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— MODULE MVCCRefinement
EXTENDS MVCC, Naturals, Sequences, FiniteSets, TLC
CONSTANTS NULL, Flip, Flop
Variables h, fateIsSet, canIssue, parity, reads, writes, ord, tenvBar
 Refinement transactions
TrR \stackrel{\triangle}{=} Tr \setminus \{T0\}
 Committed transactions
CT \stackrel{\Delta}{=} \{t \in TrR : tstate[t] = Committed\}
N \triangleq Cardinality(CT)
TypeOkR \triangleq \land TypeOk
                    \land \forall i \in \text{Domain } h : \text{Let } e \stackrel{\triangle}{=} h[i] \text{In}
                           \land e.tr \in TrR
                           \land \ e.op \in \{ \text{``r''}, \text{ ``w''}, \text{ ``c''}, \text{ ``a''} \}
                           \land e.arg \in CASE \ e.op = "r" \rightarrow Obj
                                            \square \quad e.op = \text{``w''} \rightarrow Obj \times Val
                                            \Box OTHER \rightarrow \{\langle \rangle \}
                           \land e.rval \in Val \cup \{Ok, Err\}
                           \land e.tstate \in [Tr \rightarrow \{Unstarted, Open, Committed, Aborted\}]
                           \land e.op \in \{\text{"r"}, \text{"w"}\} \Rightarrow \land \text{DOMAIN } e.wr \subseteq Obj
                                                             \land \forall obj \in DOMAIN \ e.wr : e.wr[obj] \in Val
                    \land \mathit{fateIsSet} \in \texttt{boolean}
                    \land canIssue \in Boolean
                    \land parity \in \{0, 1\}
                    \land reads \in [Tr \rightarrow \text{SUBSET } Obj]
                    \land writes \in [Tr \rightarrow \text{SUBSET } Obj]
                    \land \ tenvBar \in [CT \rightarrow [Obj \rightarrow Val]] \cup \{NULL\}
                    \land ord \in [to:[1..N \rightarrow CT] \cup \{NULL\}, benv:[1..N+1 \rightarrow [Obj \rightarrow Val]] \cup \{NULL\}]
InitR \triangleq \land Init
              \land fateIsSet = false
              \wedge parity = 0
              \wedge h = \langle \rangle
              \land canIssue = false
              \land reads = [t \in Tr \mapsto \{\}]
              \land writes = [t \in Tr \mapsto \text{if } t = T0 \text{ Then } Obj \text{ else } \{\}]
              \land ord = [to \mapsto NULL, benv \mapsto NULL]
              \land \ tenvBar = NULL
StartTransactionR(t) \triangleq \wedge StartTransaction(t)
                                      \land UNCHANGED \langle h, fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
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BeginRdR(t, obj) \triangleq \land BeginRd(t, obj)
                              \land UNCHANGED \langle h, fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
EndRdR(t, obj, val) \triangleq
     \wedge EndRd(t, obj, val)
     \wedge h' = Append(h, [tr \mapsto t, op \mapsto "r", arg \mapsto obj, rval \mapsto val,
                               tstate \mapsto tstate, wr \mapsto [o \in writes[t] \mapsto Get(t, o)]])
     \land reads' = \text{IF } obj \in writes[t] \text{ THEN } reads \text{ ELSE } [reads \text{ EXCEPT } ![t] = @ \cup \{obj\}] \text{ unwritten reads}
     \wedge parity' = 1 - parity
     \land UNCHANGED \langle fateIsSet, canIssue, writes, ord, tenvBar \rangle
BeginWrR(t, obj, val) \triangleq \land BeginWr(t, obj, val)
                                    \land UNCHANGED \langle h, fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
EndWrR(t, obj, val) \stackrel{\Delta}{=}
     \wedge EndWr(t, obj, val)
     \wedge h' = Append(h, [tr \mapsto t, op \mapsto "w", arg \mapsto \langle obj, val \rangle, rval \mapsto Ok,
                               tstate \mapsto tstate, wr \mapsto [o \in writes[t] \mapsto Get(t, o)]])
     \land writes' = [writes \ EXCEPT \ ![t] = @ \cup \{obj\}]
