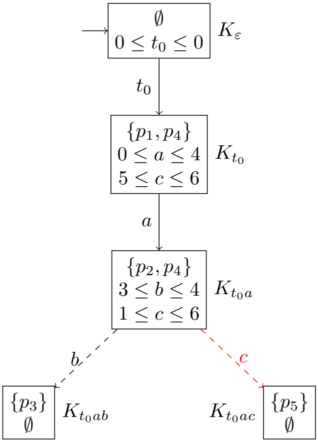


$$C = \begin{pmatrix} \{p_2, p_4\} \\ 3 \le b \le 4 \\ 1 \le c \le 6 \\ c < b \end{pmatrix} = \begin{pmatrix} \{p_2, p_4\} \\ 3 \le b \le 4 \\ 1 \le c < 3 \end{pmatrix}$$

Subset of winning states



$$C' = \begin{pmatrix} \{p_1, p_4\} \\ 0 \leq a \leq 4 \\ 5 \leq c \leq 6 \\ 1 \leq c - a < 3 \end{pmatrix} = \begin{pmatrix} \{p_1, p_4\} \\ 2 < a \leq 4 \\ 5 \leq c \leq 6 \\ 1 \leq c - a < 3 \end{pmatrix}$$

$$C = \begin{pmatrix} \{p_2, p_4\} \\ 3 \leq b \leq 4 \\ 1 \leq c \leq 6 \\ c < b \end{pmatrix} = \begin{pmatrix} \{p_2, p_4\} \\ 3 \leq b \leq 4 \\ 1 \leq c < 3 \end{pmatrix}$$

Future work

- reachability with parametric firing-time constraints: using an efficient linear programming algorithm when the dimension is small and/or fixed
- explicit firing dates semantics expressivity
- applications to concrete problems

Références

Berthomieu, Bernard and Miguel Menasche. “An Enumerative Approach For Analyzing Time Petri Nets”. In: *Proceedings IFIP*. Elsevier Science Publishers, 1983, pp. 41–46.