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– MODULE CycleDetection -
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EXTENDS Naturals
Constants N, NIL
Nodes \stackrel{\triangle}{=} 1 \dots N
-- fair algorithm Tortoise And Hare
variables
    start \in Nodes,
    succ \in [Nodes \rightarrow Nodes \cup \{NIL\}],
    cycle, tortoise, hare, done;
begin
h0: tortoise := start;
    hare := start;
    done := FALSE;
h1: while \neg done do
         h2: tortoise := succ[tortoise];
              hare := \text{LET } hare 1 \stackrel{\triangle}{=} succ[hare] \text{IN}
                       If hare1 \in DOMAIN \ succ \ THEN \ succ [hare1] \ ELSE \ NIL;
         h3: \mathbf{if} \ tortoise = NIL \lor hare = NIL \mathbf{then}
                  cycle := FALSE;
                  done := TRUE;
               elsif tortoise = hare then
                   cycle := TRUE;
                  done := TRUE;
              end if;
    end while;
end algorithm
 BEGIN TRANSLATION
Constant defaultInitValue
{\tt VARIABLES}\ start,\ succ,\ cycle,\ tortoise,\ hare,\ done,\ pc
vars \triangleq \langle start, succ, cycle, tortoise, hare, done, pc \rangle
Init \stackrel{\Delta}{=} Global variables
           \land start \in Nodes
           \land succ \in [Nodes \rightarrow Nodes \cup \{NIL\}]
           \land cycle = defaultInitValue
           \land\ tortoise = \mathit{defaultInitValue}
           \land hare = defaultInitValue
           \land done = defaultInitValue
           \wedge pc = \text{"h0"}
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h0 \stackrel{\triangle}{=} \wedge pc = \text{"h0"}
           \land \ tortoise' = start
           \wedge hare' = start
           \wedge done' = \text{False}
           \wedge pc' = \text{"h1"}
           \land UNCHANGED \langle start, succ, cycle \rangle
h1 \stackrel{\triangle}{=} \land pc = \text{"h1"}
           \wedge IF \neg done
                   THEN \wedge pc' = \text{"h2"}
                   ELSE \wedge pc' = "Done"
           ∧ UNCHANGED ⟨start, succ, cycle, tortoise, hare, done⟩
h2 \stackrel{\triangle}{=} \land pc = \text{"h2"}
           \land tortoise' = succ[tortoise]
           \wedge hare' = (\text{LET } hare1 \stackrel{\triangle}{=} succ[hare] \text{IN}
                           If hare1 \in DOMAIN \ succ \ THEN \ succ [hare1] \ ELSE \ NIL)
           \wedge pc' = \text{"h3"}
           \land UNCHANGED \langle start, succ, cycle, done \rangle
h3 \stackrel{\triangle}{=} \land pc = \text{"h3"}
           \land if tortoise = NIL \lor hare = NIL
                   Then \land cycle' = \text{false}
                              \wedge done' = \text{TRUE}
                    ELSE \land IF tortoise = hare
                                      THEN \land cycle' = \text{TRUE}
                                                 \land done' = \text{true}
                                      ELSE \land TRUE
                                                 \land UNCHANGED \langle cycle, done \rangle
           \wedge pc' = \text{"h1"}
           \land UNCHANGED \langle start, succ, tortoise, hare <math>\rangle
Next \triangleq h0 \lor h1 \lor h2 \lor h3
                  V Disjunct to prevent deadlock on termination
                     (pc = "Done" \land UNCHANGED vars)
Spec \stackrel{\Delta}{=} \wedge Init \wedge \Box [Next]_{vars}
              \wedge WF_{vars}(Next)
Termination \stackrel{\triangle}{=} \Diamond (pc = \text{``Done''})
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END TRANSLATION

Transitive closure

From https://github.com/tlaplus/Examples/blob/master/specifications/TransitiveClosure/TransitiveClosure.tla $TC(R) \triangleq$

LET
$$Support(X) \triangleq \{r[1] : r \in X\} \cup \{r[2] : r \in X\}$$

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S \triangleq Support(R) RECURSIVE TCR(\_) TCR(T) \triangleq \text{ if } T = \{\} Then R \text{ELSE LET } r \triangleq \text{CHOOSE } s \in T : \text{TRUE} RR \triangleq TCR(T \setminus \{r\}) \text{IN } RR \cup \{\langle s, t \rangle \in S \times S : \\ \langle s, r \rangle \in RR \wedge \langle r, t \rangle \in RR \} \text{IN } TCR(S) HasCycle(node) \triangleq \text{LET } R \triangleq \{\langle s, t \rangle \in Nodes \times (Nodes \cup \{NIL\}) : succ[s] = t\} \text{IN } \langle node, NIL \rangle \notin TC(R) PartialCorrectness \triangleq pc = \text{"Done"} \Rightarrow (cycle \equiv HasCycle(start))
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^{\^*} Last modified Sun Oct 15 19:33:20 PDT 2017 by lhochstein

^{*} Created Sun Oct 15 17:34:00 PDT 2017 by lhochstein