     \wedge parity' = 1 - parity
     ∧ UNCHANGED ⟨fateIsSet, canIssue, reads, ord, tenvBar⟩
AbortWrR(t, obj, val) \triangleq
     \wedge AbortWr(t, obj, val)
     \wedge h' = Append(h, [tr \mapsto t, op \mapsto \text{``a''}, arg \mapsto \langle \rangle, rval \mapsto Err,
                               tstate \mapsto [tstate \ EXCEPT \ ![t] = Aborted]])
     \land UNCHANGED \langle fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
CommitR(t) \triangleq
     Λ
             Commit(t)
             h' = Append(h, [tr \mapsto t, op \mapsto \text{``c"}, arg \mapsto \langle \rangle, rval \mapsto Ok,
                                    tstate \mapsto [tstate \ EXCEPT \ ![t] = Committed]])
             UNCHANGED (fateIsSet, canIssue, parity, reads, writes, ord, tenvBar)
AbortR(t) \stackrel{\triangle}{=}
     \wedge Abort(t)
     \wedge h' = Append(h, [tr \mapsto t, op \mapsto "a", arg \mapsto \langle \rangle, rval \mapsto Ok,
                               tstate \mapsto [tstate \ EXCEPT \ ![t] = Aborted]])
     \land UNCHANGED \langle fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
DetectDeadlockR \triangleq \land DetectDeadlock
                             \land UNCHANGED \langle h, fateIsSet, canIssue, parity, reads, writes, ord, tenvBar <math>\rangle
 Get the order in which this transactionruns
Ord(t) \stackrel{\triangle}{=} CHOOSE \ i \in DOMAIN \ ord.to : ord.to[i] = t
SetFate \triangleq \land Done
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 $\wedge fateIsSet = False$

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\wedge fateIsSet' = TRUE
                 \land ord' = \text{CHOOSE } r \in [to : [1 ... N \to CT], benv : [1 ... N + 1 \to [Obj \to Val]]]:
                       first environment must be the initialization
                      \wedge r.benv[1] = SnapInit
                       to must be a total ordering
                      \land \forall i, j \in 1 ... N : r.to[i] = r.to[j] \Rightarrow i = j
                      \land \forall i \in 1 ... N : \text{LET } t \stackrel{\triangle}{=} r.to[i] \text{IN}
                            all non-written reads have to be consistent with transaction's snapshot
                           \land \forall obj \in reads[t]: r.benv[i][obj] = GetVer(obj, vis[t] \setminus \{t\}).val
                            all writes have to be consistent with transaction's environment
                           \land \forall obj \in writes[t] : r.benv[i+1][obj] = Get(t, obj)
                            if a variable changed, there must be a corresponding write
                           \land \forall obj \in Obj : (r.benv[i+1][obj] \neq r.benv[i][obj]) \Rightarrow obj \in writes[t]
                 \wedge tenvBar' = \text{LET } ordp \stackrel{\triangle}{=} ord'
                                          benv \stackrel{\triangle}{=} ordp.benv
                                          to \stackrel{\triangle}{=} ordp.toin
                      [t \in CT \mapsto \text{LET } i \stackrel{\triangle}{=} \text{ CHOOSE } i \in \text{DOMAIN } to : to[i] = t \text{IN } benv[i]]
                 \land UNCHANGED \langle op, arg, rval, tr, db, vis, tstate, tid, deadlocked,
                                       h, canIssue, parity, reads, writes
Issue \triangleq \land h \neq \langle \rangle
             \land fateIsSet
             \wedge canIssue' = TRUE
             \wedge h' = \text{IF } canIssue \text{ THEN } Tail(h) \text{ ELSE } h
             \wedge h' \neq \langle \rangle
              tenvBar' needs to reflect the state of the *next* head in the history, not the current head
             IF tstate[e.tr] = Committed \land e.op = "w"
                                       THEN [tenvBar \ EXCEPT \ ![t][obj] = val]
                                        ELSE tenvBar
