Formula Types

Loris Jautakas

September 10, 2023

1 Different Types of Formulas

Definition 1: [L_formula]/ universal_formula

The set of universal formulas are defined as starting from quantifier free formulas, and closing under $\forall$:

$$\phi(x, \overline{y}) \in \mathcal{L}_\forall \implies \forall x \phi(x, \overline{y}) \in \mathcal{L}_\forall \tag{1}$$

Where $\mathcal{L}_A$ is initially defined as the set of quantifier free $\mathcal{L}$-formulas, and then is closed under $\forall$. A general formula will be of the form:

$$\forall_{x_1} \ldots \forall_{x_\alpha} \phi(x_1, \ldots, x_\alpha, \overline{y}) \tag{2}$$
$$\text{s.t. } \phi \text{ is quantifier free} \tag{3}$$

Definition 2: [universal_formula]/ universal_horn_formula

A universal Horn formula is a universal formula of the form:

$$\forall_{x_1, \ldots x_\alpha} \phi(x_1, \ldots, x_\alpha, \overline{y}) \implies \psi(x_1, \ldots, x_\alpha, \overline{y}) \tag{4}$$
$$\text{s.t. } \phi, \psi \text{ are quantifier free} \tag{5}$$