_formula

Introduction

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September 10, 2023

1 Introduction

add basic computability stuff

Definition 1: /complexity_theory

Complexity theory is the study of resource usage, whether that resource is time, space, entanglement, or something else.

Definition 2: [complexity_theory]/ resource

1. Time
2. Space
3. Time-space product, so if you have more space you need less time, etc.
4. Randomness
5. Entanglement

Definition 3: [complexity_theory]/ language

A language is a set $L \subseteq \Sigma^*$ for some alphabet set Σ. This can equivalently be thought of as the corresponding indicator function χ_L.

Definition 4: [resource]/ time_of_language

Given a language L and turing machine M, we say M accepts L in time $O(T(n))$ if the function:

$$\boxed{n \mapsto \left(\max_{\substack{x \in L \\ \text{len}(x)=n}} \text{time taken by } M \text{ to accept } x \right) = O(T(n))} \tag{1}$$

Definition 5: [complexity_theory]/ complexity_class

A complexity class is some class of languages.

Definition 6: [complexity_class]/ DTIME

DTIME(f(n)) is the class of languages that can be decided by a deterministic Turing machine in $O(f(n))$ time. Assume that you have the number of tapes that you need.

Definition 7: [complexity_class]/ P

P is a complexity class defined by:

$$\boxed{\begin{aligned} P &\stackrel{\text{def}}{=} \bigcup_{k \geq 0} \text{DTIME}(n^k) && (2) \\ &\stackrel{\text{def}}{=} \text{DTIME}(\text{poly}(n)) && (3) \end{aligned}}$$

where n is the length of the input.

Example 1.1: [P]/ primes

Define:

$$\boxed{\text{PRIMES} = \{x \in \omega : x \text{ is a prime number}\}} \tag{4}$$

There is a paper called PRIMES is in P.

Definition 8: [complexity_class]/ NTIME

NTIME(f(n)) is the class of languages that can be decided by a non-deterministic Turing machine in $O(f(n))$ time. Assume that you have the number of tapes that you need.

Result 1: [complexity_class]/ dtime_implies_ntime

$\text{DTIME}(f(n)) \subseteq \text{NTIME}(f(n))$

Definition 9: [complexity_class]/ NP

$$\boxed{\begin{aligned} NP &\stackrel{\text{def}}{=} \bigcup_{k \geq 0} \text{NTIME}(n^k) && (5) \\ &\stackrel{\text{def}}{=} \text{NTIME}(\text{poly}(n)) && (6) \end{aligned}}$$

where n is the length of the input. Or equivalently:

$$\boxed{\begin{aligned} L \in \text{NTIME} &\iff \exists^{\text{TM}}_M \forall^L_x \exists^\Sigma_c M(c, \cdot) && (7) \\ &\text{where both } M(c,\cdot) \text{and } |c| \text{ have polynomial complexity.} && (8) \end{aligned}}$$

where n is the length of the input.