             \land UNCHANGED \langle op, arg, rval, tr, db, vis, tstate, tid, deadlocked,
                                   fateIsSet, parity, reads, writes, ord
vv \stackrel{\triangle}{=} \langle op, arq, rval, tr, db, vis, tstate, tid, deadlocked, h, fateIsSet, canIssue,
          parity, reads, writes, ord, tenvBar
TerminationR \stackrel{\Delta}{=} \land Done
                          \wedge Tail(h) = \langle \rangle
                          \land UNCHANGED vv
NextR \triangleq \forall \exists t \in Tr, obj \in Obj, val \in Val:
                  \vee StartTransactionR(t)
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\vee BeginRdR(t, obj)
                    \vee EndRdR(t, obj, val)
                     \vee BeginWrR(t, obj, val)
                    \vee EndWrR(t, obj, val)
                    \vee AbortWrR(t, obj, val)
                    \vee CommitR(t)
                     \vee AbortR(t)
                 \lor \ DetectDeadlockR
                 \vee Issue
                 \vee SetFate
                 \vee TerminationR
SpecR \triangleq InitR \wedge \Box [NextR]_{vv}
trBar \stackrel{\triangle}{=} IF \ canIssue \ THEN \ Head(h).tr \ ELSE \ TO
opBar \stackrel{\triangle}{=} \text{ if } canIssue \text{ THEN } Head(h).op \text{ ELSE "r"}
argBar \stackrel{\triangle}{=} CASE \ canIssue \land Head(h). arg = \langle \rangle \rightarrow None
                    \Box can Issue \land Head (h). arg \neq \langle \rangle \rightarrow Head (h). arg
                    \square OTHER \rightarrow CHOOSE obj \in Obj : TRUE
rvalBar \stackrel{\triangle}{=} CASE \ canIssue \land Head(h).rval \neq Err \rightarrow Head(h).rval
                     \Box canIssue \land Head(h).rval = Err \rightarrow Ok
                     □ OTHER
                                                                           \rightarrow V0
tstateBar \stackrel{\triangle}{=} [t \in \mathit{TrR} \mapsto
                       Let s \triangleq Head(h).tstate[t]in
                       Case \neg canIssue
                                                                         \rightarrow Open
                          \Box canIssue \land s = Unstarted \rightarrow Open
                                canIssue \land s = Open
                          \rightarrow Open
                                canIssue \land s = Committed \rightarrow Committed
                          \Box canIssue \land s = Aborted
                                                                      \rightarrow Aborted
ffBar \triangleq \text{Let } Parity(hh) \triangleq Len(SelectSeq(hh, \text{Lambda } e: e.op \in \{\text{"r"}, \text{"w"}\}))\%2
                      p \triangleq Parity(h)
                      opp \triangleq Head(h).opin
     Case \neg canIssue
                                                 \rightarrow Flip
             canIssue \land opp \in \{\text{"r"}, \text{"w"}\} \land parity = p \rightarrow Flop
             canIssue \land opp \notin \{\text{"r"}, \text{"w"}\} \land parity = p \rightarrow FlipcanIssue \land opp \in \{\text{"r"}, \text{"w"}\} \land parity \neq p \rightarrow Flip
             canIssue \land opp \notin \{\text{"r"}, \text{"w"}\} \land parity \neq p \rightarrow Flop
fateBar \stackrel{\triangle}{=} \text{if } \neg fateIsSet \text{ then } NULL
                   ELSE [t \in TrR \mapsto tstate[t]]
Ser \triangleq INSTANCE SerializabilityD WITH
     Tr \leftarrow TrR,
     tr \leftarrow trBar,
     op \leftarrow opBar,
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arg \leftarrow argBar, \\ rval \leftarrow rvalBar, \\ tstate \leftarrow tstateBar, \\ fate \leftarrow fateBar, \\ to \leftarrow ord.to, \\ tenv \leftarrow tenvBar, \\ benv \leftarrow ord.benv, \\ ff \leftarrow ffBar, \\ Vinit \leftarrow V0
SerSpec \triangleq Ser!SpecD
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