MODULE *SerializabilityRefinement*

Refinement mapping to show that *Serializability* implements *Sequential*

EXTENDS *Serializability*, *Sequences*, *TLC*

VARIABLES h, $henv$, $opBar$, $argBar$, $rvalBar$, $envBar$, $ffBar$, $serialized$

$vars \triangleq \langle h, henv, opBar, argBar, rvalBar, envBar, ffBar, serialized \rangle$
$bars \triangleq \langle opBar, argBar, rvalBar, envBar, ffBar, serialized \rangle$

$$
\begin{aligned}
InitR \triangleq\ & \land Init \\
& \land h = \langle\rangle \\
& \land henv = \langle\rangle \\
& \land opBar = op \\
& \land argBar = arg \\
& \land rvalBar = rval \\
& \land envBar = benv[1] \\
& \land ffBar = Flip \\
& \land serialized = \text{FALSE}
\end{aligned}
$$

$$
\begin{aligned}
TypeOkR \triangleq\ & \land h \in Seq([tr : Tr,\ op : \{\text{“r”},\ \text{“w”}\},\ arg : Arg]) \\
& \land henv \in Seq([Obj \to Val]) \\
& \land opBar \in Op \\
& \land argBar \in Arg \\
& \land rvalBar \in Rval \\
& \land envBar \in [Obj \to Val] \\
& \land ffBar \in \{Flip,\ Flop\} \\
& \land serialized \in \text{BOOLEAN}
\end{aligned}
$$

$$Commits(t) \triangleq fate[t] = Committed$$

$$
\begin{aligned}
CommitR(t) &\triangleq Commit(t) \land \text{UNCHANGED } vars \\
AbortR(t) &\triangleq Abort(t) \land \text{UNCHANGED } vars
\end{aligned}
$$

$$
\begin{aligned}
ReadR(t,\ obj,\ val) \triangleq\ & \land Read(t,\ obj,\ val) \\
& \land h' = \text{IF } Commits(t) \\
& \qquad \text{THEN } Append(h,\ [tr \mapsto t,\ op \mapsto \text{“r”}, \\
& \qquad\qquad arg \mapsto arg',\ rval \mapsto rval']) \\
& \qquad \text{ELSE } h \\
& \land henv' = \text{IF } Commits(t) \text{ THEN } Append(henv,\ tenv'[t]) \text{ ELSE } henv \\
& \land \text{UNCHANGED } bars
\end{aligned}
$$

$$
\begin{aligned}
WriteR(t,\ obj,\ val) \triangleq\ & \land Write(t,\ obj,\ val) \\
& \land h' = \text{IF } Commits(t) \\
& \qquad \text{THEN } Append(h,\ [tr \mapsto t,\ op \mapsto \text{“w”}, \\
& \qquad\qquad arg \mapsto arg',\ rval \mapsto rval']) \\
& \qquad \text{ELSE } h
\end{aligned}
$$

$\land\ henv' = \text{IF } Commits(t) \text{ THEN } Append(henv,\ tenv'[t]) \text{ ELSE } henv$
$\land\ \text{UNCHANGED } bars$

$SerializeHistory \triangleq$
$\land\ Termination$
$\land\ \neg serialized$
$\land\ \text{LET } N \triangleq Len(h)$
$R \triangleq 1\,..\,N$
$perm \triangleq \text{CHOOSE } seq \in [R \to R] :$
$\forall\, i,\, j \in R :$
$\text{LET } si \triangleq seq[i]$
$sj \triangleq seq[j]$
$hi \triangleq h[si]$
$hj \triangleq h[sj]$
$Ti \triangleq hi.tr$
$Tj \triangleq hj.tr \text{IN}$
must be 1:1 mapping
$\land\ si = sj \Rightarrow i = j$
preserve order within transaction
$\land\ (Ti = Tj \land i < j) \Rightarrow si < sj$
$\land\ (Ti = Tj \land i > j) \Rightarrow si > sj$
respect transaction order
$\land\ Ord(Ti) < Ord(Tj) \Rightarrow i < j$
$\land\ Ord(Ti) > Ord(Tj) \Rightarrow i > j$
$\text{IN}\quad \land\ h' = [i \in R \mapsto h[perm[i]]]$
$\land\ henv' = [i \in R \mapsto henv[perm[i]]]$
$\land\ serialized' = \text{TRUE}$
$\land\ \text{UNCHANGED } \langle opBar,\ argBar,\ rvalBar,\ envBar,\ ffBar \rangle$

