_horn_formula

Composite Structures

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1 Composite Structures

Definition 1: [L_structure]/ product_structure

Let

1. $\{\mathcal{M}_i\}_{i \in I}$ be a collection of $\mathcal{L}$ structures

Then define: $\prod_{i \in I} \mathcal{M}_i$ to be the $\mathcal{L}$ structure with

1. universe:

$$\boxed{\prod_{i \in I} \operatorname{dom} \mathcal{M}_i} \tag{1}$$

2. function symbols:

$$\boxed{f^{\prod_{i \in I} \mathcal{M}_i} = \prod_{i \in I} f^{\mathcal{M}_i} = (f^{\mathcal{M}_i}(\vec{a}))_{i \in I}} \tag{2}$$

3. relation symbols:

$$\boxed{R^{\prod_{i \in I} \mathcal{M}_i} = \prod_{i \in I} R^{\mathcal{M}_i} = \{(a_i)_{i \in I} \mid \forall_i^I a_i \in R^{\mathcal{M}_i}\}} \tag{3}$$

Result 1: [universal_horn_formula, product_structure]/ universal_horn_formulas_preserved_under_products

Let $\mathcal{M}_i$ be a collection of $\mathcal{L}$ structures, and let ϕ be a universal Horn formula. Then:

$$\boxed{\prod_{i \in I} \mathcal{M}_i \models \phi \iff \forall_i^I \ \mathcal{M}_i \models \phi} \tag{4}$$

Proof.

finish

□