Languages of Unambiguous Vector Addition Systems With States

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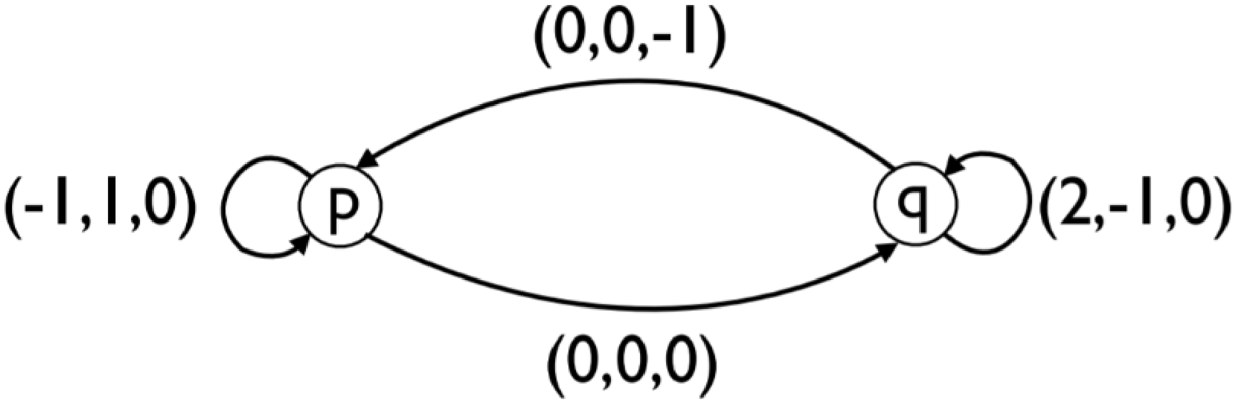
- We often know efficient algorithms for deterministic systems
- Determinization is often costly (exponential blow-up)
- Hence we often look for a middle-ground (some extensions of determinism)

VASS



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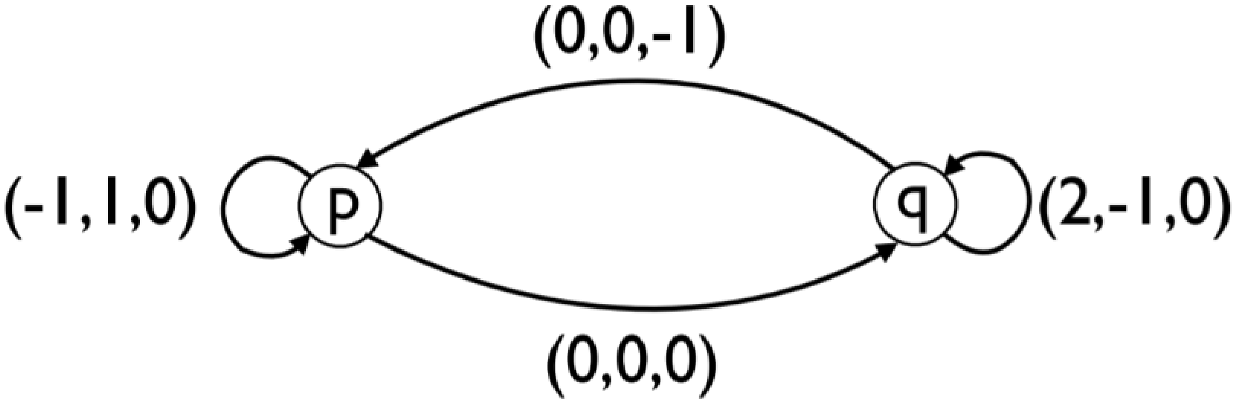

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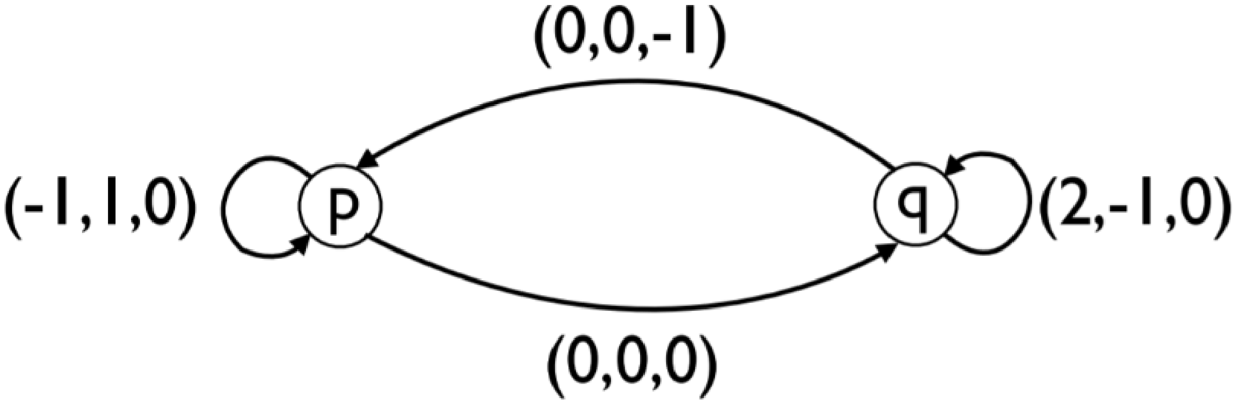

- We can equip a VASS with letters on transitions, initial and accepting configurations to recognize languages by it
- VASS is unambiguous if for each word w there is at most one accepting run over it

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- Deciding language equivalence is undecidable **(Jančar, 1995)**
- Deciding language equivalence is decidable if both VASSs are unambiguous **(Czerwiński and Hofman, 2022)**

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- For regular or context-free languages we have Pumping Lemma
- Is language $\{a^n b a^m b a^k \mid n \geq m \vee n \geq k\}$ unambiguous?
- Do we have an algorithm deciding if the language of a given VASS is unambiguous?

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- Undecidable for context-free languages **(Ginsburg and Ullian)**

Main result

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It is undecidable whether, for a given 3-dimensional VASS A, there exists an unambiguous VASS B such that $L(A) = L(B)$.

- The proof goes by a reduction from the halting problem of the 2-counter machine
- The most crucial part was to prove that a generalisation of a language $\{a^n b a^m b a^k \mid n \geq m \vee n \geq k\}$ is ambiguous

Thank You!