Baire Hierarchy

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Result 1: [baire_hierarchy]/ baire_hierarchy_rational_indicator

Let:

1. $B(\mathbb{R}, \mathbb{R})$ be the set of borel functions, i.e.

$$\boxed{B(\mathbb{R}, \mathbb{R}) = \text{closure of } C(\mathbb{R}, \mathbb{R}) \text{ under sequential pointwise limits}} \tag{1}$$

2. $T(A) : \mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$ be defined as closing under pointwise limits.

Then:

1. $\chi_{\mathbb{Q}} : \mathbb{R} \to \{0, 1\}$, the indicator function on $\mathbb{Q} \subseteq \mathbb{R}$ can be obtained in applying T 2 steps.

Proof. From $C(\mathbb{R}, \mathbb{R})$ you can get better and better approximations of χ_q for every $q \in \mathbb{Q}$. This means that $T(C(\mathbb{R}, \mathbb{R}))$ contains χ_q for any $q \in \mathbb{Q}$. Furthermore you could do the same thing for any finite subset, so χ_F for any finite $F \subseteq \mathbb{Q}$ exists in $T(C(\mathbb{R}, \mathbb{R}))$. So by simply building any increasing sequence of F, this says that $\chi_{\mathbb{Q}} \in T^2(C(\mathbb{R}, \mathbb{R}))$. $\square$