

Orderings

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1 Orderings

Definition 1: [set, relation]/ preorder

Consider

1. A set X
2. A relation $\prec \subseteq X^2$

(X, \prec) is a preorder iff:

1. \prec is reflexive

$$\forall_x^X x \prec x \quad (1)$$

2. \prec is transitive

$$\forall_{x,y,z}^X x \prec y \wedge y \prec z \implies x \prec z \quad (2)$$

Result 1: [small_category]/ preorder_is_small_category

A preorder is a small category.

Definition 2: [preorder]/ partial_order

Consider

1. Preorder $\mathcal{M} = (X, \prec)$

\mathcal{M} is a partial order iff:

1. A \mathcal{M} is antisymmetric:

$$\forall_{x,y}^X x \prec y \wedge y \prec x \implies x = y \quad (3)$$

Definition 3: [partial_order]/ total_order

A total order is a preorder \prec on a set X that is: