Question B1

(a)

Customers can leave in two possible ways:

1. A customer completes their service and exits

The system allows a maximum of 2 customers to be served simultaneously by two operators.

Each operator has a service rate of μ, so the total service rate for both is 2μ

This means that at any given time, one of the two served customers may complete their query and leave at a total rate of 2μ.

2. A waiting customer hangs up due to impatience

Since state 5 indicates a full queue (3 customers waiting, 2 being served), customers in the queue may decide to hang up.

The abandonment rate is v, meaning each of the 2 waiting customers may leave at a rate of v

Therefore, the total hang-up rate is 2v.

Thus, the **rate of transition from state 5 to state 4** is:

$P(5\to 4)=(2\mu+2v)\cdot\Delta t=2(\mu+v)\cdot\Delta t$

(b)

State 0 (P0): The system is **empty** (no customers).

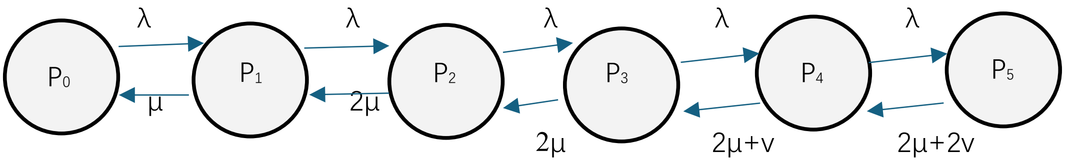
State 1 (P1): **One customer** is being served, and the queue is empty.

State 2 (P2): **Two customers** are in the system—both servers are busy, and the queue is still empty.

State 3 (P3): **Three customers** are in the system—both servers are busy, and **one customer is waiting in the queue** (position 1).

State 4 (P4): **Four customers** are in the system—both servers are busy, and **two customers are waiting in the queue** (positions 1 and 2). Customers in queue may abandon the system with probability vvv.

State 5 (P5): **Five customers** are in the system—both servers are busy, and **three customers are waiting in the queue** (positions 1, 2, and 3). Customers in queue may abandon the system with probability vvv. If a new customer arrives, they will be **rejected** as the queue is full.



$P_0\lambda=P_1\mu$

$P_1\lambda=P_2 2\mu$

$P_2\lambda=P_3 2\mu$

$P_3\lambda=P_4(2\mu+v)$

$P_4\lambda=P_5(2\mu+2v)$

$P_0+P_1+P_2+P_3+P_4+P_5=1$

(c)

P1 = (λ/μ) P0

P2 = $(\lambda^2 / 2\mu^2)$ P0

P3 = $(\lambda^3 / (2\mu)^2 \mu)$ P0

P4 = $(\lambda^4 / (2\mu)^3 (2\mu + v))$ P0

P5 = $(\lambda^5 / (2\mu)^3 (2\mu + v) (2\mu + 2v))$ P0

To find P0, we use the normalization equation:

P0 = $1 / (1 + (\lambda/\mu) + (\lambda^2 / 2\mu^2) + (\lambda^3 / (2\mu)^2 \mu) + (\lambda^4 / (2\mu)^3 (2\mu + v)) + (\lambda^5 / (2\mu)^3 (2\mu + v) (2\mu + 2v)))$

(d)

(i)

$\lambda = 4.3$ and $\mu = 2.9$ and $\nu = 0.8$ substitute into equation

P0 = $1 / (1 + (\lambda/\mu) + (\lambda^2 / 2\mu^2) + (\lambda^3 / (2\mu)^2 \mu) + (\lambda^4 / (2\mu)^3 (2\mu + v)) + (\lambda^5 / (2\mu)^3 (2\mu + v) (2\mu + 2v)))$= 0.2076

P1 = (λ/μ) P0 = 0.3078

P2 = $(\lambda^2 / 2\mu^2)$ P0 = 0.2282

P3 = $(\lambda^3 / (2\mu)^2 \mu)$ P0 = 0.1692

P4 = $(\lambda^4 / (2\mu)^3 (2\mu + v))$ P0 = 0.0551

P5 = $(\lambda^5 / (2\mu)^3 (2\mu + v) (2\mu + 2v))$ P0 = 0.0320

(ii)

The probability that an arriving request will be rejected (P5) is: 0.0320 (3.20%)

(iii)

Throughput (rate of completed requests) = $\lambda * (1 - P5)$ = 4.1623 requests per unit time

(iiii)

Using Little's Law: E[T] = L / λ_eff, where L is the expected number of customers in the system and λ_eff is the effective arrival rate (throughput).

$L = \sum_{i=0}^{5} i \cdot P_i$=1.6525

$\lambda_{eff}=\lambda(1-P_5)$=4.1623

Mean Response Time E(T) = 0.3970 time units

Question B2

(a)

Case 1: $q_1=0$ and $q_2=0$

No requests in either queue.

The processing units can be in any valid configuration:

$n_1+2n_2\leqslant 4$

Possible states:

(0,0,0,0)
(1,0,0,0)
(2,0,0,0)
(3,0,0,0)
(4,0,0,0)
(0,1,0,0)
(1,1,0,0)
(2,1,0,0)
(0,2,0,0)

Case 2: $q_1=1$ and $q_2=0$

There is a Type 1 request in the queue.

All processing units must be busy:

$n_1+2n_2=4$$n_1+2n_2=4$

Possible states:

(4,0,1,0)
(2,1,1,0)
(0,2,1,0)

Case 3: $q_1=0$$q_1=0$ and $q_2=1$$q_2=1$

There is a Type 2 request in the queue.

There cannot be two idle processing units:

$n_1+2n_2\geqslant 2$

Possible states:

(2,0,0,1)
(3,0,0,1)
(4,0,0,1)
(0,1,0,1)
(1,1,0,1)
(2,1,0,1)
(0,2,0,1)

Case 4: $q_1=1$$q_1=1$ and $q_2=1$$q_2=1$

Both queues have requests.

All processing units must be busy:

$n_1+2n_2=4$

Possible states:

(4,0,1,1)
(2,1,1,1)

(0,2,1,1)

(b)

(2,1,0,0) –λ2→ **(2,1,0,1)** ←μ1–**(1,1,0,1)**

|

| μ1

v

(1,1,0,1)

^

| μ1

(3,0,0,1) –μ1→ **(2,1,0,1)** –μ2→ **(2,0,0,1)**

|

| λ1

v

(2,1,1,1)

(c)

A Type 1 request is rejected under the following conditions:

1. The Type 1 queue is full (q1=1).
2. All processing units are occupied, meaning there are no idle processing units to immediately handle the request.

Therefore, the probability that a Type 1 request is rejected is the sum of the steady-state probabilities of all states where:

q1=1 (Type 1 queue is full).

All processing units are occupied, i.e., n1+2n2=4.

Valid States

Based on the system constraints, the states that satisfy q1=1 and n1+2n2=4 are:

1. (4,0,1,0)
2. (2,1,1,0)
3. (0,2,1,0)
4. (4,0,1,1)
5. (2,1,1,1)
6. (0,2,1,1)

So the answer is:

P(reject, Type 1)=P(4,0,1,0)+P(2,1,1,0)+P(0,2,1,0)+P(4,0,1,1)+P(2,1,1,1)+P(0,2,1,1)

(d)

A Type 2 request is rejected under the following conditions:

1. The Type 2 queue is full (q2=1).
2. There are insufficient idle processing units (fewer than 2 idle processing units).

Therefore, the probability that a Type 2 request is rejected is the sum of the steady-state probabilities of all states where:

$q2=1q2=1$ (Type 2 queue is full).

There are fewer than 2 idle processing units, i.e., $n1+2n2\geqslant 3$

Valid States

Based on the system constraints, the states that satisfy $q2=1q2=1$ and $n1+2n2\geqslant 3n1+2n2\geqslant 3$ are:

1. (3,0,0,1)
2. (4,0,0,1)
3. (1,1,0,1)
4. (2,1,0,1)
5. (0,2,0,1)
6. (4,0,1,1)
7. (2,1,1,1)
8. (0,2,1,1)

So the answer is:

P(reject, Type 2)=P(3,0,0,1)+P(4,0,0,1)+P(1,1,0,1)+P(2,1,0,1)+P(0,2,0,1)+P(4,0,1,1)+P(2,1,1,1)+P(0,2,1,1)