Issue the commands to the refinement mapping

$Issue \triangleq \text{LET } e \triangleq Head(h) \text{IN}$
$\land\ Termination$
$\land\ serialized$
$\land\ h \neq \langle\rangle$
$\land\ opBar' = e.op$
$\land\ argBar' = e.arg$
$\land\ rvalBar' = e.rval$
$\land\ envBar' = Head(henv)$
$\land\ ffBar' = Toggle(ffBar)$
$\land\ h' = Tail(h)$
$\land\ henv' = Tail(henv)$
$\land\ \text{UNCHANGED } serialized$

$vr \triangleq \langle h,\ henv,\ opBar,\ argBar,\ rvalBar,\ envBar,\ ffBar,\ serialized,\ tr,$
$op,\ arg,\ rval,\ tstate,\ fate,\ to,\ tenv,\ benv,\ eval,\ ff \rangle$

$$
\begin{array}{l}
\mathit{TerminationR} \triangleq \land\ \mathit{Termination} \\
\qquad\qquad\qquad\quad \land\ h = \langle\rangle \\
\qquad\qquad\qquad\quad \land\ \text{UNCHANGED}\ \mathit{vr}
\end{array}
$$

$$
\begin{array}{l}
\mathit{NextR} \triangleq \lor \exists\, t \in \mathit{Tr} : \\
\qquad\qquad\quad \lor\ \mathit{CommitR}(t) \\
\qquad\qquad\quad \lor\ \mathit{AbortR}(t) \\
\qquad\qquad\quad \lor \exists\, \mathit{obj} \in \mathit{Obj},\ \mathit{val} \in \mathit{Val} : \\
\qquad\qquad\qquad\quad \lor\ \mathit{ReadR}(t,\ \mathit{obj},\ \mathit{val}) \\
\qquad\qquad\qquad\quad \lor\ \mathit{WriteR}(t,\ \mathit{obj},\ \mathit{val}) \\
\qquad\quad \lor\ \mathit{SerializeHistory} \\
\qquad\quad \lor\ \mathit{Issue} \\
\qquad\quad \lor\ \mathit{TerminationR}
\end{array}
$$

$$
\begin{array}{l}
\mathit{LR} \triangleq \land\ L \\
\qquad\quad \land\ \mathrm{WF}_{\mathit{vr}}(\mathit{SerializeHistory}) \\
\qquad\quad \land\ \mathrm{WF}_{\mathit{vr}}(\mathit{Issue})
\end{array}
$$

$$
\mathit{SpecR} \triangleq \mathit{InitR} \land \Box[\mathit{NextR}]_{\mathit{vr}} \land \mathit{LR}
$$

$$
\begin{array}{l}
\mathit{Sequential} \triangleq \text{INSTANCE}\ \mathit{Sequential}\ \text{WITH} \\
\quad \mathit{op} \leftarrow \mathit{opBar}, \\
\quad \mathit{arg} \leftarrow \mathit{argBar}, \\
\quad \mathit{rval} \leftarrow \mathit{rvalBar}, \\
\quad \mathit{env} \leftarrow \mathit{envBar}, \\
\quad \mathit{ff} \leftarrow \mathit{ffBar}
\end{array}
$$

$$
\mathit{SeqSpec} \triangleq \mathit{Sequential}!\mathit{Spec}
$$

$$
\text{THEOREM}\ \mathit{Spec} \Rightarrow \mathit{SeqSpec}
$$